Optimally Computing Compressed Indexing Arrays Based on the Compact Directed Acyclic Word Graph

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The full paper: https://arxiv.org/abs/2308.02269
This slide pdf: https://ikndeva.github.io or arXiv's "Code, Data, Media/Paper with Code" section
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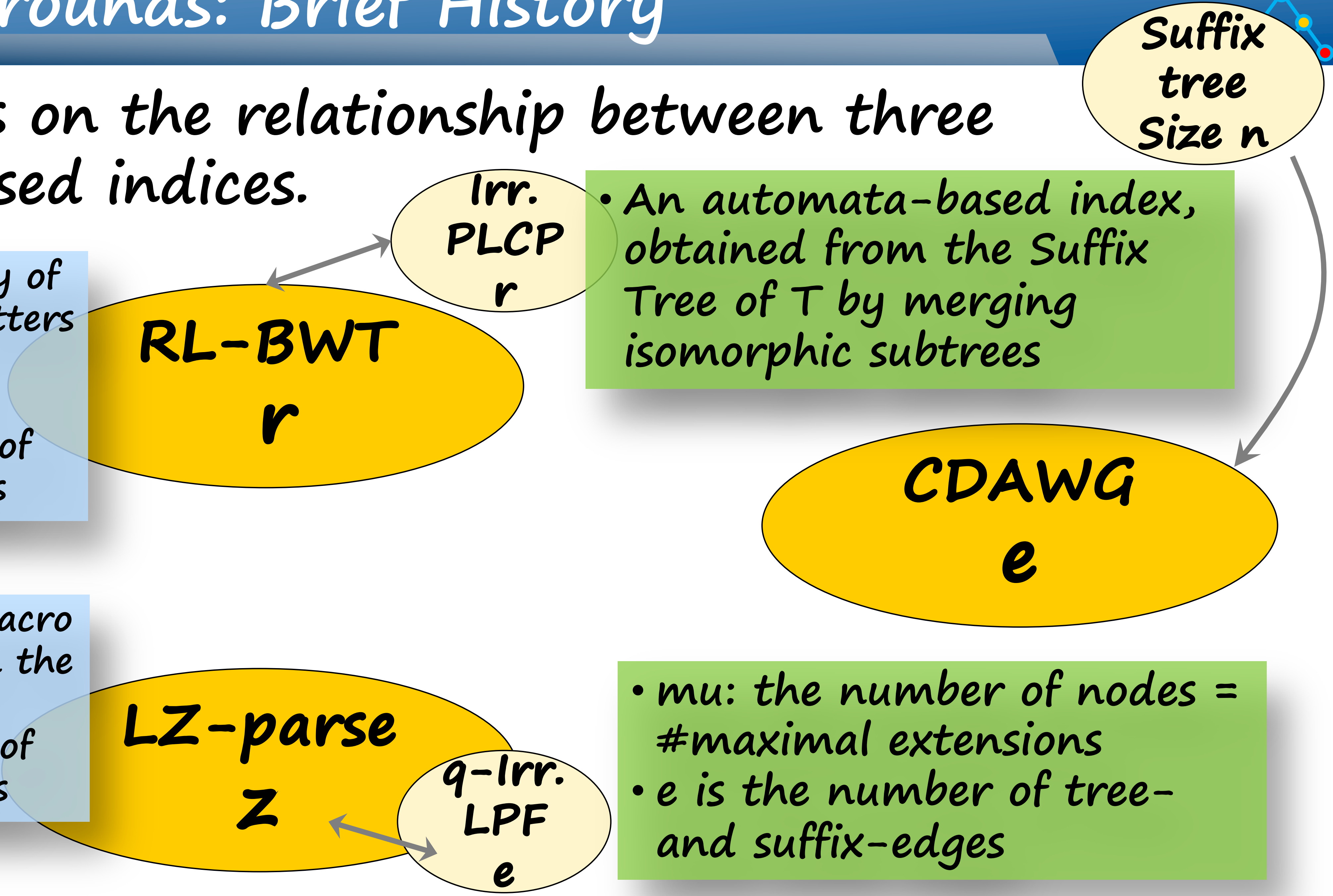
Backgrounds

- Increasing amount and types of repetitive texts
  - Markup texts (Wikipedia), Genome sequences
- Development of compressed index structures for repetitive texts attracts much attention. E.g.,
  - **RL-BWT**, irreducible PLCP arrays, Lex-parse – size r
  - **LZ-parse** (LZ76) – size z
  - **CDAWG (Compact Directed Word Graphs)** – size e

  These indices can compress highly-repetitive texts beyond the entropy bounds up to r, z, and e
- Natural questions: What is the relationships among their sizes?; what is the complexities of conversion?

