The Effectiveness of ICT-assisted Approach in Learning 3D Linear Algebra

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Abstract: *Linear algebra was one of underachieved fields of mathematics for the students of colleges of technology in Japan at the annual INCT achievement tests, which were given to all the students ten months after their learning of linear algebra. The low scores at achievement tests were partially because, in traditional lessons, the procedures of symbolic manipulation were taught without relating to the features of associated graphical objects or how they were used in real world applications. The isolated procedural knowledge did not accompany conceptual understanding and must be easily forgotten just after the term-end examinations. To compensate this situation and deepen the students' conceptual understanding, we redesigned our lesson-plan to be directed from concrete examples towards abstract mathematical ideas, from handling graphic objects and observing their characteristics towards building vector equations and manipulating them symbolically. In the new lessons, several ICT-assisted activities are included to demonstrate the close relation of graphic objects and vector equations interactively. This paper reports how the new lessons are changing our students' learning style from black-box approach to more meaningful one. Their written answers have richer explanations than their former students who learnt in traditional lessons. Their scores in the annual INCT achievement test also increased significantly in 2011.*

1. Introduction

In the college of technology the authors working for, low achievement of the students in linear algebra especially in three-dimensional (3D) vectors and matrices had been a serious issue [1]. The annual INCT (Institute of National Colleges of Technology) achievement test showed that our students' average scores were fairly high in algebra, trigonometry, and calculus but lower in 3D linear algebra and matrices.

The students were motivated to learn trigonometry and calculus because they knew that they continued to use those mathematics procedures in engineering subjects, for example, electrical circuit theory and electromagnetism. They also became to understand those mathematics concepts gradually when they continued to use the procedures in the exercises of mathematics and engineering applications.

The opposite was true to linear algebra. They were not supposed to use vector equations and matrices for a while after the learning and seemed to forget the procedures a few years later when they needed them in engineering subjects, for example, automatic control theory. They were not motivated because they could not perceive when they were to use those mathematical procedures and could not feel the reality in them. They did not deepen conceptual understanding, and the learnt procedures were rather quick to disappear. Because the annual INCT tests were given to the students about 10 months after the learning of linear algebra, the lower achievement in them were not avoidable.

The lack of the students' conceptual understanding in vector equations were easily observed in their answer sheets in term-end examinations. The answer sheets of poorly performed students had blank spaces, few drawings, and no sentences. Some of them could not tell if the equation $ax + by + cz = d$ represented a plane or a line in 3D space. Obviously, these students memorize a series of formulas without examining their features or understanding the relations of the formulas with the features of graphic objects. In another words, they had a learning style to memorize procedural knowledge as a black box without caring mathematical concepts even at the time of their learning in linear algebra.

Their learning style must be a natural result of our teaching style in linear algebra. Traditional lessons of linear algebra started from symbolic definitions of vectors, matrices, and their operations, moved to the exercises of manipulating symbolic expressions, but rarely explained the connections between symbolic and graphic representations in detail, and often lacked to introduce real-world applications familiar to the students. The students who seek *cost-effective* learning for scoring enough points to pass the examinations tended to omit conceptual considerations and stuck to blind-memorization of formulas.

To change the situation, we are redesigning our lesson plan [2] that is directed from concrete applications to abstract mathematical ideas, from handling graphic objects and identifying their characteristics toward defining symbolic expressions and manipulating them. The lesson plan is based on the MODEM theory [3] that stresses the importance of identification. Identification is the process where students recognize the tendency, patterns, or rules by observing the experienced phenomena, and gradually structure mathematical knowledge in their minds. In another word, identification is the process of personal discovery for each student even thought the knowledge is well known to the public, and the discovered knowledge is connected to rich personal experience. Such knowledge is usually more structured, and robust, and thus valuable to engineering students.

Identification should be done before production, where they are asked to apply the mathematical knowledge to solve given problems. Production is the popular process in traditional lessons but often mislead our students toward black box approach. Without well-structured knowledge, students can only memorize given procedures and apply them to solve problems in production process, and they tend to learn only the surface of the knowledge.

In the lesson plan, we provided students with the opportunity to handle 3D graphic objects by themselves and to help them to identify the relationships between three representations: graphic objects, verbal explanations, and symbolic expressions. Interactiveness and simultaneous changes of graphic objects and symbolic expressions often help the students to idenfity the close relation between them. We used several software programs to install the interactive environment including web-browser/server, a database management system, a dynamic geometry software, and a computer algebra system.

