新しいタイプの原子核の集団運動
ーNGH 空間での集団運動ー

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purpose

- Existence of Nambu-Goldstone and Higgs collective modes of alpha cluster Bose-Einstein condensation in nuclei
- Develop new theory of condensation of finite systems that respects canonical commutation relation, zero mode.

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Effective field theory of Bose-Einstein condensation of α clusters and Nambu-Goldstone-Higgs states in ^{12}C

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- 原子核の集団運動：　フェルミオン（陽子、中性子）の集まりとしての集団運動
- 3次元　ユーグリッド空間R(3)

  回転運動　　(order parameter: deformation)

  振動運動
- ゲージ　空間　U(1)　超流動（粒子数空間）

  対回転　　　(order parameter: paring gap energy)

  対振動

α粒子のゲージ空間（NGH空間）での集団運動

order parameter: density

(新しいタイプの原子核の集団運動、安定真空の存在）

位相子の運動 (phason)　　超流動

振幅子 (amplitudon)

VEV（真空期待値）≠ 0

Nambu Goldstone boson

Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).
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H. Watanabe and H. Murayama, Phys. Rev. Lett. **108**, 251602 (2012); Y. Hidaka, *ibid.* **110**, 091601 (2013).

Higgs boson

Emergence of order parameter oscillation

P. B. Littlewood and C. M. Varma, Phys. Rev. Lett. **47**, 811 (1981); Phys. Rev. B **26**, 4883 (1982).
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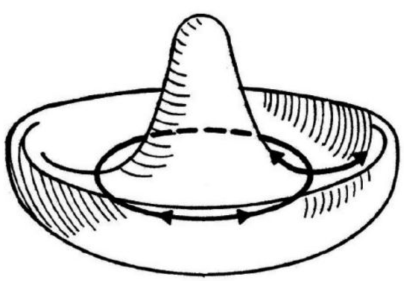
ATLAS Collaboration, Phys. Lett. B **716**, 1 (2012); CMS Collaboration, *ibid.* **716**, 30 (2012).



