

Ishiguro on Leibniz:
The Context Principle and the Infinitesimal
in Leibniz's Philosophy

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OUTLINE

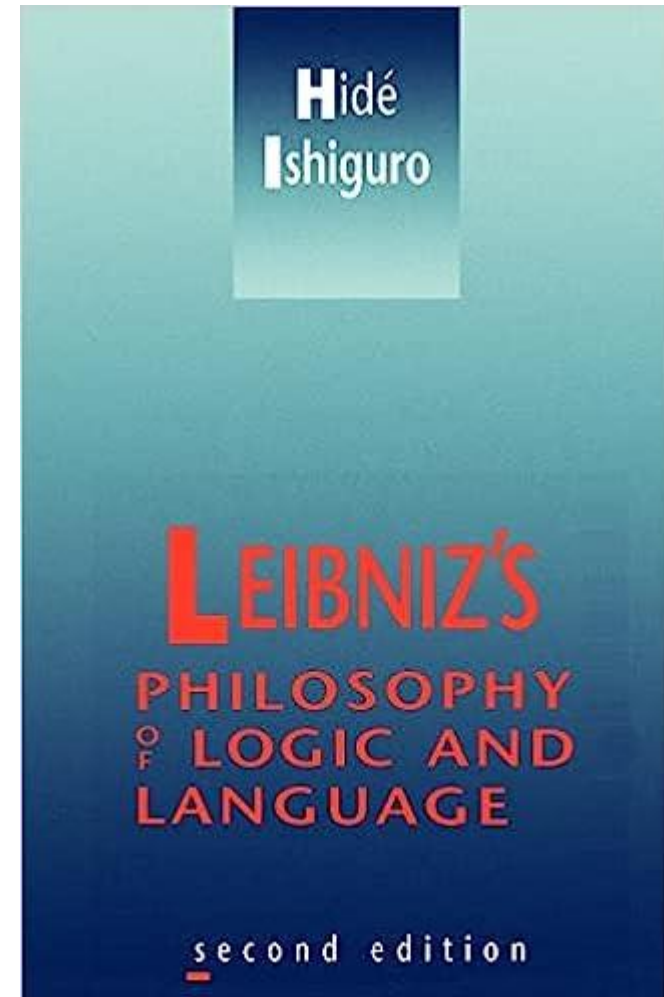
- 1 : Ishiguro's Leibniz
- 2 : Problem of Infinitesimals
- 3 : Context Principle and Confused idea
- 4 : Leibniz's infinitesimals and Russell's theory of description
- 5 : Discussion
- 6 : Conclusion

1 : Ishiguro's Leibniz

- Hidé Ishiguro (Japanese: 石黒ひで, 石黒英子; born 1935)
- The most famous Japanese philosopher in the world



- Hidé Ishiguro, *Leibniz's Philosophy of Logic and Language*, Cambridge University Press, 2nd edition, 1990.
- Discussion Leibniz from viewpoint of cotemporary philosophy
- Very well cited literature of Leibniz
- Especially, her interpretation of infinitesimal is still influential.



2: Problem of Infinitesimal

- Infinitesimal: mathematical quantity which is less than any positive quantity and is not zero.
- X is infinitesimal : def= $\exists x \in \mathbf{R} \forall y \in \mathbf{N}(|x| < 1/y \wedge x \neq 0)$
- In the middle age of Europe, there is disputes on whether infinitesimal exists in nature.
- Leibniz invented the difference calculus based on, *prima facie*, using infinitesimal, which was controversial at that time.
- Scholars of Leibniz have discussed whether Leibniz admits existence of infinitesimal.
- In this context, Ishiguro is well known as an advocator of ‘finitism’ interpretation.

Ishiguro's study of infinitesimals

- A : “La Notion Dite Confuse de l'Infinitesimal Chez Leibniz”, *Studia Leibnitiana*, 1986.
- B : “Chapter 5: Leibniz's notion of the infinitesimal” in *Leibniz's Philosophy of Logic and Language*, 2nd edition, 1990.
- C : 『ライプニッツの哲学』 (Japanese Translation of *Leibniz's Philosophy of Logic and Language*), 2003.

