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Sampling Brownian house-moving

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Abstract

We numerically generate a stochastic process called "Brownian house-moving," which is a Brownian bridge that stays between its starting point and its terminal point. To construct this process, statements are prepared on the weak convergence of conditioned Brownian bridges. We also study the sample path properties of Brownian house-moving and the decomposition formula for its distribution. Using this decomposition formula and a Monte Carlo sampling technique for a BES(3)-bridge, we are able to numerically generate Brownian house-moving at discrete times.

Keywords barrier options, Greeks, Brownian bridge, BES(3)-bridge, Brownian house-moving

Research Activity Group Mathematical Finance

1. Introduction

Recently, [1] developed a chain rule for Wiener path integrals between two curves that arise in the computation of first-order Greeks for barrier options, and demonstrated the effectiveness of this chain rule through numerical examples. In this chain rule, a BES(3)-bridge and a Brownian meander played an important role. We are currently investigating higher-order chain rules for computing higher-order Greeks of barrier options, and we expect a stochastic process called "Brownian house-moving" to play an important role in their computation.

This paper is organized as follows. Sections 2 and 3 review the results of [2]. In Section 2, we construct the Brownian house-moving as the one-dimensional Brownian bridge conditioned to stay between its starting point and its terminal point. In section 3, we extend the notion of the Brownian house-moving and construct the Brownian house-moving between two curves. After applying the results in this section, we obtain the decomposition formula for the distribution of the Brownian house-moving. Using this decomposition formula, we numerically generate Brownian house-moving in Section 4. Section 5 presents our conclusions.

2. Notations and main results

Let $C([0,1],\mathbb{R})$ be a class of $\mathbb{R}$-valued continuous functions defined on $[0,1]$, and let

$$d_\infty(w,w') = \sup_{0\le t\le 1} |w(t)-w'(t)| \ \ (w,w'\in C([0,1],\mathbb{R})).$$

$\mathcal{B}(C([0,1],\mathbb{R}))$ denotes the Borel σ-algebra with respect to the topology generated by the metric d_∞. For $0\le t_1<t_2\le 1$ and $f,g\in C([0,1],\mathbb{R})$, we define

$$K_{[t_1,t_2]}(f,g) := \{w=\{w(t)\}_{t\in[t_1,t_2]} \in C([t_1,t_2],\mathbb{R}) \mid f(t)\le w(t)\le g(t),\ t_1\le t\le t_2\},$$

$$K^-_{[t_1,t_2]}(g) := \bigcup_{n=1}^{\infty} K_{[t_1,t_2]}(-n,g).$$

Assume that

$$Y:(\Omega,\mathcal{F},P)\to (C([0,1],\mathbb{R}),\mathcal{B}(C([0,1],\mathbb{R})))$$

is a random variable and $\Lambda\in\mathcal{B}(C([0,1],\mathbb{R}))$ satisfies $P(Y\in\Lambda)>0$. Then, we define the probability measure $P_{Y^{-1}(\Lambda)}$ on $(Y^{-1}(\Lambda), Y^{-1}(\Lambda)\cap\mathcal{F})$ by

$$P_{Y^{-1}(\Lambda)}(A) := \frac{P(A)}{P(Y\in\Lambda)},$$
$$\left(A\in Y^{-1}(\Lambda)\cap\mathcal{F} := \left\{Y^{-1}(\Lambda)\cap F \mid F\in\mathcal{F}\right\}\right).$$

Throughout this paper, $P_{Y^{-1}(\Lambda)}(Y|_\Lambda\in\Gamma)$ is often written as $P(Y|_\Lambda\in\Gamma)$.

For $r>0$ and $c<d$, we define

$$n_r(x) := \frac{1}{\sqrt{2\pi r}}\exp\left(-\frac{x^2}{2r}\right), \quad N_r(c,d) := \int_c^d n_r(x)dx.$$

If $\{X_n\}_{n=1}^\infty$ converges to X in distribution, then we denote $X_n \xrightarrow{\mathcal{D}} X$. Additionally, we write $X \overset{\mathcal{D}}{=} Y$ for random variables X,Y that obey the same distribution.

