超流動クラスター模型による軽い核・中重核におけるαクラスターのボーズアインシュタイン凝縮と南部ゴールドストーンソフトモード

S. Ohkubo
大久保茂男

（RCNP　大阪大学核物理研究センター)
/高知県立大(U. of Kochi)

Organization of this talk

- Part I 池田図に魅せられて～池田さんを追って半世紀

  池田図の実体化と普遍化をめざして

- Ｐａｒｔ Ⅱ 池田図とαクラスターの超流動状態

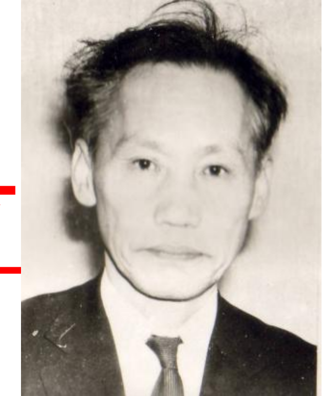
  クラスター集団運動と自発的対称性の破れ

Ｐart　Ⅰ

池田図に魅せられて～半世紀

池田図の実体化と普遍化をめざして

PTP　Suppl. 1968年
小林稔 先生 還暦特別号



The Systematic Structure-Change into the Molecule-like Structures in the Self-Conjugate 4*n* Nuclei

Kiyomi IKEDA,*) Noboru TAKIGAWA and Hisashi HORIUCHI

Department of Physics, University of Tokyo, Tokyo

(Received November 6, 1968)

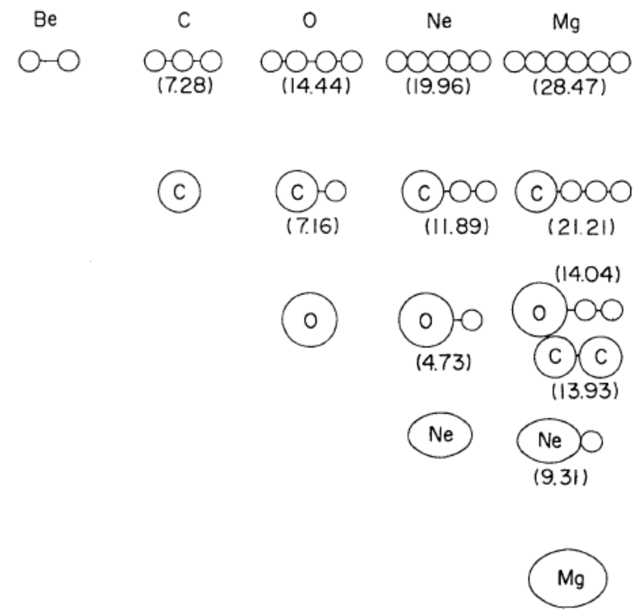
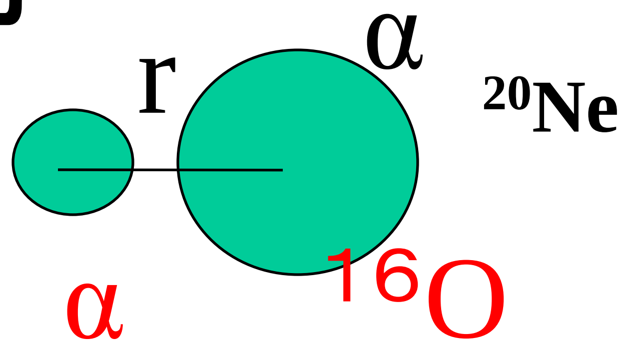


Fig. 1. Threshold energy for each decay mode. In the figure, the threshold energy for each decay mode is given in MeV. The systematics suggests the possible molecular nature around each energy. Some of the molecular states are already found and are represented in Fig. 2.

池田清美　34歳

1968年6月ー1970年3月　Ikeda Kiyomi
Abroad　NBI,　**Dubna**

池田図の　新しい　概念
新しい集団運動



池田図の　新しい　概念

教科書に新鮮な見方

新しい集団運動　**相対運動の励起**　**higher nodal state**

Bohr Mottelson　**振動運動**　　**平均場の運動**
　　　　　　　　　　回転運動

S.Ohkubo (博士論文、京都大1977)

Mayer &Jensen　**一粒子運動**

池田図は殻模型的理解に対峙

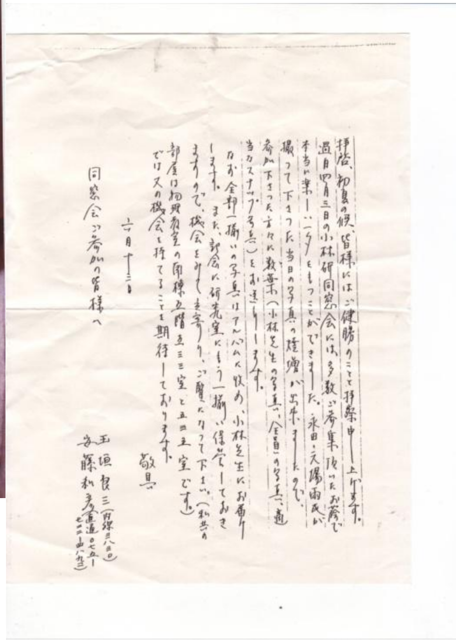
閾値＝結節点　**散乱状態のクラスター構造（殻模型の複雑さ：捨象）**

クラスター構造のレーゾンデートル

小林稔　研究室同窓会　1985.4.3　京都

1985年4月3日小林研同窓会での小林稔先生
(京都大学で日本物理学会第40回年会のおりに

大久保茂男、永田忍



拝啓、初夏の候、皆様にはご健勝のことと拝察申し上げます。
過日四月三日の小林研同窓会には多数ご参集頂いたお蔭で
本当に楽しい一ト時をもつことができました。永田・大場両氏が
撮って下さった当日の写真の焼増が出来ましたので、
参加下さった方々に数葉（小林先生の写真一、全員の写真一、適
当なスナップ写真）をお送りします。
なお全部揃いの写真はアルバムに収め、小林先生にお届け
しました。また、新たに研究室にもう一揃い保管しておき
ますので、機会をみてお寄り、ご覧になって下さい。（松井の
部屋は物理教室の南棟五階五三三室と五四五室です。）
では又の機会を持てることを期待しております。
敬具
六月十二日
玉垣良三（内線三八三〇）
安藤和彦（直通〇七五一七二二一四八九三）
同窓会ご参加の皆様へ

永田忍先生からの　励まし
「クラスターで頑張ってください」

小林稔先生からの　励まし「クラスターで頑張ってください」
小林研　同窓会

18/10/2 11時40分

^{44}Ti　のクラスターへ

PRL 57 (1986) 1215

小林先生からの　励まし

1985年5月から　Oxford　Brink
　ベルギー　 Mons　　　　　　　　へ遊学

1985年5月　Brink　fusionとクラスターの研究

1985年9月　ベルギーで
^{44}Ti N=13負回転バンドの存在の着想
(38歳)　　散乱状態のクラスター構造（殻模型の複雑さ：捨象）

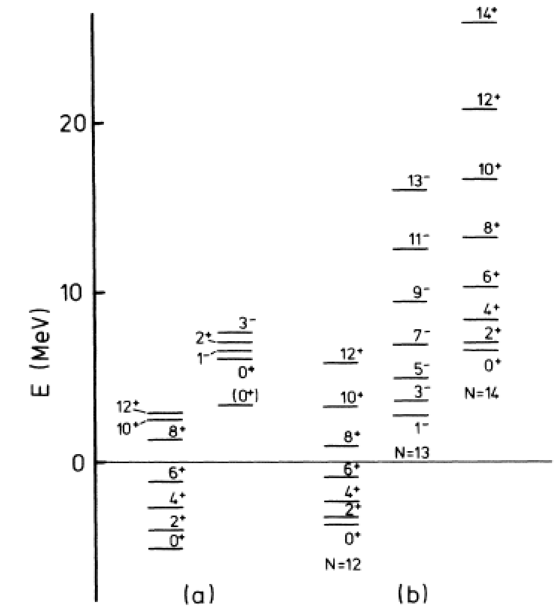


FIG. 1. (a) Experimental ground-state band (Ref. 3) and α-particle cluster-state candidates (Refs. 4–6) in ^{44}Ti; (b) ^{44}Ti $N=12$, $N=13$, and $N=14$ states supported by the local potential (the potential depth is fixed to the average value $U_0=180$ MeV). Energies are given with respect to the $\alpha+{}^{40}$Ca threshold.

ican Physical Society　　1215

VOLUME 57, NUMBER 10　　PHYSICAL REVIEW LETTERS　　8 SEPTEMBER 1986

Evidence for Alpha-Particle Clustering in the ^{44}Ti Nucleus

F. Michel
Faculté des Sciences, Université de l'Etat, B-7000 Mons, Belgium

G. Reidemeister
Physique Nucléaire Théorique, CP229, Université Libre de Bruxelles, B-1050 Bruxelles, Belgium

and

S. Ohkubo
Department of Applied Science, Kochi Women's University, Kochi 780, Japan

(Received 27 March 1986)

The α-particle cluster structure of ^{44}Ti, which is of considerable interest for investigation of the persistence of α clustering in the region of the *sd*-shell closure, is studied within the frame of a local-potential approach. It is shown that the model leads to specific predictions if continuity with existing unique $\alpha+{}^{40}$Ca optical potentials is insisted upon; in particular, the existence of an as yet experimentally unknown α-cluster negative-parity band, starting just above the $\alpha+{}^{40}$Ca threshold, is strongly suggested.

18/10/2 11時55分

Guy Reidemeister, Francis Michel，Shigeo Ohkubo

^{44}Ti　fp殻　池田図の領域のクラスター　へ

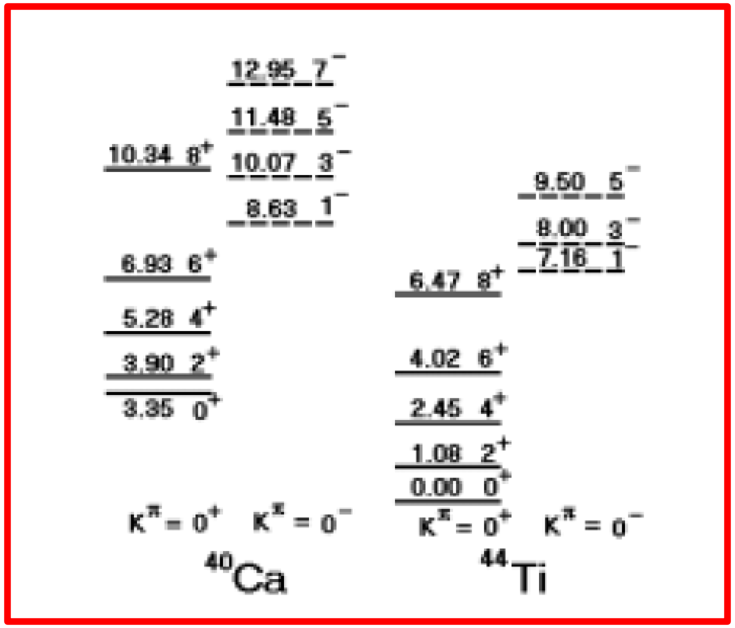


Fig. 24. Parity doublet bands in ^{40}Ca and ^{44}Ti. Broken lines indicate that they are the energy centroids of fragmented levels of each spin.

fromT. Yamaya et al
PTP Suppl. 132 (1998) 73.

