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Stable Fixed Point Assignment Problems in Neural Networks

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***Abstract*—The problem of assigning a prescribed set of vectors to locally asymptotically stable fixed points of a system arises in implementing associative memory using a neural network. This paper discusses this problem for neural networks of the discrete state space type in the framework of systems and control theory. Although the so-called *orthogonal projection method* is reasonably powerful and widely used to construct such a network for associative memory, there is yet another important problem to be investigated. That is the problem of how to avoid fictitious fixed points created around desired fixed points or how to enlarge and /or adjust the domains of attraction of desired fixed points. Firstly *a generalized orthogonal projection method* is studied, and secondary introducing a state feedback structure in to a neural network it is shown that it is possible to design a control law such that without changing the already assigned fixed points each fixed point achieves a maximum convergence margin to improve the capability as associative memory. Finally, to illustrate the results, numerical examples for associative memory are worked out.**

***Keywords*— neural network, stability, stability margin, associative memory**

I. INTRODUCTION

IN the dynamical systems theory, the stability analysis of fixed points is a very important problem. On the other hand, in the neural network theory, the problem of assigning arbitrarily given vectors to fixed points of a neural network may become a main concern. In fact, in implementing associative memory or pattern recognition using a dynamical neural network, the desired true information is memorized as a locally asymptotically stable fixed point of the network. Then, the memorized information can be recalled by only giving an incomplete content or a portion of the memorized information, which is taken as an initial state of the network, so that its state converges asymptotically to the fixed point that contains the desired information. This type of memory is quite different from the memory used in ordinary digital computers in which information is stored in a memory device with a unique address for each memory unit and the content of information is ecalled by merely specifying the address.

There have been studied the two types of neural networks, i.e., the discrete state space with discrete time and the continuous state space with discrete or continuous time. In implementing associative memory, the fixed point assignment problem has attracted a great deal of attention, however there is another important problem to be studied, that is, the problem of how to enlarge and/or adjust the convergence margin of each assigned stable fixed point in order to improve the capability of associative memory. However, this problem has not been thoroughly studied. In addition to this, there is still another challenging but extremely difficult problem, that is, the problem of how to handle information corrupted by structural deformation.

For the discrete type, a variety of methods for assigning given vectors to locally asymptotically stable fixed points has been studied and further the problem of enlarging the convergence margin of each fixed point has been examined, see e.g., [1]-[9]. In particular, the papers [8]-[9] proposed and studied methods for improving or maximizing the convergence margins of assigned stable fixed points, and this problem has been fairly understood.

For the continuous type, there have also appeared a number of investigations, see, e.g., [10]-[13] and the references there. However, there are still a number of essential problems unsolved, including even the fixed-point assignment problem and the other problems mentioned above. For instance, when dealing with two-dimensional images corrupted by shape deformation, the problem becomes extremely difficult [13].

This paper deals with neural networks of the discrete state space type, more specifically, those described by the *McCulloch-Pitts model* [1]. For this type a variety of methods for assigning a set of prescribed vectors as asymptotically stable fixed points have been studied [2]-[9]. Among them, the so-called *orthogonal projection method* is reasonably powerful and widely used. However, for this method there is yet another important problem to be investigated. It is the problem of how to avoid fictitious fixed points created around desired fixed points or how to enlarge and /or adjust the domains of attraction of desired fixed points because the capability of recalling information as associative memory is dependent on the domains of attraction. In particular, it has been pointed out [8] [9] that, although any given vectors can be assigned to stable fixed points of a given neural network by means of the orthogonal projection method, some fictitious fixed points may be also created in vicinities of the assigned stable fixed points. Therefore, this may cause a fatal problem that not only the recalling process leads to expected information but also the convergence margin becomes unexpectedly small.

This paper first proposes and studies a generalized or-

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thogonal projection method, and then discusses a method for maximizing the convergence margins of the desired fixed points by introducing a state feedback into the neural networks. Finally, to illustrate the theoretical results obtained, some numerical examples of simple neural networks implementing associative memory are presented.

