# NONLINEAR OBSERVERS APPEARING IN DYNAMICAL MACHINE VISION

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# I. INTRODUCTION

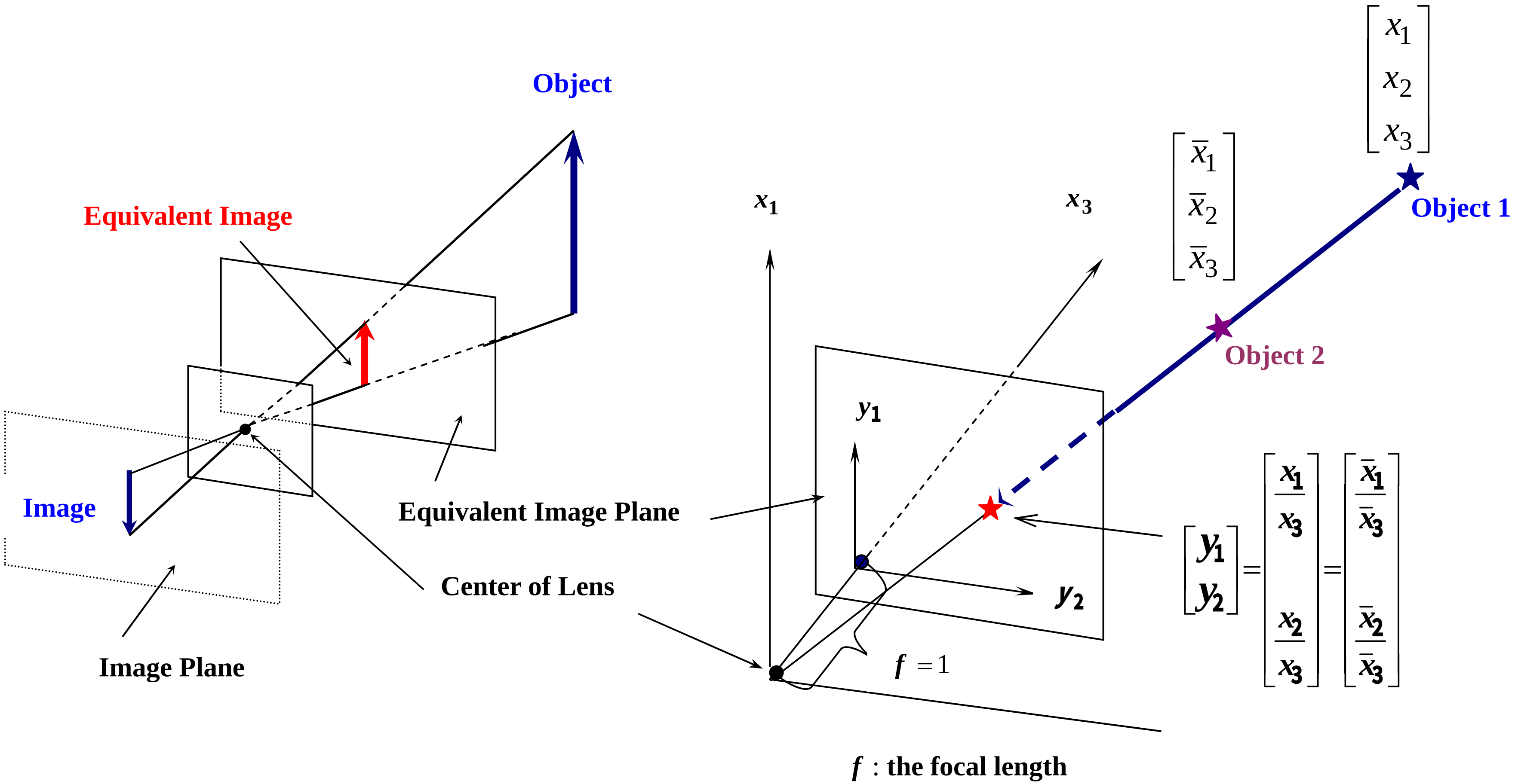
The essential problem in ***dynamical machine vision*** is how to determine the position and the shape of a moving rigid body from knowledge of the associated ***optical flow***.

A ***perspective dynamical system*** arises from such a ***machine vision problem***, and this essential problem is to estimate the unknown state and to identify the unknown parameters for such a system based on ***perspective observation*** (optical flow).

This talk presents a generalization of our previous result on nonlinear observers for perspective linear systems with a ***single observing point*** to those with a ***multiple observing points***.

