Workshop on Nonlinear Hyperbolic PDEs

On the occasion of 60th birthday of Professor Yi Zhou

Date : 24 (Fri.) $\sim$ 25 (Sat.) November 2023.

Venue : Kawai Hall, Tohoku University.
http://www.math.tohoku.ac.jp/images/campus1.jpg

Access : Aobayama Station (Subway).
http://www.math.tohoku.ac.jp/english/access/access-e.html

Note : This workshop will be held by hybrid style of face-to-face and Zoom.

Program

24 November (Friday)

13:30–13:40 Opening Address

13:40–14:30 **Kunio Hidano (Mie Univ.)**
"Combined effect" and global existence of small solutions to critical-power nonlinear wave equations

14:40–15:30 **Chengbo Wang (Zhejiang Univ.)**
Sharp local well-posedness for quasilinear wave equations with spherical symmetry

15:50–16:40 **Kyouhei Wakasa (National Col. Kushiro)**
On the critical decay for the wave equation with a cubic convolution in three space dimensions

16:50–17:30 **Jie Shao (Fudan Univ., PD)**
Local wellposedness for the quasilinear Schrödinger equations via the generalized energy method

18:30– Welcome Party

25 November (Saturday)

10:30–11:10 **Shunsuke Kitamura (Tohoku Univ., D2)**
The lifespan of classical solutions of semilinear wave equations with weights of space-time variables in one space dimension

11:20–12:00 **Lyu Cai (Fudan Univ., D3)**
Lifespan estimate for compressible Euler equations

14:00–14:30 Shu Takamatsu (Tohoku Univ. M2)
Improvement of the general theory for one dimensional nonlinear wave equations related to the combined effect

14:40–15:30 Kosuke Kita (Waseda Univ.)
On a weighted estimate for the solution to the damped wave equation in 3D

15:50–16:40 Ning-An Lai (Zhejiang Normal Univ.)
Finite time blow up results and lifespan estimate to compressible Euler and MHD system

16:50–17:40 Makoto Nakamura (Osaka Univ.)
On the Klein-Gordon equation under the quartic potential in the de Sitter spacetime

17:40-17:50 Closing address

19:00– Banquet

Organizing Committee :

Hiroyuki Takamura (Chair, Tohoku Univ.), e-mail : hiroyuki.takamura.a1@tohoku.ac.jp,
Ning-An Lai (Zhejiang Normal Univ.),
Soichiro Katayama (Osaka Univ.),
Hideo Kubo (Hokkaido Univ.),
Kyouhei Wakasa (National Col. Kushiro)

Acknowledgment :

This workshop is supported by JSPS KAKENHI Grant Number 22H00097,
Grant-in-Aid for Scientific Research(A).

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Abstract

Kunio Hidano (Mie Univ.)

A survey talk will be given on the Li-Zhou method and the combined effect in the lifespan of small solutions to nonlinear wave equations. It will focus on global existence of small solutions to 3D (systems of) critical-power nonlinear wave equations. This talk is based on joint research with YOKOYAMA Kazuyoshi (Hokkaido University of Science) and WANG Chengbo (Zhejiang University)
Contents:

1. Li-Zhou method and the Strauss conjecture for $\Box u = |u|^q$
2. Combined effect in $\Box u = |u_t|^p + |u|^q$ and global existence for the critical-power case
3. Combined effect in the system

$$\Box u = |v|^p, \qquad \Box v = |u_t|^{(n+1)/(n-1)} + |u|^q$$

   and global existence for the critical-power case
4. Application to a class of systems satisfying the weak null condition

Chengbo Wang (Zhejiang Univ.)

In this talk, I will present a sharp local well-posed result for spherically symmetric solutions to quasilinear wave equations with rough initial data, when the spatial dimension is three or higher. Our approach is based on Morawetz type local energy estimates with fractional regularity for linear wave equations with variable C^1 coefficients, which rely on multiplier method, weighted Littlewood-Paley theory, duality and interpolation. Together with weighted linear and nonlinear estimates (including weighted trace estimates, Hardy's inequality, fractional chain rule and fractional Leibniz rule) which are adapted for the problem, the well-posed result is proved by iteration. In addition, our argument yields almost global existence for n=3 and global existence for dimension 4 and higher, when the initial data are small, spherically symmetric with almost critical Sobolev regularity.

Kyouhei Wakasa (National Col. Kushiro)

In this talk, we consider the Cauchy problem for the wave equation with a cubic convolution in three space dimensions. We assume slowly decaying conditions on the initial data. The nonlinearity and the decay rate of the initial data are critical, which is determined from the scale invariance of the solution. In this case, H.Kubo (2004) conjectured that the solution blows up in finite time. Our purpose is to verify this conjecture by obtaining the optimal lifespan estimates of the solutions. This is a joint work with Prof. Tomoyuki Tanaka (Doshisha University).