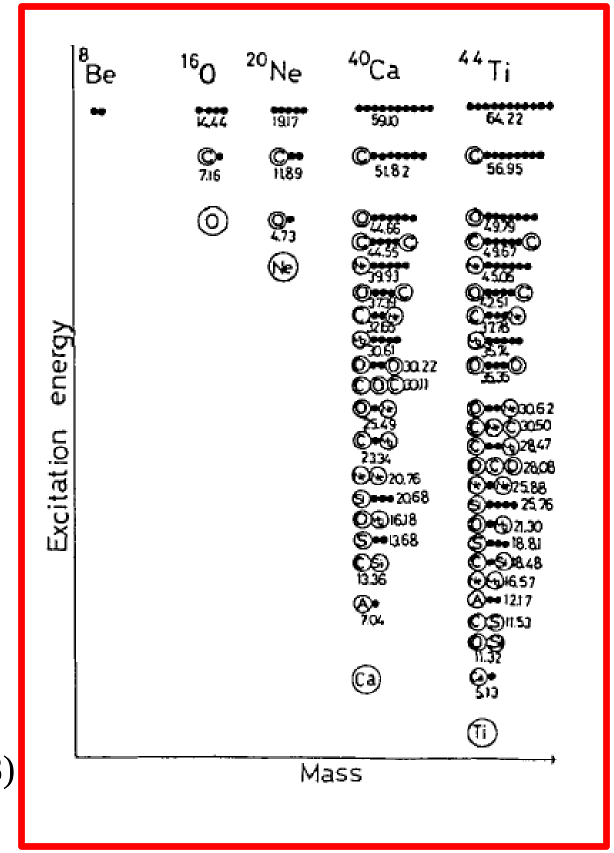


Fig.11 Diagram of molecular viewpoint extended to the fp-shell region. The threshold energies are also written (in MeV).

S.Ohkubo et al,
Developments of nuclear cluster dynamics
World Scientific (1988)
p.114
(田中一先生退官札幌クラスター国際会議1988)

PTP Suppl.（1968）小林記念号
元祖池田図から20年

1⁻、3⁻、5⁻

T. Yamaya et al　PRC **42**,1935(1990).　RCNP　AVF cyclotron
C.Y. Kim and T. Udagawa , PRC **46**,532 (1992),P. Guazzoni et al NPA **564,** 425(1993) Berlin　cyclotron

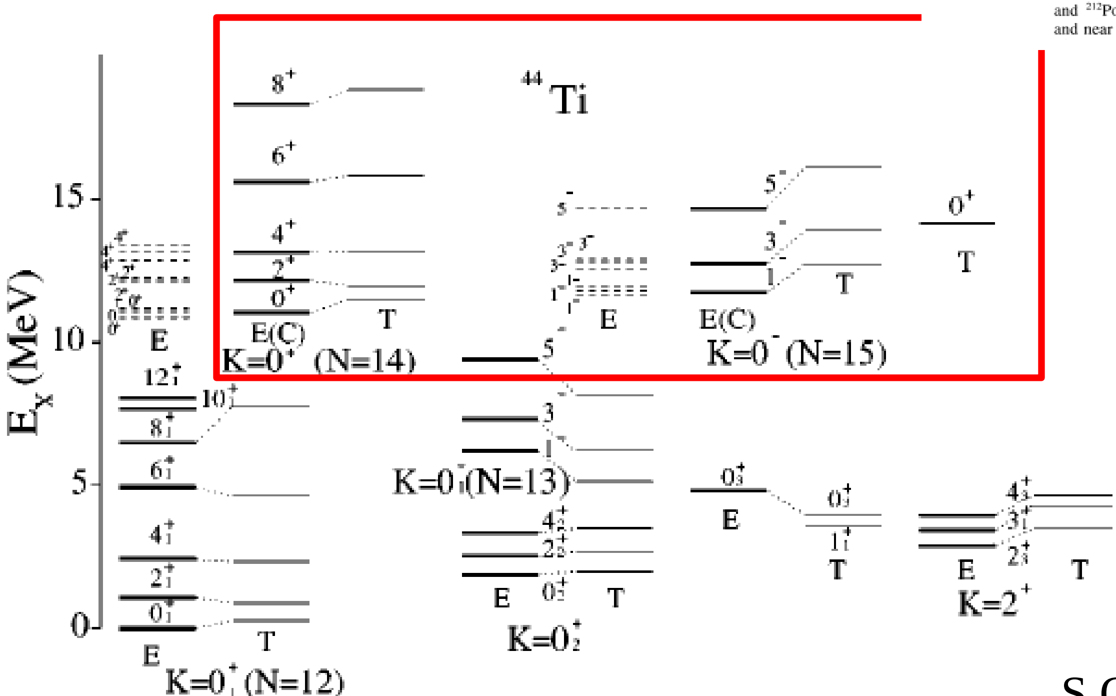
7⁻　M.Fukada et al　PRC **80, 064613 (2009)** 京大物理　Tandem　加速器

α structure in ^{44}Ti cluster & medium-weight nuclei

^{44}Ti Higher nodal states

N=14, N=15

相対運動の　励起



VOLUME 74, NUMBER 12　　PHYSICAL REVIEW LETTERS　　20 MARCH 1995

Alpha Clustering and Structure of ^{94}Mo and ^{212}Po

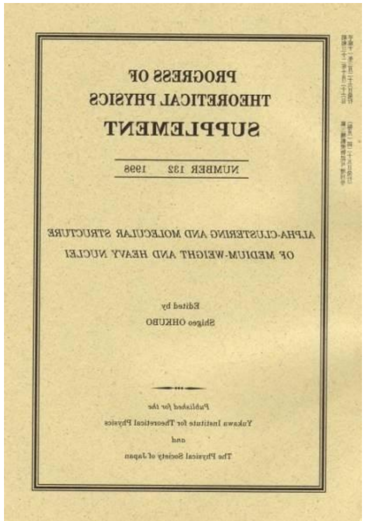
Shigeo Ohkubo

Department of Applied Science, Kochi Women's University, Kochi 780, Japan

(Received 26 July 1994; revised manuscript received 27 October 1994)

The α-particle clustering structure of ^{94}Mo and ^{212}Po, which is of interest for investigating the persistence of α clustering in heavy nuclei, is studied within the framework of a local potential approach using a double folding model. It is shown that the model, which describes α scattering from ^{90}Zr well, locates the ground state of ^{94}Mo at the energy corresponding to experiment. A similar result was obtained for the α + ^{208}Pb system. It is found that the model gives not only the ground band of ^{94}Mo and ^{212}Po as compact α-cluster states but also predicts other developed genuine α-cluster states below and near the Coulomb barrier.

S.Ohkubo et al　PRL 74, 2176(1995).



S.Ohkubo et al　PTP Suppl. 132(1998).

S.Ohkubo et al　PRC **57**,2760(1998).

PTP Suppl.（1968）小林記念号

元祖池田図から30年

1999

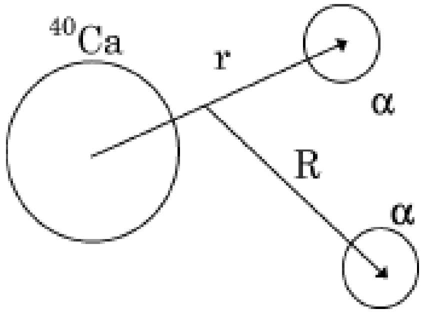


Fig. 1. Relative coordinates of the ^{40}Ca + α + α system.

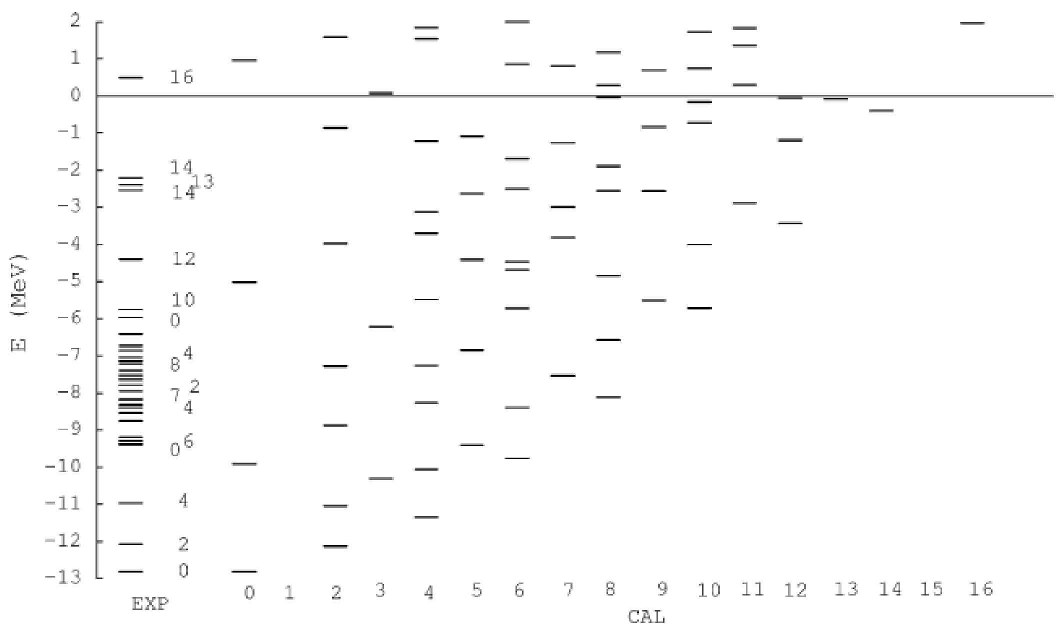
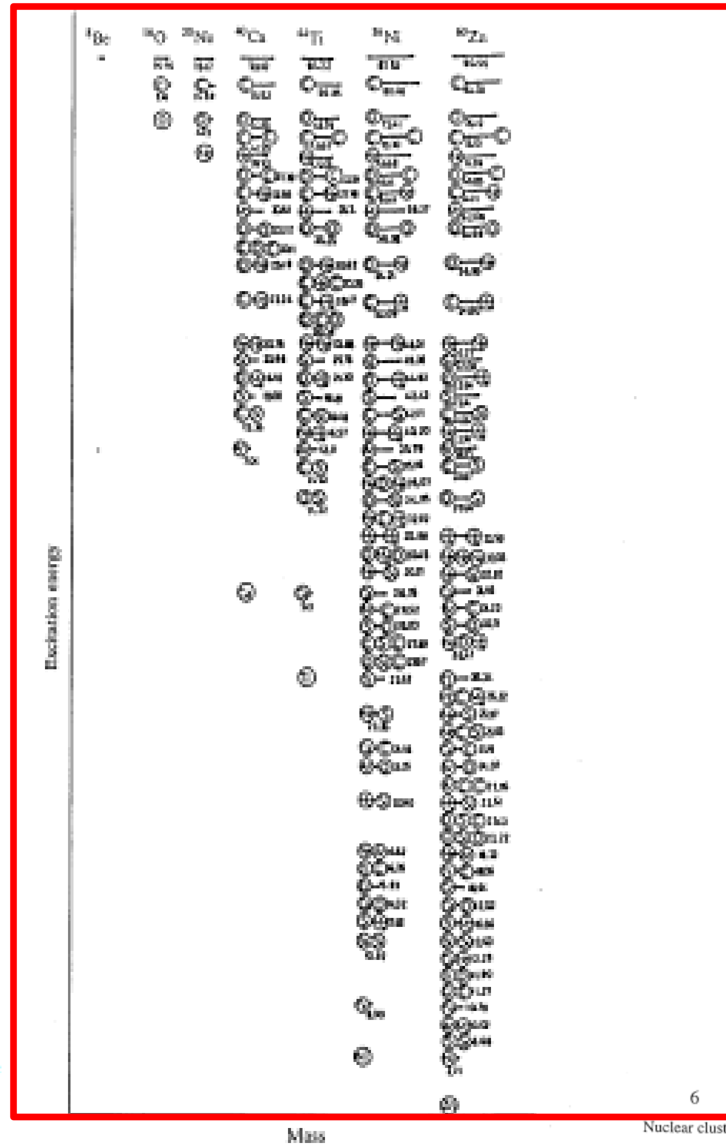