$$\psi(\mathbf{k}) = |\psi(\mathbf{k})| \exp(i\phi).$$

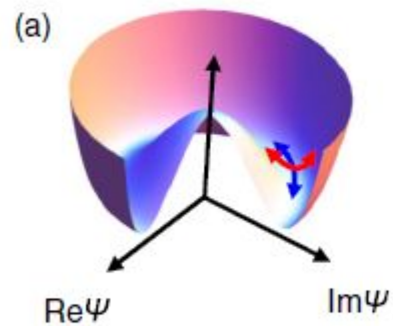
Higgs Boson in Superconductors

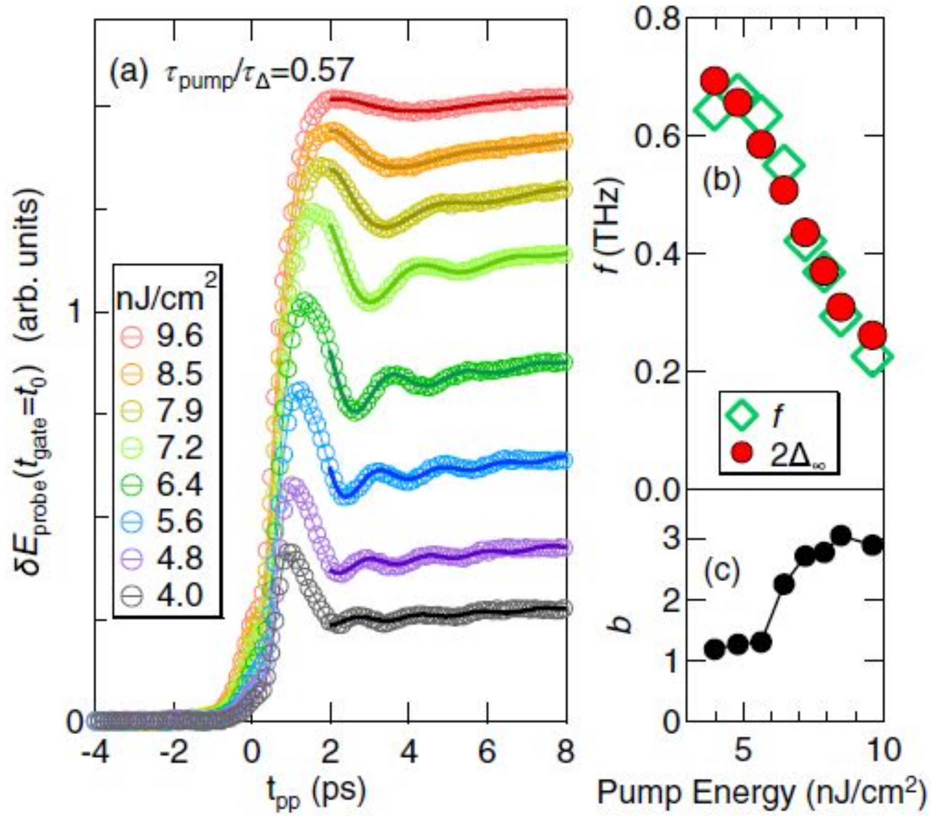
C.M. Varma

R. Matsunaga, Y. I. Hamada, K. Makise, Y. Uzawa, H. Terai, Z. Wang, and R. Shimano, Phys. Rev. Lett. **111**, 057002 (2013).

Higgs Amplitude Mode in the BCS Superconductors $Nb_{1-x}Ti_xN$ Induced by Terahertz Pulse Excitation

Ryusuke Matsunaga,[1] Yuki I. Hamada,[1] Kazumasa Makise,[2] Yoshinori Uzawa,[3] Hirotaka Terai,[2] Zhen Wang,[2] and Ryo Shimano[1]





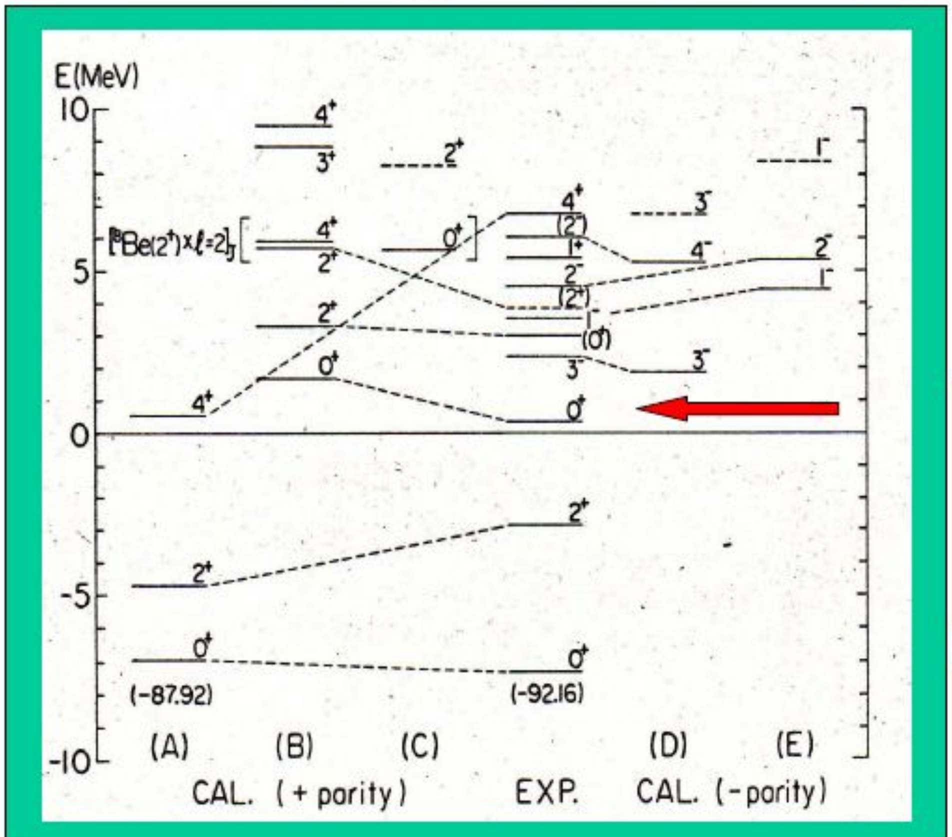
Uegaki et al 1977

^{12}C gas-like alpha-cluster states ”new phase”

1977

Uegaki 3 alpha cluster model

Energy level ^{12}C (PTP 57,1262(1977)) GCM



3 α Hoyle state

Uegaki et al
gas state of α particles
PTP 57,1262(1977)

α粒子ガス状態の存在

Tohsaki et al
condensate PRL **87**, 192501(2001)