We construct a stochastic process called "Brownian house-moving" $H^{0\to b}$ $(b>0)$ as the weak limit of conditioned Brownian bridges.

Theorem 1 *Let $b>0$, $B^{0\to b}=\{B^{0\to b}(t)\}_{t\in[0,1]}$ be the one-dimensional Brownian bridge from 0 to b on $[0,1]$. There exists an $\mathbb{R}$-valued continuous Markov process $H^{0\to b}=\{H^{0\to b}(t)\}_{t\in[0,1]}$ that satisfies*

$$B^{0\to b}|_{K_{[0,1]}(-\varepsilon,b+\varepsilon)} \xrightarrow{\mathcal{D}} H^{0\to b}, \qquad \varepsilon\downarrow 0. \tag{1}$$

Moreover, for $0<s<t<1$ and $x,y\in(0,b)$, the law for $H^{0\to b}$ is given by

$$P\left(H^{0\to b}(t)\in dy\right) = \frac{J_1^{(b)}(t,y)\ J_2^{(b)}(1-t,y)}{J^{(b)}(b)}dy, \tag{2}$$

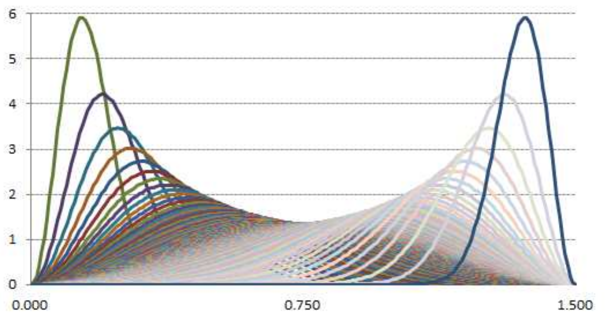

Fig. 1. Densities of $H^{0\to b}(t)$ ($b = 1.5$) for $t = 0.01, 0.02, \ldots, 0.99$.

$$P\left(H^{0\to b}(t) \in dy \mid H^{0\to b}(s) = x\right) = \frac{J_2^{(b)}(1-t, y)\ J_3^{(b)}(s, x, t, y)}{J_2^{(b)}(1-s, x)} dy, \tag{3}$$

where

$$J_1^{(\eta)}(r, z) := \sum_{k=-\infty}^{\infty} \frac{2(z + 2k\eta)}{r} n_r(z + 2k\eta),$$

$$J_2^{(\eta)}(r, z) := J_1^{(\eta)}(r, \eta - z),$$

$$J_3^{(\eta)}(s, x, t, y) := \sum_{k=-\infty}^{\infty} (n_{t-s}(y - x + 2k\eta) - n_{t-s}(y + x + 2k\eta)),$$

$$J_4^{(\eta)}(r, z) := \frac{\partial}{\partial \eta} J_1^{(\eta)}(r, z), \quad J^{(\eta)}(z) := J_4^{(\eta)}(1, z).$$

Moreover, the sample path properties of the Brownian house-moving $H^{0\to b}$ are studied. It is shown that the Brownian house-moving does not hit b on the time interval $[0, 1)$.

Proposition 2 *Let $b > 0$. For $t_0 \in (0, 1)$, it holds that*

$$P\left(\max_{0\le u\le t_0} H^{0\to b}(u) < b\right) = 1.$$

The regularity of the sample path of the Brownian house-moving is also established.

