# VIRTUAL GAME FOR DEEPENING STUDENTS' CONCEPTUAL UNDERSTANDING OF 3D VECTOR OPERATIONS

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## KEYWORDS

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## ABSTRACT

A simple virtual application is aimed to provide young engineering students with more meaningful learning of linear algebra, which is the least performed field of pre-college mathematics in Japan. The application uses 3D vectors to show the rules and mechanism of a virtual game graphically, so the students have graphical image of vectors and feel the reality in them before they start learning vectors and their operations in traditional classes.

In this presentation, we also would like to discuss the necessity and possibility of such virtual applications for more meaningful learning of abstract mathematics. With the help of 3D graphics, mathematics classroom could become more fun to learn for younger engineering students.

## BACKGROUND

At the college the authors work for, linear algebra, especially three-dimensional (3D) vector equations and their operations, is the most poorly performed field of pre-college level mathematics in annual INCT (Institute of National College of Technology, Japan) achievement tests, which has been given to all the third-year students (18 years old) of colleges of technology in Japan since 2007 (Nishizawa & Yoshioka, 2008).

The suspected major reason of the poor performance is surface learning (Fujisawa, 2002) spreading among Japanese students even in the learning of mathematics as a *cost effective* learning style to pass entrance examinations to universities with higher score but fewer learning.

Surface-learners are easily recognized by their answer sheets, which have more blank spaces, fewer drawings, and fewer explanatory sentences. They blindly memorize as many formulas and solutions as possible to be ready for the examinations of limited target fields, but forget most of them after the examinations. Since INCT achievement tests are given at least half a year after their learning of target fields, and the knowledge of 3D linear algebra is not used in engineering subjects for the third-year students, such isolated pieces of knowledge do not remain at the time of the achievement tests. Surface learning is also a fatal learning style to engineering education because it prevents students from thinking by themselves and hinders the growth of their problem solving ability. It must be altered in early college education before they start to learn mathematics-based engineering subjects.

One of the causes of our students' surface learning is a typical teaching style of linear algebra in Japan. It is taught deductively as abstract ideas starting from definition, followed by manipulating rules, and some calculating exercises.

The relations between symbolic expressions and their graphic representations are rarely mentioned especially in three dimensions, partly because of the difficulty of drawing 3D graphic objects on chalkboard. It is quite difficult for engineering students to learn such abstract mathematical ideas without knowing actual applications in their minds.

For guiding the students to more meaningful learning of linear algebra, we changed our lesson plan to start from a virtual game using 3D graphics (Nishizawa, Zgraggen, Yoshioka, 2009) based on the MODEM theory (Haapasalo, 2003). In the game, students join a tournament composed of various battles using 3D vectors.

Because the winner of each battle is decided by 3D vector operations and shown in 3D graphic images, students are accustomed to use the idea of vector operations before they learn the mathematical definition or their symbolic manipulations.

After the experiences in the game, students have the opportunities to use DGS (Dynamic Graphics Software) to handle 3D graphic objects on their computer screens by themselves, and to identify the relationships between three representations: graphic objects, verbal explanations, and symbolic expressions.

The virtual game is supported by software modules programmed with a computer algebra system Mathematica and a database management system PostgreSQL.

Because the rule of the game is expressed with multi-dimensional vectors, students are gradually accustomed to their graphical and symbolic expressions, and their operations. After an experience of the game, they are ready to learn linear algebra based on their experiences in the game.

## THE LESSON PLAN

Our lesson plan of linear algebra is based on the MODEM theory that stresses the importance of identification. Identification is the process where students recognize the tendency, patterns, or rules by observing the experienced phenomena, and gradually structure mathematical knowledge by themselves.

In another word, identification is the process of personal discovery for each student even thought the knowledge is well known to the public, and the discovered knowledge is connected to rich personal experience. Such knowledge is usually more structured, and robust, and thus valuable to engineering students.

Identification should be done before production, where they are asked to apply the mathematical knowledge to solve given problems. Production is popular but often misleads process in mathematics lessons.

Without well-structured knowledge, students can only memorize given procedures and apply them to solve problems in production process, and they tend to learn only the surface of the knowledge.

Virtual reality helps students to start identifying such patterns in 3D linear algebra. 3D graphic images show students simple but actual examples, which are not easily visualized in traditional classrooms with chalkboards, pencils, and papers. They also give students simple examples, which show close relations between symbolic operations and the resulting transformations of graphic objects.

By observing the examples, students become familiar not only to 3D graphic objects but also the related symbolic operations.

## A VIRTUAL GAME

We selected a virtual game as the introductory application of our linear algebra lessons because it appeals to most young Japanese students who have been playing video games regularly and felt some reality in them. Affinity to three or more dimensions is another benefit of the game as the start-up application of linear algebra.

## TOURNAMENT AS THE GAME

The virtual game consists of a tournament (see Figure 1), where an avatar of a student or a team of avatars compete each other in different kind of battles in the virtual world at every stage of the tournament, for example, boxing or Sumo wrestling matches, solving puzzles or chess matches, or singing competitions or musical auditions.