- As for argument, $A \doteq B$, but $B \neq C$

- Ishiguro 1990 p.99
- “Leibniz, however, like Frege, denied that the notion expressed by any word was an image, and thought that we cannot define any notion except through propositions and truths. Thus what he says about infinitesimals constitutes an integral and coherent part of his theory of ideas and of meaning.”

- 石黒 2003 p.117 (= Ishiguro 1990 p.99)
- “Leibniz, however, like Frege, denied that the notion expressed by any word was an image, and thought that we cannot define any notion except through propositions and truths. Thus what he says about infinitesimals constitutes an integral and coherent part of his theory of ideas and of meaning. (Whether what can be true or false is expressed in words, numbers or symbols makes no difference to Leibniz.)”

論理計算のための記号と操作符号をそれらの意味から抽象されたものと見做すとさえ書いている。⁽²⁸⁾

しかしながら、フレーゲと同じくライブニッツは、何であれ言葉によって表現された観念が像であることを否定し、命題と真理を通ず以外にはいかなる観念も定義され得ないと考えていた。だから、無限小について彼が述べていることは、彼の観念と意味の理論と緊密に結びついている。(真か偽であり得るものが言葉で表現されようが、数字や記号でされようが、ライブニッツにとって違いはない。)

第二のそして最後の点は、一般的な形而上学的なものである。ライブニッツは全ての大きさや数が実体の可能な存在に依存している二次的な存在であり、観念的对象(enta rationis)だと考えていた。しかしこの事実、厳密に言えば、全ての数と大きさは有限だ、という主張にライブニッツがあたえた重要性をいささかも減じるものではない。ライブニッツが観念的对象と呼ぶもの、つまり理性的対象(enta rationis, êtres de raison)と、虚構つまり虚構的对象(enta fictium)とを我々は区別しなければならない。観念的对象である数や関係を指示するとき、我々は対象を指示している。たとえそれらが可能な個体の性質や属性に対応し、あるいは、可能な個体の複数の集合やそれらの等値な集合に対応して理性が構成したものであるとしても、それらは、客観的基準で構成された指示できる(抽象)対象である。一方、虚構は恣意的に作られたものである。さらには別の標準的な種類の対象について述べることに還元可能な話法と相互関係を持つものである。数学的虚構への指示は標準的な数学的对象についての表現で言い直すことができる。我々は、モノド主義者であり得る一方で数学においては有限主義者であり続けることができるのであり、また、モノドを現実世界の唯一の構成要素と考えるのであれば、無限小についての実在論者であると同時にモノド主義者でもあり得るだろう。すべての数は観念的对象であるという見解をたえとったとしても、観念としての対象についても客観的真理があるし、虚構について述べるのが観念的对象の話に還元可能で

- **No difference thesis (ND):** Whether what can be true or false is expressed in words, numbers or symbols makes no difference.
- ND seems to imply the so-called Leibniz's isomorphism between symbols and world. (cf. *Quid sit idea?*)
- But it is a question why ND was inserted at Japanese translation.
- Our answer: Ishiguro goes to acknowledge the existence of infinitesimals, which seems to contradict the finitism thesis but match the practice of mathematicians.

- In Ishiguro 1990, she argues that Leibniz has **the finitism interpretation of infinitesimal (FII)**.
- According to FII, the word ‘infinitesimal’ has no reference. Instead, it means that for any number x , we can take number y which is less than $1/x$.
- FII claims that $\forall x \in \mathbf{N} \exists y \in \mathbf{R} (|y| < 1/x \wedge y \neq 0)$.
- This interpretation has strong influence not only among scholar of Leibniz but also philosopher of mathematics.
- Some scholar criticize her interpretation on the ground that her argument lacks textual evidence. (e.g., Bascelli et. alia 2016)
- So, is it correct to say that Ishiguro’s finitism is wrong?
- We discuss this question through another question: what is the relationship between ND and FII?