Proposition 3 *For every $\gamma \in (0, \frac{1}{2})$, the path of $H^{0\to b}$ ($b > 0$) on $[0, 1]$ is locally Hölder-continuous with exponent γ:*

$$P\left(\bigcup_{n=1}^{\infty}\left\{\sup_{\substack{t,s\in[0,1]\\ 0<|t-s|\le \frac{1}{n}}} \frac{\left|H^{0\to b}(t) - H^{0\to b}(s)\right|}{|t-s|^{\gamma}} < \infty\right\}\right) = 1.$$

3. Construction of Brownian house-moving between two curves

With regards to Theorem 1, the Brownian house-moving can be considered as a one-dimensional Brownian bridge conditioned to stay between two constant levels. In this section, we use this viewpoint to extend the notion of Brownian house-moving to construct the one-dimensional Brownian bridge conditioned to stay between two curves.

Let $0 \le t_1 < t_2 \le 1$. Throughout this section, we use the following notation.

For $a, b \in \mathbb{R}$, $c \ge 0$, and $d > 0$, $W_{[t_1,t_2]}$, $W^+_{[t_1,t_2]}$, $B^{a\to b}_{[t_1,t_2]}$, and $r^{c\to d}_{[t_1,t_2]}$ denote a Brownian motion, a Brownian meander, a Brownian bridge from a to b, and a BES(3)-bridge from c to d defined on $[t_1, t_2]$, respectively.

Let g^- and g^+ be C^2-functions on $[0, 1]$ satisfying

$$\min_{0\le t\le 1}(g^+(t) - g^-(t)) > 0.$$

According to the values $g^-(t_1) \le a \le g^+(t_1)$ and $g^-(t_2) \le b \le g^+(t_2)$, the continuous process $X^{a,b,(g^-,g^+)}_{[t_1,t_2]}$ on $[t_1, t_2]$ is defined as follows (see also Lemma 4 below):

- In the case $a = g^-(t_1), b < g^+(t_2)$, the weak limit of $B^{a\to b}_{[t_1,t_2]}|_{K_{[t_1,t_2]}(g^- -\varepsilon, g^+)}$ as $\varepsilon \downarrow 0$.
- In the case $a > g^-(t_1), b = g^+(t_2)$, the weak limit of $B^{a\to b}_{[t_1,t_2]}|_{K_{[t_1,t_2]}(g^-, g^+ +\varepsilon)}$ as $\varepsilon \downarrow 0$.

For a continuous process X on $[t_1, t_2]$ and an $\mathbb{R}$-valued C^2-function g on $[t_1, t_2]$, we define

$$Z^g_{[t_1,t_2]}(X) := \exp\bigg(g'(t_2)X(t_2) - g'(t_1)X(t_1) - \int_{t_1}^{t_2} X(u)g''(u)du - \frac{1}{2}\int_{t_1}^{t_2} g'(u)^2 du\bigg).$$

Therefore, if X is $W_{[t_1,t_2]}$, then it follows from Itô's formula that

$$Z^g_{[t_1,t_2]}(W_{[t_1,t_2]}) = \exp\left\{\int_{t_1}^{t_2} g'(u)dW_{[t_1,t_2]}(u) - \frac{1}{2}\int_{t_1}^{t_2} g'(u)^2 du\right\}.$$

For convenience later, we define

$$\widetilde{Z}^g_{[t_1,t_2]}(X) := Z^g_{[t_1,t_2]}(X + g).$$

For $f \in C([t_1, t_2], \mathbb{R})$, $\overleftarrow{f}$ denotes the function

$$\overleftarrow{f}(t) := f(t_1 + t_2 - t) \qquad (t_1 \le t \le t_2).$$

Let $t_0 \in (t_1, t_2)$. For $w_1 \in C([t_1, t_0], \mathbb{R})$ and $w_2 \in C([t_0, t_2], \mathbb{R})$ that satisfy $w_1(t_0) = w_2(t_0)$, we define $w_1 \oplus_{t_0} w_2 \in C([t_1, t_2], \mathbb{R})$ by

$$(w_1 \oplus_{t_0} w_2)(t) := \begin{cases} w_1(t) & (t_1 \le t \le t_0), \\ w_2(t) & (t_0 \le t \le t_2). \end{cases}$$