Fig. 3. Calculated and experimental positive-parity states of ^{48}Cr. The energy scale is measured from the ^{40}Ca + α + α threshold ($E_{\mathrm{th}} = 12.82$ MeV).

T. Sakuda and S. Ohkubo, NPA 712, 59(2002)



Nuclear clusters

S. Ohkubo, T. Yamaya, and P. E. Hodgson

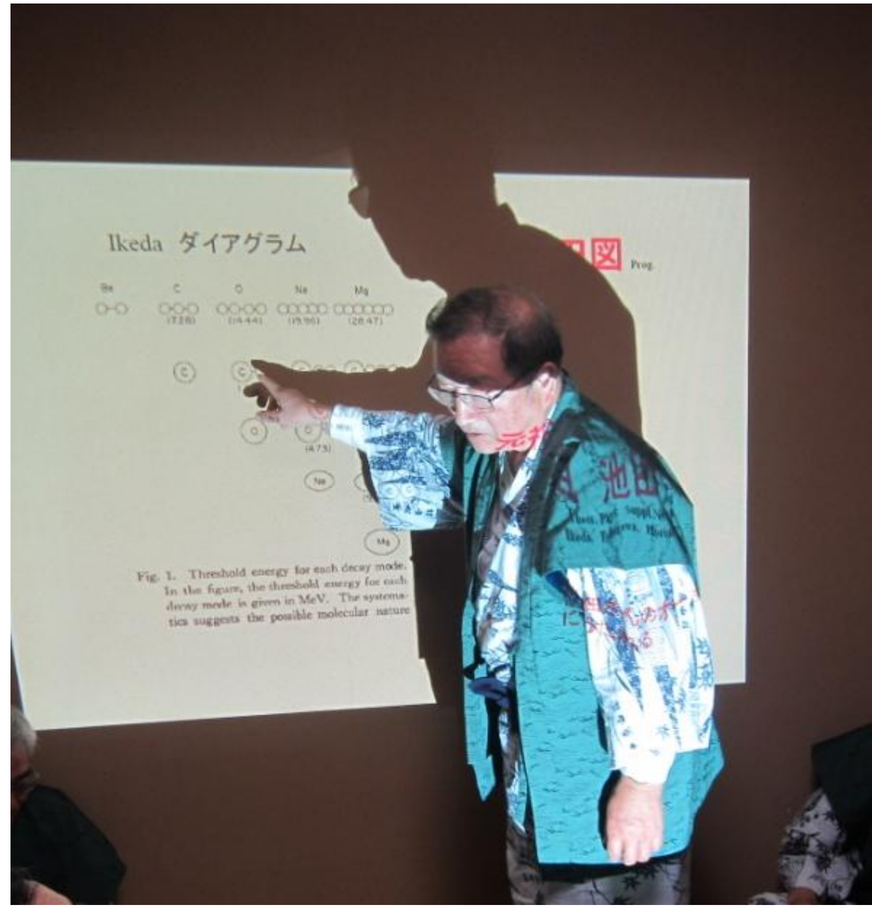
Fig. 6.22 Diagram of molecular viewpoint extended to the fp-shell region. T... ... into subunit clusters are given in MeV. (Taken from Ohkubo *et al.* [94].)

OXFORD STUDIES
IN
NUCLEAR PHYSICS

GENERAL EDITOR
P. E. HODGSON

Oxford studies in Nuclear Physics Nucleon－Hadron Many－body sytem（Oxford Univ. Press, 1999）

元祖が語る池田図
2012．3．27　大原山荘



Part Ⅱ

池田図とαクラスターの超流動状態

Ｐ．　Ｓｃｈｕｃｋ　の質問

1999　Ｃｌｕｓｔｅｒ　conference at Rab Island, Croatia

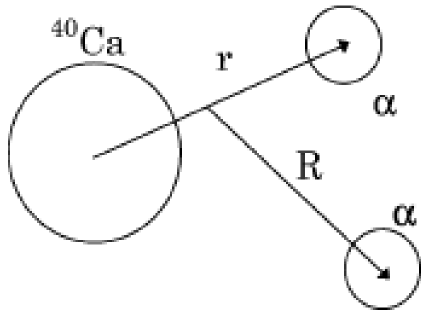


Fig. 1. Relative coordinates of the ^{40}Ca + α + α system.

^{48}Cr　で　2個のα　クラスターは　凝縮しているか

Alpha- cluster condensation
J. Eichler (Hahn Meitner Institute, Berlin)
Eichler & Yamamura Nucl. Phys. A 182, 33 (1972)
qudrupling　and pairing in the shell model
Eichlerと議論　１９７４？

Uegaki et al **1977**

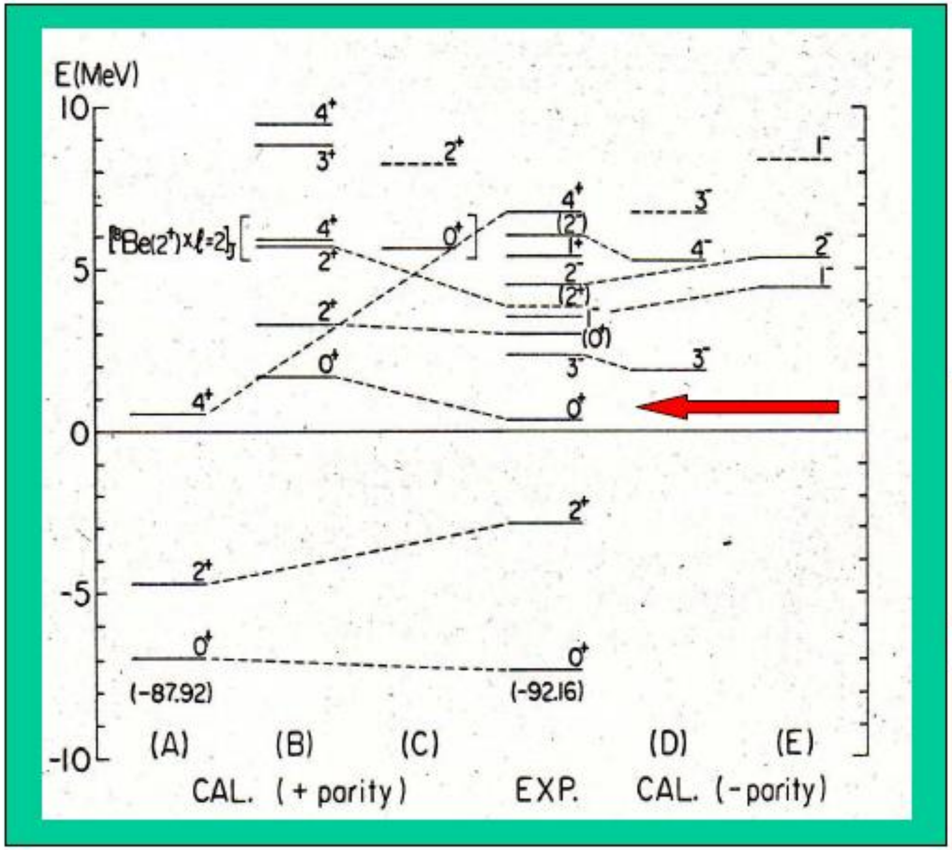
^{12}C **gas-like alpha-cluster states** ”new phase”

1977

Uegaki 3 alpha cluster model

GCM

Energy level ^{12}C (PTP 57,1262(1977))



3α粒子ガス状態の存在を指摘

3α Hoyle state

Uegaki et al

gas state of α particles

PTP 57,1262(1977)

Tohsaki et al

condensate PRL **87**, 192501(2001)