II. PROCEDURE FOR PAPER SUBMISSION

First, basic definitions and notations used in the sequel are introduced. Let $\mathbb{B} := \{-1, 1\}$ and consider a general dynamical system over $\mathbb{B}^n$ described in the following form:

$$x(k+1) = f(x(k)), \quad x(0) = x_0 \in \mathbb{B}^n . \tag{1}$$

Denote the solution of (1) by $x(k; x_0)$ or simply $x(k)$. Then, a vector $\xi \in \mathbb{B}^n$ is said to be a *fixed point* or an *equilibrium solution* of (1) if $f(\xi) = \xi$ or equivalently $x(k;\xi) = \xi$ for all $k \ge 0$. The *domain of attraction* of a fixed point $\xi \in \mathbb{B}^n$ is defined as

$$\mathcal{D}(\xi) := \{x_0 \in \mathbb{B}^n \,|\, \exists k \ge 0 \text{ such that } x(k; x_0) = \xi\} . \tag{2}$$

Next let the Hamming distance between ξ and ζ in $\mathbb{B}^n$ be denoted by $d_H(\zeta, \xi)$, and the δ-ball centered at $\xi \in \mathbb{B}^n$ by $\mathcal{N}_\delta(\xi)$, i.e.,

$$\mathcal{N}_\delta(\xi) := \{\zeta \in \mathbb{B}^n \,|\, d_H(\zeta, \xi) \le \delta\}, \ \ \delta \ge 0 .$$

Then, a fixed point $\xi \in \mathbb{B}^n$ is said to be *locally asymptotically stable* (or simply *stable*) if $\mathcal{D}(\xi) \supset \mathcal{N}_1(\xi)$. Finally, for a fixed point $\xi \in \mathbb{B}^n$, define

$$r(\xi) := \max\{\delta \ge 0 \,|\, \mathcal{N}_\delta(\xi) \subset \mathcal{D}(\xi)\} , \tag{3}$$

which will be used as a measure of *convergence margin* of the fixed point.

Next, consider a neural network of the *McCulloch-Pitts model* [1] defined over the discrete state space $\mathbb{B}^{\boldsymbol{n}}$ as

$$\text{MP:} \quad \begin{cases} x(k+1) = \mathrm{Sgn}\{Wx(k) - h\}, \\ x(0) = x_0 \in \mathbb{B}^n \end{cases} \tag{4}$$

where $x(k) \in \mathbb{B}^{\boldsymbol{n}}$ is the state, $W \in \mathbb{R}^{n\times n}$ the connection matrix, $h \in \mathbb{R}^n$ the threshold vector and $\mathrm{Sgn}(\cdot)$ designates the vector-valued sign function, i.e.,

$$\mathrm{Sgn}(\xi) := [\mathrm{sgn}\,\xi_1 \ \ \cdots \ \ \mathrm{sgn}\,\xi_n]^{\mathrm{T}}$$

where $\xi = [\xi_1 \ \ \cdots \ \ \xi_n]^{\mathrm{T}} \in \mathbb{R}^n$ and

$$\mathrm{sgn}\,\eta := \begin{cases} 1, & \eta > 0 \\ -1, & \eta \le 0. \end{cases}$$

Further, denote the solution of MP by $x_{MP}(k; x_0)$ or simply. Then it is clear that all the properties of solution $x_{MP}(k; x_0)$, or equivalently, of dynamical neural network MP are determined by the *parameter set* (W, h), and hence any design problem of such a network can be described as a problem of choosing an appropriate parameter set (W, h). Finally let $r < n$ and $\mathcal{P} := \{\xi^{(1)}, \cdots, \xi^{(r)}\} \subset \mathbb{B}^n$ be a set of distinct vectors, called *a set of prototype vectors* or simply *a prototype set*, which represents the set of information to be stored in a neural network. Further, introduce a Lyapunov function for MP model (4) by

$$E(x) := -\frac{1}{2} x^T W x + x^T h . \tag{5}$$

Then we cite the following theorems [2], [5], [6], which provide a method for assigning a given set of vectors to its asymptotically stable fixed points of a neural network.