Lemma 4 *$X^{a,b,(g^-,g^+)}_{[t_1,t_2]}$ exists and its distribution is given as follows. For a bounded continuous function F on $C([t_1, t_2] : \mathbb{R})$,*

(1) *If $a = g^-(t_1)$, $b < g^+(t_2)$, then*

$$E\left[F(X^{a,b,(g^-,g^+)}_{[t_1,t_2]})\right] \tag{4}$$
$$= E\bigg[F\Big(r^{0\to b-g^-(t_2)}_{[t_1,t_2]}|_{K^-_{[t_1,t_2]}(g^+-g^-)} + g^-\Big) \times \widetilde{Z}^{g^- - a}_{[t_1,t_2]}\Big(r^{0\to b-g^-(t_2)}_{[t_1,t_2]}|_{K^-_{[t_1,t_2]}(g^+-g^-)}\Big)^{-1}\bigg] \times \left(E\left[\widetilde{Z}^{g^- - a}_{[t_1,t_2]}\Big(r^{0\to b-g^-(t_2)}_{[t_1,t_2]}|_{K^-_{[t_1,t_2]}(g^+-g^-)}\Big)^{-1}\right]\right)^{-1}.$$

(2) *If $a > g^-(t_1)$, $b = g^+(t_2)$, then*

$$E\left[F(X^{a,b,(g^-,g^+)}_{[t_1,t_2]})\right] \tag{5}$$
$$= E\Bigg[F\Big(g^+ - \overleftarrow{r}^{\,0\to g^+(t_1)-a}_{[t_1,t_2]}|_{K^-_{[t_1,t_2]}(\overleftarrow{g}^+ - \overleftarrow{g}^-)}\Big)$$
$$\times \widetilde{Z}^{b-\overleftarrow{g}^+}_{[t_1,t_2]}\left(r^{0\to g^+(t_1)-a}_{[t_1,t_2]}|_{K^-_{[t_1,t_2]}(\overleftarrow{g}^+ - \overleftarrow{g}^-)}\right)^{-1}\Bigg]$$
$$\times \left(E\left[\widetilde{Z}^{b-\overleftarrow{g}^+}_{[t_1,t_2]}\left(r^{0\to g^+(t_1)-a}_{[t_1,t_2]}|_{K^-_{[t_1,t_2]}(\overleftarrow{g}^+ - \overleftarrow{g}^-)}\right)^{-1}\right]\right)^{-1}.$$

For $0 < t < 1$, $0 < t_1 < t_2 < 1$, $y \in (g^-(t), g^+(t))$, $y_1 \in (g^-(t_1), g^+(t_1))$, and $y_2 \in (g^-(t_2), g^+(t_2))$, we define

$$h(t,y) := E\left[\widetilde{Z}^{g^-}_{[0,t]}(r^{0\to y-g^-(t)}_{[0,t]})^{-1}\ ;\ r^{0\to y-g^-(t)}_{[0,t]} \in K^-_{[0,t]}(g^+ - g^-)\right] \times P(W^+_{[0,t]}(t) \in dy - g^-(t))/dy,$$

and

$$h(t_1,y_1,t_2,y_2)$$
$$:= E\left[\widetilde{Z}^{b-\overleftarrow{g}^+}_{[t_2,1]}(r^{0\to g^+(t_2)-y_2}_{[t_2,1]})^{-1}\ ;\ r^{0\to g^+(t_2)-y_2}_{[t_2,1]} \in K^-_{[t_2,1]}(\overleftarrow{g}^+ - \overleftarrow{g}^-)\right]$$
$$\times P\left(W^+_{[t_2,1]}(1) \in db - y_2 - (b - g^+(t_2))\right)/dy_2$$
$$\times P\left(y_1 + W_{[t_1,t_2]} \in K_{[t_1,t_2]}(g^-, g^+),\ y_1 + W_{[t_1,t_2]}(t_2) \in dy_2\right).$$

We establish the existence of the weak limit of $B^{0\to b}_{[0,1]}|_{K_{[0,1]}(g^- -\varepsilon, g^+ +\varepsilon)}$ as ε tends to 0, where g^-, g^+ satisfy $g^-(0) = 0$ and $g^+(1) = b$.