# II. PERSPECTIVE LINEAR SYSTEMS

## What is *perspective observation*?



# What is a ***perspective linear system***?

Consider the following simple example in which only one point is observed:

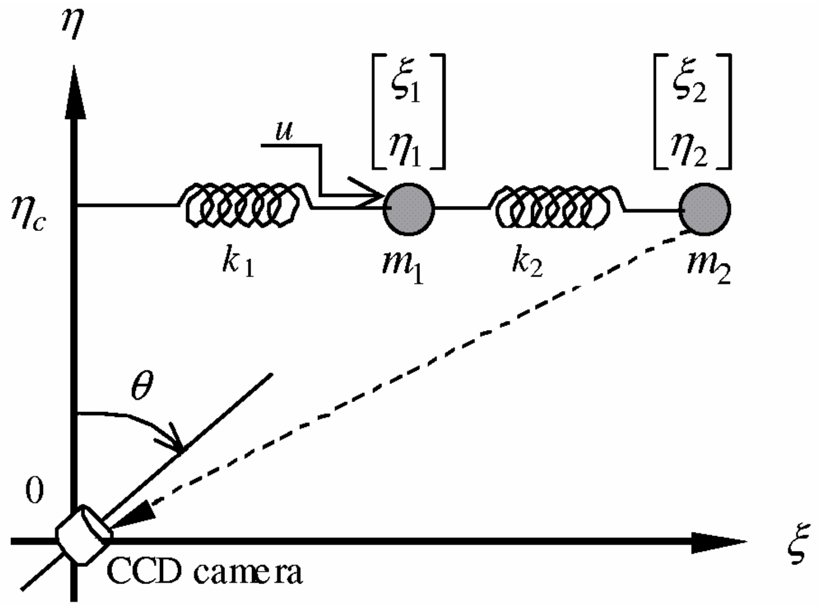



Fig. 2. A two-degree-of-freedom system

Introduce the state vector $x = [x_1 \cdots x_5]^T$ as

$$x_1 := \xi_1, x_2 := \dot{\xi}_1, x_3 := \xi_2, x_4 := \dot{\xi}_2, x_5 := \eta_1 = \eta_2 \equiv \eta_c.$$

Then this system can be described in the form

$$\text{PLS:}\ \begin{cases} \dot{x}(t) = Ax(t) + v(t), \ \ x(0) = x_0 \in \mathbb{R}^5 \\ y(t) = h(Cx(t)) \end{cases}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\dfrac{k_1+k_2}{m_1} & 0 & \dfrac{k_1}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \dfrac{k_2}{m_2} & 0 & -\dfrac{k_2}{m_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad v(t) = \begin{bmatrix} 0 \\ \dfrac{k_1}{m_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$C = \begin{bmatrix} \hat{C} \\ \tilde{C} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cos\theta & 0 & -\sin\theta \\ 0 & 0 & \sin\theta & 0 & \cos\theta \end{bmatrix},$$

and the observation obtained by the CCD is one-dimensional, and is given as a rational function of the state variables $x_1, \cdots, x_5$ of the form

$$y = h(Cx) = \frac{\hat{C}x}{\tilde{C}x} = \frac{x_3 \cos\theta - x_5 \sin\theta}{x_3 \sin\theta + x_5 \cos\theta}.$$

Such a system is called a ***perspective linear system.***

Now consider a ***perspective linear system*** with multiple observing points on a moving body. Then assuming that the motion of the moving body is described as a linear differential equation, our ***perspective linear system*** with $p$ observing points is described as

$$\text{PLSM:} \begin{cases} \dot{x}(t) = Ax(t) + v(t), \;\; x(0) = x_0 \in \mathbb{R}^n \\ y(t) = H(Cx(t)) \in \mathbb{R}^{2p} \end{cases}$$

where

$x(t) \in \mathbb{R}^n$: the entire state of the moving bodies

$v(t) \in \mathbb{R}^n$: the external input, and $A \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{(3p)\times n}$
$y(t) \in \mathbb{R}^{2p}$: the ***perspective observation vector***
$H : \mathbb{R}^n \to \mathbb{R}^{2p}$: a function of the form

$$y(t) = H(Cx(t)) = \begin{bmatrix} h(C^{(1)}x(t)) \\ \vdots \\ h(C^{(p)}x(t)) \end{bmatrix} = \begin{bmatrix} y^{(1)}(t) \\ \vdots \\ y^{(p)}(t) \end{bmatrix} \in \mathbb{R}^{2p},$$

each $y^{(k)}(t) = h(C^{(k)}x(t))$ representing the perspective observation generated by the $k$-th observing point, and

$$h(\xi) := \begin{bmatrix} \frac{\xi_1}{\xi_3} & \frac{\xi_2}{\xi_3} \end{bmatrix}^T, \;\; \xi = \begin{bmatrix} \xi_1 & \xi_2 & \xi_3 \end{bmatrix}^T, \quad \xi_3 \neq 0$$

$$C = [C^{(1)T} \quad \cdots \quad C^{(p)T}]^T \;\; \text{with} \; C^{(k)} \in \mathbb{R}^{3\times n}.$$

The objective of this talk is to show that, under suitable conditions on a given perspective linear system PLSM, including

(i) PLSM is ***Lyapunov stable***,

(ii) PLSM satisfies ***some sort of detectability condition***,

it is possible to construct a ***Luenberger-type nonlinear observer*** whose estimation error converges ***exponentially to zero***.