α粒子凝縮

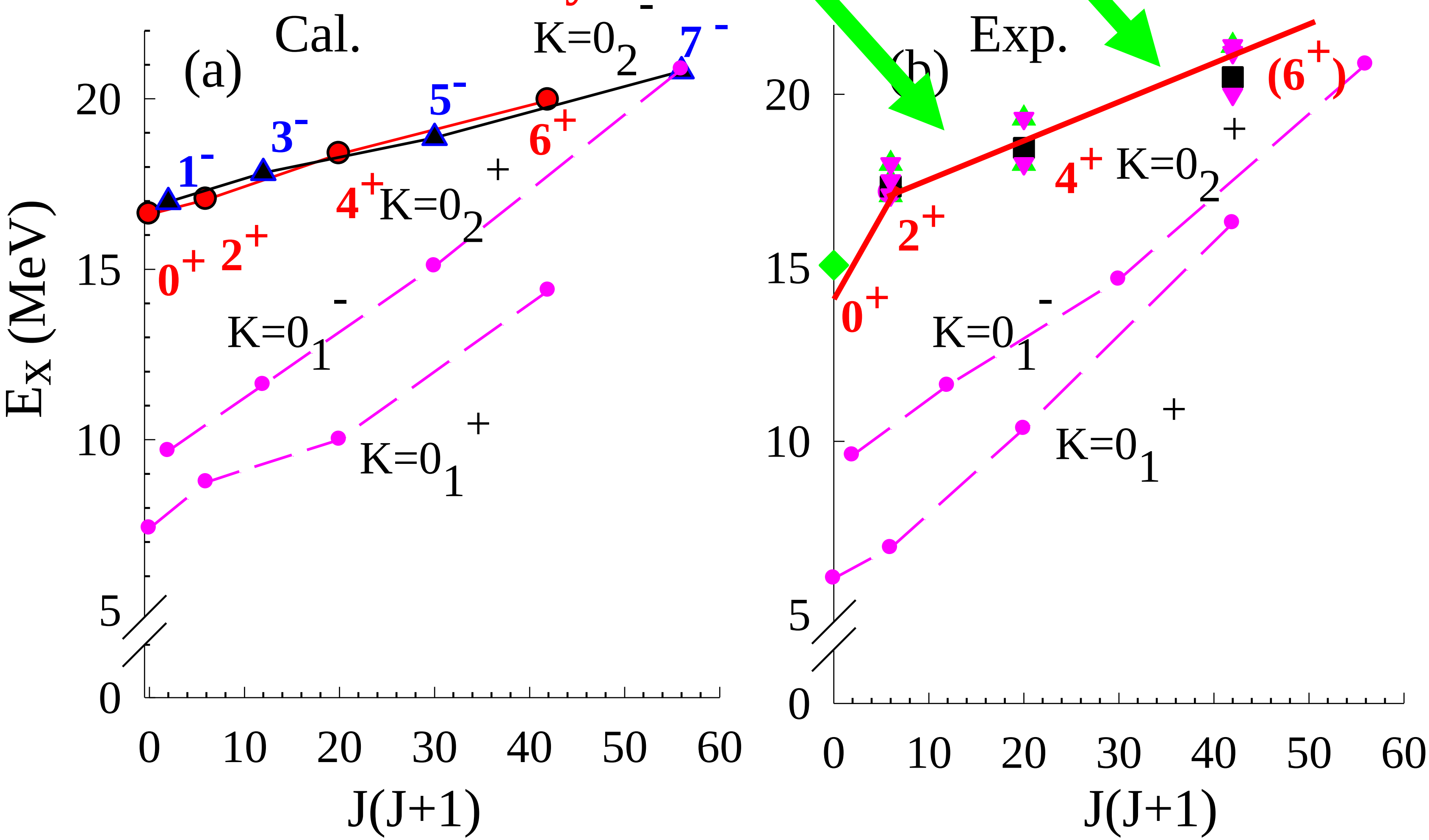
No order parameter

^{16}O: α＋^{12}C(Hoyle) cluster structure

PLB684,127（2010）

S. Ohkubo and Y. Hirabayashi

歴史的にまた通常は4α linear chain と考えられている



arXiv:1301.7485v1 [nucl-th] 31 Jan 2013

Evidence of α particle condensation in ^{12}C and ^{16}O and Nambu-Goldstone boson

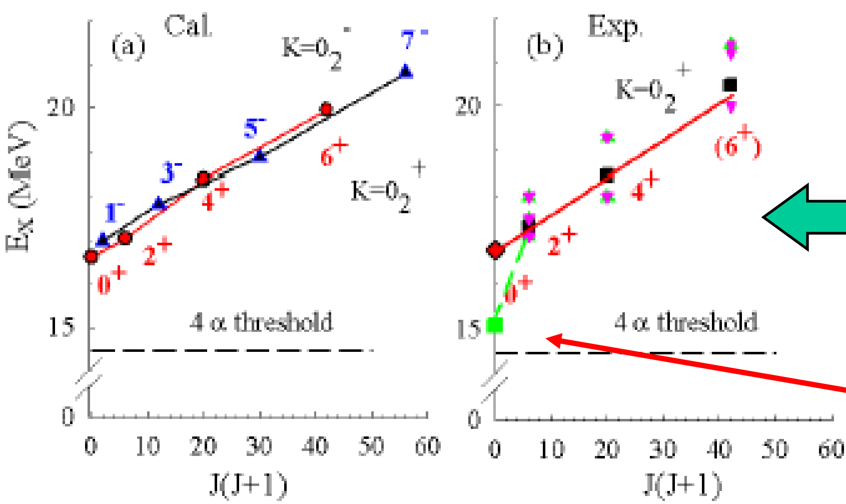
S. Ohkubo[a]

[a]University of Kochi , Kochi 780-8515, Japan and
Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka 567-0047, Japan

Abstract

It is suggested that direct evidence of Bose-Einstein condensation of α particles is obtained by observing a phase mode (Nambu-Goldstone boson) with long wavelength even when characteristic features such as superfluidity is diffucult to observe. For the 7.65 MeV 0_2^+ Hoyle state in ^{12}C and 15.1 MeV 0^+ state in ^{16}O, which are candidates for an α particle condensate, it is suggested that the emergent band head 0^+ state of the $K = 0_2^+$ rotational band with a very large moment of inertia is considered to be a Nambu-Goldstone boson.

Keywords: α particle condensation, Nambu-Goldstone boson, spontaneous symmetry breaking, α cluster structure, ^{12}C; ^{16}O
PACS: 21.60.Gx, 27.20.+n, 03.75.Nt



Nambu-Goldstone boson 、

4α 超流動状態

相転移(NG相)＝VEV（真空期待値)≠ 0

NG相の特徴

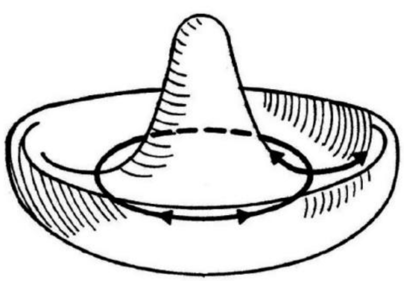
Nambu Goldstone boson

Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).
J. Goldstone, Nuovo Cimento **19**, 154 (1961).

H. Watanabe and H. Murayama, Phys. Rev. Lett. **108**, 251602 (2012); Y. Hidaka, *ibid.* **110**, 091601 (2013).

Higgs boson

Emergence of order parameter oscillation



P. B. Littlewood and C. M. Varma, Phys. Rev. Lett. **47**, 811 (1981); Phys. Rev. B **26**, 4883 (1982).
C. M. Varma, J. Low Temp. Phys. **126**, 901 (2001).

ATLAS Collaboration, Phys. Lett. B **716**, 1 (2012); CMS Collaboration, *ibid.* **716**, 30 (2012).

$$\psi(\mathbf{k}) = |\psi(\mathbf{k})| \exp(i\phi).$$

Higgs Boson in Superconductors

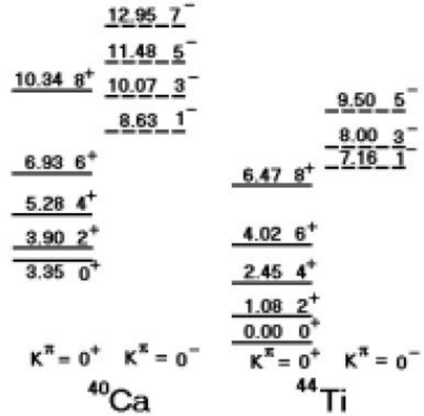
C.M. Varma

23

原子核の自発的対称性の破れによる集団運動と南部・ゴールドストンモード

- **4重極集団運動**（回転対称性の破れ：
  3次元　ユーグリッド空間R(3)）

回転運動　(order parameter: 変形deformation)



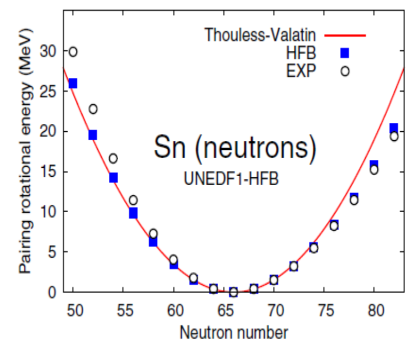
回転バンドの観測　→原子核の変形

（南部ゴールドストーンボソン）

- **対相関集団運動**（ゲージ位相固定
  粒子数空間　ゲージ　空間　U(1)　）

対回転　(order parameter: paring gap energy)

粒子数空間空間での集団運動



Hinohara and Nazarewicz
PRL 116,1525012(2016)

エネルギ準位におけるギャップ　ー＞超流動

（南部ゴールドストーンボソン）

集団運動と対称性の破れ

気体のボーズ凝縮による集団運動と原子核の相違

cold atom BEC
超流動、量子渦の存在 ———> 原子核 ^{12}C
Dilute α gas condensation
（ほぼ確からしいが確証はない）

南部/goldstone mode (phonon)は
見つかっていない

南部Goldstone mode ？
離散的準位で観測にかかりやすい

クラスター模型:
真空の相転移と模型と秩序パラメータ

クラスター構造を記述する模型

1. GCM,RGM,OCM:,
2. 微視的ボソン模型:
3. 局所ポテンシャル模型:
4. 気体模型:
5. ab initio no-core shell model AMD/FMD

他

粒子描像

秩序パラメータ 含まず
真空概念なし

αクラスターのBose-Einstein 凝縮の相転移を記述するには秩序パラメータ含むクラスター理論の必要性

場の理論

安定な真空概念
秩序パラメータ 、

ゼロ(NG)モードを厳密に扱う場の理論が必要

1．ボゴリウボフ凝縮場理論：演算子に正準共役性を破る（体積V大では問題にならないが少数系の凝縮理論では問題になり使えない）

2．少数粒子系でも厳密に正準共役性を満たす場の理論が必要

3．崩壊しない安定な真空の存在の必要性

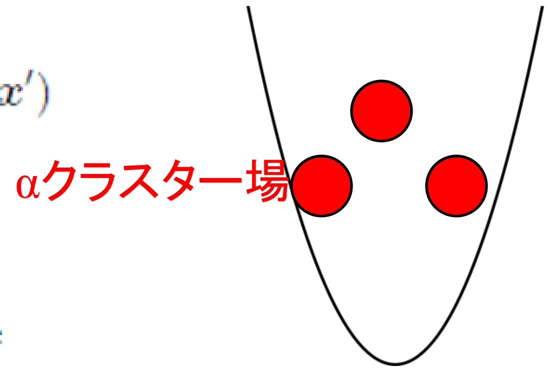
The Heisenberg equation

$$i\hbar\frac{\partial}{\partial t}\psi_\alpha(x) = \left(-\frac{2m}{\hbar^2}\nabla^2 - \mu + V_{\rm ext}(x)\right)\psi_\alpha(x) + \int d^3x'\,\psi^\dagger_\alpha(x')V(|x-x'|)\psi_\alpha(x')\psi_\alpha(x)$$

canonical commutation relation for $t=t'$

$$\left[\psi_\alpha(x,t)\,,\,\psi^\dagger_\alpha(x',t)\right] = \delta(x-x')$$

For stationary system (independent of t)



$$\psi_\alpha(x) = \xi(x) + \varphi_\alpha(x)$$

Here $\xi(x)$ condensate

$\varphi_\alpha(x)$ operator for excitation field

$$\left[\varphi_\alpha(x,t)\,,\,\varphi^\dagger_\alpha(x',t)\right] = \delta(x-x')$$

Goldstone theorem (Ward Takahashi identity) is respected.