Theorem 5 *Assume that g^-, g^+ satisfy $g^-(0) = 0$ and $g^+(1) = b$. There exists an $\mathbb{R}$-valued continuous Markov process $H = \{H(t)\}_{t\in[0,1]}$ that satisfies*

$$E[F(H)] = \lim_{\varepsilon\downarrow 0} E[F(B^{0\to b}_{[0,1]}|_{K_{[0,1]}(g^- -\varepsilon, g^+ +\varepsilon)})] \tag{6}$$
$$= \int_{g^-(t_0)}^{g^+(t_0)} E\left[F(X^{0,y,(g^-,g^+)}_{[0,t_0]} \oplus_{t_0} X^{y,b,(g^-,g^+)}_{[t_0,1]})\right] \times P(H(t_0) \in dy), \tag{7}$$

for every bounded continuous function F on $C([0,1],\mathbb{R})$ and $0 < t_0 < 1$, where the $X^{0,y,(g^-,g^+)}_{[0,t_0]}$ and $X^{y,b,(g^-,g^+)}_{[t_0,1]}$ that appear in (7) are chosen to be independent. Moreover, for $0 < t_1 < t_2 < 1$, $y_1 \in (g^-(t_1), g^+(t_1))$ and $y_2 \in (g^-(t_2), g^+(t_2))$, the law for H is given by

$$P(H(t_1) \in dy_1) = \frac{h(t_1,y_1)\int_{g^-(t_2)}^{g^+(t_2)} h(t_1,y_1,t_2,z_2)dz_2}{\int_{g^-(t_2)}^{g^+(t_2)}\int_{g^-(t_1)}^{g^+(t_1)} h(t_1,z_1)h(t_1,z_1,t_2,z_2)dz_1\,dz_2}dy_1,$$

$$P(H(t_1) \in dy_1, H(t_2) \in dy_2) = \frac{h(t_1,y_1)h(t_1,y_1,t_2,y_2)}{\int_{g^-(t_2)}^{g^+(t_2)}\int_{g^-(t_1)}^{g^+(t_1)} h(t_1,z_1)h(t_1,z_1,t_2,z_2)dz_1\,dz_2}\,dy_1dy_2.$$

Applying Theorem 5 (7), we obtain the next proposition.

Proposition 6 *The stochastic process H defined in Theorem 5 satisfies*

$$P\left(g^-(t) < H(t) < g^+(t) \text{ for } t \in (0,1)\right) = 1.$$

Applying Theorem 5 (7) for $g^- \equiv 0$ and $g^+ \equiv b$, we obtain the next decomposition formula for the distribution of $H^{0\to b}$.

Corollary 7 *It holds for every bounded continuous function F on $C([0,1],\mathbb{R})$ that*

$$E\left[F(H^{0\to b})\right]$$
$$= \int_0^b E\left[F\left(r^{0\to y}_{[0,t_0]}|_{K^-_{[0,t_0]}(b)} \oplus_{t_0}\left(b - \overleftarrow{r}^{\,0\to b-y}_{[t_0,1]}|_{K^-_{[t_0,1]}(b)}\right)\right)\right] \times P\left(H^{0\to b}(t_0) \in dy\right), \qquad 0 < t_0 < 1,$$

where $r^{0\to y}_{[0,t_0]}|_{K^-_{[0,t_0]}(b)}$ and $\overleftarrow{r}^{\,0\to b-y}_{[t_0,1]}|_{K^-_{[t_0,1]}(b)}$ are chosen to be independent.

