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Output Feedback Control for a Class of Nonlinear Systems

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Abstract: This paper studies the global stabilization problem by an output controller for a family of uncertain nonlinear systems satisfying some relaxed triangular-type conditions and with dynamics which may not be exactly known. Using a feedback domination design method, we explicitly construct a dynamic output compensator which globally stabilizes such an uncertain nonlinear system. The usefulness of our result is illustrated with an example.

Keywords: Nonlinear system, global stabilization, output feedback.

1 Introduction

The problem of controlling nonlinear systems by output feedback is one of the most important problems in the field of nonlinear control. Unlike in the case of linear systems, the separation principle generally does not hold for general nonlinear systems[1]. Due to this, the problem has been more difficult and challenging for the systems and control community.

In recent years, many important results for the problem have been obtained. However, as investigated in [1], some extra growth conditions on the immeasurable states of a system are usually necessary for the global stabilization of nonlinear systems via output feedback. Since this work, a great deal of subsequent research work has focused on the output feedback stabilization of nonlinear systems under various structural or growth conditions[2~8]. For example, it was assumed that the nonlinear terms of a given system satisfy triangular conditions in [2] and [8], or some global Lipschitz-like condition in [7], *etc.*

In this paper, we consider essentially the same class of nonlinear systems as treated in [2,5~8]. So far, it seems that one of most relaxed conditions imposed on the nonlinear terms of a given system, is a triangular-type condition as far as output feedback control is concerned, as shown in [2,5,6].

Most recently, as a new way of understanding observers, a backstepping-like design procedure for observers was introduced in [2] and [8], in which global stabilization was achieved by a linear output feedback controller under a triangular condition.

The main purpose of this paper is to develop a global stabilizer for a class of uncertain nonlinear systems by linear output feedback under a far more relaxed condition on the nonlinear terms of a given system than the triangular-type condition. In fact, we consider the following class of uncertain nonlinear systems

$$\begin{aligned}\dot{x}_1 &= x_2 + \delta_1(t,x,u)\\ \dot{x}_2 &= x_3 + \delta_2(t,x,u)\\ &\ \vdots\\ \dot{x}_n &= u + \delta_n(t,x,u)\\ y &= x_1\end{aligned} \tag{1}$$

where $x = [x_1, x_2, \cdots, x_n]^{\mathrm{T}} \in R^n$ is the state, and $u \in R$ and $y \in R$ are input and output of the system, respectively. One feature of this paper is that our design method for global stabilizing controllers does not require a detailed structure for the nonlinear terms $\delta_i : R \times R^n \times R \to R$, $i = 1, \cdots, n$, including a triangular-type condition (see (3) below), except that they are Lipschitz continuous and satisfy the following assumptions.

Assumption 1. For system (1), there exist some constants $c > 0$ and $0 < \alpha \leqslant 1$ such that for any $s \in (0, \alpha)$, the inequality

$$\sum_{i=1}^{n} s^{i-1}|\delta_i(t,x,u)| \leqslant c\sum_{i=1}^{n} s^{i-1}|x_i| \tag{2}$$

is satisfied.

It is not difficult to see that if a triangular condition is satisfied for $\delta_i(t,x,u)$ as in [1,2,4,5,6,8], i.e.,

$$|\delta_i(t,x,u)| \leqslant c\sum_{j=1}^{i}|x_j| \tag{3}$$

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then Assumption 1 is always satisfied, but not vice versa. In fact, if condition (3) is satisfied, then for any $s \in (0, \alpha)$

$$\begin{aligned}\sum_{i=1}^{n} s^{i-1}|\delta_i(t,x,u)| \leqslant\ & c|x_1| + cs(|x_1|+|x_2|) + \\ & \cdots + cs^{n-1}(|x_1|+\cdots+|x_n|) \leqslant \\ & c(1+s+\cdots+s^{n-1})|x_1| + \\ & cs(1+s+\cdots+s^{n-2})|x_2| + \cdots + cs^{n-1}|x_n| \leqslant \\ & c\left(\sum_{i=1}^{n} s^{i-1}\right)\sum_{i=1}^{n} s^{i-1}|x_i|\end{aligned}$$

and hence Assumption 1 is satisfied. However, it is clear that the converse may not always hold true.