古典凝縮場ξ　は超流動のグロス・ピタエスキー方程式を満たす

$$V_{\rm ex}(r) = \frac{1}{2}m\Omega^2 r^2\,, \tag{1}$$

and the α–α interaction is given by the Ali–Bodmer potential [50],

$$U(|\boldsymbol{x}-\boldsymbol{x}'|) = V_r e^{-\mu_r^2|\boldsymbol{x}-\boldsymbol{x}'|^2} - V_a e^{-\mu_a^2|\boldsymbol{x}-\boldsymbol{x}'|^2}\,. \tag{2}$$

The repulsive Coulomb potential affects numerical results very little and is suppressed in this work.

Let $\psi(x)$ ($x = (\boldsymbol{x}, t)$) be the field operator of the α cluster, and the model Hamiltonian is

$$\hat{H} = \int d^3x\, \hat{\psi}^\dagger(x)\left(-\frac{\nabla^2}{2m} + V_{\rm ex}(\boldsymbol{x}) - \mu\right)\hat{\psi}(x) + \frac{1}{2}\int d^3x\, d^3x'\, \hat{\psi}^\dagger(x)\hat{\psi}^\dagger(x')U(|\boldsymbol{x}-\boldsymbol{x}'|)\hat{\psi}(x')\hat{\psi}(x)\,, \tag{3}$$

$$\hat{H} = \hat{H}_2 + \hat{H}_{3,4}$$

$$\hat{H}_2 = \frac{1}{2}\int d^3x\, d^3x'\, \hat{\bar{\Phi}}(x)\mathcal{T}(\boldsymbol{x},\boldsymbol{x}')\hat{\Phi}(x')\,, \tag{4}$$

$$\hat{H}_{3,4} = \frac{1}{2}\int d^3x\, d^3x'\, U(|\boldsymbol{x}-\boldsymbol{x}'|) \times \left[\{2\xi(\boldsymbol{x}') + \hat{\varphi}^\dagger(x')\}\,\hat{\varphi}^\dagger(x)\hat{\varphi}(x)\hat{\varphi}(x') + \text{h.c.}\right]\,, \tag{5}$$

with $t = t'$, and

$$V_H(\boldsymbol{x}) = \int d^3x'\, U(|\boldsymbol{x}-\boldsymbol{x}'|)\xi^2(\boldsymbol{x}')\,, \tag{6}$$

$$\hat{\Phi}(x) = \begin{pmatrix}\hat{\varphi}(x)\\ \hat{\varphi}^\dagger(x)\end{pmatrix}\,,\quad \hat{\bar{\Phi}}(x) = \hat{\Phi}^\dagger(x)\sigma_3\,, \tag{7}$$

$$\mathcal{T}(\boldsymbol{x},\boldsymbol{x}') = \begin{pmatrix}\mathcal{L}(\boldsymbol{x},\boldsymbol{x}') & \mathcal{M}(\boldsymbol{x},\boldsymbol{x}')\\ -\mathcal{M}(\boldsymbol{x},\boldsymbol{x}') & -\mathcal{L}(\boldsymbol{x},\boldsymbol{x}')\end{pmatrix}\,, \tag{8}$$

$$\mathcal{M}(\boldsymbol{x},\boldsymbol{x}') = U(|\boldsymbol{x}-\boldsymbol{x}'|)\xi(\boldsymbol{x})\xi(\boldsymbol{x}')\,, \tag{9}$$

$$\mathcal{L}(\boldsymbol{x},\boldsymbol{x}') = \delta(\boldsymbol{x}-\boldsymbol{x}')\{-\nabla^2/2m + V_{\rm ex}(\boldsymbol{x}) - \mu + V_H(\boldsymbol{x})\} + \mathcal{M}(\boldsymbol{x},\boldsymbol{x}')\,, \tag{10}$$

where σ_i ($i = 1, 2, 3$) is the Pauli matrix. We have the Gross–Pitaevskii (GP) equation [51]

$$\{-\nabla^2/2m + V_{\rm ex}(\boldsymbol{x}) - \mu + V_H(\boldsymbol{x})\}\,\xi(\boldsymbol{x}) = 0\,, \tag{11}$$

$$\psi = \xi + \hat{\varphi},$$

$$\langle 0|\,\psi\,|0\rangle = \xi.$$

$$\int d^3x\, |\xi(\boldsymbol{x})|^2 = N_0.$$

superfluid density [59–61] is given by $|\xi(r)|^2/N_0$.

$$\int d^3x'\ \mathcal{T}(\boldsymbol{x},\boldsymbol{x}')\boldsymbol{y_n}(\boldsymbol{x}') = \omega_{\boldsymbol{n}}\boldsymbol{y_n}(\boldsymbol{x})\,, \tag{12}$$

$$\boldsymbol{y_n}(\boldsymbol{x}) = \begin{pmatrix} u_{\boldsymbol{n}}(\boldsymbol{x}) \\ v_{\boldsymbol{n}}(\boldsymbol{x}) \end{pmatrix}\,. \tag{13}$$

The index $\boldsymbol{n} = (n, \ell, m)$ is a triad of the main, azimuthal, and magnetic quantum numbers for isotropic ξ. Similarly as the BdG equation is introduced for fermionic systems, Eq. (13) is the BdG equation for bosonic systems. The bosonic eigenfunction is normalized as $\int d^3x\ (|u_{\boldsymbol{n}}|^2 - |v_{\boldsymbol{n}}|^2) = 1$ (see Eq. (16)) for the commutation relations, while the fermionic eigenfunction is normalized as $\int d^3x\ (|u_{\boldsymbol{n}}|^2 + |v_{\boldsymbol{n}}|^2) = 1$ for the anti-commutation relations.

Another eigenfunction, denoted by $\boldsymbol{z_n}$, is introduced:

$$\int d^3x'\ \mathcal{T}(\boldsymbol{x},\boldsymbol{x}')\boldsymbol{z_n}(\boldsymbol{x}') = -\omega_{\boldsymbol{n}}\boldsymbol{z_n}(\boldsymbol{x})\,, \tag{14}$$

$$\boldsymbol{z_n}(\boldsymbol{x}) = \sigma_1\boldsymbol{y}^*_{\boldsymbol{n}}(\boldsymbol{x}) = \begin{pmatrix} v^*_{\boldsymbol{n}}(\boldsymbol{x}) \\ u^*_{\boldsymbol{n}}(\boldsymbol{x}) \end{pmatrix}\,. \tag{15}$$

The inner product is defined as $((\boldsymbol{a},\boldsymbol{b})) \equiv \int d^3x\ \ \boldsymbol{a}^\dagger(\boldsymbol{x})\sigma_3\boldsymbol{b}(\boldsymbol{x})$, and the orthonormal relations are

$$((\boldsymbol{y_n},\boldsymbol{y_{n'}})) = -((\boldsymbol{z_n},\boldsymbol{z_{n'}})) = \delta_{\boldsymbol{nn'}}\,, \tag{16}$$

$$((\boldsymbol{y_n},\boldsymbol{z_{n'}})) = 0\,. \tag{17}$$

We also have the eigenfunction with zero eigenvalue,

$$\int d^3x'\ \mathcal{T}(\boldsymbol{x},\boldsymbol{x}')\boldsymbol{y}_0(\boldsymbol{x}') = 0\,,\quad \boldsymbol{y}_0(r) = \begin{pmatrix} \xi(\boldsymbol{x}) \\ -\xi(\boldsymbol{x}) \end{pmatrix}\,, \tag{18}$$

which is orthogonal to all the eigenfunctions including itself,

$$((\boldsymbol{y}_0,\boldsymbol{y}_0)) = ((\boldsymbol{y}_0,\boldsymbol{y_n})) = ((\boldsymbol{y}_0,\boldsymbol{z_n})) = 0\,. \tag{19}$$

For the completeness of the set of BdG eigenfunctions, the adjoint eigenfunction $\boldsymbol{y}_{-1}$ is necessary,

$$\int d^3x'\ \mathcal{T}(\boldsymbol{x},\boldsymbol{x}')\boldsymbol{y}_{-1}(\boldsymbol{x}') = I\,\boldsymbol{y}_0(\boldsymbol{x})\,, \tag{20}$$

$$\boldsymbol{y}_{-1}(\boldsymbol{x}) = \begin{pmatrix} \eta(\boldsymbol{x}) \\ \eta(\boldsymbol{x}) \end{pmatrix}\,, \tag{21}$$

$$((\boldsymbol{y}_{-1},\boldsymbol{y}_{-1})) = ((\boldsymbol{y}_{-1},\boldsymbol{y_n})) = ((\boldsymbol{y}_{-1},\boldsymbol{z_n})) = 0\,, \tag{22}$$

where the constant I is determined by the condition,

$$((\boldsymbol{y}_{-1},\boldsymbol{y}_0)) = 1\,. \tag{23}$$

The function $\eta(\boldsymbol{x})$ and the constant I can also be calculated as

$$\eta(\boldsymbol{x}) = \frac{\partial\xi(\boldsymbol{x})}{\partial N_0}\,,\qquad I = \frac{\partial\mu}{\partial N_0}\,. \tag{24}$$