THEOREM 1. Consider a dynamical neural network of the MP model (4). If the connection matrix $W \in \mathbb{R}^{n\times n}$ is nonnegative definite over the set $\{-1, 0, 1\}^n$, then for any initial state $x_{MP}(0) = x_0 \in \mathbb{B}^n$

(i) $x_{MP}(k+1; x_0) \ne x_{MP}(k; x_0)$
$$\Rightarrow E(x_{MP}(k+1; x_0)) < E(x_{MP}(k; x_0))$$

(ii) the trajectory $x(k; x_0)$ converges to an asymptotically stable fixed point of the MP model with finite steps. □

THEOREM 2 (The Orthogonal Projection Method, OPM). Consider a dynamical neural network of the MP model (4) with the parameter set (W, h) given by

$$\begin{cases} W := \Xi\Xi^\dagger \in \mathbb{R}^{n\times n}, \\ \Xi := [\xi^{(1)} \ \ \cdots \ \ \xi^{(r)}] \in \mathbb{B}^{n\times r} \\ h := [h_1 \ \ \cdots \ \ h_n]^T, \ \ |h_k| \le 1 \end{cases} \tag{6}$$

where $\Xi^\dagger \in \mathbb{B}^{r\times n}$ denotes the Moore-Penrose generalized inverse.

Then the following statements hold:

(i) Each $\xi^{(i)} \in \mathcal{P}$ is a fixed point of MP.

(ii) $E(\xi^{(1)}) = E(\xi^{(2)}) = \cdots = E(\xi^{(r)}) \le E(x), \quad \forall x \in \mathbb{B}^n$

(iii) MP has no limit cycles. □

The name *"the Orthogonal Projection Method* (*OPM*)*"* comes from the fact that the matrix $W := \Xi\Xi^\dagger$ represents the orthogonal projection operator from $\mathbb{R}^n$ onto the subspace spanned by the column vectors in Ξ (hence it is nonnegative definite).

III. THE GENERALIZED ORTHOGONAL PROJECTION METHOD

This section proposes and studies a generalized orthogonal Projection method. To begin with, the following fact is cited.

THEOREM 3. Let $X \in \mathbb{R}^{n\times m}$ be a matrix. Then the Moore-Penrose inverse $X^\dagger \in \mathbb{R}^{m\times n}$ satisfies

$$\begin{aligned} X^\dagger &= \lim_{\lambda\to+0} X^T(\lambda I_n + XX^T)^{-1} \\ &= \lim_{\lambda\to+0} X^T(\lambda I_m + X^T X)^{-1}. \end{aligned} \tag{7}$$

□

Now the next theorem holds, but its proof is omitted here.

THEOREM 4. Let $\mathcal{P} := \{\xi^{(1)},\cdots,\xi^{(r)}\} \subset \mathbb{B}^n$ be a set of given prototype vectors with $r < n$ and define $\Xi := [\xi^{(1)} \ \cdots \ \xi^{(r)}] \in \mathbb{B}^{n\times r}$ .Further, choose an integer d satisfying $1 \le d < (n-\sqrt{n})/2$ and a positive function $q:\{0,\cdots,d\} \to (0,\infty)$. Moreover for each $i = 1,\cdots,r$, define $p_i(x) := q(d_H(x,\xi^{(i)}))$ and $\mathcal{D}_i := N_d(\xi^{(i)})$. Finally introduce a nonnegative function $E:\mathbb{R}^{n\times n} \to [0,\infty)$ by

$$E(W) := \sum_{i=1}^{r} \sum_{x\in\mathcal{D}_i} p_i(x)\left\| Wx - \xi^{(i)} \right\|^2 . \tag{8}$$

Then, there exists a unique matrix $W \in \mathbb{R}^{n\times n}$ that minimizes $E(W)$ and is given as

$$W = \alpha\gamma\Xi\left[\frac{\beta}{\gamma} I_r + \Xi^T\Xi\right]^{-1}\Xi \tag{9}$$

where

$$\begin{aligned} \alpha &= \sum_{s=0}^{d} q(s)\frac{n-2s}{n}\,{}_nC_s \\ \beta &= \sum_{s=0}^{d} q(s)\frac{4s(n-s)}{n(n-1)}\,{}_nC_s \\ \gamma &= \sum_{s=0}^{d} q(s)\left\{1-\frac{4s(n-s)}{n(n-1)}\right\}{}_nC_s. \end{aligned}$$