3: Context Principle and Confused idea

- Gottlob Frege (1848-1925): German mathematician, logician and philosopher;
 - Inventor of first-order logic, quantificational logic
 - Distinction of Sinn (sense) and Bedeutung (reference)
 - Exploring philosophical foundation of mathematics
- In her entire study, Ishiguro seeks to interpretate Leibniz as the pioneer of Frege.



Context principle

- Ishiguro argues that Leibniz has the idea of the context principle of Frege.
- **Context principle (CP)** is the central idea of Frege's programs of grounding mathematics on logic.
- CP: DO NOT ask the meaning of a word in isolation, but only in the context of a proposition.
- Frege argues that we can know the reference of abstract words through CP.

- Example: What is the reference of “the direction of a line” ?
- Frege’s answer: We can not know the reference itself of “the direction of a line”, however we can identify it.
- In *Die Grundlagen der Arithmetik* (1884), Frege proposes that the reference of an abstract word can be known via substitution.
- If “the direction of a line A” = “the direction of a line B”, we can substitute everywhere the former for the latter.
- This method guarantees that “the direction of a line A” has the reference. (This is originated in Euclid’s *Elements*)

- Ishiguro claims that Leibniz has the idea as the same as CP.
- *Generales Inquisitiones de Analyysi Notionum et Veritatum* 1684:
- “Idem autem esse A ipsi B, significat alterum alteri substitui posse in propositione quacunque salva veritate. Nam respectus illi per propositiones sive veritates explicantur [For those respects are explained by means of propositions, i.e.truth].” (A.VI.4. 746)
- Referring this passage, Ishiguro claims that Leibniz has CP. This interpretation is controversial, but here we would not examine this interpretation.

- Ishiguro thinks that Leibniz's 'confused idea' which is equivalent to 'be able to recognize it but not to enumerate the marks' also means that we sometimes could know a reference of a term even if we could not explain a sense of it.
- As for this claim, what Ishiguro has as typical examples of confused idea are the idea of 'blue' and 'red'. We could not explain what 'blue' is but could distinguish 'blue' and 'red'.
- We could include the idea of infinitesimal in those examples. In fact, we think it is difficult to explain what infinitesimal is, however, we could easily understand the difference itself between the concept of infinite and the concept of finite. (cf. Esquisabel and Quintana 2021)
- Leibniz's confused idea can be thought as reflection of his mathematical practice, that is, even if he does not know much about the reference of infinitesimals, he is sure that it works in mathematical research.
- Ishiguro does not examine the relationship between 'confused idea' and infinitesimals, but we claim that such relationship implies reconsideration of FII thesis.

4: Leibniz's infinitesimals and Russell's theory of description

- Ishiguro writes that Leibniz is close to Russell's theory of description than to Frege's CP.
- However, she also points that there is difference between Leibniz and Russell on uniqueness of reference.
- According to Russell's theory, there is a unique object that satisfies a definite description. On the other hand, Leibniz's finitism does not accept the unique existence of infinitesimals. Instead, the existence of infinitesimals can be justified indirectly via FII. (Ishiguro 1990 p.98)

- Ishiguro justifies FII of syncategorematic infinitesimals based on Frege's CP.
- She writes, "what he says about infinitesimals constitutes an integral and coherent part of his theory of ideas and of meaning". This justifies to apply CP and the theory of description to the infinitesimals.
- In other word, we could say that examining Ishiguro's thesis on Leibniz's infinitesimals need to consider not only CP and FII, but also Russellian contextual definition and ND.

5: Discussion

- Ishiguro does not argue the connection between her interpretation of infinitesimal and that of confused idea.
- We would claim that the distinction of sense and reference could be applied to the discussion of philosophical debates of infinitesimals. Ishiguro's Fregeanized Leibniz would have such connection.
- In fact, Frege himself discusses that CP guarantees only the sense of a word. (Not always the reference) However, he thinks that this justifies that abstract words have a reference.
- If Leibniz holds CP and the distinction of sense and reference, then he also must admit the existence of infinitesimal and therefore does not need to avoid some problem of infinitesimals.
- The motivation of Ishiguro's thesis of infinitesimal seems to avoid the problematic idea of allowing the existence of infinitesimal and instead adopt infinitesimals as a process which justifies that we can always take a less number for given non-zero number.
- However, her study suggests that Ishiguro's Leibniz implicitly admit the existence of infinitesimal.