The completeness relation reads as

The completeness relation reads as

$$\sigma_3\delta(x-x') = y_0(x)y_{-1}^\dagger(x') + y_{-1}(x)y_0^\dagger(x') + \sum_n \{y_n(x)y_n^\dagger(x') - z_n(x)z_n^\dagger(x')\}\,, \qquad (25)$$

and the field operators are expanded as

$$\hat\Phi(x) = -i\hat Q(t)y_0(x) + \hat P(t)y_{-1}(x) + \sum_n \{\hat a_n(t)y_n(x) + \hat a_n^\dagger(t)z_n(x)\}\,, \qquad (26)$$

where the commutation relations,

$$[\hat Q\,,\,\hat P] = i\,, \quad [\hat a_n\,,\,\hat a_{n'}^\dagger] = \delta_{nn'}\,, \qquad (27)$$

$$\hat H_u^{QP} = -(\delta\mu + 2C_{2002} + 2C_{1111})\,\hat P + \frac{I-4C_{1102}}{2}\hat P^2 + 2C_{2011}\hat Q\hat P\hat Q + 2C_{1102}\hat P^3 + \frac{1}{2}C_{2020}\hat Q^4 - 2C_{2011}\hat Q^2 + C_{2002}\hat Q\hat P^2\hat Q + \frac{1}{2}C_{0202}\hat P^4\,, \qquad (29)$$

where

$$C_{iji'j'} = \int d^3x d^3x'\, U(|x-x'|) \times \{\xi(x)\}^i\{\eta(x)\}^j\{\xi(x')\}^{i'}\{\eta(x')\}^{j'}\,, \qquad (30)$$

and $\delta\mu$ is a counter term that the criterion $\langle 0|\,\hat\psi\,|0\rangle = \xi$ determines. The Hamiltonian H_u^{QP} is obtained from gathering all the terms consisting only of $\hat Q$ and $\hat P$ in $\hat H_2$ and $\hat H_{3,4}$. We set up the eigenequation of H_u^{QP},

$$\hat H_u^{QP}\,|\Psi_\nu\rangle = E_\nu\,|\Psi_\nu\rangle \qquad (\nu = 0, 1, \cdots)\,. \qquad (31)$$

NG モードとBDGの直積空間

$$|S\rangle = |\Psi_\nu\rangle\,|\cdot\rangle_{ex}\,, \qquad (32)$$

NG モード（ゼロモード） 固有値方程式

2016　αクラスター場　Higgsモード

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Effective field theory of Bose-Einstein condensation of α clusters and Nambu-Goldstone-Higgs states in ^{12}C

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An effective field theory of α-cluster condensation is formulated as a spontaneously broken symmetry in quantum field theory to understand the *raison d'être* and the nature of the Hoyle and α-cluster states in ^{12}C. The Nambu-Goldstone and Higgs mode operators in infinite systems are replaced with a pair of canonical operators whose Hamiltonian gives rise to discrete energy states in addition to the Bogoliubov–de Gennes excited states. The calculations reproduce well the experimental spectrum of the α-cluster states. The existence of the Nambu-Goldstone-Higgs states is demonstrated and crucial. The γ-decay transitions are also obtained.

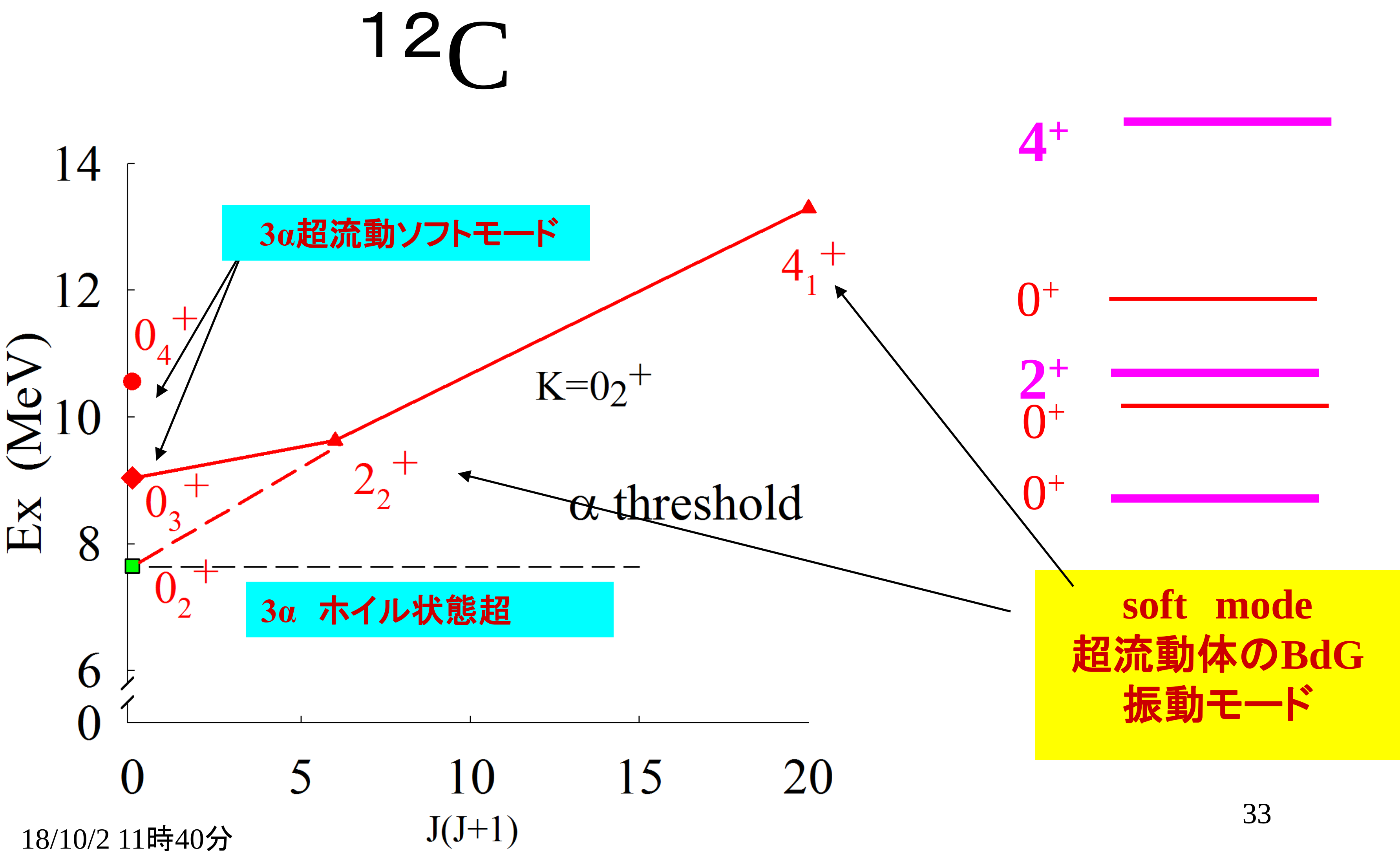
クラスター構造

DOI: 10.1103/PhysRevC.94.014314

粒子描像から場の描像へ

^{12}C集団運動状態

回転運動バンド vs 振動運動バンド？



凝縮率の変化とエネルギー準位 ^{12}C

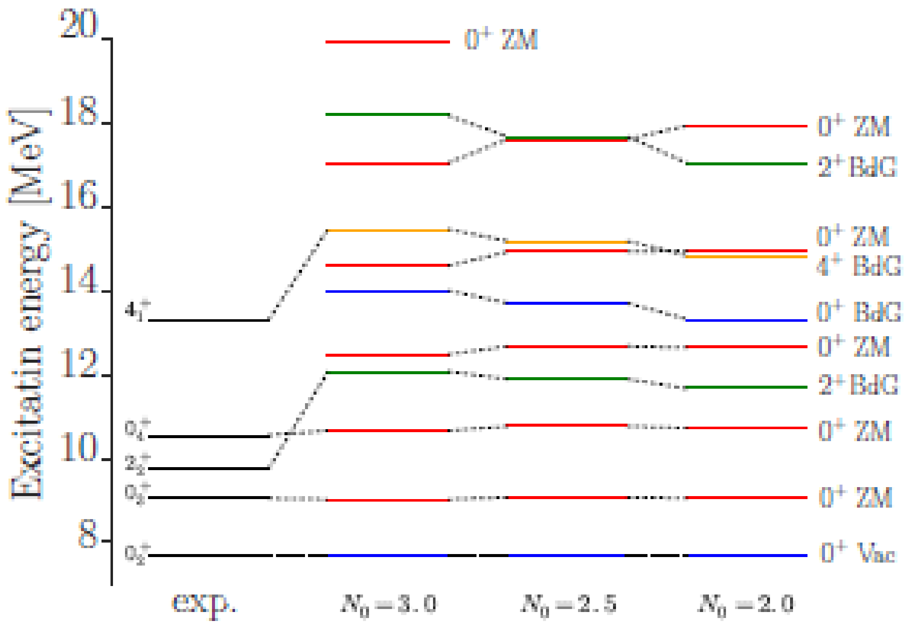


FIG. 6. (Color online) The energy levels of ^{12}C calculated with the parameters in Table III for the three condensation rates, $N_0 = 3.0$ (100%), 2.5 (83%) and 2.0 (67%), with fixed $\bar{r} = 3.8$ fm.

TABLE III. The fitted parameters of Ω and V_r used in the calculations with different three condensation rates of $N_0 = 3.0, 2.5, 2.0$ in ^{12}C with fixed $\bar{r} = 3.8$ fm.

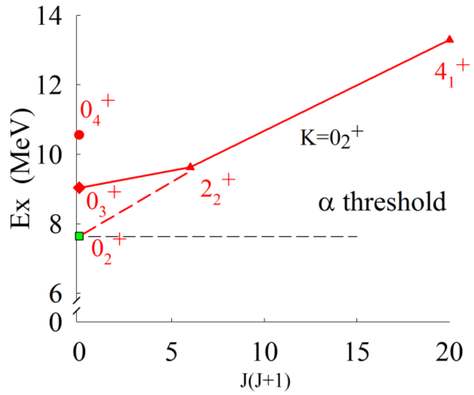
N_0	Ω [MeV]	V_r [MeV]	common parameters
3.0	2.62	403	$V_a = 130$ MeV
2.5	2.53	410	$\mu_a = 0.475\,\text{fm}^{-1}$
2.0	2.40	417	$\mu_r = 0.7\,\text{fm}^{-1}$

18/10/2 11時40分

凝縮率の変化と遷移確率 ^{12}C

TABLE II. Calculated reduced transition probabilities $B(\mathrm{E2}: 2 \to 0)$ and monopole transition probabilities $M(\mathrm{E0}: 0 \to 0)$ in ^{12}C with 70% and 100% (see Subsec. VI B) condensation in units of $e^2\,\mathrm{fm}^4$ and fm^2, respectively, are displayed in comparison with Ref. [35] and Ref. [24].