□

Now, notice that for any $a > 0$

$$\mathrm{Sgn}(Wx-h) = \mathrm{Sgn}\{a(Wx-h)\} = \mathrm{Sgn}(aWx-ah) .$$

Therefore, THEOREM 4 may imply that it is meaningful to set

$$\begin{cases} W_\varepsilon = \Xi\left[\varepsilon I_r + \Xi^T\Xi\right]^{-1}\Xi, \quad \varepsilon > 0 \\ h := [h_1 \ \cdots \ h_n]^T, \quad |h_k| \le 1 \end{cases} \tag{10}$$

as a *parameter set* (W,h) of the MP model (4) because it follows from THEOREM 3 that

$$W_0 := \lim_{\varepsilon\to+0} W_\varepsilon = \Xi\Xi^\dagger . \tag{11}$$

Based on the above arguments, (10) will be called "*the Generalized Orthogonal Projection Method* (*GOPM*)" comparing with (6) in THEOREM 2. In implementing the MP model (4), choose a connection matrix W_ε with sufficiently small $\varepsilon > 0$ so that

$$\mathrm{Sgn}(W_\varepsilon\xi^{(i)} - h) = \mathrm{Sgn}(\Xi\Xi^\dagger\xi^{(i)} - h) = \xi^{(i)}, \quad i = 1,\cdots,r ,$$

while some of fictitious fixed points can disappear..

IV. MAXIMIZATION OF CONVERGENCE MARGIN

First, we consider the MP model (4) with an arbitrary parameter set (W,h), and introduce to this system an input vector $u(k) \in \mathbb{R}^n$ and a state feedback with a special form to define the *McCulloch-Pitts Model with feedback* (MPF) as follows:

$$\text{MPF:} \quad \begin{cases} x(k+1) = \mathrm{Sgn}\{Wx(k) - h + u(k)\},\ x(0) = x_0 \in \mathbb{B}^n \\ u(k) = F\mathrm{Sgn}\{Vx(k) - g\} + \theta \end{cases} \tag{12}$$

or equivalently in the closed loop form

$$\text{MPF:} \quad \begin{cases} x(k+1) = \mathrm{Sgn}[Wx(k) - h + F\mathrm{Sgn}\{Vx(k) - g\} + \theta] \\ x(0) = x_0 \in \mathbb{B}^n \end{cases} \tag{13}$$

where $F \in \mathbb{R}^{n\times m}$ with $m \le n$, $V \in \mathbb{R}^{m\times n}$, $g \in \mathbb{R}^m$ and $\theta \in \mathbb{R}^n$. (m,F,V,g,θ) forms a *feedback parameter set* to be chosen so as to improve the convergence margin without changing its pre-assigned fixed points. Let the solution of MPF be denoted by $x_{MPF}(k;x_0)$ or simply $x_{MPF}(k)$ and consider a prototype vector set $\mathcal{P} := \{\xi^{(1)},\cdots,\xi^{(r)}\} \subset \mathbb{B}^n$ with $r < n$. Now, for each $i = 1,\cdots,r$, denote by d_i the minimum distance from $\xi^{(i)} \in \mathcal{P}$ to the others $\xi^{(j)}$ for $j \ne i$, i.e.,

$$d_i := \min\{d_H(\xi^{(i)},\xi^{(j)}) \mid j = 1,\cdots,r \text{ and } j \ne i\}. \tag{14}$$

Then, the following theorem holds [8].

THEOREM 5. Let $\mathcal{P} := \{\xi^{(1)},\cdots,\xi^{(r)}\} \subset \mathbb{B}^n$ be a prototype set of distinct vectors, and define

$$\Xi := [\xi^{(1)} \ \cdots \ \xi^{(r)}] \in \mathbb{B}^{n\times r} .$$

Consider the MP model (4) with an arbitrary parameter set (W,h) and the MPF model (12) or (13) with the feedback parameter set given by

$$\begin{cases} m := r \\ F := a\Xi, \quad V := \Xi^T \\ g := [g_1 \quad \cdots \quad g_r]^T, \quad g_i := n - d_i \\ \theta := a\sum_{j=1}^{r} \xi^{(j)} = a\Xi[1 \quad \cdots \quad 1]^T \end{cases} \tag{15}$$

where, denoting $W = (w_{ij})$ and $h = (h_i)$, $a > 0$ is chosen to be any constant satisfying

$$a > \frac{1}{2} \max_{1 \le i \le n} \left\{ \sum_{j=1}^{n} |w_{ij}| + h_i \right\}. \tag{116}$$

Then, letting $x_0 \in \mathbb{B}^n$ be an initial state and $k \ge 0$ be an integer, the trajectories $x_{MP}(k)$ of MP model and $x_{MMP}(k)$ of MPF model satisfy the following properties:

(i) If $x_{MPF}(k) \notin \bigcup_{j=1}^{r} \mathcal{N}_{(d_i-1)/2}(\xi^{(j)})$, then
$$x_{MPF}(k+1) = x_{MP}(k+1).$$

(ii) If $x_{MPF}(k) \in \mathcal{N}_{(d_i-1)/2}(\xi^{(i)})$ for some $k \ge 0$ and some $\xi^{(i)}$, then
$$x_{MPF}(k+1) = \xi^{(i)}.$$

(iii) Every $\xi^{(i)} \in \mathcal{P}$ is a fixed point for MPF model, that is, for all $i \ge 0$
$$\begin{cases} \xi^{(i)} = \mathrm{Sgn}\{W\xi^{(i)} - h + u\} \\ \quad u = F\mathrm{Sgn}\{V\xi^{(i)} - g\} + \theta. \end{cases}$$

□

Further, the following corollary can be easily verified using THEOREM 5.

COROLLARY 1. Let all the notations be the same as those in the previous THEOREM 5. Then, the following statements are satisfied:

(i) For every $\xi^{(i)} \in \mathcal{P}$,

$$\mathcal{D}(\xi^{(i)}) = \{\xi \in \mathbb{B}^n \mid \exists k \ge 0 \text{ such that } x_{MP}(k;\xi) \in \mathcal{N}_{(d_i-1)/2}(\xi^{(i)}) \text{ and } x_{MP}(l;\xi) \notin \bigcup_{j=1}^{r} \mathcal{N}_{(d_j-1)/2}(\xi^{(j)}),\ \forall\, l < k\}.$$

(ii) For every $\xi^{(i)} \in \mathcal{P}$,
$$\mathcal{N}_{(d_i-1)/2}(\xi^{(i)}) \subset \mathcal{D}(\xi^{(i)}).$$

(iii) If $d_i \ge 3$, then $\xi^{(i)}$ is a locally asymptotically stable fixed point.

(iv) If the MP model has no limit cycles, then the MPF model has also no limit cycles. □

Finally, the following theorem can be easily verified using THEOREM 5 and COROLLARY 1.

THEOREM 6. Let all the notations be the same as those in the previous THEOREM 5 except an arbitrary parameter set (W, h) is replaced with the parameter set constructed the Orthogonal Projection Method as in THEOREM 2.

Then, if $d_i \ge 3$ for all $i = 1, \cdots r$,

(i) each assigned fixed point $\xi^{(i)}$ is locally asymptotically stable

(ii) there is no other fixed point in $\mathcal{N}_{(d_i-1)/2}(\xi^{(i)})$ and its convergence margin is given as $r(\xi^{(i)}) = (d_i - 1)/2$, that is, each fixed point $\xi^{(i)}$ has the maximum convergence margin. □

Figure 1 explains the result of Theorem 6 together with THEOREM 5, hat is, on the outside of the convergence margin $(d_i - 1)/2 \ge 1$ the two trajectories $x_{MPF}(k)$ and $x_{MP}(k; x_0)$ are exactly the same until they reach the convergence margin and then at the next moment $x_{MPF}(k)$ immediately moves to the fixed point $\xi^{(i)}$ but $x_{MP}(k)$ may travel more and eventually reach a fictitious fixed point ξ^* created by MP model.

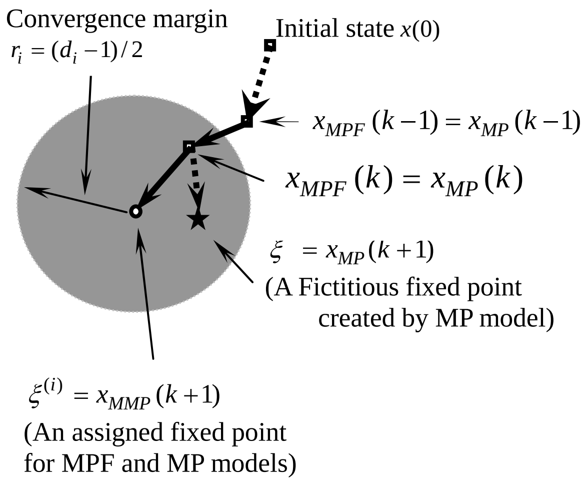

Figure 1. The Trajectories of MP and MPF models

V. NUMERICAL EXAMPLES

Some numerical examples were performed for *the generalized orthogonal projection method* obtained in Section III, and the results showed that some fictitious fixed points are removed and the capability for associative memory is definitely improved. However, the improvement seems much less than the method proposed in Section IV. Due to the shortage of space, all the numerical results are omitted, and only those results obtained using the method developed in Section IV are presented here.