2 Global stabilization by output feedback

In this section, we prove that there exists a dynamic output compensator of the form

$$\dot{\xi} = f(\xi, y),\ u = h(\xi, y) \tag{4}$$

such that closed-loop system (1) with dynamic output compensator (4) satisfies

$$\lim_{t\to\infty}(x(t), \xi(t)) = (0,0).$$

That is to say that the system (1) is stabilized by the dynamic output compensator (4). The dynamic output compensator we propose is made of a linear high gain observer and a linear high gain controller as follows:

Theorem 1. Under Assumption 1, there is a dynamic output compensator of the form (4), which solves the global stabilization problem for an uncertain nonlinear system of the form (1).

Proof. We begin by introducing the following dynamic system

$$\begin{aligned}\dot{\hat{x}}_1 &= \hat{x}_2 + ra_1(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 &= \hat{x}_3 + r^2 a_2(x_1 - \hat{x}_1) \\ &\ \vdots \\ \dot{\hat{x}}_n &= u + r^n a_n(x_1 - \hat{x}_1)\end{aligned} \tag{5}$$

where $r \geqslant 1$ is a gain parameter to be determined later, and $a_i (i = 1, \cdots, n)$ are the coefficients of any Hurwitz polynomial $\rho^n + a_1\rho^{n-1} + \cdots + a_{n-1}\rho + a_n$.

Next, treat (5) as an observer for system (1), and consider the estimation error

$$e_i = x_i - \hat{x}_i,\ 1 \leqslant i \leqslant n. \tag{6}$$

Then it follows from (1) and (5) that

$$\begin{aligned}\dot{e}_1 &= e_2 - ra_1e_1 + \delta_1(t,x,u) \\ \dot{e}_2 &= e_3 - r^2a_2e_1 + \delta_2(t,x,u) \\ &\ \vdots \\ \dot{e}_n &= -r^n a_n e_1 + \delta_n(t,x,u).\end{aligned} \tag{7}$$

Further, introduce the scaled estimation error ε by

$$\varepsilon_i = \frac{1}{r^{i-1}}e_i,\ 1 \leqslant i \leqslant n \tag{8}$$

and set

$$\varepsilon = [\varepsilon_1\ \ \varepsilon_2\ \ \cdots\ \ \varepsilon_n]^{\mathrm{T}} \in R^n. \tag{9}$$

Then one obtains

$$\begin{aligned}\dot{\varepsilon}_1 &= r(\varepsilon_2 - a_1\varepsilon_1) + \delta_1(t,x,u) \\ \dot{\varepsilon}_2 &= r(\varepsilon_3 - a_2\varepsilon_1) + \frac{1}{r}\delta_2(t,x,u) \\ &\ \vdots \\ \dot{\varepsilon}_n &= -ra_n\varepsilon_1 + \frac{1}{r^{n-1}}\delta_n(t,x,u)\end{aligned} \tag{10}$$

or equivalently

$$\dot{\varepsilon} = rA\varepsilon + \Phi_1 \tag{11}$$

where

$$A = \begin{pmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ -a_2 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_{n-1} & 0 & 0 & \cdots & 1 \\ -a_n & 0 & 0 & \cdots & 0 \end{pmatrix}$$

$$\Phi_1 = [\delta_1(t,x,u), \frac{1}{r}\delta_2(t,x,u), \cdots, \frac{1}{r^{n-1}}\delta_n(t,x,u)]^{\mathrm{T}}. \tag{12}$$

Now consider the quadratic function given as

$$V_1 := \varepsilon^{\mathrm{T}} P \varepsilon \tag{13}$$

where P is a positive definite symmetric matrix satisfying

$$A^{\mathrm{T}}P + PA = -I. \tag{14}$$

Then it follows from (10) and (14) that the time derivative of V_1 along the solution of (11) satisfies

$$\begin{aligned}\dot{V}_1 =& r\varepsilon^{\mathrm{T}}(A^{\mathrm{T}}P + PA)\varepsilon + 2\varepsilon^{\mathrm{T}}P\Phi_1 \leqslant \\ & -r\|\varepsilon\|^2 + 2\varepsilon^{\mathrm{T}}P\Phi_1.\end{aligned} \tag{15}$$

