LUENBERGER-TYPE NONLINEAR OBSERVERS FOR PERSPECTIVE LINEAR SYSTEMS IN MACHINE VISION*

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Presented at Korea Advanced Institute of Science and Technology
15 October 2002

* This work was supported in part by the Grant-Aid for General Scientific Research of the Japanese Ministry of Education, Science and Culture under Grant C-13650497, and in part by the Research Institute for Science and Technology of Tokyo Denki University under Grant Q01J-05.

CONTENTS

I. INTRODUCTION

II. PRELIMINARIES

III. NONLINEAR OBSERVERS OF THE LUENBERGER-TYPE

IV. COMPUTER SIMULATIONS

V. CONCLUDING REMARKS

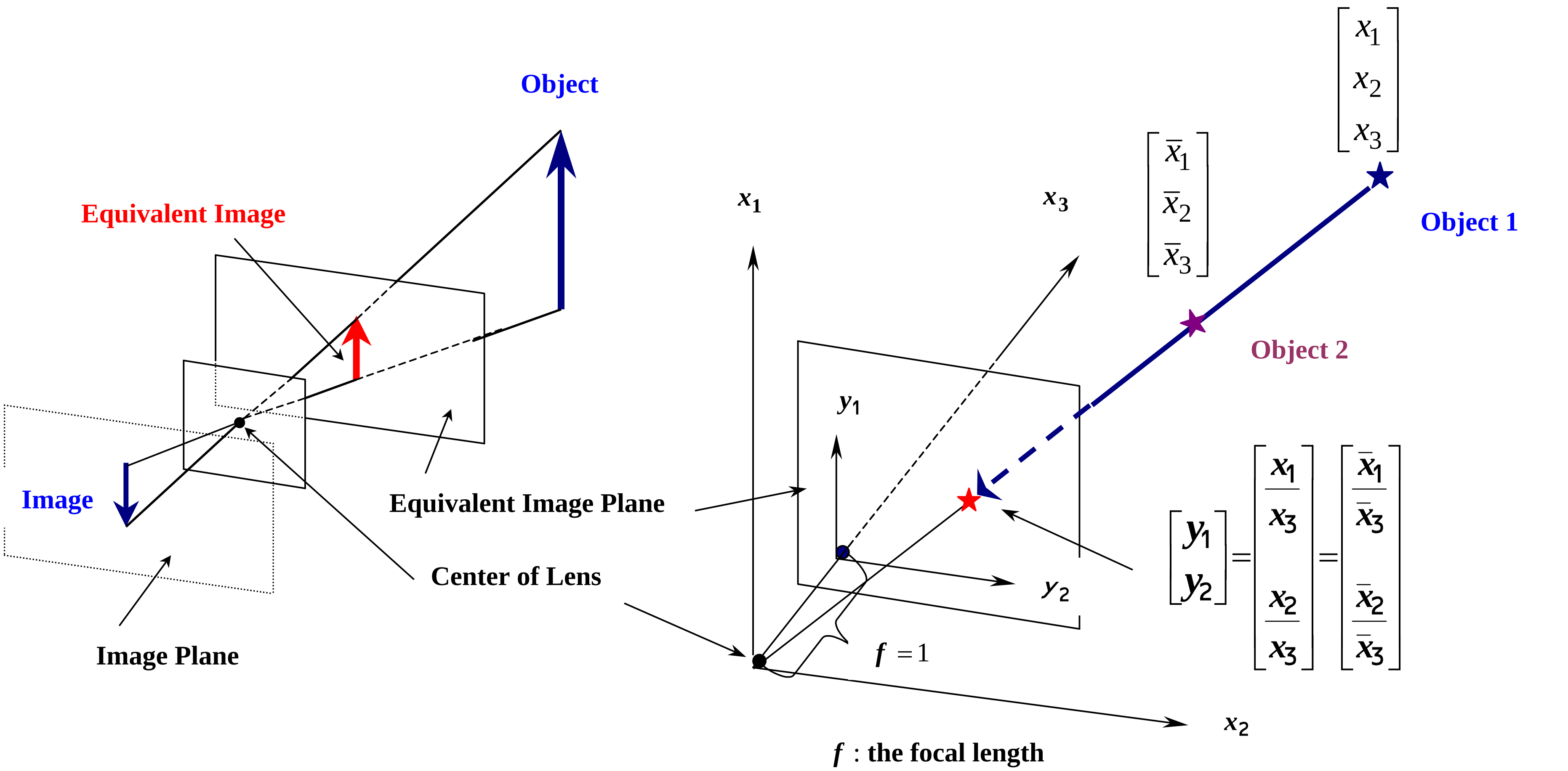
I. Introduction

(1) The essential problem in ***dynamical machine vision*** is how to determine the position and the shape of a moving rigid body from knowledge of the associated ***optical flow***.