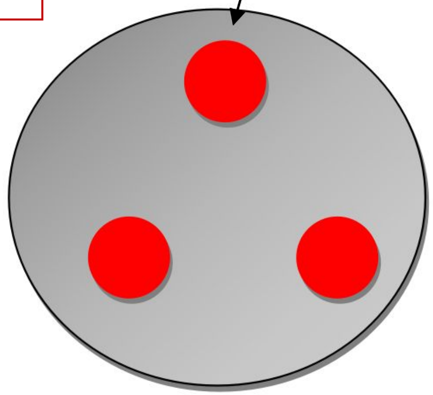
α粒子凝縮
No order parameter

Dilute alpha particle gas structure and alpha particle condensation in ^{12}C: Hoyle state

Energy level ^{12}C

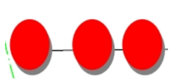
- all the α cluster states appear above the threshold and the Hoyle state

α particle gas



3α Hoyle state: alpha condensate

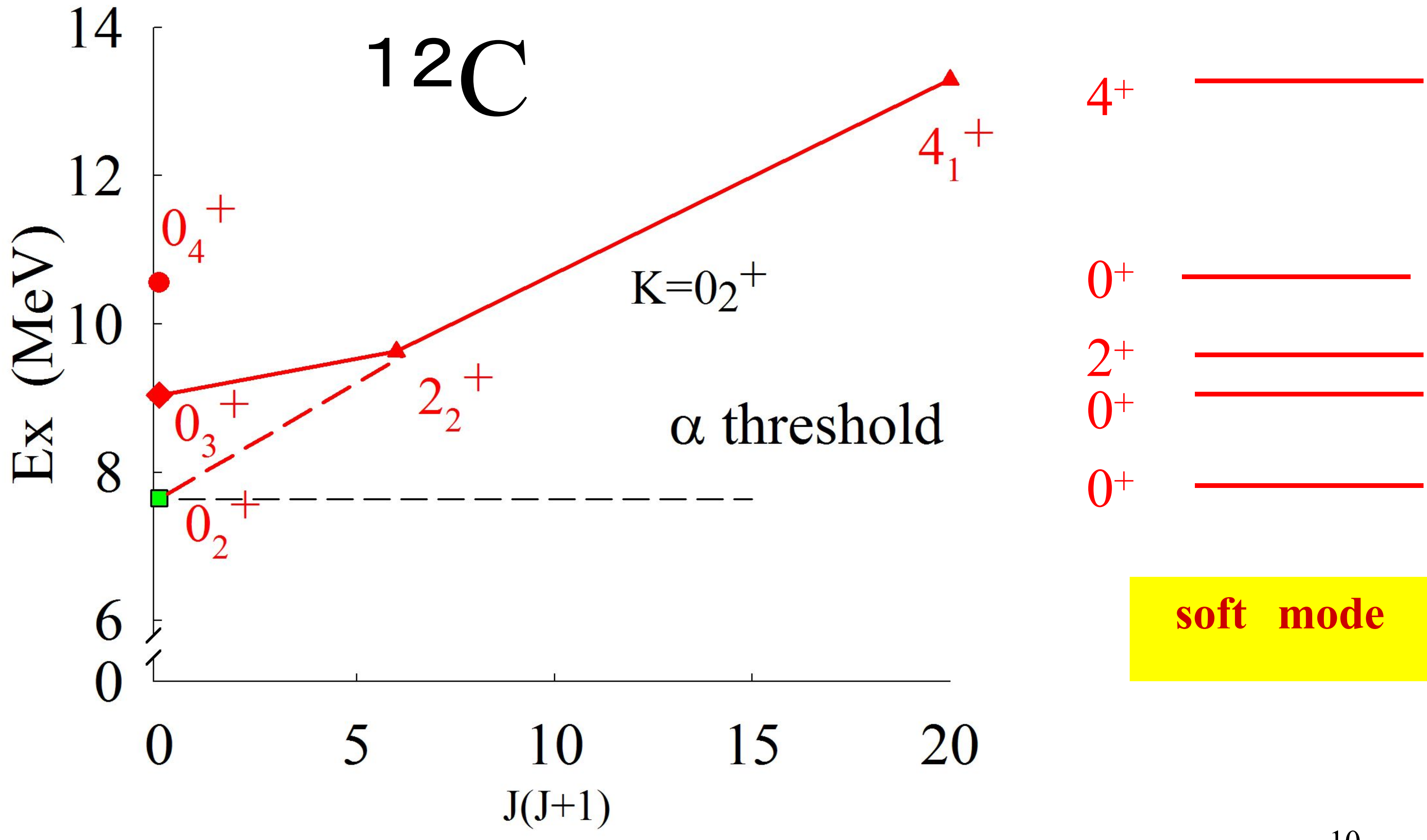
3 α particle



lowest 0s state

集団運動状態

回転運動バンド vs 振動運動バンド？



Bose-Einstein condensation

- Spontaneous symmetry of the global phase due to Bose-Einstein
- All the cluster models, ab initio calculations and other theories don't treat global phase U(1) symmetry and particle number fluctuation
- Conventional BEC theory (Bogoliubov) based on field theory does not respect gauge invariance (C-number approximation): valid for macroscopic systems but not for small particle number systems
- Strict treatment of zero modes is necessary

Field theory of alpha condensation

Large number of dilute alpha particles trapped in an external potential, for example h.o. potential

asymptotic field in the Heisenberg picture

ψ_H : *Heisenberg field*

ψ : *asmptotic field*

Effective field theory

$$\psi_{Hp}\psi_{Hn} \sim \psi_p\psi_n + \psi_D + \text{higher orders}$$

$$\psi_{Hp}\psi_{Hp}\psi_{Hn}\psi_{Hn} \sim \psi_p\psi_p\psi_n\psi_n + \psi_p\psi_n\psi_D + \psi_D\psi_D + \psi_\alpha + \text{higher orders}$$

alpha field and the model

$$H = \int d^3x \left[\psi_\alpha^\dagger(x) \left(-\frac{\hbar^2}{2m}\nabla^2 - \mu + V_{ext}(x) \right) \psi_\alpha(x) \right] + \frac{1}{2}\int\int d^3x d^3x' \, \psi_\alpha^\dagger(x)\psi_\alpha^\dagger(x') V(|x - x'|)\psi_\alpha(x')\psi_\alpha(x)$$