From Assumption 1 and the fact that $r \geqslant 1$, one can obtain

$$\begin{aligned}\|\Phi_1\| \leqslant& (|\delta_1| + \frac{1}{r}|\delta_2| + \cdots + \frac{1}{r^{n-1}}|\delta_n|) \leqslant \\ & c\sum_{j=1}^{n}\left(\frac{1}{r}\right)^{j-1}\sum_{i=1}^{n}\left(\frac{1}{r}\right)^{i-1}|x_i| \leqslant \\ & nc\sum_{i=1}^{n}\frac{1}{r^{i-1}}|x_i|.\end{aligned}$$

Further, a simple computation with (6) and (8) gives

$$2\varepsilon^{\mathrm{T}}P\Phi_1 \leqslant 2\|\varepsilon\|\|P\|nc\sum_{i=1}^{n}\frac{1}{r^{i-1}}|x_i| \leqslant$$
$$2\|\varepsilon\|\|P\|nc\sum_{i=1}^{n}\left(\frac{1}{r^{i-1}}|\hat{x}_i|+|\varepsilon_i|\right)=$$
$$2\|\varepsilon\|\|P\|nc\left(\sum_{i=1}^{n}\frac{1}{r^{i-1}}|\hat{x}_i|+\sum_{i=1}^{n}|\varepsilon_i|\right)\leqslant$$
$$2\|\varepsilon\|\|P\|nc\left(\sum_{i=1}^{n}\frac{1}{r^{i-1}}|\hat{x}_i|+\sqrt{n}\|\varepsilon\|\right)\leqslant$$
$$2\|P\|nc\left(\sum_{i=1}^{n}\frac{1}{r^{i-1}}|\hat{x}_i|\|\varepsilon\|+\sqrt{n}\|\varepsilon\|^2\right)\leqslant$$
$$2\|P\|nc\Big(\sum_{i=1}^{n}\frac{1}{2}\left(\frac{1}{r^{2(i-1)}}\hat{x}_i|^2+\|\varepsilon\|^2\right)+\sqrt{n}\|\varepsilon\|^2\Big)\leqslant$$
$$n(2\sqrt{n}+n)c\|P\|\sum_{i=1}^{n}\frac{1}{r^{2(i-1)}}\hat{x}_i|^2+$$
$$n(2\sqrt{n}+n)c\|P\|\|\varepsilon\|^2\leqslant$$
$$k_1\|\varepsilon\|^2+k_1\sum_{i=1}^{n}\frac{1}{r^{2(i-1)}}\hat{x}_i^2$$

where $k_1=cn(2\sqrt{n}+n)\|P\|$ is a constant, independent of r. Then from (15), one obtains

$$\dot{V}_1\leqslant-(r-k_1)\|\varepsilon\|^2+k_1\sum_{i=1}^{n}\frac{1}{r^{2(i-1)}}\hat{x}_i^2. \tag{16}$$

Next introduce

$$\xi_i=\frac{\hat{x}_i}{r^{i-1}},\ 1\leqslant i\leqslant n$$

and set $\xi=[\xi_1\quad\xi_2\quad\cdots\quad\xi_n]^{\mathrm{T}}\in R^n$. Then,

$$\begin{aligned}\dot{\xi}_1&=r\xi_2+ra_1\varepsilon_1\\ \dot{\xi}_2&=r\xi_3+ra_2\varepsilon_1\\ &\vdots\\ \dot{\xi}_n&=r\left(\frac{1}{r^n}u\right)+ra_n\varepsilon_1\end{aligned} \tag{17}$$

and hence inequality (17) can be written as

$$\dot{V}_1\leqslant-(r-k_1)\|\varepsilon\|^2+k_1\|\xi\|^2. \tag{18}$$

Now we design a compensator of the form

$$u=-r^n(b_n\xi_1+b_{n-1}\xi_2+\cdots+b_1\xi_n) \tag{19}$$

where b_i are the coefficients of any Hurwitz polynomial $\rho^n+b_1\rho^{n-1}+\cdots+b_{n-1}\rho+b_n$. Then, it is easy to verify that ξ-subsystem (17) with controller (19) can be expressed as

$$\dot{\xi}=rB\xi+r\varepsilon_1[a_1,a_2,\cdots,a_n]^{\mathrm{T}} \tag{20}$$

where

$$B=\begin{pmatrix}0&1&\cdots&0\\ \vdots&\vdots&\ddots&\vdots\\ 0&0&\cdots&1\\ -b_n&-b_{n-1}&\cdots&-b_1\end{pmatrix}.$$