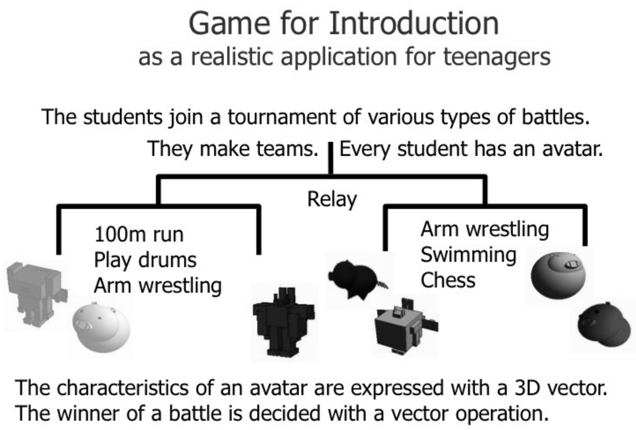


Figure 1. The tournament.

Every student in the class owns an avatar for the game selected from four species, and tunes its characteristics before joining the tournament. The graphical icon of an avatar is decided by its species: two types of robot, pig, or dog. And a vector of three independent elements: physical strength, thinking ability, and musical skills, expresses the characteristics of an avatar.

The characteristic vector is roughly featured by the avatar's species, and tuned by the owner. According to the characteristic vector, the avatar changes the detailed shape of its graphical icon. For example, an avatar of high physical strength has longer arms and feet. Thinking ability is expressed by the size of the head, and musical skills are expressed by the size of the body of an avatar.

The tournament is fought between teams, and all the students in the class are registered to one of the teams beforehand. Two teams compete each other in a battle, where the type of battle and characteristics of battlefield differ from one to another. For example, if a tennis match is set at a stage of the tournament, three players each selected from a team play three games in the courts of slightly different condition. The team that wins two games or more proceeds to the next stage of the tournament.

The type of battle is revealed after the team gathering and designing avatars but before each stage of the tournament, so each team can decide its tactics based on the given information and the characteristics of the avatars in the team.

For every stage of the tournament, surviving teams decide their tactics in the coming battle along with the representing avatars. The battles are conducted in a lesson one by one and shown on the projected screen in classroom. Winning teams of battles are decided from the first to the final stage.

## 3D VECTORS IN THE GAME

A 3D vector expresses the characteristics of an avatar, where physical strength, thinking ability, and musical skills are three independent basic elements of a player. For example, a Sumo wrestler is to have high physical strength but modest thinking ability and musical skills as an avatar. An opera singer has high musical skills but modest physical strength.

Each student designs his/her avatar by rotating the unit characteristic vector of the player in 3D space (see Figure 2) before attending one of the teams in the tournament.

The actual length of an avatar's vector is decided randomly according to a normal probability distribution N (10, 1) just after the team-assembling to avoid the intentional occurrence of a super team.

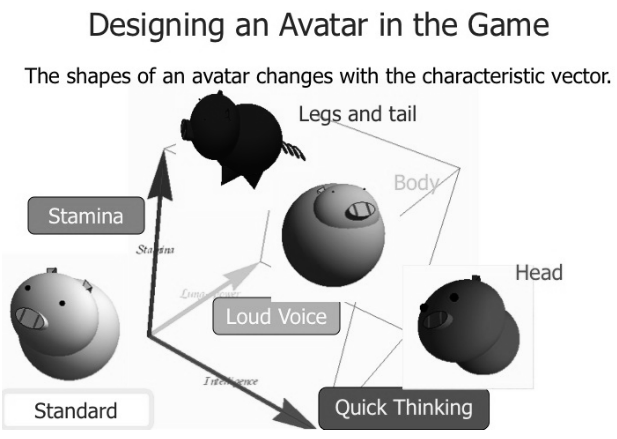


Figure 2. Designing an avatar in 3D characteristic field.

The 3D space shows that the value of a basic element of the characteristic vector, for example the thinking ability, is graphically expressed as the length of the shadow projected to the "Thinking ability" axis.

The rulebook describes it using symbolic expression that the value is calculated by a dot product of the characteristic vector and a unit vector on the "Thinking ability" axis.

When the type of battles in each stage at the tournament are shown with an explanation of the battlefields, each team discuss the strategy and selects the participating avatars for the stages, for example 3 players out of 5 candidates in the team for the tennis match of semi-final stage.

A characteristic vector of a battlefield is also expressed as a 3D vector of the same elements but as a unit vector. Its direction is decided by the teacher to express the feature of each battlefield. For example, if it is a game of basketball, musical skills are of little use.

In a one-to-one battle, the strength of each avatar is calculated by the dot product of the avatar's characteristic vector and the battlefield's characteristic vector, which is a unit vector, and the winner of the battle is decided by comparing projected lengths to the battlefield vector from two competitor vectors.

In 3D space, the projected length shows the performance of each avatar in the battlefield (see Figure 3). The avatar whose projected length is longer is the winner. 3D graphics with its capability to change the viewpoint help us to distinguish the winner clearly, telling the potential benefit of 3D vectors to students.