$$V(r) = V^{Nucl}(r) + V^{Coulomb}(r)$$

The Heisenberg equation

$$i\hbar\frac{\partial}{\partial t}\psi_\alpha(x) = \left(-\frac{2m}{\hbar^2}\nabla^2 - \mu + V_{\rm ext}(x)\right)\psi_\alpha(x) + \int d^3x'\, \psi_\alpha^\dagger(x')V(|x-x'|)\psi_\alpha(x')\psi_\alpha(x)$$

canonical commutation relation for $t=t'$

$$[\psi_\alpha(x,t)\,,\,\psi_\alpha^\dagger(x',t)] = \delta(x-x')$$

For stationary system (independent of t)

$$\psi_\alpha(x) = \xi(x) + \varphi_\alpha(x)$$

Here $\xi(x)$ condensate

$\varphi_\alpha(x)$ operator for excitation field

$$[\varphi_\alpha(x,t)\,,\,\varphi_\alpha^\dagger(x',t)] = \delta(x-x')$$

Goldstone theorem (Ward Takahashi identity) is respected.

Total Hamiltonian is now given by

$$H = H_0 + H_{int}$$
$$H_0 = H_1 + H_2$$

where

$$H_1 = \int d^3x \left[\left\{ \varphi_\alpha^\dagger(x) \left(-\frac{\hbar^2}{2m}\nabla^2 - \mu + V_{ext}(x) + \int d^3x' \, |\xi(x')|^2 V(|x-x'|) \right) \xi(x) \right\} + \text{h.c.} \right]$$

$$H_2 = \int d^3x \left[\varphi_\alpha^\dagger(x) \left(-\frac{\hbar^2}{2m}\nabla^2 - \mu + V_{ext}(x) \right) \varphi_\alpha(x) \right.$$
$$+ \int d^3x' \left\{ \varphi_\alpha^\dagger(x) \, |\xi(x')|^2 V(|x-x'|)\varphi_\alpha(x) + \varphi_\alpha^\dagger(x)\xi^*(x')V(|x-x'|)\varphi_\alpha(x')\xi(x) \right\}$$
$$\left. + \frac{1}{2}\int d^3x' \, \{ \varphi_\alpha^\dagger(x)\varphi_\alpha^\dagger(x')V(|x-x'|)\xi(x')\xi(x) + \text{h.c.} \} \right]$$

$$H_{int} = \int\int d^3x d^3x' \, \left[\{ \varphi_\alpha^\dagger(x)\xi(x')V(|x-x'|)\varphi_\alpha(x')\varphi_\alpha(x) + \text{h.c.} \} \right.$$
$$\left. + \frac{1}{2}\varphi_\alpha^\dagger(x)\varphi_\alpha^\dagger(x')V(|x-x'|)\varphi_\alpha(x')\varphi_\alpha(x) \right]$$