Further, choose a quadratic function of the form

$$V_2:=\xi^{\mathrm{T}}Q\xi \tag{21}$$

where Q is a positive definite symmetric matrix satisfying

$$B^{\mathrm{T}}Q+QB=-2I. \tag{22}$$

Then one can easily obtain the inequality

$$\begin{aligned}\dot{V}_2=&\dot{\xi}^{\mathrm{T}}Q\xi+\xi^{\mathrm{T}}Q\dot{\xi}=\\ &r\xi^{\mathrm{T}}(B^{\mathrm{T}}Q+QB)\xi+2r\xi^{\mathrm{T}}Qa\varepsilon_1\leqslant\\ &-2r\|\xi\|^2+2r\xi^{\mathrm{T}}Qa\varepsilon_1.\end{aligned} \tag{23}$$

And similarly

$$\begin{aligned}2r\xi^{\mathrm{T}}Qa\varepsilon_1&\leqslant 2r\|\xi\|\|Q\|\|[a_1,\cdots,a_n]^{\mathrm{T}}\|\|\varepsilon\|\\ &\leqslant 2r\left(\frac{1}{2}\|\xi\|^2+\frac{1}{2}(\|Q\|\|[a_1,\cdots,a_n]^{\mathrm{T}}\|\|\varepsilon\|)^2\right)\\ &\leqslant r\|\xi\|^2+r(\|Q\|\|[a_1,\cdots,a_n]^{\mathrm{T}}\|)^2\|\varepsilon\|^2\\ &\leqslant r\|\xi\|^2+rk_2\|\varepsilon\|^2\end{aligned}$$

where $k_2(\|Q\|\|[a_1,\cdots,a_n]^{\mathrm{T}}\|)^2$ is a constant, independent of r. Therefore, inequality (23) can be written as

$$\dot{V}_2\leqslant-r\|\xi\|^2+rk_2\|\varepsilon\|^2. \tag{24}$$

Next, we observe that the closed-loop system (1) with (5) and (19) can be treated as an interconnection of a ε-subsystem and ξ-subsystem. Now, consider the function

$$V:=(k_2+1)V_1+V_2=(k_2+1)\varepsilon^{\mathrm{T}}P\varepsilon+\xi^{\mathrm{T}}Q\xi. \tag{25}$$

It easily follows from (18) and (24) that

$$\begin{aligned}\dot{V}=&(k_2+1)\dot{V}_1+\dot{V}_2\leqslant-(r-k_1)(k_2+1)\|\varepsilon\|^2+\\ &k_1(k_2+1)\|\xi\|^2-r\|\xi\|^2+rk_2\|\varepsilon\|^2\leqslant\\ &-(r-k_1(k_2+1))\|\varepsilon\|^2-(r-k_1(k_2+1))\|\xi\|^2=\\ &-(r-k_1(k_2+1))(\|\varepsilon\|^2+\|\xi\|^2).\end{aligned} \tag{26}$$

Clearly, if we choose the gain parameter r to be

$$r\geqslant 1+k_1(k_2+1)$$

then

$$\dot{V}_2\leqslant-(\|\varepsilon\|^2+\|\xi\|^2).$$

This implies

$$\varepsilon(t) \to 0,\ \xi(t) \to 0 \text{ as } t \to \infty$$

and hence that closed-loop system (1) with (5) and (19) is globally asymptotically stable. This completes the proof. □

The new approach proposed does not need to go through a recursive design procedure as in [8]. It can determine all the observer and controller parameters in one step, rather than n-steps[2,8].

Example. To check the effectiveness of the result, consider the following simple system

$$\begin{aligned} \dot{x}_1 &= x_2 + \frac{x_1}{(1-c_1x_2)^2 + x_2^2} \\ \dot{x}_2 &= u + \ln(1 + (x_2^2)^{c_2}) \\ y &= x_1 \end{aligned} \tag{27}$$

where c_1 and $c_2 \geqslant 1$ are constant. It is easy to check that system (27) satisfies Assumption 1. Therefore, by Theorem 1, a globally stabilizing output dynamic compensator can be constructed. To construct such a compensator by following the proof of Theorem 1, choose the coefficients of the two Hurwitz polynomials to be $a_1 = a_2 = 1$ and $b_1 = 11/4$, $b_2 = 20$. Then, the compensator given by (19) can now be described as

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + r(y - \hat{x}_1) \\ \dot{\hat{x}}_2 &= u + r^2(y - \hat{x}_1) \\ u &= -r(b_2r\hat{x}_1 + b_1\hat{x}_2). \end{aligned} \tag{28}$$

For our numerical simulation, we choose $r \geqslant 8339$, and the initial states to be

$$(x_1(0), x_2(0), \hat{x}_1(0), \hat{x}_2(0)) = (1, 5, 3, 5).$$

Simulation results are shown in Fig. 1. They show the effectiveness of output dynamic compensator (28).