(2) A ***perspective dynamical system*** arises from such a ***machine vision problem***, and this essential problem is described as the state estimation and parameter identification problem for such a system based on ***perspective observation*** (optical flow).

What is ***perspective observation***?

PERSPECTIVE OBSERVATION



(3) A ***perspective linear system*** we introduce in this paper is described as

$$(1.1)\qquad \begin{cases} \dot{x}(t) = Ax(t) + v(t), \quad x(0) = x_0 \in \boldsymbol{R}^n \\ y(t) = h(Cx(t)) \end{cases}$$

where $x(t) \in \boldsymbol{R}^n$ is the state, $v(t) \in \boldsymbol{R}^n$ the external input, $y(t) \in \boldsymbol{R}^m$ the ***(generalized) perspective observation***, $A \in \boldsymbol{R}^{n\times n}, C \in \boldsymbol{R}^{(m+1)\times n}$ are matrices with $m < n$, and finally $h : \boldsymbol{R}^{m+1} \to \boldsymbol{R}^m$ is a function of the form

$$(1.2)\qquad \begin{cases} h(\xi) := \begin{bmatrix} \dfrac{\xi_1}{\xi_{m+1}} & \cdots & \dfrac{\xi_m}{\xi_{m+1}} \end{bmatrix}^T, \quad \xi_{m+1} \neq 0, \\ \xi = \begin{bmatrix} \xi_1 & \cdots & \xi_m & \xi_{m+1} \end{bmatrix}^T \in \boldsymbol{R}^{m+1} \end{cases}.$$

A simplified 3-dimensional machine vision problem has an observation of the form

$$y = [y_1 \ y_2]^T = [x_1/x_3 \quad x_2/x_3]^T =: h(Cx),$$

where $n = 3, m = 2$ and $C = I_3$ (the identity matrix). Therefore, y is represented as a perspective observation of the form (1.1).

(4) This problem has been formulated and studied in a number of approaches. One of the interesting approaches is to formulate such a problem by introducing a notion of ***implicit observation*.**

(5) In the present paper, we propose a ***Luenberger-Type nonlinear observer*** for ***perspective linear systems*** without using the notion of implicit observation, and investigate its convergence problem.

More precisely, it is shown that, under suitable conditions on a given perspective linear systems, including

(a) System (1.1) is ***Lyapunov stable***,

(b) it satisfies ***some sort of detectability condition***,

it is possible to construct a nonlinear observer of Luenberger-type whose estimation error converges ***exponentially to zero***.

(6) Finally, using some simple examples appearing in machine vision, some numerical results are presented to illustrate the proposed nonlinear observer. The numerical results show that ***the observer proposed works well***.

II. PRELIMINARIES

BASIC NOTATIONS

$\boldsymbol{R}, \boldsymbol{C}$: the fields of real and complex numbers, respectively.

$\boldsymbol{R}^n, \boldsymbol{C}^n$: the Euclidean vector spaces with norm $\|x\|$.

For a matrix $M \in \boldsymbol{R}^{m\times n}$ or $M \in \boldsymbol{C}^{m\times n}$,

(2.1) $$\|M\| := \max\{\|Mx\| \mid \|x\| = 1\},$$

M^*: the conjugate transpose of M.

FUNCTION SPACES

The ***Lebesgue space*** $L_p[0,\infty)$ is the set of functions $f:[0,\infty)\to \boldsymbol{R}\,(\text{or}\,\boldsymbol{C})$ with the norm given by

$$\|f\|_p := \begin{cases} \left\{\int_0^\infty |f(t)|^p\,dt\right\}^{1/p}, & p\in[1,\infty) \\ \sup\{|f(t)|\,|\,t\in[0,\infty)\}, & p=\infty. \end{cases} \tag{2.2}$$

The m-dimensional Lebesgue space is denoted by $L_p^m[0,\infty)$, i. e.,

$$L_p^m[0,\infty) := \left\{ f = \begin{bmatrix} f_1 \\ \vdots \\ f_m \end{bmatrix} \,\middle|\, f_k \in L_p[0,\infty), k=1,\cdots,m \right\}. \tag{2.3}$$

Similarly $L_p^{m\times n}[0,\infty)$ is defined.

In particular, the following norms are used later:

$$(2.4)\qquad \begin{cases} \|f\|_2 := \left\{\int_0^\infty \|f(t)\|^2 dt\right\}^{\frac{1}{2}}, & f \in L_2^m[0,\infty) \\ \|f\|_\infty := \sup\limits_{t\geq 0}\{\|f(t)\|\}, & f \in L_\infty^m[0,\infty). \end{cases}$$

GENERALIZED EIGENSPACES

For a complex matrix $M \in \boldsymbol{C}^{m\times n}$, let

$\sigma(M)$: the set of all the eigenvalues of M,

$\sigma(M) = \{\lambda_1, \cdots, \lambda_q\} \subset \boldsymbol{C}$,

$W_k \subset \boldsymbol{C}^n$: the ***generalized eigenspace*** with respect to λ_k.