From the condition $H_1 = 0$ $<\varphi_\alpha>=0$

the Gross-Pitaevski equation is derived as follows:

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V_{ext}(x) + \int d^3x' \, |\xi(x')|^2 \, V(|x-x'|)\right)\xi(x) = \mu\xi(x)$$

If we take the unperturbed Hamiltonian H_0=H_2 in the interaction picture, the equation of motion for the field operator is given in the matrix form as follows:

$$i\hbar\frac{\partial}{\partial t}\Phi_\alpha(x) = (\mathcal{T}\Phi_\alpha)(x)$$

where

$$\Phi_\alpha(x) = \begin{pmatrix}\varphi_\alpha(x)\\ \varphi_\alpha^\dagger(x)\end{pmatrix}$$

$$(\mathcal{T}\Phi_\alpha)(x) = \int dy \, T(x,y)\Phi_\alpha(y)$$

2x2 matrix is given by

$$T_{11}(x,y) = \left\{ -\frac{\hbar^2}{2m}\nabla^2 - \mu + V_{ext}(x) + \int d^3x' \, |\xi(x')|^2 \, V(|x-x'|) \right\} \delta(x-y) + \xi^*(y) V(|x-y|) \xi(x)$$

$$T_{12}(x,y) = V(|x-y|)\xi(x)\xi(y)$$

$$T_{21}(x,y) = -\xi^*(x)\xi^*(y)V(|x-y|)$$

$$T_{22}(x,y) = -\left\{ -\frac{\hbar^2}{2m}\nabla^2 - \mu + V_{ext}(x) + \int d^3x' \, |\xi(x')|^2 \, V(|x-x'|) \right\} \delta(x-y) - \xi^*(x) V(|x-y|) \xi(y)$$

To solve the equation we expand the field operator in the complete set

$$\varphi_\alpha(x) = \sum_n a_n(t) w_n(x)$$

The complete set wave functions satisfy the following completeness condition

$$\sum_n w_n(x) w_n(x') = \delta(x - x')$$

From the canonical commutation relations

$$[a_n(t), a^{\dagger}_{n'}(t)] = \delta_{nn'}$$

The eigenvalue equation of Bogoliubov-de-Gennes is given by

$$(\mathcal{T}Y_n)(x) = \varepsilon_n Y_n(x)\,, \qquad Y_n = \begin{pmatrix} u_n \\ v_n \end{pmatrix}$$

From the symmetry property of the eigenvalue equation, the following function

$$Z_n(x) = \sigma_1 Y_n^*(x)$$

σ_1: Pauli matrix

is also an eigenfunction with $-\varepsilon_n$

For the zero mode with $\varepsilon_n = 0$

we can define the function $Y_{-1}(x)$

$$(\mathcal{T}Y_0)(x) = IY_{-1}(x) \qquad (I = \text{constant})$$

The completeness condition is given by

$$Y_0(x)Y_{-1}^\dagger(x') + Y_{-1}(x)Y_0^\dagger(x') + \sum_{n\neq 0}\{Y_n(x)Y_n^\dagger(x') - Z_n(x)Z_n^\dagger(x')\} = \sigma_3\delta(x-x')$$

By using the complete set $\{Y_0, Y_{-1}, Y_n, Z_n\}$
the wave function Φ_α
is expanded as follows;

$$\Phi_\alpha(x) = \begin{pmatrix} \varphi_\alpha(x) \\ \varphi_\alpha^\dagger(x) \end{pmatrix}$$

$$\Phi_\alpha(x) = -iq(t)Y_0(x) + p(t)Y_{-1}(x) + \sum_{n\neq 0}\{b_n(t)Y_n(x) + b_n^\dagger(t)Z_n(x)\}$$

for $n=0$ we used $q(t)$ and $p(t)$ instead of b_0 and b_0^+

From the canonical commutation relation for $\varphi_\alpha(x)$
the operators $\{q(t), p(t), b_n(t), b_n^\dagger(t)\}$ satisfy