# III. LUENBERGER-TYPE NONLINEAR OBSERVERS

Now, consider a full-order ***observer*** for a perspective linear system of the form

$$\text{PLSM:}\ \begin{cases} \dot{x}(t) = Ax(t) + v(t),\ \ x(0) = x_0 \in \mathbb{R}^n \\ y(t) = H(Cx(t)) \in \mathbb{R}^{2p} \end{cases}.$$

First, notice that a full-order state observer for PLS generally has the form

$$\frac{d}{dt}\hat{x}(t) = \varphi(\hat{x}(t), v(t), y(t)), \qquad \hat{x}(0) = \hat{x}_0 \in \mathbb{R}^n$$

which satisfies that for any $v(\bullet)$

$$\hat{x}(0) = x(0) \Rightarrow \hat{x}(t) = x(t),\ \ \forall t \geq 0.$$

Thus, we may assume that $\varphi(\hat{x},v,y)$ has the form

$$\varphi(\hat{x},v,y) = A\hat{x} + v + r(\hat{x},y)$$

where $r(\hat{x},y)$ is any function satisfying $r(x,h(Cx)) = 0, \forall x \in \mathbb{R}^n$. Further, for such a function $r(\hat{x},y)$, we may take

$$r(\hat{x},y) = K(y,\hat{x})[y - h(C\hat{x})]$$

where $K(y,\hat{x})$ is any sufficiently smooth matrix-valued function.

These choices of functions lead to a ***nonlinear observer of the Luenberger-type***:

$$\text{NLO: } \frac{d}{dt}\hat{x}(t) = A\hat{x}(t) + v(t) + K(y(t),\hat{x}(t))[y(t) - h(C\hat{x}(t))],$$

where $\hat{x}(0) = \hat{x}_0 \in \mathbb{R}^n$ and $K : \mathbb{R}^{2p} \times \mathbb{R}^n \to \mathbb{R}^{n\times(2p)}$ is called an ***observer gain matrix***.

In what follows, let us consider a suitable form of the gain matrix $K(y,\hat{x})$. First, introducing the following notations

$$Cx = \begin{bmatrix} C^{(1)} \\ \vdots \\ C^{(p)} \end{bmatrix} x$$

$$= \begin{bmatrix} \underbrace{\xi_1^{(1)} \quad \xi_2^{(1)} \quad \xi_3^{(1)}}_{=(C^{(1)}x)^T} & \cdots & \underbrace{\xi_1^{(p)} \quad \xi_2^{(p)} \quad \xi_3^{(p)}}_{=(C^{(p)}x)^T} \end{bmatrix}^T,$$

$$= \xi \in \mathbb{R}^{3p}$$

and similarly $C\hat{x} =: \hat{\xi} \in \mathbb{R}^{3p}$, and use them to simplify the term $y - H(C\hat{x})$ as follows:

$$y - H(C\hat{x}) = H(Cx) - H(C\hat{x})$$

$$= \begin{bmatrix} \dfrac{\xi_1^{(1)}}{\xi_3^{(1)}} - \dfrac{\hat{\xi}_1^{(1)}}{\hat{\xi}_3^{(1)}} \\ \dfrac{\xi_1^{(1)}}{\xi_3^{(1)}} - \dfrac{\hat{\xi}_1^{(1)}}{\hat{\xi}_3^{(1)}} \\ \vdots \\ \dfrac{\xi_1^{(1)}}{\xi_3^{(1)}} - \dfrac{\hat{\xi}_1^{(1)}}{\hat{\xi}_3^{(1)}} \\ \dfrac{\xi_1^{(p)}}{\xi_3^{(p)}} - \dfrac{\hat{\xi}_1^{(p)}}{\hat{\xi}_3^{(p)}} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{\hat{\xi}_3^{(1)}} \begin{bmatrix} 1 & 0 & -\dfrac{\xi_1^{(1)}}{\xi_3^{(1)}} \\ 0 & 1 & -\dfrac{\xi_2^{(1)}}{\xi_3^{(1)}} \end{bmatrix} \begin{bmatrix} \xi_1^{(1)} - \hat{\xi}_1^{(1)} \\ \xi_2^{(1)} - \hat{\xi}_2^{(1)} \\ \xi_3^{(1)} - \hat{\xi}_3^{(1)} \end{bmatrix} \\ \vdots \\ \dfrac{1}{\hat{\xi}_3^{(p)}} \begin{bmatrix} 1 & 0 & -\dfrac{\xi_1^{(p)}}{\xi_3^{(p)}} \\ 0 & 1 & -\dfrac{\xi_2^{(p)}}{\xi_3^{(p)}} \end{bmatrix} \begin{bmatrix} \xi_1^{(p)} - \hat{\xi}_1^{(p)} \\ \xi_2^{(p)} - \hat{\xi}_2^{(p)} \\ \xi_3^{(p)} - \hat{\xi}_3^{(p)} \end{bmatrix} \end{bmatrix}$$

$$
= \begin{bmatrix} \frac{1}{\hat{\xi}_3^{(1)}} \begin{bmatrix} I_2 & -y^{(1)} \end{bmatrix} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{\hat{\xi}_3^{(p)}} \begin{bmatrix} I_2 & -y^{(p)} \end{bmatrix} \end{bmatrix} (\xi - \hat{\xi})
$$