From the design procedure of Theorem 1, it is clear that there is a linear output feedback controller (5)~(19) which makes the entire family of nonlinear systems (1) simultaneously asymptotically stable, as long as they satisfy Assumption 1.

The global stabilization idea above can be extended to a family of nonlinear systems of the following form

$$\begin{aligned} \dot{z} &= f(z) + g(t, z, x, u) \\ \dot{x}_1 &= x_2 + \delta_1(t, z, x, u) \\ \dot{x}_2 &= x_3 + \delta_2(t, z, x, u) \\ &\vdots \\ \dot{x}_n &= u + \delta_n(t, z, x, u) \\ y &= x_1 \end{aligned} \tag{29}$$

where $u, y \in R$ are input and output respectively, and $(z, x) \in R^m \times R^n$ is the state, as long as the following assumption is satisfied.

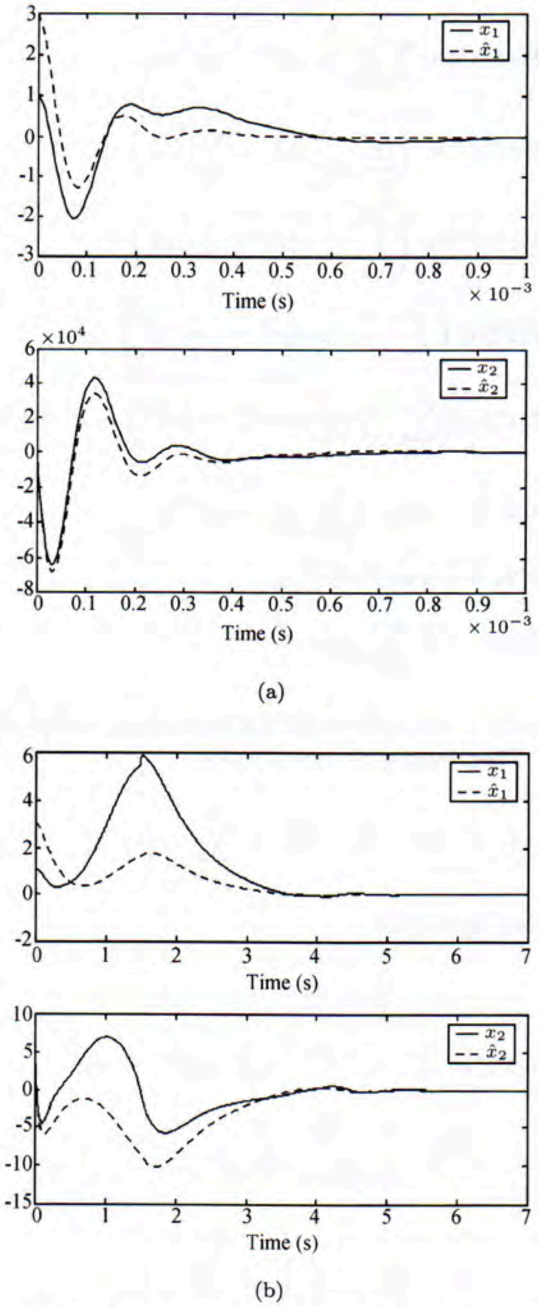

Fig. 1 State trajectories: (a) shows the results by the proposed method and (b) the results by the method in [8]

Assumption 2. For system (29), suppose that

i) $\dot{z} = f(z)$ is globally exponentially stable at $z = 0$

ii) There exist some constants $\hat{c} > 0$, $\tilde{c} > 0$, and $0 < \alpha \leqslant 1$, such that for any $s \in (0, \alpha)$, the inequalities

$$|g(t, z, x, u)| \leqslant \hat{c}|x_1|$$

$$\sum_{i=1}^{n} s^{i-1}|\delta_i(t,z,x,u)| \leqslant \tilde{c}\sum_{i=1}^{n} s^{i-1}(\|z\|+|x_i|) \tag{30}$$

are satisfied.