Remark 8 *Let A be a closed subset of $C([0,1],\mathbb{R})$ and*

$$\phi(t) := 1 - \int_0^1 1_{(-\infty,t]}(u)du, \quad t \in \mathbb{R}.$$

Then we have

$$F_n(w) := \phi(n d_\infty(w,A)) \downarrow 1_A(w), \quad n \to \infty.$$

Therefore, the dominated convergence theorem and Dynkin's π-λ theorem imply that Corollary 7 holds true for $F = 1_B$, where $B \in \mathcal{B}(C([0,1],\mathbb{R}))$ is a measurable subset of $C([0,1],\mathbb{R})$.

Remark 8 implies the following lemma.

Lemma 9 *For $0 < z \le x \le b$ and $t_0 \in (0,1)$, we have*

$$P\left(\max_{u\in[0,t_0]} H^{0\to b}(u) = x\right) = 0,$$
$$P\left(\max_{u\in[0,t_0]} H^{0\to b}(u) \le x, H^{0\to b}(t_0) \le z\right) = \int_0^z \frac{J_1^{(x)}(t_0,y)J_2^{(b)}(1-t_0,y)}{J^{(b)}(b)}dy.$$

Remark 10 *Let $t_0 \in (0,1)$. Lemma 9 implies that*

$$P\left(\max_{u\in[0,t_0]} H^{0\to b}(u) = b\right) = 0,$$
$$P\left(\max_{u\in[0,t_0]} H^{0\to b}(u) \le b\right) = P\left(\max_{u\in[0,t_0]} H^{0\to b}(u) \le b, H^{0\to b}(t_0) \le b\right) = \int_0^b P\left(H^{0\to b}(t_0) \in dy\right) = 1.$$

Therefore, $P\left(\max_{u\in[0,t_0]} H^{0\to b}(u) < b\right) = 1$ holds and Proposition 2 is obtained. Propositions 6 and 2 imply that the Brownian house-moving $H^{0\to b}$ does not hit b on the time interval $[0,1)$.

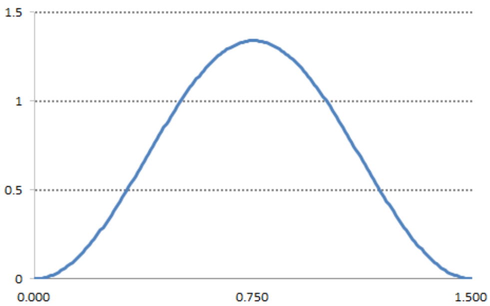

Fig. 2. Density of $H^{0\to b}(0.5)$ $(b = 1.5)$.

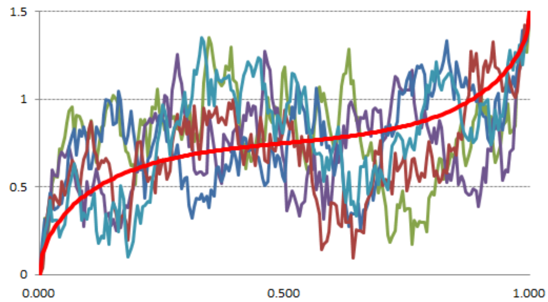

Fig. 3. Five sample paths for $\{H^{0\to b}(t)\}_{t\in[0,1]}$, and the graph of $\{E[H^{0\to b}(t)]\}_{t\in[0,1]}$ $(b = 1.5)$.

4. Numerical methods

A possible numerical strategy consists of directly sampling Brownian house-moving paths with discrete time steps. This strategy is demanding, since the CDFs for $H^{0\to b}$ and its inverse are not known analytically. Hence, such simulations were restricted to a small number of discrete time steps. Using Corollary 7 for $t_0 = 0.5$, we were able to generate $H^{0\to b}$ with long discrete steps, since the inverse CDF of $H^{0\to b}(0.5)$ can be obtained beforehand and the CDF of the BES(3)-bridge is known analytically.

First, we can generate a random number $y \in (0, b)$ from the inverse CDF for the distribution of $H^{0\to b}(0.5)$.