$$
= E(\hat{x}) B(y) C \rho
$$

where $I_2$ indicates the $2 \times 2$ identity matrix and

$$\begin{cases} E(\hat{x}) := \begin{bmatrix} \frac{1}{C_3^{(1)}\hat{x}} I_2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{C_3^{(p)}\hat{x}} I_2 \end{bmatrix} \in \mathbb{R}^{2p} \\ B(y) := \begin{bmatrix} \begin{bmatrix} I_2 & -y^{(1)} \end{bmatrix} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \begin{bmatrix} I_2 & -y^{(p)} \end{bmatrix} \end{bmatrix} \in \mathbb{R}^{(2p)\times(3p)} \end{cases}$$

$$\rho := x - \hat{x}.$$

Then one obtains

$$K(y,\hat{x})[y - H(C\hat{x})] = K(y,\hat{x})E(\hat{x})B(y)C\rho$$

and hence to eliminate from this expression all the denominators $C_3^{(k)}\hat{x}$ appearing in $E(\hat{x}(t))$, one can choose a gain matrix $K(y,\hat{x})$ of the form:

$$K(y,\hat{x}) = P^{-1}C^{*}B^{*}(y)E^{-1}(\hat{x})$$

where $C^{*}$ indicates the complex conjugate transpose of $C$ and $P \in \mathbb{R}^{n\times n}$ is an appropriately chosen matrix, which is considered to be a ***free parameter*** for the gain matrix. And with this choice for $K(y,\hat{x})$, the Luenberger-type nonlinear observer becomes

$$\text{NLO:} \quad \frac{d}{dt}\hat{x}(t) = A\hat{x}(t) + v(t) + P^{-1}C^{*}B^{*}(y(t))E^{-1}(\hat{x}(t))[y(t) - H(C\hat{x}(t))], \quad \hat{x}(0) = \hat{x}_0 \in \mathbb{R}^n$$

where $P \in \mathbb{R}^{n\times n}$ is an appropriately chosen ***free parameter matrix***.

Now, for what follows, introduce the following notations:

$$\sigma_s(A) := \{\lambda \in \sigma(A) \mid \operatorname{Re}\lambda < 0\},\ \ \sigma_{us}(A) := \{\lambda \in \sigma(A) \mid \operatorname{Re}\lambda \geq 0\},$$

$W_s,\ W_{us}$ : the generalized eigenspaces corresponding to $\sigma_s(A),\ \ \sigma_{us}(A)$, respectively

$\pi_s : \mathbb{C}^n \to W_s$: the projection operator along$W_{us}$

$\pi_{us} : \mathbb{C}^n \to W_{us}$ : the projection operator operators along $W_s$.

Next, we make various conditions on PLSM, which seem to be necessary and/or reasonable from the viewpoint of machine vision.

**ASSUMPTION.**

(i) PLSM is ***Lyapunov stable***, i.e.,

$$\sigma(A) = \sigma_s(A) \cup \sigma_{us}(A)$$

where

$\sigma_s(A)$: the set of eigenvalues with strictly negative real part

$\sigma_{us}(A)$: the set of eigenvalues with zero real part.

(ii) $y(t)$ is a ***continuous and bounded function***, that is,

$$y(\cdot) \in C^m[0,\infty) \cap L^m_\infty[0,\infty).$$

(iii) Let

$W_s, \; W_{us} \subset \boldsymbol{C}^n$: the generalized eigenspaces corresponding to $\sigma_s(A)$ and $\sigma_{us}(A)$ respectively,

$E_{us} = [\xi_1 \quad \cdots \quad \xi_r]$: a basis matrix for $W_{us}$ with $r := \dim W_{us}$.

Then, $\exists T > 0$ and $\exists \varepsilon > 0$ such that

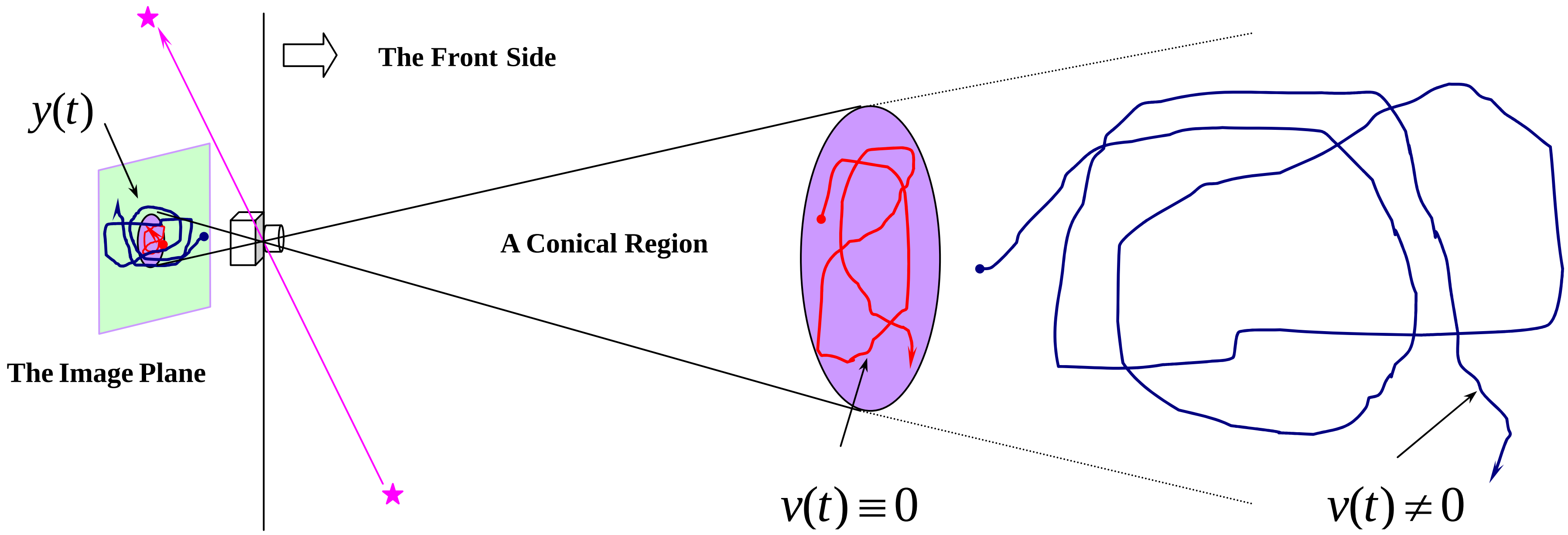
$$\int_0^T E_{us}^* e^{A^*\tau} C^* B^*(y(t+\tau)) B(y(t+\tau)) C e^{A\tau} E_{us} d\tau \geq \varepsilon I_r, \quad \forall t \geq 0. \quad \square$$