2. New lesson plan

Our new lesson plan consists of seven steps described in Table 1 [2] and takes 5.65 hours of lesson time plus additional outside-class activities. It introduces 3D vectors in a real-world application, moves to handle them with real objects in classroom and with graphic objects in 3D virtual space, and discusses the symbolic representations and their mechanisms at the later steps. It also blends traditional classroom activities with technology-based ones.

Table 1 New lesson plan of 3D linear algebra for engineering students

Step		Activity	Lesson time
1	Start from actual application	Students recognize the usefulness of linear algebra to solve engineering problems and are motivated to learn.	0.75h + outside class
2	Handling real objects in class	Students handle simple real objects and are accustomed to the idea of parallel and perpendicular.	0.4h
3	Observing and constructing graphic objects in 3D virtual space	Students observe graphically how 3D objects can be constructed with vectors in interactive webpages developed with dynamic geometry software. They concentrate on the relation of graphic objects, but not on the symbolic representations.	0.75h+ outside class
4	Identifying the relation of different representations	Students identify the relation of graphic objects, explanatory sentences, and symbolic expressions by the simultaneously changing representations in the computer screens.	0.75h + outside class
5	Deducing formulas	The teacher explains the deducing process of formulas using the chalkboard.	1.5h
6	Exercises with pencil-and-paper	Students solve traditional pencil-and-paper problems using more descriptive explanation of the solving processes.	0.5h * 3 + outside class
7	Mutual mini-lectures	Students explain their own solutions to the other students in a small group.	outside class

On the first step, we introduced 3D vectors as the building blocks of constructing a virtual game, which helps engineering students to feel the reality in vectors through playing the game [1]. However, since only a few students involved in this study actually played the virtual game, we do not expect the effect yet.

On the second step, we let the students use real objects: thin wooden sticks and a plastic plate before virtual 3D graphic objects. We set a corner of each student's desk as the origin of 3D space with two sides of the desktop as x and y axes, and a stick fixed to stand vertically from the corner as z axis. In this *real* 3D space, we ask the students to hold the plastic plate at a certain position and angle over their desks using sticks and pieces of clay. They recognize that they need three sticks to hold a plate. We encourage them to find as much facts as possible by themselves in the activity. This step makes a good introduction for the students before handling 3D graphic objects in a virtual space because the experience helps them to imagine similar objects in their minds.

On the third step, the students use the webpages which embed interactive graphic objects constructed with Cabri 3D [5] to observe the relation among graphic objects. For example, an animation of a shrinking and stretching vector shows that the end-points form a straight line in 3D space (Figure 1). They recognize the regularity of the points by rotating the viewpoint by mouse operations. In this step, we use vectors and their symbolic expressions on the webpages, and add some explanatory texts that tell the construction of symbolic vector equations. However, we do not expect the majority of the students to understand the relation between the graphic objects and the vector equation yet in this step. The aim of the pages is rather to clarify the fact that simple graphic objects can be defined with a set of vectors graphically. We also expect our students to recognize that there are two types of vectors, position vectors and the other usual vectors during the activities.

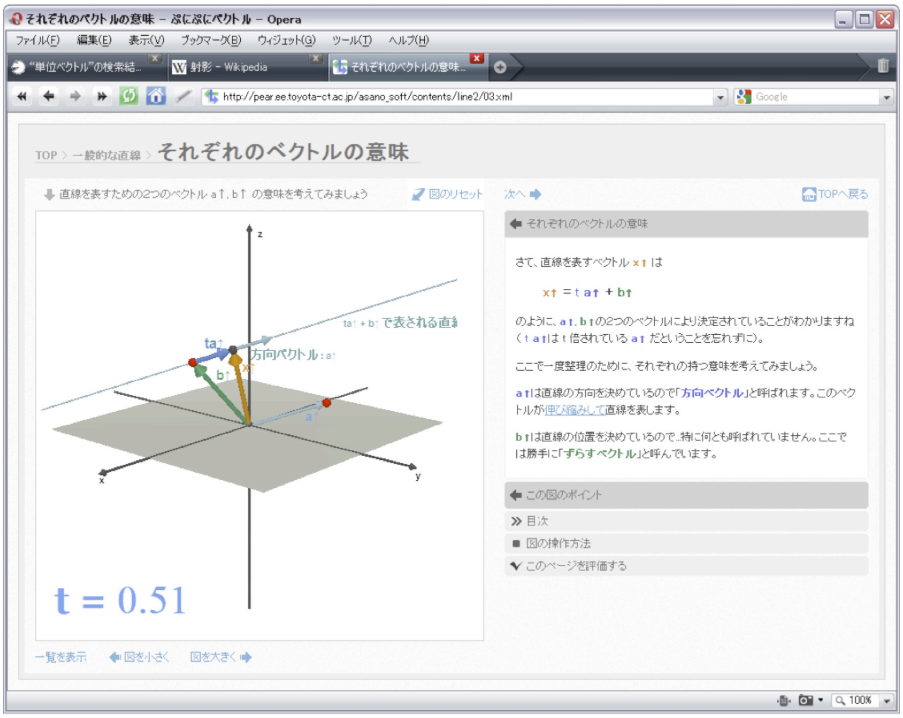


Figure 1 Example webpage embedding interactive graphic objects [4]