$$[q(t), p(t)] = i\,, \quad [b_n(t), b_{n'}^\dagger(t)] = \delta_{nn'}$$

By putting $\Phi_\alpha(x)$ into the Hamiltonian,

we obtain the diagonalized Hamiltonian H_0 as follows:

$$H_0 = \frac{I}{2}p^2(t) + \sum_{n\neq 0} \varepsilon_n b_n^\dagger(t) b_n$$

Goldstone theorem is respected

The vacuum for the operator b_n is defined by $b_n|0\rangle = 0$

Bogoliubov-de-Genne mode

$$|0\rangle = |\Psi\rangle \otimes |0\rangle_b \, , \Psi(q) = \langle q|\Psi\rangle$$

Now the non-perturbative field operator, the vacuum, non-perturbative hamiltonian and the interaction potential hamiltonian are given.

NG mode sector is modified to include higher power terms of NG quantum coordinate *p, q*

$$\hat{H}_u^{QP} = -\left(\delta\mu + 2C_{2002} + 2C_{1111}\right)\hat{P} + \frac{I - 4C_{1102}}{2}\hat{P}^2$$
$$+ 2C_{2011}\hat{Q}\hat{P}\hat{Q} + 2C_{1102}\hat{P}^3 + \frac{1}{2}C_{2020}\hat{Q}^4 - 2C_{2011}\hat{Q}^2$$
$$+ C_{2002}\hat{Q}\hat{P}^2\hat{Q} + \frac{1}{2}C_{0202}\hat{P}^4\,, \qquad (10)$$

where $C_{iji'j'} = \int d^3x d^3x' U(r)\xi^i(\boldsymbol{x})\eta^j(\boldsymbol{x})\xi^{i'}(\boldsymbol{x}')\eta^{j'}(\boldsymbol{x}')$ with $r = |\boldsymbol{x} - \boldsymbol{x}'|$, and $\delta\mu$ is to be determined self-consistently to satisfy the criterion $\langle 0|\hat{\psi}|0\rangle = \xi$. The

$$\eta(x) = \frac{\partial}{\partial N_0}\xi(x), \qquad \int d^3x\,[\xi^*\eta + \eta^*\xi] = 1$$

$$H_u^{QP}\,|\Psi_\nu >= E_\nu\,|\Psi >$$

V : (n, ℓ, m)

Modes due to SSB

- $\alpha - \alpha$ nuclear interaction

Ali-Bodmer potential : determined from $\alpha - \alpha$ scattering to fit the s-wave phase shift fit potential

$$V^{nucl}(r) = V_r \exp[-\mu_r^2 r^2] + V_a \exp[-\mu_a^2 r^2]$$

$$V_r = 500 MeV, \mu_r = 0.7 fm^{-1}, V_a = -130 MeV, \mu_a = 0.474 fm^{-1}$$

- $\alpha - \alpha$ Coulomb interaction :

$\alpha - \alpha$ folding potential $V^{\mathrm{Coul}}_{\alpha-\alpha}(r) = (4e^2/r)\mathrm{erf}(\sqrt{3}r/2b)$