**REMARK.** All the conditions given in Assumption are reasonable requirements from the viewpoint of machine vision.

(i) Assumption (i) is imposed to ensure that if $v(t) \equiv 0$ then the motion of a moving body take places within ***a bounded region***.

(ii) Assumption (ii) is imposed to ensure that the motion $x(t)$ described by PLS is smooth enough and takes place inside ***a conical region centered at the camera*** so as to produce a continuous and bounded measurement $y(t)$ on the image plane. In particular, it is assumed that the motion never crosses the plane$C_{m+1}x = 0$, and hence takes place ***only on one side of the camera***.

(iii) Assumption (iii) ensures ***some sort of detectability*** of the perspective system PLS, and ***the external input being not identically zero***. These facts will be cited in the following proposition. □


The Front Side
$y(t)$
A Conical Region
The Image Plane
$v(t) \equiv 0$
$v(t) \neq 0$

**PROPOSITION.** Assume that PLSM is Lyapunov stable, let $A_{us}$ denote the unstable part of the matrix $A$ and set $C_{us} := CE_{us}$. If Assumption (iii) is satisfied, then the following statements hold true.

(i) $(C, A)$ is ***a detectable pair***, that is, the unstable part $(C_{us}, A_{us})$ of $(C, A)$ is observable.

(ii) The external input $v(t)$ is ***never identically zero***. □

# MAIN THEOREM

**THEOREM (LUENBERGER-TYPE NONLINEAR OBSERVERS).**

Assume that PLSM satisfies the Assumption and consider a nonlinear observer of the Luenberger-type, i.e.,

$$\text{NLO: } \frac{d}{dt}\hat{x}(t) = A\hat{x}(t) + v(t)$$
$$+ P^{-1}C^{*}B^{*}(y(t))E^{-1}(\hat{x}(t))[y(t) - H(C\hat{x}(t))],$$
$$\hat{x}(0) = \hat{x}_0 \in \mathbb{R}^n$$

and the differential equation for the estimation error $\rho(t) := x(t) - \hat{x}(t)$, i.e.,

$$\frac{d}{dt}\rho(t) = [A - P^{-1}C^{*}B^{*}(y(t))B(y(t))C]\rho(t),$$
$$\rho(0) = x(0) - \hat{x}(0) \in \mathbb{R}^n.$$

Further, let $\pi_{-}:\mathbb{C}^{n}\to W_{-}$, $\pi_{0}:\mathbb{C}^{n}\to W_{0}$ denote the matrix representations of the projection operators along $W_{0}, W_{-}$, respectively, and $P\in\mathbb{R}^{n\times n}$ be a symmetric positive definite matrix satisfying the Lyapunov inequality

$$A^{*}P+PA\leq -a\pi_{-}^{*}\pi_{-}$$

where $a>0$ is a constant.

Then, $\rho(t)$ converges exponentially to zero, that is, there exist $\alpha>0,\beta>0$ such that

$$\|\rho(t)\|:=\|x(t)-\hat{x}(t)\|\leq \beta e^{-\alpha t}\|\rho(0)\|,\ \forall t\geq 0. \quad \square$$