On the fourth step, students are to identify the relations of graphic objects, verbal explanation of them, and vector equations. A program coded with MATHEMATICA [6] shows simultaneous change of graphic objects and related symbolic expressions (Figure 2). When a student changes a parameter in an equation with one of the *manipulators* on the left side of the screen, the vector equation shown below the screen changes the related parameter, and the graphic object displayed at the center of the screen also changes the position or direction in the 3D-space simultaneously. The students are expected to identify the relation by the simultaneous changes of graphics and symbolic equation, although there are also some explanatory texts on the right side of the screen.

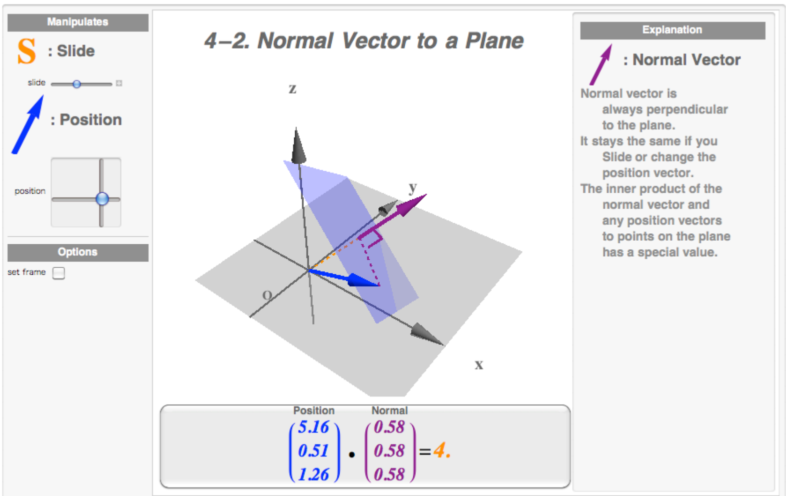


Figure 2 Example of simultaneous changing graphic objects and vector equations

At the step five, after the experience in real and virtual 3D space, the students learn vector equations of lines and planes in traditional lectures. They are shown that there are several types of symbolic expressions to express a line or a plane, and they can be rewritten to each other. The lectures especially stress that a feature of a symbolic expression is linked to a certain feature of the graphic object, and the link should be explained verbally. We often reuse the 3D graphics objects already shown to the students during the experiments in virtual space in this step. The students, who know real and virtual graphic objects, accept vector equations easier than the former students learnt with traditional lesson plan.

On the sixth step, we also set up pencil-and-paper exercises after the lectures because it is important for the students to express their own thoughts on papers if they thoroughly understand. In the exercises, they are encouraged to start from a drawing of target objects, add necessary vectors, and express them in symbolic expressions at the final step. We forbid them to use formulas written in their textbooks as the starting point of their solutions because it hardly explains necessary mathematical ideas by itself. If they need symbolic manipulations, they have to write every step of the rewriting process on their paper, too.

The seventh step has not yet conducted for the students of this study. All the students participated in the class activities, and slower learners were invited to the out-of-class activities in the computer laboratory, where a pair of students used a computer terminal operating the software and discussing the results displayed in the screen. Teaching assistants resided in the laboratory and were ready to help if any student had difficulty operating the machine, but it seldom happened. When the students did not need help, the assistants observed the students' activities and noted their behaviors.

3. Preliminary result

We have been changing our lessons gradually for recent five years as shown in Table 2.

The group of students who received the 2007 INCT test is the reference among the experimental groups. They had only step 6: pencil-and-paper exercises of the lesson plan and did not have any activities of steps 3 and 4, which are supported by information and communication technologies (ICT).