Then, it is well known that

(a) $\{W_1, \cdots, W_q\}$ is a set of linearly independent subspaces,

(b) $W_1 \oplus \cdots \oplus W_q = \boldsymbol{C}^n$,

(c) W_k is A-invariant, i.e., $AW_k \subset W_k$.

Next, for a matrix $A \in \boldsymbol{C}^{n \times n}$, let

(2.5) $\sigma_s(A) := \{\lambda \in \sigma(A) | \operatorname{Re}\lambda < 0\}$, $\sigma_{us}(A) := \{\lambda \in \sigma(A) | \operatorname{Re}\lambda \geq 0\}$.

W_s, W_{us}: the generalized eigenspaces corresponding to $\sigma_s(A), \sigma_{us}(A)$, respectively,

π_s, π_{us} : the matrix representations of the projection operators $\boldsymbol{C}^n \to W_s$, $\boldsymbol{C}^n \to W_{us}$ along with W_{us}, W_s, respectively.

Then:

(a) $\boldsymbol{C}^n = W_s \oplus W_{us}$,

(b) $\pi_s W_s \subset W_s$, $\pi_{us} W_{us} \subset W_{us}$,

(c) $A\pi_s = \pi_s A$ and $A\pi_{us} = \pi_{us} A$.

NECESSARY LEMMAS

(2.6) LEMMA. Let $A \in \boldsymbol{C}^{n\times n}$, and consider

$$\dot{x}(t) = Ax(t), \quad x(0) = x_0 \in \boldsymbol{C}^n. \qquad (*)$$

Then:

(i) System $(*)$ is asymptotically stable

$\Leftrightarrow \quad \forall Q \in \boldsymbol{C}^{n\times n}$ with $Q^* = Q > 0$, $\exists P \in \boldsymbol{C}^{n\times n}$ with $P^* = P > 0$ satisfying the Lyapunov equation

$$PA + A^*P = -Q.$$

(ii) System $(*)$ is Lyapunov stable

$\Leftrightarrow \exists P \in \boldsymbol{C}^{n\times n}$ with $P^* = P > 0$ satisfying

$$PA + A^*P \le 0.$$

(iii) (1) is Lyapunov stable

$$\Rightarrow \forall a > 0,\ \exists P \in \boldsymbol{C}^{n\times n} \text{ satisfying } P^* = P > 0 \text{ and}$$

$$PA + A^*P \le -a\pi_s^*\pi_s. \tag{4}$$

$\square$

Notice that $Q := a\pi_s^*\pi_s \in \boldsymbol{R}^{n\times n}$ is a nonnegative and Hermitian matrix.

(2.7) LEMMA. Let $A(\cdot) \in L_{\infty}^{n \times n}(\boldsymbol{R})$ and assume that $A(t)$ is continuously differentiable. Let $t_0 \in \boldsymbol{R}$ and consider

$$\dot{x}(t) = A(t)x(t), \quad x(t_0) = x_0 \in \boldsymbol{R}^n. \qquad (\ast)$$

Then:

System $(\ast)$ is exponentially stable, i.e., $\exists \alpha > 0$ and $\exists \beta > 0$ such that $\forall t_0 \geq 0$ and $\forall x(t_0) \in \boldsymbol{R}^n$

$$\|x(t)\| \leq \beta \|x(t_0)\| e^{-\alpha(t-t_0)}, \quad \forall t \geq t_0$$

$\Leftrightarrow$ $\exists \gamma > 0$ such that

$$\int_{t_0}^{\infty} \|x(t)\|^2 dt \leq \gamma \|x(t_0)\|^2, \quad \forall t_0 \geq 0 \text{ and } \forall x(t_0) \in \boldsymbol{R}^n. \quad \square$$

III. NONLINEAR OBSERVERS OF THE LUENBERGER-TYPE

In this section, we propose and study an ***observer*** for

(1.1) $$\begin{cases} \dot{x}(t) = Ax(t) + v(t), \quad x(0) = x_0 \in \boldsymbol{R}^n \\ y(t) = h(Cx(t)) \end{cases}$$

Notice that a full-order state observer for (1.1) generally has the form

(3.1) $$\frac{d}{dt}\hat{x}(t) = \varphi(\hat{x}(t), v(t), y(t)), \quad \hat{x}(0) = \hat{x}_0 \in \boldsymbol{R}^n$$

and it must satisfy that for any $v(\bullet)$

$$\hat{x}(0) = x(0) \Rightarrow \hat{x}(t) = x(t), \quad \forall t \geq 0.$$

Thus, we may assume that $\varphi(\hat{x}, v, y)$ has the form

$$\varphi(\hat{x}, v, y) = A\hat{x} + v + r(\hat{x}, y)$$

where $r(\hat{x}, y)$ is any function satisfying $r(x, h(Cx)) = 0, \forall x \in \boldsymbol{R}^n$. Further, as such a function $r(\hat{x}, y)$, we may take

$$r(\hat{x}, y) = K(y, \hat{x})[y - h(C\hat{x})],$$

where $K(y, \hat{x})$ is any sufficiently smooth function.