# IV. Computer Simulations

## Example 1.

The first example we consider is the system with the following data:

$$A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix},$$

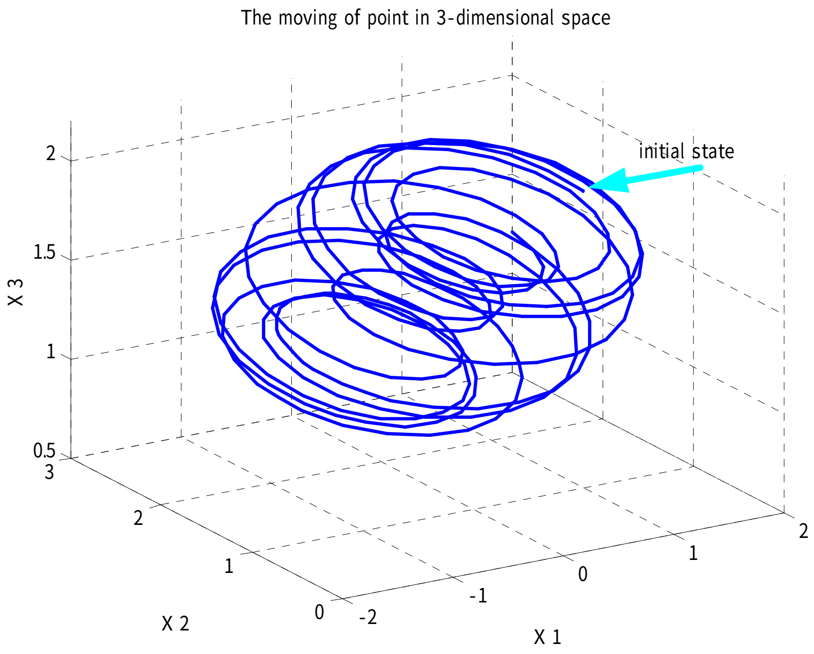
$$v(t) = 2\pi \begin{bmatrix} -\sin(2\pi t) & \cos(2\pi t) & 0 \end{bmatrix}^T$$

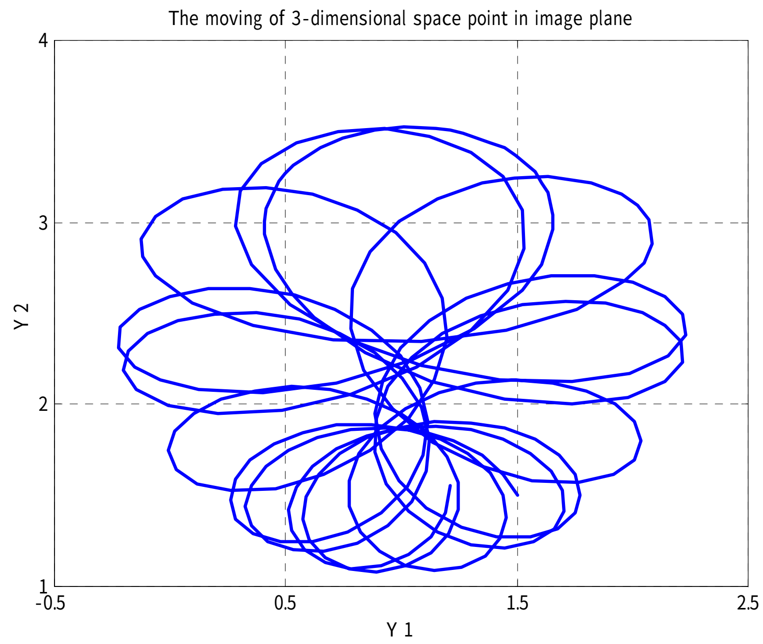
$$x_0 = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix}^T.$$

Then, the trajectories of the state $x(t)$ and the perspective observation $y(t)$ given as

$$y(t) = \begin{bmatrix} y_1(t) & y_2(t) \end{bmatrix}^T = \begin{bmatrix} (x_1(t) + x_3(t))/x_3(t) & (x_2(t) + x_3(t))/x_3(t) \end{bmatrix}^T$$

are depicted below:

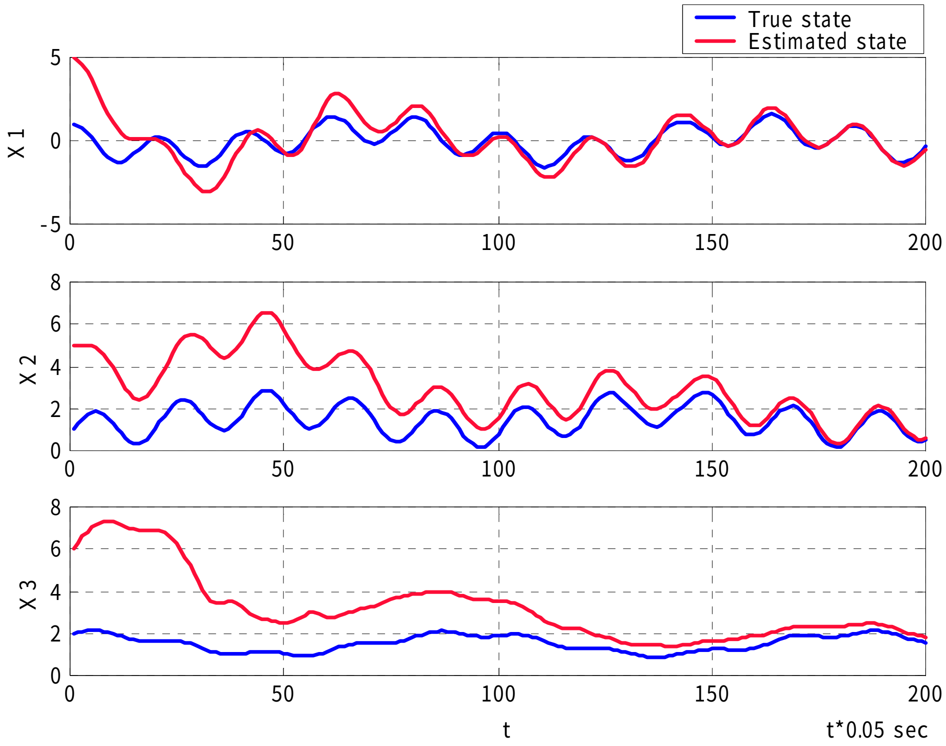


Next, for the observer, we set the following data:

$$\hat{x}_0 = \begin{bmatrix} 5 & 5 & 6 \end{bmatrix}^T, \; P^{-1} = \mathrm{diag}\left\{ \; 30, \; 30, \; 30 \right\}.$$

The result is depicted below:



## EXAMPLE 2 .

The second we consider is the following two-degree-of freedom system:

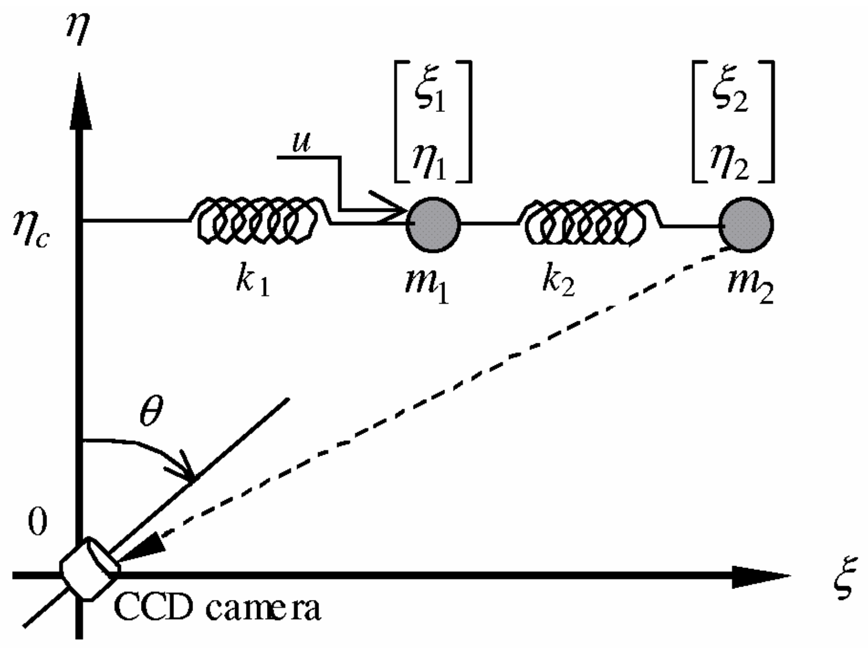



Fig. 2. A two-degree-of-freedom system

Introduce the state vector $x = [x_1 \cdots x_5]^T$ as

$$x_1 := \xi_1, x_2 := \dot{\xi}_1, x_3 := \xi_2, x_4 := \dot{\xi}_2, x_5 := \eta_1 = \eta_2 \equiv \eta_c.$$

Then this system can be described in the form

$$\begin{cases} \dot{x}(t) = Ax(t) + v(t), \;\; x(0) = x_0 \in \boldsymbol{R}^5 \\ y(t) = h(Cx(t)) \end{cases}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\dfrac{k_1 + k_2}{m_1} & 0 & \dfrac{k_1}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \dfrac{k_2}{m_2} & 0 & -\dfrac{k_2}{m_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & \cos\theta & 0 & -\sin\theta \\ 0 & 0 & \sin\theta & 0 & \cos\theta \end{bmatrix}, v(t) = \begin{bmatrix} 0 \\ \dfrac{k_1}{m_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t)$$

and the observation is given in the form

$$y = h(Cx) = \frac{x_3 \cos\theta - x_5 \sin\theta}{x_3 \sin\theta + x_5 \cos\theta}.$$

Next, the numerical values for simulation are set as follows: For the perspective system,

$$m_1 = 2, m_2 = 1, k_1 = 2, k_2 = 1, \theta = 0.5\,[\text{rad}], \omega = 1\,[\text{rad}],$$

$$q = 1, x(0) = [0.1 \quad 0.2 \quad 0.3 \quad 0.1 \quad 0.5]^T$$

and for the observer,

$$\hat{x}(0) = [5 \quad 5 \quad 5 \quad 5 \quad 5]^T$$

$$P^{-1} = \begin{bmatrix} 1.1665 & -0.0000 & -0.6905 & 0.0000 & 0 \\ -0.0000 & 2.0950 & -0.0000 & -1.8571 & 0 \\ -0.6905 & -0.0000 & 1.6425 & -0.0000 & 0 \\ 0.0000 & -1.8571 & -0.0000 & 2.3330 & 0 \\ 0 & 0 & 0 & 0 & 0.2562 \end{bmatrix}$$

where $P$ is a suitably chosen solution of the matrix equation $A^{*}P + PA = 0$. The time evolutions of each component of the estimation error $\rho(t) = x(t) - \hat{x}(t)$ are depicted in Fig. 3, and the results show that the observer works well.