Table 2 Learning activities done by the experimental groups

Year of INCT test	Average at term-end test [2]	No. of students	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6	Step 7
2007 (reference)	6.4	45						○	
2008	7.7	48					○	○	
2009	7.2	42					○	○	
2010	7.5	48		○	△		○	○	
2011	9.2	41		○	◇	□	○	○	

○: for all the students in class, △: extra work outside the class for 1/3 of the students,
◇: extra work outside the class for 1/3 of the students with *improved* interface (right picture of Figure 4),
□: pilot activities outside the class for some students who had difficulty in pencil-and-paper exercises

The lessons for experimental groups who participated in 2008 and 2009 INCT tests additionaly included step 5: deducing formulas. The lessons for experimental groups who participated in 2010 and 2011 INCT tests also included step 2: real objects in classroom. One third of the students in experimental groups 2010 and 2011, who were slow learners in these two groups, did the extra work described as step 3: observing and handling graphic objects in 3D virtual space using our Web-based learning system outside class. The interface of the learning system was improved for the 2011 group to have more structured webpages and richer graphic content rather than lengthy explanatory sentences. Some of the slow learners in 2011 group attended additional session of step 4: identifying and producing the relation of graphic and symbolic representations.

At the term-end examinations, the 2008 and 2009 groups had higher average scores than the 2007 group (Table 2). The 2010 group had a fewer students who scored zero point at the test, and the 2011 group increased the average score a little more. The 2011 group's answer sheets at the term-end examination were fairly richer in description than the ones of the 2007 group. There were less blank sheets, more drawings and graphs, and more sentences. The interviews to the students of the 2011 group showed that the activities of step 3 and 4 help them recognize the importance of connecting various representations, especially graphic representation to symbolic one, for the first time [2].

We measured the long-lasting effect of step 2 to 5 of the lesson plan by the students' scores at the INCT tests. The test has been conducted annually since 2007 for all the third-year students in national colleges of technology. More than 7,000 students participated in each test. The target field of this paper is "3D vectors and matrices" section of the test. The INCT tests were conducted 10 months after the teaching and learning of 3D vectors and matrices.

Because the average scores of all the students in INCT tests fluctuate year to year, we normalized the scores by the average *a* and standard deviation *s* of all the students (Figure 3). We normalized the scores so that the average score *a* is 50 and the standard deviation *s* is 10 after the normalization. Normalized 60 equals to $a + s$, and normalized 70 equals to $a + 2s$. The normalized score shows the relative performance of a student relative to all the students who participated in the INCT test.

We compared two groups of about 40 students as the experimental group who received our new lessons and the control group who participated only in traditional lessons. The both groups belong to the same college, and they have quite similar learning history other than the new lessons.

According to Figure 3, the average score of 2008 experimental group were not significantly higher than the 2007 reference. It was even lower than the average score of the control group of 2008, although the difference was not statistically significant. The average scores of 2009 and 2010 experimental groups were not significantly higher than the one of 2007 reference either. They are a little higher than the average scores of the control groups of the same years, but the differences were not statistically significant. The first average score, which was significantly higher than the one of the control group, was of 2011. The score distributions in Figure 3 show the cause of higher average score. There were only 17% of the students in the 2011 experimental group who scored lower than the national average. It was a drastic decrease from 44% of the students in 2008 experimental group. There were fewer students who could not answer to any problem in "3D vectors and matrices" field.