Transition	70%	100%	Ref. [35] (Kanada－Enyo PTP2007)	Ref. [24] (Funaki PRC2015)	Set A	set B
$B(\mathrm{E2}: 2_2^+ \to 0_2^+)$	121	158	100	295-340	(280) (290)	142 (204)
$B(\mathrm{E2}: 2_2^+ \to 0_3^+)$	76	62	310	88-220	(296) (342) old result	104) (1 87) old
$M(\mathrm{E0}: 0_2^+ \to 0_3^+)$	1.59	2.34	34.5	2.0		
$M(\mathrm{E0}: 0_2^+ \to 0_4^+)$	0.072	0.145	0.57	—		



ゼロモード　の波動関数　100%凝縮

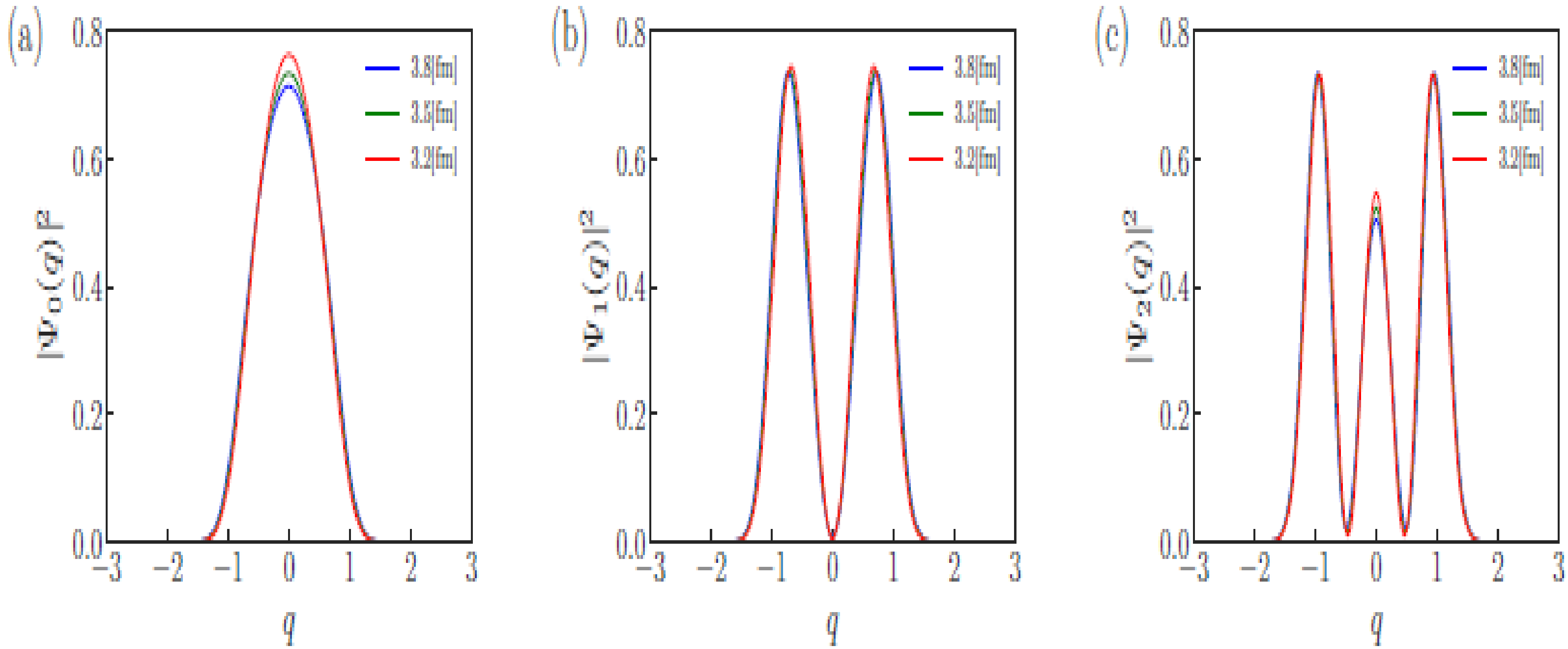


FIG. 12. (Color online) The squares of numerically calculated wavefunctions of the zero mode states, (a) $|\Psi_0(q)|^2$, (b) $|\Psi_1(q)|^2$, and (c) $|\Psi_2(q)|^2$, with $\bar{r} = 3.8$, 3.5, 3.2, fm for100% condensation ($N_0 = N$).

低い凝縮率とNGエネルギー準位^{12}C

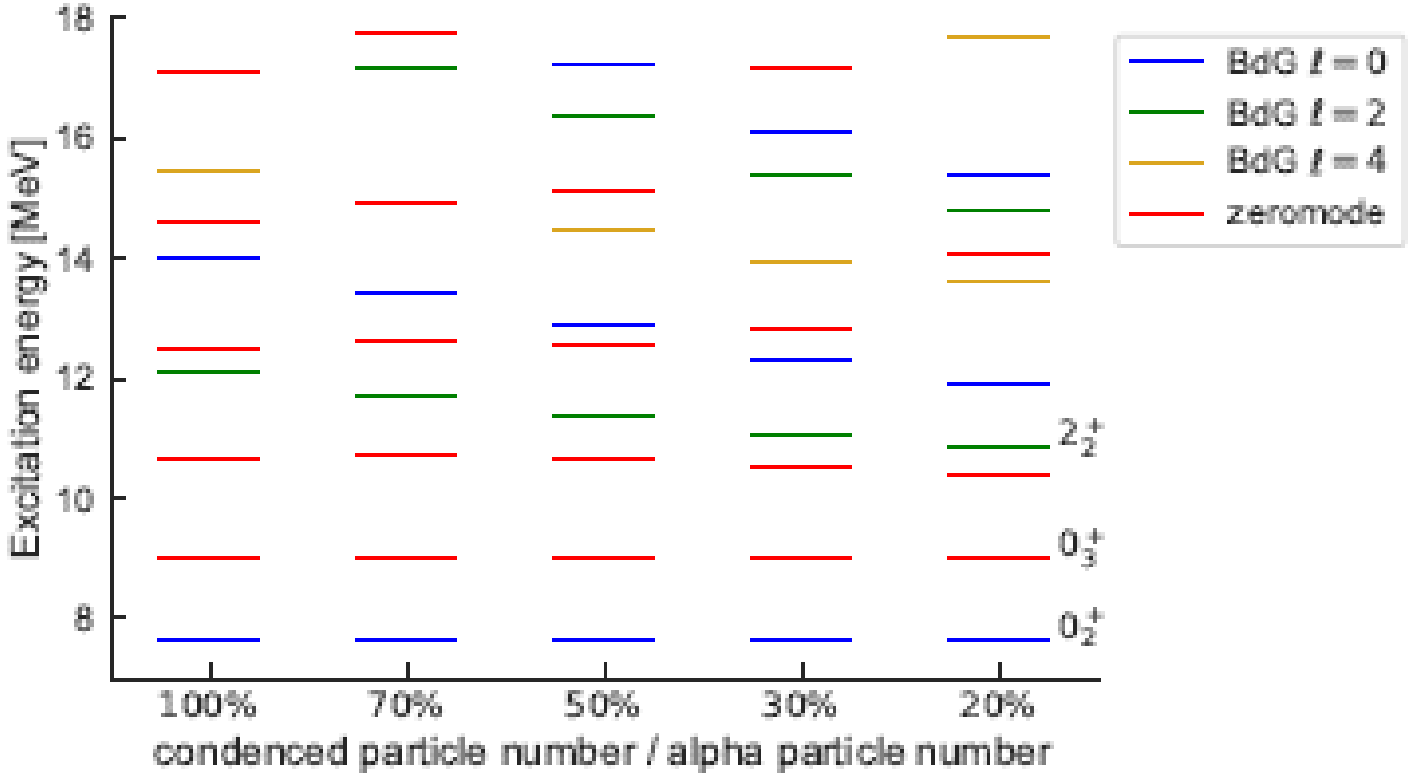


FIG. 14. (Color online) The energy levels of ^{12}C calculated with different condensation rates with $\bar{r} = 3.8$ fm.

^{12}C-^{52}FeのNGソフトモード　100%凝縮

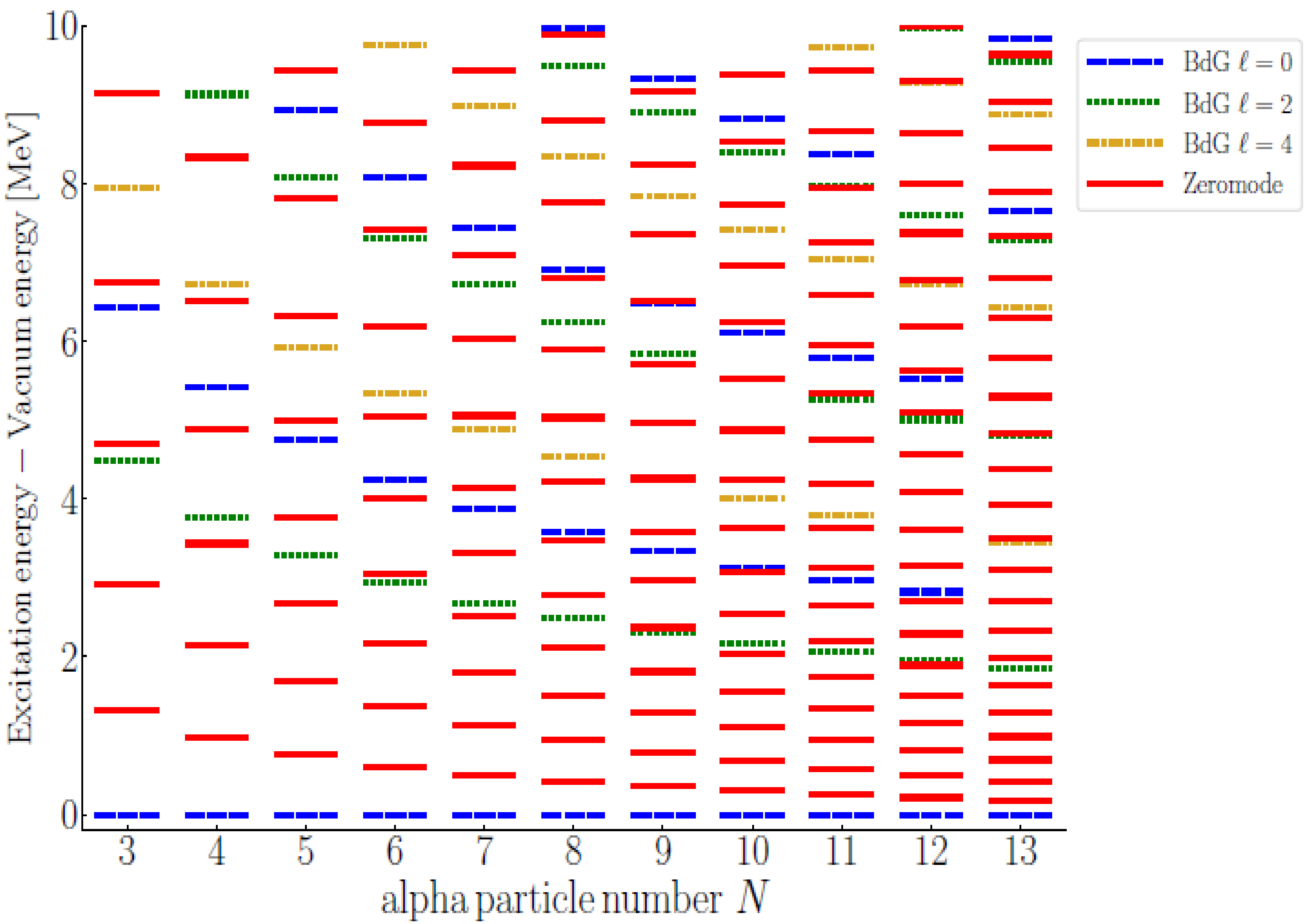


FIG. 17. (Color online) The energy levels calculated for N = 3–13 (^{12}C–^{52}Fe) with 100% condensation using N-dependent Ω given in Fig. 16. Excitation energy is measured from the Hoyle-analog vacuum, i.e., the N-alpha condensate state near the N-alpha threshold.