These choices of functions lead to a ***nonlinear observer of the Luenberger-type*** **(**D. G. Luenberger, An introduction to observers, *IEEE Trans. Automatic Control*, Vol. AC-16, 569-603, 1971**)**:

(3.2)

$$\frac{d}{dt}\hat{x}(t) = A\hat{x}(t) + v(t) + K(y(t), \hat{x}(t))[y(t) - h(C\hat{x}(t))], \quad \hat{x}(0) = \hat{x}_0 \in \boldsymbol{R}^n$$

where $K(y, \hat{x})$ is a suitably chosen matrix-valued function, called an ***observer gain matrix***.

In what follows, let us consider a suitable form of the gain matrix $K(y, \hat{x})$.

First, noticing that

$$\xi = \begin{bmatrix} \xi_1 \\ \vdots \\ \xi_m \\ \xi_{m+1} \end{bmatrix} := Cx, \quad \hat{\xi} = \begin{bmatrix} \hat{\xi}_1 \\ \vdots \\ \hat{\xi}_m \\ \hat{\xi}_{m+1} \end{bmatrix} := C\hat{x}, \quad C = \begin{bmatrix} C_1 \\ \vdots \\ C_m \\ C_{m+1} \end{bmatrix} \in \boldsymbol{R}^{(m+1)\times n},$$

we easily obtain

$$(3.3) \quad y - h(C\hat{x}) = h(Cx) - h(C\hat{x}) = \begin{bmatrix} \dfrac{\xi_1}{\xi_{m+1}} \\ \vdots \\ \dfrac{\xi_m}{\xi_{m+1}} \end{bmatrix} - \begin{bmatrix} \dfrac{\hat{\xi}_1}{\hat{\xi}_{m+1}} \\ \vdots \\ \dfrac{\hat{\xi}_m}{\hat{\xi}_{m+1}} \end{bmatrix} = \begin{bmatrix} \dfrac{\xi_1}{\xi_{m+1}} - \dfrac{\hat{\xi}_1}{\hat{\xi}_{m+1}} \\ \vdots \\ \dfrac{\xi_m}{\xi_{m+1}} - \dfrac{\hat{\xi}_m}{\hat{\xi}_{m+1}} \end{bmatrix}$$

$$
= \frac{1}{\hat{\xi}_{m+1}} \begin{bmatrix} 1 & 0 & \cdots & 0 & -\dfrac{\xi_1}{\xi_{m+1}} \\ 0 & 1 & \cdots & 0 & -\dfrac{\xi_2}{\xi_{m+1}} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -\dfrac{\xi_m}{\xi_{m+1}} \end{bmatrix} \begin{bmatrix} \xi_1 - \hat{\xi}_1 \\ \xi_2 - \hat{\xi}_2 \\ \vdots \\ \xi_{m+1} - \hat{\xi}_{m+1} \end{bmatrix}
$$

$$
= \frac{1}{C_{m+1}\hat{x}} \begin{bmatrix} 1 & 0 & \cdots & 0 & -y_1 \\ 0 & 1 & \cdots & 0 & -y_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -y_m \end{bmatrix} \begin{bmatrix} C_1(x-\hat{x}) \\ C_2(x-\hat{x}) \\ \vdots \\ C_{m+1}(x-\hat{x}) \end{bmatrix}
$$

$$
= \frac{1}{C_{m+1}\hat{x}} B(y)C(x-\hat{x}) = \frac{1}{C_{m+1}\hat{x}} B(y)C\rho,
$$

where

(3.4)
$$\begin{cases} B(y) := [I_m \quad -y] \in \mathbf{R}^{m\times(m+1)} \\ \rho := x - \hat{x}. \end{cases}$$

The structure of (3.3) suggests a gain matrix $K(y,\hat{x})$ of the form

(3.5)
$$K(y,\hat{x}) = C_{m+1}\hat{x}P^{-1}C^{*}B^{*}(y),$$

so that the Luenberger-Type nonlinear observer becomes

(3.6)
$$\frac{d}{dt}\hat{x}(t) = A\hat{x}(t) + v(t) + C_{m+1}\hat{x}P^{-1}C^{*}B^{*}(y)[y(t) - h(C\hat{x}(t))], \quad \hat{x}(0) = \hat{x}_0 \in \mathbf{R}^n$$

where $P \in \mathbf{R}^{n\times n}$ is an appropriately chosen ***free parameter matrix***.