Theorem 2. Under Assumption 2, there is a dynamic output compensator of the form (4), which solves the global stabilization problem for an uncertain nonlinear system of the form (29).

Proof. Since system (29) satisfies Assumption 2, by the converse theorem of globally exponential stability[9], there is a positive and radically unbounded function $V(z)$ such that

$$\frac{\partial V(z)}{\partial z}f(z) \leqslant -\|z\|^2$$
$$\left\|\frac{\partial V(z)}{\partial z}\right\| \leqslant \bar{c}\|z\| \text{ with } \bar{c}>0.$$

This in turn implies

$$\frac{\partial V(z)}{\partial z}(f(z)+g(z,x,u)) \leqslant -\|z\|^2+\left\|\frac{\partial V(z)}{\partial z}\right\||g(z,x,u)| \leqslant -\|z\|^2+\bar{c}\hat{c}\|z\||x_1| \leqslant -\frac{3}{4}\|z\|^2+(\bar{c}\hat{c})^2|x_1|^2. \tag{31}$$

Now, one can construct a dynamic system (5) with a gain parameter r to be determined later.

Next, assume that (5) is an observer for system (29), and consider the estimation error

$$e_i = x_i - \hat{x}_i,\ 1\leqslant i\leqslant n. \tag{32}$$

Then, it follows from (29) and (5) that

$$\begin{aligned}\dot{e}_1 &= e_2 - ra_1e_1 + \delta_1(t,z,x,u)\\ \dot{e}_2 &= e_3 - r^2a_2e_1 + \delta_2(t,z,x,u)\\ &\ \vdots\\ \dot{e}_n &= -r^na_ne_1 + \delta_n(t,z,x,u).\end{aligned} \tag{33}$$

Further, introduce the scaled estimation error ε by

$$\varepsilon_i = \frac{1}{r^{i-1}}e_i,\ 1\leqslant i\leqslant n \tag{34}$$

and

$$\varepsilon = [\varepsilon_1\ \ \varepsilon_2\ \ \cdots\ \ \varepsilon_n]^{\mathrm{T}} \in R^n. \tag{35}$$

Then one obtains

$$\dot{\varepsilon} = rA\varepsilon + \varPhi_1 \tag{36}$$

where

$$\varPhi_1 = [\delta_1(t,z,x,u), \frac{1}{r}\delta_2(t,z,x,u),\cdots,\frac{1}{r^{n-1}}\delta_n(t,z,x,u)]^{\mathrm{T}} \tag{37}$$

$$A = \begin{pmatrix} -a_1 & 1 & 0 & \cdots & 0\\ -a_2 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ -a_{n-1} & 0 & 0 & \cdots & 1\\ -a_n & 0 & 0 & \cdots & 0\end{pmatrix}.$$

Now, consider the function

$$V_1 := V(z) + \varepsilon^{\mathrm{T}}P\varepsilon \tag{38}$$

where P is a positive definite symmetric matrix satisfying

$$A^{\mathrm{T}}P + PA = -I. \tag{39}$$

Then, it follows from (31), (36), and (39) that the time derivative of V_1 along the solution of (36) satisfies

$$\dot{V}_1 = \frac{\partial V(z)}{\partial z}\dot{z} + r\varepsilon^{\mathrm{T}}(A^{\mathrm{T}}P+PA)\varepsilon + 2\varepsilon^{\mathrm{T}}P\varPhi_1 \leqslant -\frac{3}{4}\|z\|^2 + (\bar{c}\hat{c})^2|x_1|^2 - r\|\varepsilon\|^2 + 2\varepsilon^{\mathrm{T}}P\varPhi_1. \tag{40}$$

From Assumption 2 (ii) and the fact that $r\geqslant 1$, one can obtain

$$\|\varPhi_1\| \leqslant \left(|\delta_1| + \frac{1}{r}|\delta_2| + \cdots + \frac{1}{r^{n-1}}|\delta_n|\right) \leqslant \tilde{c}\sum_{j=1}^{n}\left(\frac{1}{r}\right)^{j-1}\sum_{i=1}^{n}\left(\frac{1}{r}\right)^{i-1}(\|z\|+|x_i|) \leqslant n^2\tilde{c}\|z\| + n\tilde{c}\sum_{i=1}^{n}\frac{1}{r^{i-1}}|x_i|.$$