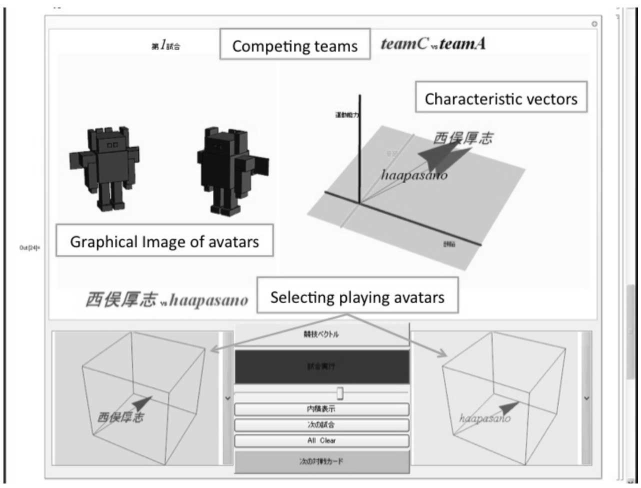


Figure 3. Displaying the winner of a battle graphically in 3D space.

In a team-battle, the characteristic vector of a team is calculated as the vector sum of all the avatar's characteristic vectors, and the team vector is used instead of the avatar's vector in one-to-one battle.

As the result of a battle, the winner grows 20% in the length of its characteristic vector, the loser shrinks 20%, and the waiting players stay the same length. In the case of team battle, all the players in the winning team grow 20% in length in their characteristic vectors.

## VECTOR OPERATIONS AS THE RULE DESCRIPTION

In the game, we use most of vector operations; dot product to calculate the value of each element of a characteristic vector and the strength of each player in a battle, vector addition to calculate the characteristic vector of each team in a team battle, and absolute value of a vector to set the strength of a player.

A scalar multiplication is used to stretch/shrink the player's vectors as the result of a battle. These operations are first shown graphically in the game, and later explained using their symbolic operations in the rulebook, which becomes the introduction to formal lessons of vector operations.

Eager students learn the symbolic operations earnestly as the rule of the game to win the battles, and teach them to their teammates. On this process, they sometimes learn unknowingly all the knowledge before the lessons, and their teammates also become familiar to the vector operations.

## SYSTEM STRUCTURE

The game is conducted by a client/server system in the college LAN environment. The students' modules, which work as front-end modules, are programmed with a computer algebra system Mathematica (ver. 7) using its graphic interface, a built-in function called *Manipulate*, its database-link function, and its programming language.

*Manipulate* is the function which enables immediate changes of graphic objects in the screen of a student's module triggered by the users' operations.

All the data, which are characteristics of battlefield, member-lists of each team, and characteristic vectors of all the avatars, are stored in a database server and accessed from front-end modules through the database-link function.

Students use the system through front-end modules installed to the computers in the college laboratory. They design their avatars on the computers and register to one of the teams before the actual lesson of 3D vector operations.

The activity doesn't require any prior knowledge of vectors. After the registration of the students, the system decides the matchups in the tournament, and holds all the information necessary to conduct the virtual game in its database.

The teacher uses another front-end module shown in figure 3 to demonstrate the battles to the students in class. He shows the matchup, calls the captain of each team, and lets he/she select the player for each battle. After displaying the selected avatars on the screen, the teacher shows the 3D vectors and rotates them to the viewpoint from where the winning player is visible to the class.

## DISCUSSION

Surface learning usually happens when the teacher try to explain abstract mathematics ideas deductively. In mathematics, a learning material is composed of simple definitions and their applied results to various examples.

The material is highly structured, and each component is logical, symmetrical, and sometimes quite beautiful. It is easy to teach but difficult to learn. It is especially difficult to learn when the teacher directs his/hear lesson starting from the definition and moving toward applications as most Japanese mathematics lessons because the direction is opposite from the one of discovering patterns, symmetries, and underlying mathematical ideas.

It also snatches the opportunity to discover mathematical ideas away from students. The damage is bigger, the higher the abstractness of the material, such as liner algebra. Curious students are sacrificed the most in the lessons.

We may avoid the situation by teaching inductively at least partially at introduction. We redirect our lessons starting from an actual application, letting the students to have some experiences in using the knowledge, and finally arriving at the abstract ideas.

3D vectors are introduced at first as the means to express the rule of the virtual game. The students who want to win the game analyse the rule, and find the patterns and underlying mechanisms, which are the mathematical ideas. They find the patterns through 3D graphics, and hope to describe the rules generally. Here come symbolic expressions as convenient tools to express the general rules simply.

Virtual reality has the potential to serve vivid graphical applications of 3D vector operations, and in many fields of mathematics. Hopefully, it should also reveal typical features or hidden mechanism related to the abstract ideas we try to teach.

## CONCLUSIONS

A virtual tournament is designed for young engineering students to experience an application of 3D linear algebra before their traditional lessons.

It shows graphical image of 3D vectors and their operations in the virtual 3D space, and the students become familiar to those graphic image and be ready to learn symbolic expressions and their operations in traditional classes.

It is one of the authors' efforts to teach highly abstract linear algebra by starting the lessons from actual applications closely related to engineering. They believe that the learning has much reality in this direction, and is more meaningful to the engineering students.

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