- It is remarkable that Ishiguro thinks that philosophical framework we need to understand Leibniz's idea of infinitesimals is not that of Frege, but Russell.
- Russell's theory of description justifies that we could paraphrase the sentence in which 'the author of *Théodicée*' appears into the sentence in which such phrase do not appear.
- By paraphrasing the sentence, we would make ontological reduction.
- Ishiguro claims that by paraphrasing the sentence which mentions the infinitesimals into the sentence which do not mention the infinitesimals, Leibniz's infinitesimals can be thought as not referring to the real object. (Fictionalism)
- We emphasize that paraphrasing and ontological reduction are different. In fact, we could keep realism of infinitesimals accepting Russellian paraphrasing.
- This is the core of her thesis of finitism. Insofar as this thesis, Frege is not involved.

	Fregean Leibniz	Russellian Leibniz
The existence of infinitesimals	Justified weakly via CP	Finely justified via theory of description
The existence of limit	Justified via confused idea and ND	Unjustified?

- Here we could understand what ND means for Leibniz and Ishiguro.
- ND implies that we should take consideration that mathematicians usually rely on not only language but also other symbols such that diagram and number.
- If Leibniz accept ND, he would also accept the existence of infinitesimals without philosophical consideration.
- This reading is not a regression. Further, it suggests that philosophical research of examining mathematical practice is important to understand Leibniz better.

- Reading Leibniz's *Nova Methodus*, Jacob Bernoulli says 'ce qui était une énigme plutôt qu'une explication'. (GM III 5)
- This statement expresses that if we use mathematical concept which lead us to mathematical results, but we do not know the mechanism of such leading, we find some problem in that concept.
- But, according to 'confused idea', we could do well in such situations, that is, limit and infinitesimals are considered as 'confused'.

6: Conclusion

- Ishiguro's study of Leibniz seems to be old-fashioned because of its anachronistic stance and textual situation at that time.
- In fact, recent studies of Leibniz's infinitesimal criticize this point.
- If we would like to save Ishiguro's infinitesimal from criticism, we need to examine her interpretation in more detail.
- Ishiguro's idea of infinitesimals is product combined Frege's theory of language (CP and distinction of sense and reference) and Russell's theory of description.
- In addition to this, if we consider ND, then we could grasp Ishiguro's idea more effectively and more appealing to understand mathematician's practice.

Reference

- Tiziana Bascelli, Piotr Blaszczyk, Vladimir Kanovei, Karin U. Katz, Mikhail G. Katz, David M. Schaps, and David Sherry, ‘Leibniz versus Ishiguro: Closing a Quarter Century of Syncategoremania’, *HOPOS: The Journal of the International Society for the History of Philosophy of Science*, Volume 6, Number 1, 2016, pp.117-147.
- Oscar M. Esquisabel, Federico Raffo Quintana, “Fiction, possibility and impossibility: three kinds of mathematical fictions in Leibniz’s work”, *Archive for History of Exact Sciences*, 2021, 75, pp.613–647.
- Hidé Ishiguro, “La Notion Dite Confuse de l’Infinitesimal Chez Leibniz”, *Studia Leibnitiana*, 1986, pp.183-196.
- Hidé Ishiguro, *Leibniz’s Philosophy of Logic and Language*, Cambridge University Press, 2nd edition, 1990.
- 石黒ひで、『ライブニッツの哲学—論理と言語を中心に 増補改訂版』、岩波書店、2003年。(Japanese translation of Ishiguro 1990)
- Hidé Ishiguro, ‘Leibniz et la distinction frégréenne entre «sens» et «référence»’, *Leibniz et les puissances du langage*, ed. par Dominique Berlioz et Frédéric Nef, Vrin, pp.201-10, 2005.
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Thank you for your attention!