Now, we make various conditions on System (1.1), which seem to be necessary and/or reasonable from the viewpoint of machine vision.

(3.7) ASSUMPTION.

(i) System (1.1) is ***Lyapunov stable***, i.e.,

$$\sigma(A) = \sigma_s(A) \cup \sigma_{us}(A)$$

where

$\sigma_s(A)$: the set of eigenvalues with strictly negative real part

$\sigma_{us}(A)$: the set of eigenvalues with zero real part.

(ii) $y(t)$ is a ***continuous and bounded function***, that is,

$$y(\cdot) \in C^m[0,\infty) \cap L_\infty^m[0,\infty).$$

(iii) Let

$W_s, W_{us} \subset \boldsymbol{C}^n$: the generalized eigenspaces corresponding to $\sigma_s(A)$ and $\sigma_{us}(A)$ respectively,

$E_{us} = [\xi_1 \quad \cdots \quad \xi_r]$: a basis matrix for W_{us} with $r := \dim W_{us}$.

Then, $\exists T > 0$ and $\exists \varepsilon > 0$ such that

$$\int_0^T E_{us}^* e^{A^* \tau} C^* B^* (y(t+\tau)) B(y(t+\tau)) C e^{A\tau} E_{us} d\tau \geq \varepsilon I_r, \quad \forall t \geq 0.$$

$\square$

(3.8) REMARK. All the conditions given in Assumption (3.7) are reasonable requirements from the viewpoint of machine vision.

(i) The condition (i) is imposed to ensure that if $v(t) \equiv 0$ then the motion of a moving body take places within ***a bounded region***.

(ii) The condition (ii) is imposed to ensure that the motion $x(t)$ described by (1) is smooth enough and takes place inside ***a conical region centered at the camera*** so as to produce a continuous and bounded measurement $y(t)$ on the image plane. In particular, it is assumed that the motion never crosses the plane $C_{m+1}x = 0$, and hence takes place ***only on one side of the camera***.

(iii) The condition (iii) ensures ***some sort of detectability*** of the perspective system (1.1), and further ***the external input being identically zero***. These facts will be cited in Proposition 3.9. $\square$

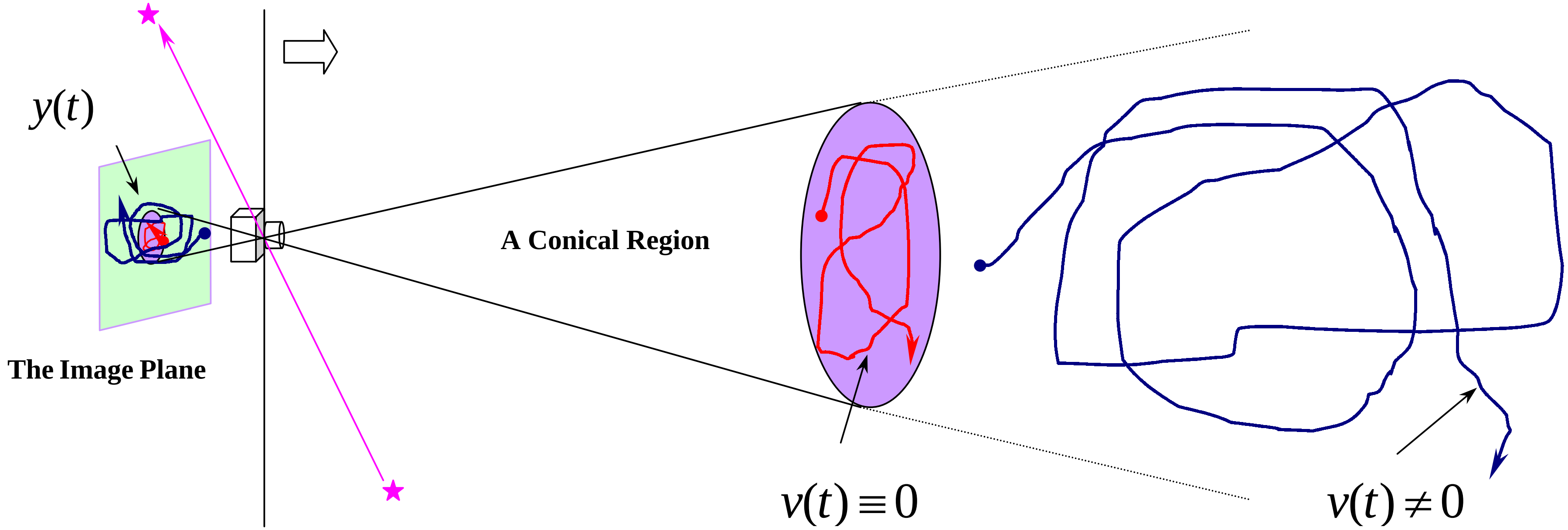
Assumption 3.7

(i) Sytem (1.1) is Lyapunov stable (i.e., A is Lyapunov stable).