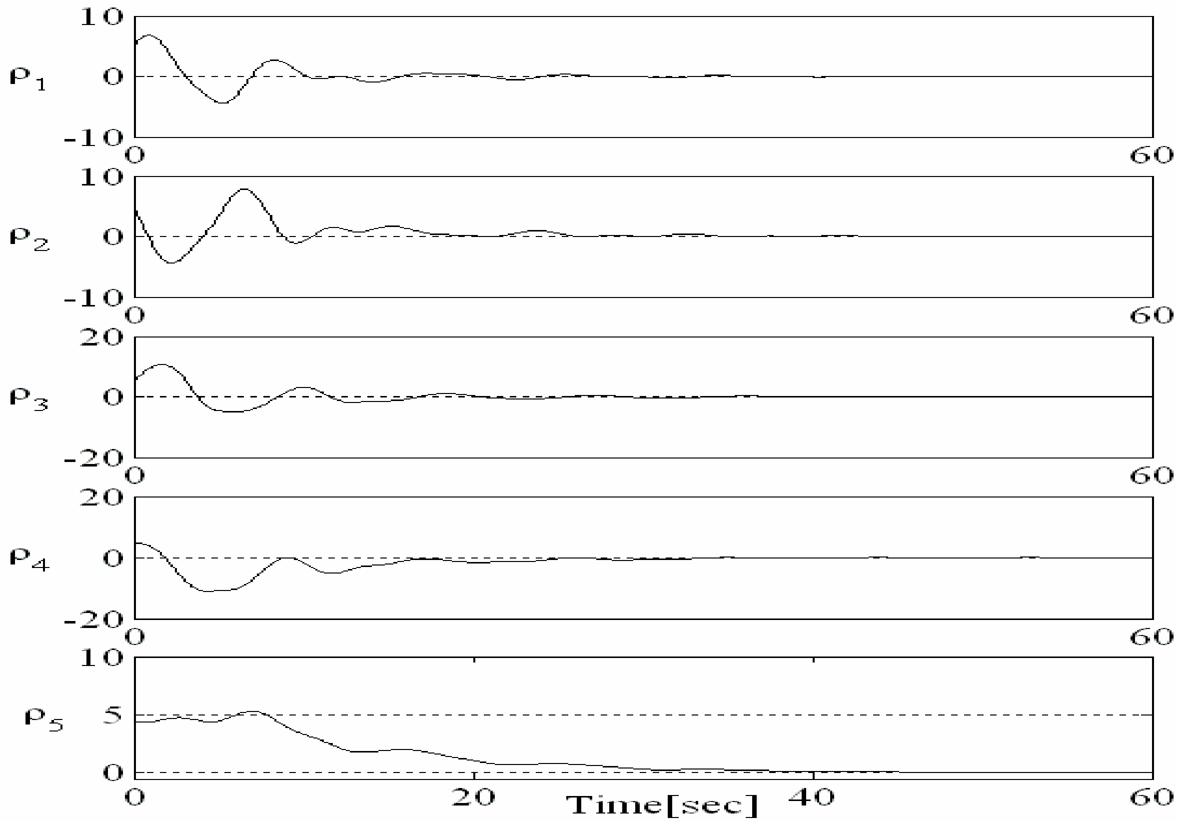



Fig.3 Estimation errors $\rho(t)$.

It should be noted that theoretically the matrix $P$ above can be replaced with any matrix of the form $\mu P$ with $\mu > 0$ to obtain a different convergence property of the estimation error, and further that it can be also replaced with any solution of the Lyapunov inequality $A^* P + PA \leq 0$. However, various numerical simulation results with different $\mu > 0$ show that either case $\mu \gg 1$ or $0 < \mu \ll 1$ does not seem to provide a nicer convergence property. Thus more details of this point should be studied as a future problem.

# V. CONCLUDING REMARKS

This paper studied the state estimation problem for a ***perspective linear system with multiple observing points*** arising in machine vision.

(1) ***A Luenberger-type nonlinear observer*** was proposed, and it was shown that under some reasonable assumptions on a perspective system it is possible to construct such a nonlinear observer whose estimation error converges ***exponentially to zero***.

(2) There are several ***future problems*** to be studied.

   (a) First, although Assumption (iii) is obviously related to the ***detectability condition***, the detail should be investigated. Furthermore, ***how to check the condition (iii)*** is an important future problem to be studied.

   (b) Further, in constructing the proposed nonlinear observer, there is a free matrix parameter $P > 0$ to be chosen. This parameter seems to essentially determine the ***speed of error convergence*** of the observer, but no explicit discussion has been given to this problem.

(c) Another important future problem is to investigate the sensitivity of the proposed observer to ***noisy observation***.

(d) Finally, it is natural to consider the problem of extending the proposed observer to a perspective ***time-varying*** linear system of the form

$$\begin{cases} \dot{x}(t) = A(t)x(t) + v(t), \ \ x(0) = x_0 \in \boldsymbol{R}^n \\ y(t) = h(C(t)x(t)). \end{cases}$$

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