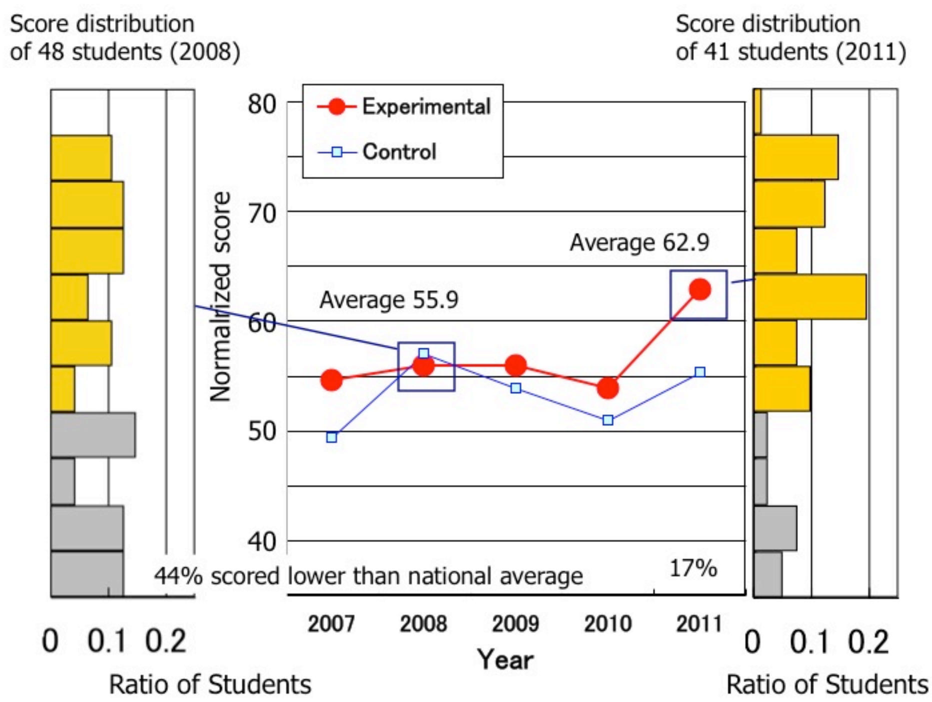
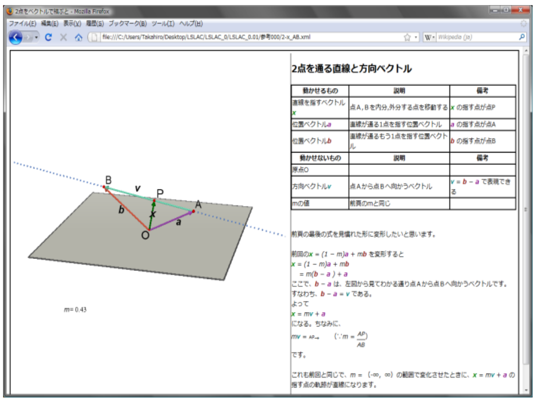


Figure 3 Students' scores of "3D vectors and matrices" in the annual INCT tests

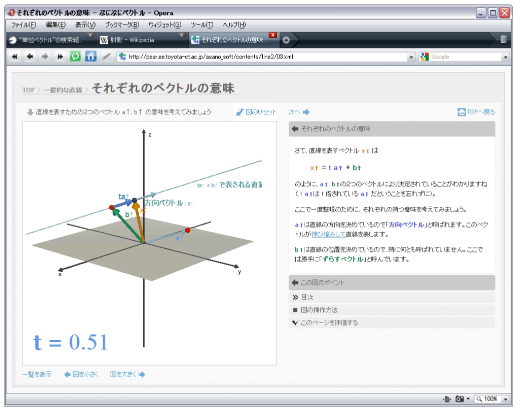
4. Discussion

The result of the INCT tests in 2008 and 2009 suggests that the knowledge learnt through deducing formulas and pencil-and-paper exercises have relatively shorter lifetime. Although those activities increased the students' average scores in the term-end examinations as shown in Table 2, they did not affect the average scores much in the INCT tests, which took place 10 months later the term-end examinations. It is possible that the procedural knowledge could not survive the 10-months period if it was not understood conceptually by closely linked the procedural knowledge to graphic representations.

The result of the 2010 and 2011 INCT tests conflict each other. In both years, the students of experimental groups participated in the activities of steps 2 and 3 in the lesson plan. However, the students of 2010 group scored not so highly at INCT test, and the students of 2010 group scored significantly higher. The difference might be caused by the quality of activities in step 3, which use the interactive webpages (Figure 4). Both pages have embedded Cabri-3D graphic objects that could be modified interactively and explanatory sentences. The pages for 2011 group have shorter but more structured explanatory sentences, and they have more explanations around the graphic objects rather than in the sentences. The pages for 2011 group have better interface and are easier to use.



Original interface used for 2010 group



Improved interface used for 2011 group

Figure 4 Example webpage for observing graphic objects in 3D virtual space

The increased average score of the 2011 INCT test implies the long-lasting effect of adding steps 2 to 4 to the lessons when the activities are well organized. Because those steps give the students a big picture before the learning of detailed symbolic manipulations, it is possible that more students learnt symbolic procedures by connecting them with the feature of graphic objects.

We intend to confirm the effectiveness of adding steps 2 to 4 further in the 2012 INCT test, and also the effectiveness of adding step 1: actual application as the introduction to the lessons in the 2013 and 2014 INCT tests.

5. Conclusion

The long-lasting effect of adding ICT-assisted activity is confirmed by the INCT achievement tests. The students' scores in the 3D vectors and matrices part increased significantly at the 2011 test. This result implies that the students' procedural knowledge remained after a blank period of 10 months after the learning of linear algebra when they learnt 3D vector equations with the close relation to the features of graphic objects and confirmed the strong connection between them, and thus deepened conceptual understanding. With the help of ICT, the students could identify the features of 3D graphic objects and relate them to the symbolic expressions more easily.

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