(ii) The perspective observation $y(t)$ is continuous and bounded.

(iii) Some sort of detectability is satisfied on the trajectory $x(t)$.

(1.1) $\dot{x}(t) = Ax(t) + v(t), \quad x(0) = x_0 \in \boldsymbol{R}^n, \quad y(t) = h(Cx(t))$



(3.9) PROPOSITION. Assume that System (1.1) is Lyapunov stable, let A_{us} denote the unstable part of the matrix A and set $C_{us} := CE_{us}$. If Assumption 3.7 (iii) is satisfied, then the following statements hold true.

(i) (C, A) is ***a detectable pair***, that is, the unstable part (C_{us}, A_{us}) of (C, A) is observable.

(ii) The external input $v(t)$ is ***never identically zero***. $\square$

MAIN THEOREM

(3.10) THEOREM (LUENBERGER-TYPE NONLINEAR OBSERVERS).

Assume that System (1.1) satisfies Assumption 3.7 and consider a nonlinear observer of the Luenberger-type, that is,

$$\frac{d}{dt}\hat{x}(t) = A\hat{x}(t) + v(t) + K(y(t), \hat{x}(t))[y(t) - h(C\hat{x}(t))], \qquad (1)$$

$$\hat{x}(0) = \hat{x}_0 \in \boldsymbol{R}^n$$

where the gain matrix is given by

$$K(y, \hat{x}) := C_{m+1}\hat{x}\, P^{-1} C^* B^*(y), \quad B(y) := [I_m \quad -y] \in \boldsymbol{R}^{m\times(m+1)}. \qquad (2)$$

Further, let

$\pi_s : \boldsymbol{C}^n \to W_s$ denote the projection operator,

$P \in \boldsymbol{R}^{n\times n}$ be a symmetric positive definite matrix satisfying

$$A^{*}P + PA \leq -a\pi_s^{*}\pi_s, \quad a > 0. \tag{3}$$

Then, the following statements hold.

(i) The estimation error

$$\rho(t) := x(t) - \hat{x}(t) \tag{4}$$

satisfies the differential equation

$$\frac{d}{dt}\rho(t) = [A - P^{-1}C^{*}B^{*}(y(t))B(y(t))C]\rho(t), \quad \rho(0) \in \boldsymbol{R}^{n}. \tag{5}$$

(ii) $\rho(t)$ converges ***exponentially to zero***, that is, there exist $\alpha > 0, \beta > 0$ such that

$$\|\rho(t)\| := \|x(t) - \hat{x}(t)\| \leq \beta e^{-\alpha t}\|\rho(0)\|, \quad \forall t \geq 0. \tag{6}$$

SKETCH OF PROOF: The statement (i) can be easily verified. Therefore, only the statement (ii) is briefly proved.

To prove (ii), it suffices to show by virtue of Lemma 2.7 that

$$\int_{t_0}^{\infty} \|\rho(t)\|^2 dt \le \gamma \|\rho(0)\|^2 \tag{7}$$

where $\gamma > 0$ is a constant, independent of $\rho(0)$.

Using the notations in Theorem 3.10, define

$$\rho_s(t) := \pi_s \rho(t), \quad \rho_{us}(t) := \pi_{us} \rho(t), \qquad \forall t \ge 0. \tag{8}$$

Then, since $\boldsymbol{C}^n = W_s \oplus W_{us}$, one has

$$\rho(t) = \rho_s(t) + \rho_{us}(t), \tag{9}$$

which together with (5), (3) and (9) easily gives

$$\frac{d}{dt}(\rho(t)^* P\rho(t)) \le -a\|\rho_s(t)\|^2 - 2\|B(y(t))C\rho(t)\|^2.$$

Hence, for any $t \geq 0$,

$$0 \leq \rho^*(t)P\rho(t)$$
$$\leq \rho^*(0)P\rho(0) - a\int_0^t \|\rho_s(s)\|^2 ds - 2\int_0^t \|B(y(s))C\rho(s)\|^2 ds,$$

which leads to the following inequalities:

$$\begin{cases} \rho^*(t)P\rho(t) \leq \rho^*(0)P\rho(0), \quad \forall t \geq 0 \\ a\int_0^t \|\rho_s(s)\|^2 ds \leq \rho^*(0)P\rho(0), \quad \forall t \geq 0 \\ 2\int_0^t \|B(y(s))C\rho(s)\|^2 ds \leq \rho^*(0)P\rho(0), \quad \forall t \geq 0. \end{cases} \tag{10}$$