NG　の生成機構

$$\hat{H}_u^{QP} = -\left(\delta\mu + 2C_{2002} + 2C_{1111}\right)\hat{P} + \frac{I - 4C_{1102}}{2}\hat{P}^2 + 2C_{2011}\hat{Q}\hat{P}\hat{Q} + 2C_{1102}\hat{P}^3 + \frac{1}{2}C_{2020}\hat{Q}^4 - 2C_{2011}\hat{Q}^2 + C_{2002}\hat{Q}\hat{P}^2\hat{Q} + \frac{1}{2}C_{0202}\hat{P}^4\,, \qquad (29)$$

where

$$C_{iji'j'} = \int d^3x d^3x'\, U(|x - x'|) \times \{\xi(x)\}^i\{\eta(x)\}^j\{\xi(x')\}^{i'}\{\eta(x')\}^{j'}\,, \qquad (30)$$

and $\delta\mu$ is a counter term that the criterion $\langle 0|\hat{\psi}|0\rangle = \xi$ determines. The Hamiltonian H_u^{QP} is obtained from gathering all the terms consisting only of $\hat{Q}$ and $\hat{P}$ in $\hat{H}_2$ and $\hat{H}_{3,4}$. We set up the eigenequation of H_u^{QP},

$$\hat{H}_u^{QP}|\Psi_\nu\rangle = E_\nu|\Psi_\nu\rangle \qquad (\nu = 0, 1, \cdots)\,. \qquad (31)$$

$$\hat{H}_0^{QP} = \frac{I}{2}\hat{P}^2 + \frac{1}{2}C_{2020}\hat{Q}^4 \qquad (53)$$

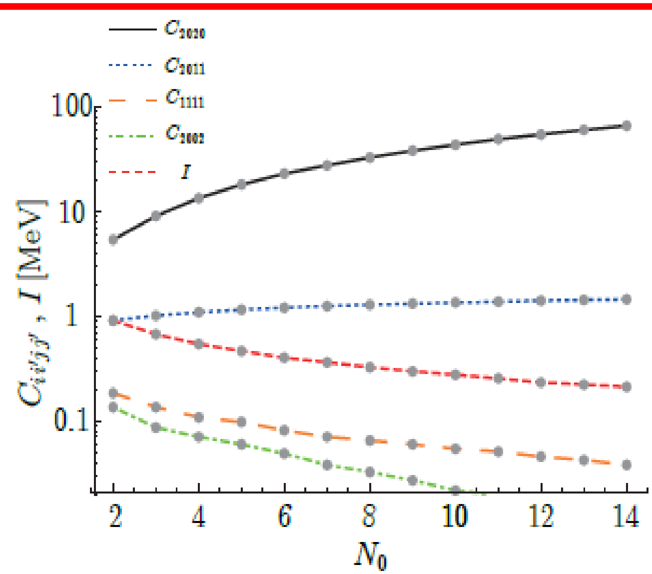


FIG. 20. (Color online) The N_0-dependences of $C_{iji'j'}$ and I with the parameters $V_r = 403$[MeV], $\Omega = 2.62$[MeV] and for $N_0 = 2$–14 are shown. The two coefficients C_{1102} and C_{0202} that are smaller than C_{1111} are not plotted.

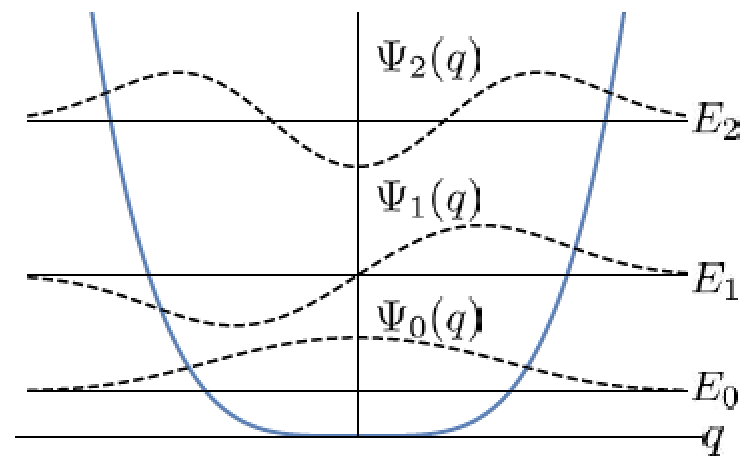


FIG. 21. (Color online) The q^4 potential and the eigenfunctions $\Psi_\nu(q)$ belonging to the eigenvalues E_ν $(\nu = 0, 1, 2)$ for $\hat{H}_0^{QP}$.

^{12}C-^{52}FeのNGソフトモード　70%凝縮

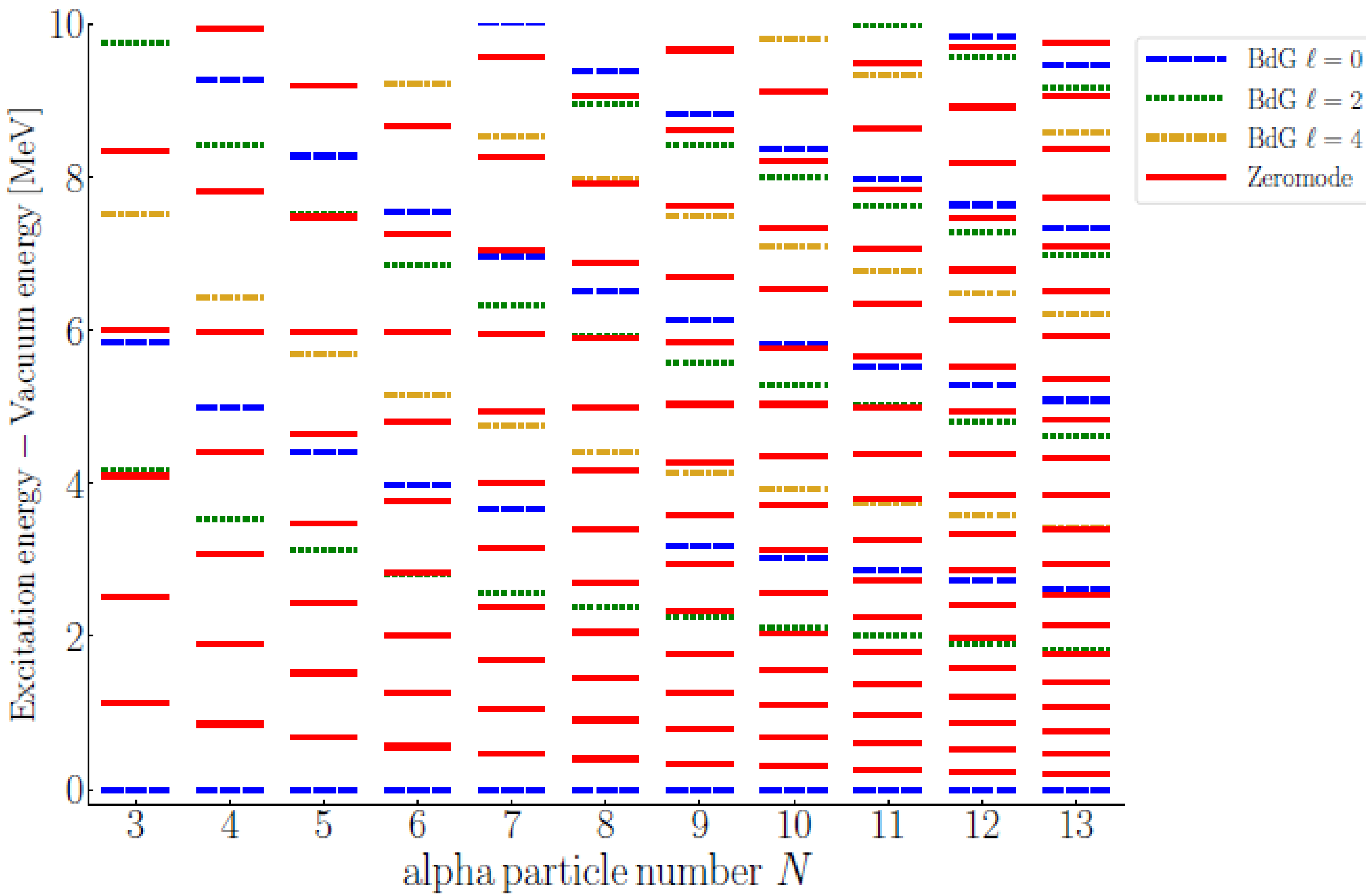


FIG. 19. (Color online) The energy levels calculated for N =3–13 (^{12}C–^{52}Fe) in case of 70% condensation ($N_0 = 0.7N$) using the interaction with N-dependent Ω in Fig. 18. Excitation energy is measured from the Hoyle-analog vacuum, i.e., the N-alpha condensate state near the N-alpha threshold.

まとめ

1. 秩序変数を持つ　超流動クラスター模型で　^{12}C-^{52}Feのα粒子のボーズ・アインシュタイン凝縮を調べた。秩序変数は超流動密度

2. ^{12}Cのホイル状態の上に立つαクラスター構造が超流動クラスター模型で理解できることが分かった。 0^+状態は南部ゴールドストーンモードのソフトモード

3. 凝縮率は100%でなくても、あるいはあまり大きくなくても南部ゴールドストーンモードのソフトモードが発生する。

4. ^{12}C —^{52}Feにおいても一定の凝縮率で南部ゴールドストーンモードによるソフトモードが真空から低いエネルギーに出ることが分かった。

5. 南部ゴールドストーンモードの観測でαクラスターのボーズアインシュタイン凝縮を検証できる可能性

collaborator

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