Shao Jie (Fudan Univ., PD)

In this talk, we investigate the initial value problem of the quasilinear Schrödinger equation with cubic nonlinearities or quadratic nonlinearities and introduce the generalized energy method, which is different from those of the series work of Kenig et al. (Invent. Math., 2004; Adv. Math., 2005; Adv. Math., 2006) and the work of Marzuola et al. (Adv. Math., 2012; Kyoto J. Math., 2014; Arch. Ration. Mech. Anal., 2021). We find that the momentum type estimates can be equally important as the energy estimates. By combining these two type bounds, we eventually close the estimates, which will lead to the desired results by artificial viscosity method. The key of the analysis is to find suitable weight to multiply momentum type conservation law equality and produce some good terms that can help the momentum type estimates and the energy estimates to control the bad terms in each other. For cubic interaction problem, we obtain the same low regularity local wellposedness results as those of Marzuola et al. (Kyoto J. Math., 2014). For quadratic interaction problem, we derive the lower regularity local wellposedness of small initial data in the same function spaces as those in the series work of Kenig et al. This talk is based on a joint work with Yi Zhou.

Shunsuke Kitamura (Tohoku Univ., D2)

The general theory of nonlinear wave equations is that, for initial value problems with smooth and sufficiently small initial data, where the smooth nonlinear term consists of the derivative of the unknown function and itself and has polynomial order near the origin, the maximum existence time of the solution, lifespan, is estimated from below by a function whose variables are the inverse of the size of the initial data. The one-dimensional case revealed by T.-T. Li & X. Yu & Y. Zhou(1991,1992) mostly and S. Takamatsu(2023) improved some of them. On the other hand, one of the scale-invariant damped wave equations can be attributed to a nonlinear term with time decay by the Liouville transform, which was analyzed by M. Kato & H. Takamura & K. Wakasa(2019) and found that the lifespan can be classified as wave-like in the case considered heat-like. In line with this fact, it is necessary to generalize the nonlinear term to the nonautonomous case. In order to consider the conditions on space-time variables in this generalization, we analyzed the nonlinear terms in the model equations for the optimality of the general theory with weights for space-time variables and obtained the results of S. Kitamura & K. Morisawa & H. Takamura(2022, 2023), S. Kitamura & H. Takamura & K. Wakasa(2023), S. Kitamura(preprint). These results will be presented in this talk.
There are a few previous works on space-time variable weighted nonlinear wave equations. In the space-variable weighted case, A. Suzuki(2010) presented a condition for the existence of a time global classical solution when the nonlinear term changes sign, the initial data are odd and decay, and the initial support is not compact, and the condition was improved by H. Kubo & A. Osaka & M. Yazici(2013). In addition, E. Belchev & M. Kepka & Z. Zhou(2001), X. Liu & Y. Zhou(2007) showed the blow-up of solution with the condition to the weight of the nonlinear term that vanishes by a geometric transformation of variables.

Lyu Cai (Fudan Univ., D3)

I will talk about some recent lifespan estimate results for compressible Euler system, including the upper bound of lifespan to the initial boundary value problem in exterior domain and the lower bound of lifespan to the Cauchy problem with time dependent damping. These results are based on joint works with Ning-An Lai, Wenze Su and Yi Zhou.

Shu Takamatsu (Tohoku Univ. M2)

We focus on the general theory to the Cauchy problem for one dimensional nonlinear wave equations with small initial data. In the general theory, we aim to obtain the lower bound estimate of the lifespan of classical solution. In this talk, we improve it in some case related to the "combined effect", which was expected complete more than 30 years ago.

Kosuke Kita (Waseda Univ.)

In this talk, we derive a weighted L^∞-estimate of the solution to the damped wave equation in three space dimensions. The pioneering work of weighted estimates developed by F. John (1979) is well known for the wave equation. Roughly speaking, these estimates take into account characteristic weights of the wave equation and play an essential role in their application to nonlinear problems. On the other hand, even though the damped wave equation is of the same hyperbolic type as the wave equation, an estimate that focuses on dissipativity ("parabolicity") is often considered, and it seems not to be considered "hyperbolicity" well. We here will establish a John-type estimate for the damped wave equation. Our proof uses a concrete representation formula of the solution to the damped wave equation that does not rely on the Fourier transform or the energy method. This talk is a joint work with Professor V. Georgiev (University of Pisa).

Ning-An Lai (Zhejiang Normal Univ.)

In this talk I will present some finite time blow up results and lifespan estimate for compressible Euler system with time dependent damping and axially symmetric compressible MHD wave in 3-D. These results are based on the joint work with Lyu Cai and Nico Michele Schiavone.

Makoto Nakamura (Osaka Univ.)

The Cauchy problem for the Klein-Gordon equation under the quartic potential is considered in the de Sitter spacetime. The existence of global solutions for small rough initial data is shown based on the mechanism of the spontaneous symmetry breaking for the small positive Hubble constant. The effects of the spatial expansion and contraction on the problem are considered.