Further, a simple computation with (32) and (34) gives

$$\begin{aligned}
&2\varepsilon^{\mathrm{T}}P\varPhi_1 \leqslant 2\|\varepsilon\|\|P\|\left(n^2\tilde{c}\|z\| + n\tilde{c}\sum_{i=1}^{n}\frac{1}{r^{i-1}}|x_i|\right) \leqslant\\
&2\|\varepsilon\|\|P\|n^2\tilde{c}\|\|z\| + 2\|\varepsilon\|\|P\|n\tilde{c}\sum_{i=1}^{n}\left(\frac{1}{r^{i-1}}|\hat{x}_i| + |\varepsilon_i|\right) \leqslant\\
&\frac{1}{4}\|z\|^2 + c^2\|\varepsilon\|^2 +\\
&2\|\varepsilon\|\|P\|n\tilde{c}\left(\sum_{i=1}^{n}\frac{1}{r^{i-1}}|\hat{x}_i| + \sum_{i=1}^{n}|\varepsilon_i|\right) \leqslant\\
&\frac{1}{4}\|z\|^2 + c^2\|\varepsilon\|^2 +\\
&2\|\varepsilon\|\|P\|n\tilde{c}\left(\sum_{i=1}^{n}\frac{1}{r^{i-1}}|\hat{x}_i| + \sqrt{n}\|\varepsilon\|\right) \leqslant\\
&\frac{1}{4}\|z\|^2 + c^2\|\varepsilon\|^2 +\\
&2\|P\|n\tilde{c}\left(\sum_{i=1}^{n}\frac{1}{r^{i-1}}|\hat{x}_i|\|\varepsilon\| + \sqrt{n}\|\varepsilon\|^2\right) \leqslant\\
&\frac{1}{4}\|z\|^2 + c^2\|\varepsilon\|^2 +
\end{aligned}$$

$$2\|P\|n\tilde{c}\left(\sum_{i=1}^{n}\frac{1}{2}\left(\frac{1}{r^{2(i-1)}}|\hat{x}_i|^2+\|\varepsilon\|^2\right)+\sqrt{n}\|\varepsilon\|^2\right)\leqslant$$
$$\frac{1}{4}\|z\|^2+c^2\|\varepsilon\|^2+n(2\sqrt{n}+n)\tilde{c}\|P\|\sum_{i=1}^{n}\frac{1}{r^{2(i-1)}}|\hat{x}_i|^2+$$
$$n(2\sqrt{n}+n)\tilde{c}\|P\|\|\varepsilon\|^2\leqslant$$
$$\frac{1}{4}\|z\|^2+k_1\|\varepsilon\|^2+k_1\sum_{i=1}^{n}\frac{1}{r^{2(i-1)}}\hat{x}_i^2$$

where $k_1=c^2+\tilde{c}n(2\sqrt{n}+n)\|P\|$ is a constant, independent of r. Then, from (40) one obtains

$$\dot{V}_1\leqslant-\frac{1}{2}\|z\|^2-(r-k_1)\|\varepsilon\|^2+(\bar{c}\tilde{c})^2|x_1|^2+k_1\sum_{i=1}^{n}\frac{1}{r^{2(i-1)}}\hat{x}_i^2. \quad (41)$$

Next, introduce $\xi=[\xi_1 \quad \xi_2 \quad \cdots \quad \xi_n]^{\mathrm{T}}\in R^n$ by

$$\xi_i=\frac{\hat{x}_i}{r^{i-1}},\ 1\leqslant i\leqslant n.$$

Then,

$$\begin{aligned}\dot{\xi}_1&=r\xi_2+ra_1\varepsilon_1\\ \dot{\xi}_2&=r\xi_3+ra_2\varepsilon_1\\ &\vdots\\ \dot{\xi}_n&=r\left(\frac{1}{r^n}u\right)+ra_n\varepsilon_1\end{aligned} \quad (42)$$

and hence inequality (41) can be written as

$$\dot{V}_1\leqslant-\frac{1}{2}\|z\|^2-(r-k_1)\|\varepsilon\|^2+k_1\|\xi\|^2. \quad (43)$$