size parameter of the α particle b is 1.44 fm

- Number of α clusters: N_0

$$\int |\xi|^2 dx = N_0 = 3$$

- External field potential : harmonic oscillator

$$V_{ex}(x) = m\Omega^2 x^2 / 2, \ \Omega = 2MeV /$$

Ω : parameter

Hoyle state: rms and density distribution

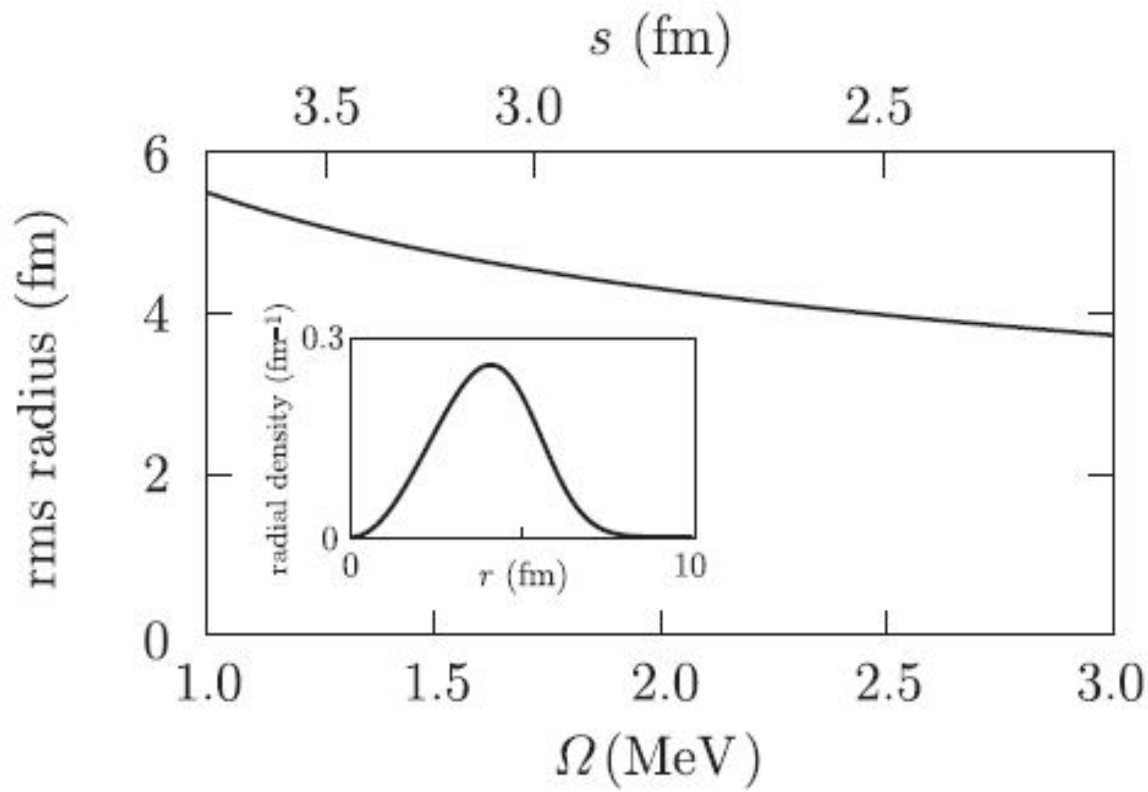


FIG. 1. Calculated $\bar{r}$ as a function of Ω and the radial density distribution for $\Omega = 2.14$ MeV (inset) of the Hoyle state. The upper horizontal axis indicates the rms radius $s = \sqrt{3/2m\Omega}$ of the 0s orbit of the external harmonic oscillator potential.

Cal. I

Ω=2.14 MeV, Vr=422 MeV

(rms radius=4.21 fm of the Hoyle state)

Energy levels

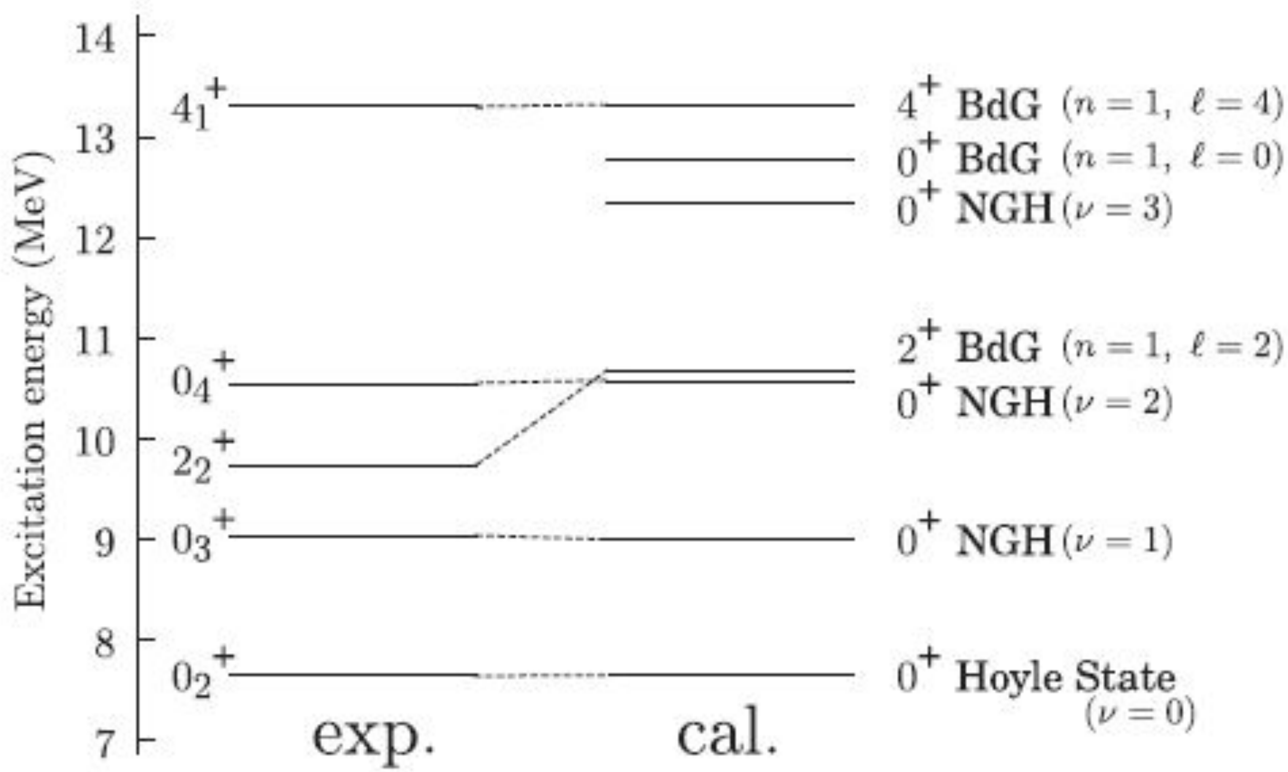
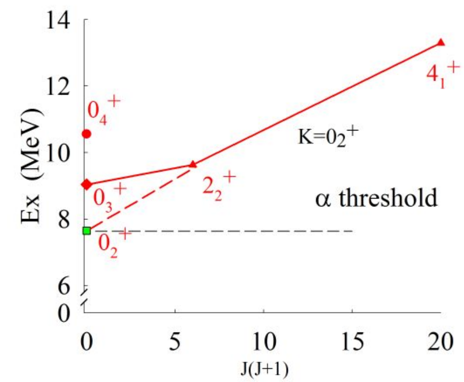