Next, combining the scaling identity

$$r^{0\to y}_{[0,t_0]}(\cdot) \overset{\mathcal{D}}{=} \sqrt{t_0} \times r^{0\to \widehat{y}}_{[0,1]}\left(\frac{\cdot}{t_0}\right) \qquad \left(\widehat{y} := \frac{y}{\sqrt{t_0}}\right)$$

and samples of the BES(3)-bridge $r^{0\to\widehat{y}}_{[0,1]}(\cdot)$ on $[0, 1]$, we are able to obtain samples of the BES(3)-bridge $r^{0\to y}_{[0,t_0]}(\cdot)$ on $[0, t_0]$ and conditioned samples of the BES(3)-bridge $r^{0\to y}_{[0,t_0]}|_{K^-_{[0,t_0]}(b)}$. Similarly, conditioned samples of the BES(3)-bridge $\overleftarrow{r}^{0\to b-y}_{[t_0,1]}|_{K^-_{[t_0,1]}(b)}$ can be obtained.

Here, the CDFs for the BES(3)-bridge $r^{0\to\widehat{y}} := r^{0\to\widehat{y}}_{[0,1]}$ can be obtained analytically as follows:

$$\begin{aligned}
&P\left(r^{0\to\widehat{y}}\left(\frac{1}{n}\right) \le z\right) \\
&\quad = N_{\frac{n-1}{n^2}}\left(-\frac{\widehat{y}}{n}, z - \frac{\widehat{y}}{n}\right) + N_{\frac{n-1}{n^2}}\left(\frac{\widehat{y}}{n}, z + \frac{\widehat{y}}{n}\right) \\
&\qquad + \frac{n-1}{n\widehat{y}}\left(n_{\frac{n-1}{n^2}}\left(z + \frac{\widehat{y}}{n}\right) - n_{\frac{n-1}{n^2}}\left(z - \frac{\widehat{y}}{n}\right)\right),
\end{aligned}$$

$$\begin{aligned}
&P\left(r^{0\to\widehat{y}}\left(\frac{k}{n}\right) \le z \mid r^{0\to\widehat{y}}\left(\frac{k-1}{n}\right) = x\right) \\
&= \frac{\Phi^{x,\widehat{y}}_{\frac{1}{n},\frac{n-k}{n}}(z) - \Phi^{-x,\widehat{y}}_{\frac{1}{n},\frac{n-k}{n}}(z) - \Phi^{x,-\widehat{y}}_{\frac{1}{n},\frac{n-k}{n}}(z) + \Phi^{-x,-\widehat{y}}_{\frac{1}{n},\frac{n-k}{n}}(z)}{n_{\frac{n-k+1}{n}}(\widehat{y} - x) - n_{\frac{n-k+1}{n}}(\widehat{y} + x)}
\end{aligned}$$

where

$$\begin{aligned}
&\Phi^{c_1,c_2}_{r_1,r_2}(z) \\
&\quad := n_{r_1+r_2}(c_1 - c_2) \\
&\qquad \times N_{\frac{r_1 r_2}{r_1+r_2}}\left(-\frac{c_1 r_2 + c_2 r_1}{r_1 + r_2}, z - \frac{c_1 r_2 + c_2 r_1}{r_1 + r_2}\right).
\end{aligned}$$

Thus, using this analytical representation and the binary search method, sample paths for $r^{0\to\widehat{y}}$ can be generated.

In Fig. 3, five sample paths for $\{H^{0\to b}(t)\}_{t\in[0,1]}$, and the graph of $\{E[H^{0\to b}(t)]\}_{t\in[0,1]}$, are shown for $b = 1.5$ and 100 discrete time steps.

5. Conclusion and discussions

We introduced a stochastic process called Brownian house-moving to compute higher-order Greeks of barrier options. We also introduced the decomposition formula for the distribution of Brownian house-moving. Using this decomposition formula, we were able to generate Brownian house-moving with discrete time steps.

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