Now one can easily obtain from (10) that

$$\int_0^\infty \|\rho_s(t)\|^2 dt \leq \frac{\|P\|}{a}\|\rho(0)\|^2 =: \gamma_s \|\rho(0)\|^2, \tag{11}$$

where $\gamma_s > 0$ is a constant, independent of $\rho(0)$. Further, after lengthy and cumbersome technical arguments, one can also obtain a similar inequality for $\rho_{us}(t)$, which is given as

$$\int_0^\infty \|\rho_{us}(t)\|^2 dt \le \gamma_{us} \|\rho(0)\|^2, \tag{12}$$

where $\gamma_{us} > 0$ is again a constant, independent of $\rho(0)$. Thus it follows from (9), (11) and (12) that

$$\begin{aligned}\int_0^\infty \|\rho(t)\|^2 dt &\le 2\int_0^\infty \|\rho_s(t)\|^2 dt + 2\int_0^\infty \|\rho_{us}(t)\|^2 dt \\ &\le 2\gamma_s \|\rho(0)\|^2 + 2\gamma_{us}\|\rho(0)\|^2 = 2(\gamma_s + \gamma_{us})\|\rho(0)\|^2 =: \gamma\|\rho(0)\|^2\end{aligned}$$

where $\gamma > 0$ is a constant, independent of $\rho(0)$. $\square$

IV. COMPUTER SIMULATIONS

Some simple examples are used to illustrate the result obtained. The simulation result seems to indicate the proposed nonlinear observer works quite well.

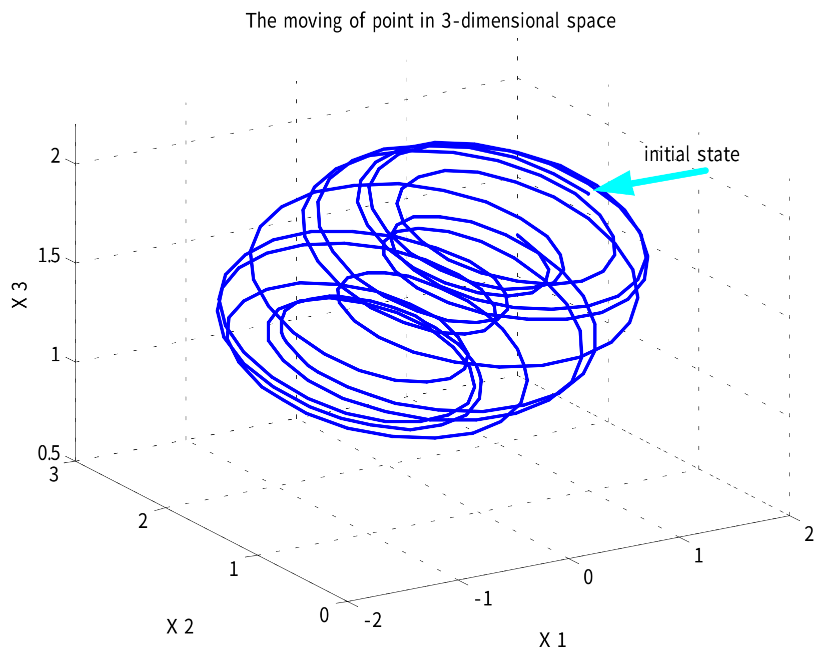
For the example we consider is the system with the following data:

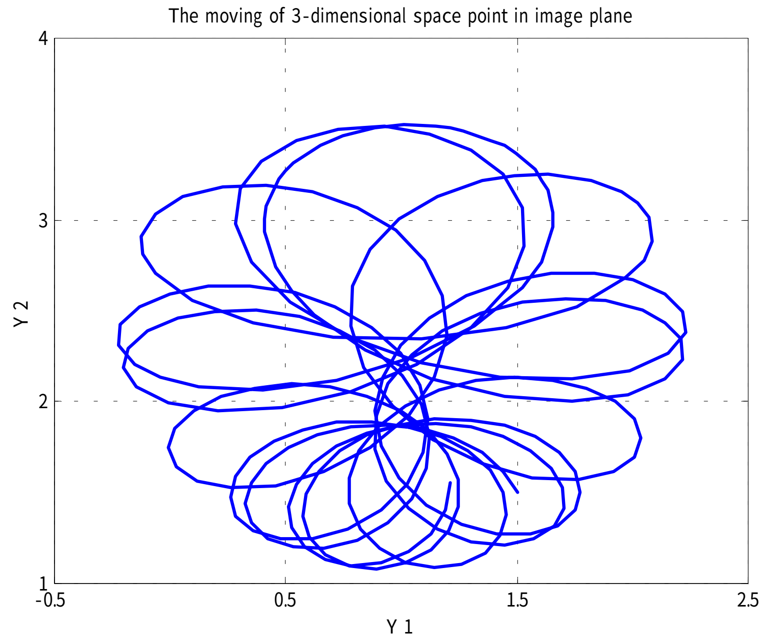
$$A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix},$$

$$v(t) = 2\pi\begin{bmatrix} -\sin(2\pi t) & \cos(2\pi t) & 0 \end{bmatrix}^T$$

$$x_0 = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix}^T.$$

Then, the state trajectory $x(t)$ and the perspective observation trajectory $y(t)$ are given as follows:

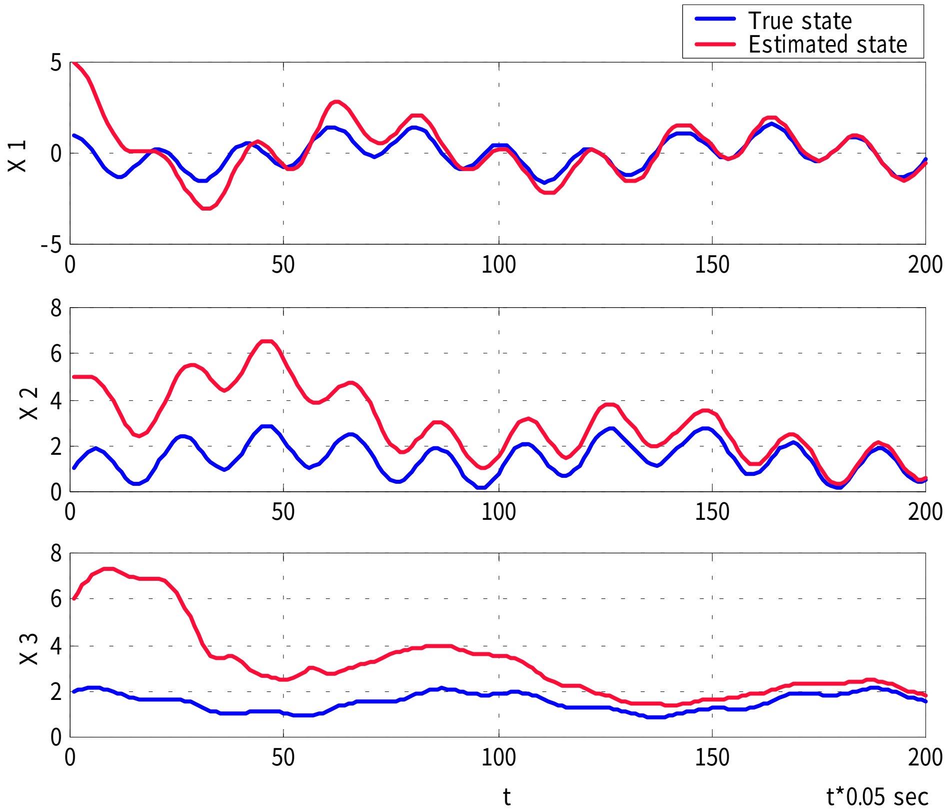




where $$y(t) = \left[(x_1(t) + x_3(t)) / x_3(t) \quad (x_2(t) + x_3(t)) / x_3(t) \right]^T.$$

Next, for the observer, we set the following data:

$$\hat{x}_0 = [5 \quad 5 \quad 6]^T, \quad P^{-1} = \mathrm{diag}\{30, 30, 30\}.$$



IV. CONCLUDING REMARKS

This paper discussed a ***nonlinear observer*** for ***perspective linear systems*** arising in machine vision.

(1) ***A Luenberger-type nonlinear observer*** was proposed, and under some reasonable assumptions on a perspective system, it was shown that it is possible to construct a nonlinear observer whose estimation error converges ***exponentially to zero***.

(2) Further, computer simulations using typical examples in machine vision were performed, and the results indicate that the proposed nonlinear observer works well.

(3) There are several ***future problems*** to be studied.

(a) First, although Assumption (3.7) (iii) is obviously related to the ***detectability condition***, the detail should be investigated. Furthermore, ***how to check the condition (iii)*** is an important future problem to be studied.

(b) Further, in constructing the proposed nonlinear observer, there is a free matrix parameter $P > 0$ to be chosen. This parameter seems to essentially determine the ***speed of error convergence*** of the observer, but no explicit discussion has been given to this problem.

(c) Another important future problem is to investigate the sensitivity of the proposed observer to ***noisy observation***.

(d) Finally, it is natural to consider the problem of extending the proposed observer to a perspective linear ***time-varying*** system of the form

$$\begin{cases} \dot{x}(t) = A(t)x(t) + v(t), \quad x(0) = x_0 \in \boldsymbol{R}^n \\ y(t) = h(C(t)x(t)). \end{cases}$$

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