EXAMPLE 1. First, we consider an associative memory for memorizing English alphabets $A, B, \cdots, Z$ and blank □.

Each character is divided into 100 ($=10\times10$) pixels and each pixel is represented by 1 or -1 according to black or white, forming the set $\mathcal{P}=\{\xi^{(A)},\xi^{(B)},\cdots,\xi^{(Z)},\xi^{(\square)}\}$ of 27 prototype vectors in $\mathbb{B}^{100}$. The parameter set (W,h) of the MP model is constructed by the Orthogonal Projection Method described in THEOREM 2 with $h=0$, and the feedback parameter set (m,F,V,g,θ) of the MPF model is determined according to THEOREM 5.

First, it is checked that, for these prototype vectors, all the minimum distances are computed and turn out to satisfy $d_A,d_B,\cdots,d_Z,d_\square\geq4$. Therefore, it follows from THEOREM 6 that all the prototype vectors are assigned to locally asymptotically stable fixed points of the MPF model.

Figure 2 depicts the trajectories of both MP and MPF models starting from the same initial state $x_0=x(0)\in\mathbb{B}^{100}$ obtained from $\xi^{(A)}$ by adding 15 % noises. The result shows that the MPF model trajectory $x_{MPF}(k)$ converges to the expected prototype vector $\xi^{(A)}$, but surprising enough the MP model trajectory $x_{MP}(k;x_0)$ converges to a fictitious fixed point ξ^*, which is created by the Orthogonal Projection Method and differs only at one pixel, that is, $d_H(\xi^{(A)},\xi^*)=1$. That is to say that the Orthogonal Projection Method ensures to assign all the prototype vectors to fixed points but simultaneously may produce a fictitious fixed point just next to a prototype vector as demonstrated in this example.

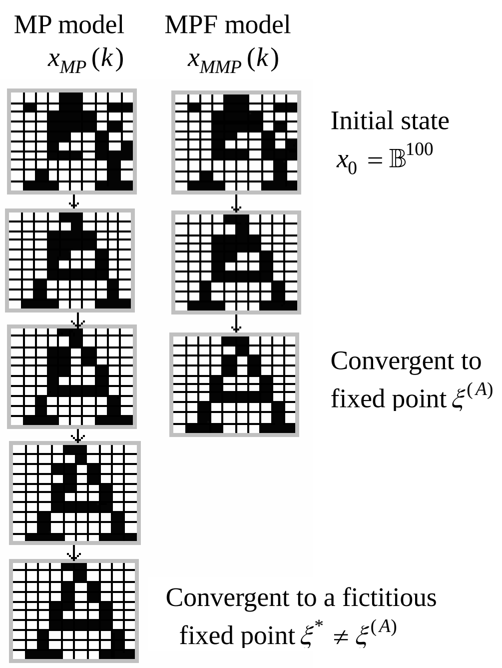

Figure 2. Trajectories of MP and MPF models

EXAMPLE 2. Next, we consider an associative memory for memorizing English words. A word to be memorized is composed of at most five (5) alphabets and the following 30 words are memorized:

CAT	APPLE	MOON	MOUSE	PEACH
EARTH	TIGER	LEMON	SUN	WOLF
MELON	VENUS	WHALE	GRAPE	MARS
JAPAN	SHIP	ROSE	CHINA	TRAIN
LILY	INDIA	PLANE	PANSY	SPAIN
BIKE	TULIP	ITALY	BOAT	OLIVE

In this time, each character is divided into $24\times24(=576)$ pixels, so that each character is represented by a vector in $\mathbb{B}^{576}$. Therefore each word consisting at most 5 characters is represented by a vector in $\mathbb{B}^{2880}$ and the 30 prototype vectors are denoted as $\xi^{(\mathrm{CAT})},\cdots,\xi^{(\mathrm{OLIVE})}$ in $\mathbb{B}^{2880}$.