FIG. 3. The calculated energy levels for parameter set A (Ω = 2.14 MeV, V_r = 422 MeV), compared with the observed α-cluster states in ^{12}C [14–19].



Here note that $\langle\Psi_0|\hat{Q}|\Psi_0\rangle = \langle\Psi_0|\hat{P}|\Psi_0\rangle = 0$

$$\langle\Psi_1|\hat{Q}|\Psi_0\rangle, \langle\Psi_1|\hat{P}|\Psi_0\rangle \neq 0,$$

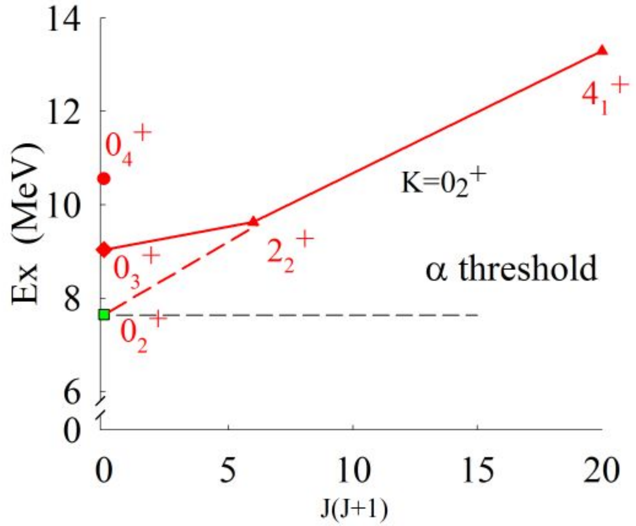


TABLE I. Calculated reduced transition probabilities $B(\mathrm{E2}: 2 \to 0)$ in unit of $e^2\,\mathrm{fm}^4$: Ref. [9], Ref. [59], and our results for the parameter sets A and B.

Transition	Ref. [9]	Ref. [59]	Ours (A)	Ours (B)
$2_2^+ \to 0_2^+$	100	295-340	290	204
$2_2^+ \to 0_3^+$	310	88-220	342	187

summary

1. Formulated effective field theory of Bose-Einstein condensation of finite number α particles.

Treated zero modes correctly.

3. The energy levels of alpha cluster states of ^{12}Care reproduced well and transition probabilities are predicted.

4. Alpha cluster 0^+ above the Hoyle states are Nambu-Goldstone and Higgs modes due to the phase locking of the condensate

5. 2^+, 4^+ above the Hoyle state are not a rotational band but a BdG excitation of the condensate