Now, we design a compensator of the form

$$u=-r^n(b_n\xi_1+b_{n-1}\xi_2+\cdots+b_1\xi_n) \quad (44)$$

where b_i are the coefficients of any Hurwitz polynomial $\rho^n+b_1\rho^{n-1}+\cdots+b_{n-1}\rho+b_n$. Then, it is easy to verify that ξ-subsystem (42) with controller (44) can be expressed as

$$\dot{\xi}=rB\xi+r\varepsilon_1[a_1,a_2,\cdots,a_n]^{\mathrm{T}} \quad (45)$$

where

$$B=\begin{pmatrix}0&1&\cdots&0\\ \vdots&\vdots&\ddots&\vdots\\ 0&0&\cdots&1\\ -b_n&-b_{n-1}&\cdots&-b_1\end{pmatrix}.$$

Further, choose a quadratic function of the form

$$V_2:=\xi^{\mathrm{T}}Q\xi \quad (46)$$

where Q is a positive definite symmetric matrix satisfying

$$B^{\mathrm{T}}Q+QB=-2I. \quad (47)$$

Then, one can easily obtain the inequality

$$\begin{aligned}\dot{V}_2=&\dot{\xi}^{\mathrm{T}}Q\xi+\xi^{\mathrm{T}}Q\dot{\xi}=\\ &r\xi^{\mathrm{T}}(B^{\mathrm{T}}Q+QB)\xi+2r\xi^{\mathrm{T}}Qa\varepsilon_1\leqslant\\ &-2r\|\xi\|^2+2r\xi^{\mathrm{T}}Qa\varepsilon_1\end{aligned} \quad (48)$$

and similarly

$$\begin{aligned}2r\xi^{\mathrm{T}}Qa\varepsilon_1\leqslant&2r\|\xi\|\|Q\|\|[a_1,\cdots,a_n]^{\mathrm{T}}\|\|\varepsilon\|\leqslant\\ &r\|\xi\|^2+r(\|Q\|\|[a_1,\cdots,a_n]^{\mathrm{T}}\|)^2\|\varepsilon\|^2\leqslant\\ &r\|\xi\|^2+rk_2\|\varepsilon\|^2\end{aligned}$$

where $k_2=(\|Q\|\|[a_1,\cdots,a_n]^{\mathrm{T}}\|)^2$ is a constant, independent of r. Therefore, inequality (48) can be written as

$$\dot{V}_2\leqslant-r\|\xi\|^2+rk_2\|\varepsilon\|^2. \quad (49)$$

Next, we observe that closed-loop system (29) with (5) and (44) can be treated as an interconnection of a ε-subsystem and ξ-subsystem. Now, consider the function

$$V:=(k_2+1)V_1+V_2=(k_2+1)(V(z)+\varepsilon^{\mathrm{T}}P\varepsilon)+\xi^{\mathrm{T}}Q\xi. \quad (50)$$

It easily follows from (43) and (49) that

$$\begin{aligned}\dot{V}=&(k_2+1)\dot{V}+\dot{V}_2\leqslant\\ &-\frac{1}{2}\|z\|^2-(r-k_1)(k_2+1)\|\varepsilon\|^2+\\ &k_1(k_2+1)\|\xi\|^2-r\|\xi\|^2+rk_2\|\varepsilon\|^2\leqslant\\ &-\frac{1}{2}\|z\|^2-(r-k_1(k_2+1))\|\varepsilon\|^2-\\ &(r-k_1(k_2+1))\|\xi\|^2=\\ &-\frac{1}{2}\|z\|^2-(r-k_1(k_2+1))(\|\varepsilon\|^2+\|\xi\|^2).\end{aligned} \quad (51)$$

Clearly, if we choose the gain parameter r to be

$$r\geqslant1+k_1(k_2+1)$$

then

$$\dot{V}_2\leqslant-\left(\frac{1}{2}\|z\|^2+\|\varepsilon\|^2+\|\xi\|^2\right).$$

This implies

$$\varepsilon(t)\to0,\ \xi(t)\to0 \text{ as } t\to\infty$$

and hence closed-loop system (29) with (5) and (44) is globally asymptotically stable. This completes the proof. □

3 Conclusions

We presented a new result for the global stabilization of a class of uncertain nonlinear systems using a dynamic output compensator. By integrating the idea of the use of an output feedback domination design method[2], we gave an explicit method for constructing a globally stabilizing output dynamic compensator for a family of uncertain nonlinear systems.

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