For the associative memory for these words, we also construct two types of neural networks, using MP model

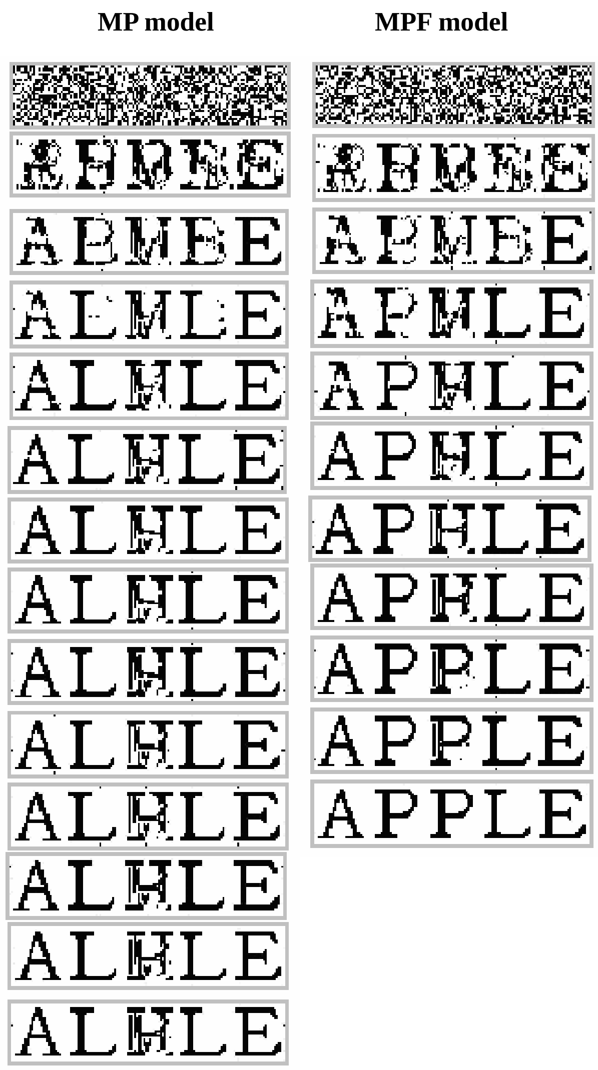

Figure 3. Associative memories for English words using MP model and MPF model

and MPF model, respectively. Each type network is composed of five MP (MPF) models, each of which is constructed as in EXAMPLE 1 to memorize $\xi^{(A)}, \xi^{(B)}, \cdots,$ $\xi^{(Z)}, \xi^{(\square)}$ in $\mathbb{B}^{576}$. Then these five networks are connected each other, through a newly introduced hidden network to each individual network, in such a way that each word is assigned to an asymptotically stable fixed point in the space $\mathbb{B}^{2880}$ of the total network but the pre-assigned fixed points in each individual network for memorizing $\xi^{(A)}, \xi^{(B)}, \cdots, \xi^{(Z)}, \xi^{(\square)}$ are unchanged. A detailed description for this construction is given in [7].

Figure 3 depicts the computer simulation results of the two associative memories for English words using MP model and MPF model. As seen from the figure, the initial state is a very nosy information which is generated from the prototype vector $\xi^{(\mathrm{APPLE})}$ of "APPLE" by adding 45 % noises. It is seen that in the associative memory using the MP model a fictitious fixed point (i.e., a meaningless word) is created near the word "APPLE" and the correct information cannot be recalled, while in the associative memory using the MPF model the correct information "APPLE" is recalled.

VI. CONCLUDING REMARKS

This paper dealt with the problem of assigning a given set of points to the locally asymptotically stable fixed points in a neural network. In particular, a generalized orthogonal projection method was proposed. Further from the viewpoint of systems and control theory a method for maximizing the convergence margin of each assigned fixed point was studied, which is vital for implementing an associative memory by a neural network to improve the capability of associative memory. In fact introducing a state feedback structure into a neural network it was shown that it is possible to design a state feedback law such that the convergence margin of each fixed point in its closed loop system is maximized without changing all the pre-assigned asymptotically stable fixed points. Finally, to show the effectiveness of the result obtained, some numerical examples for associative memory were presented.

More sophisticated neural networks were considered by introducing limit cycles to memorize information [6]. Further, recently neural networks of continuous state spaces with continuous time have been considered in the framework of the systems and control theory [13]. For these networks, the same problem studied in this paper should be investigated.

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