Backgrounds: Brief History

- We focus on the relationship between three compressed indices.



- BWT is the array of the preceding letters at the starting positions in SA
- r is the number of equal-letter runs

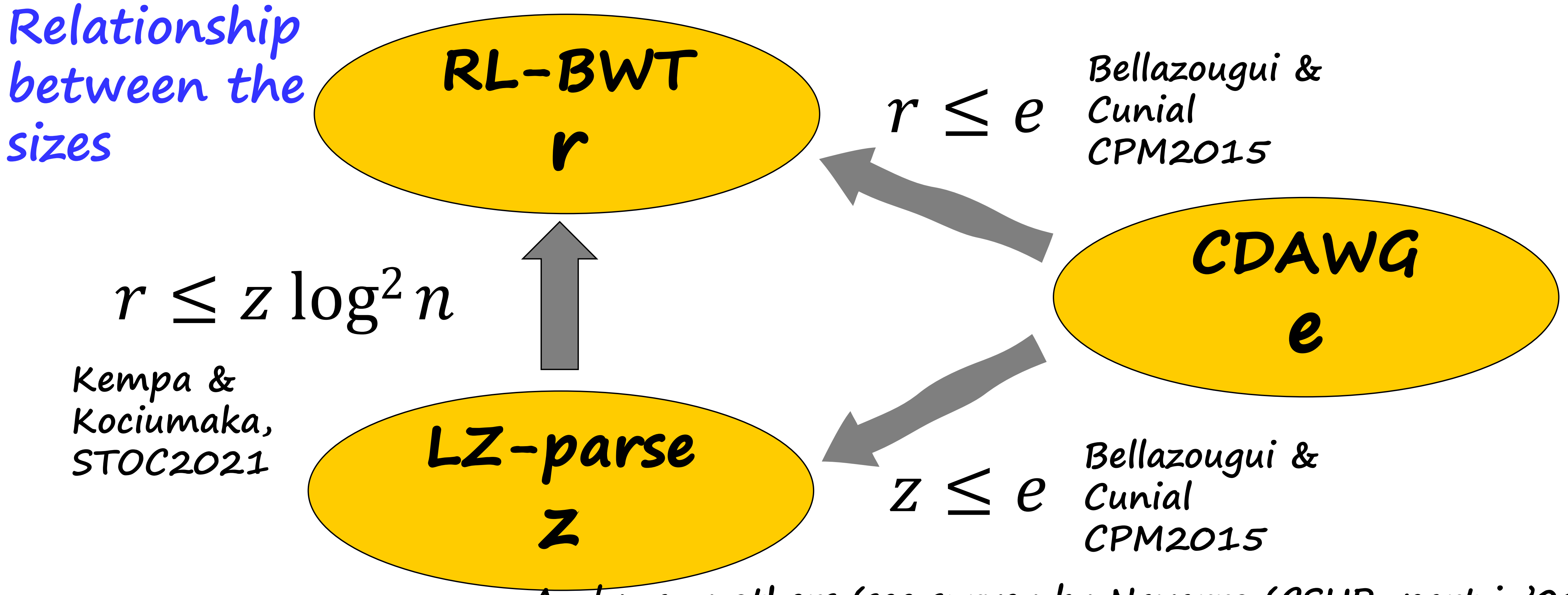
- An automata-based index, obtained from the Suffix Tree of T by merging isomorphic subtrees

- LZ-parse is a macro scheme based on the previous factors.
- z is the number of equal-letter runs

- mu: the number of nodes = #maximal extensions
- e is the number of tree- and suffix-edges

Backgrounds: Brief History

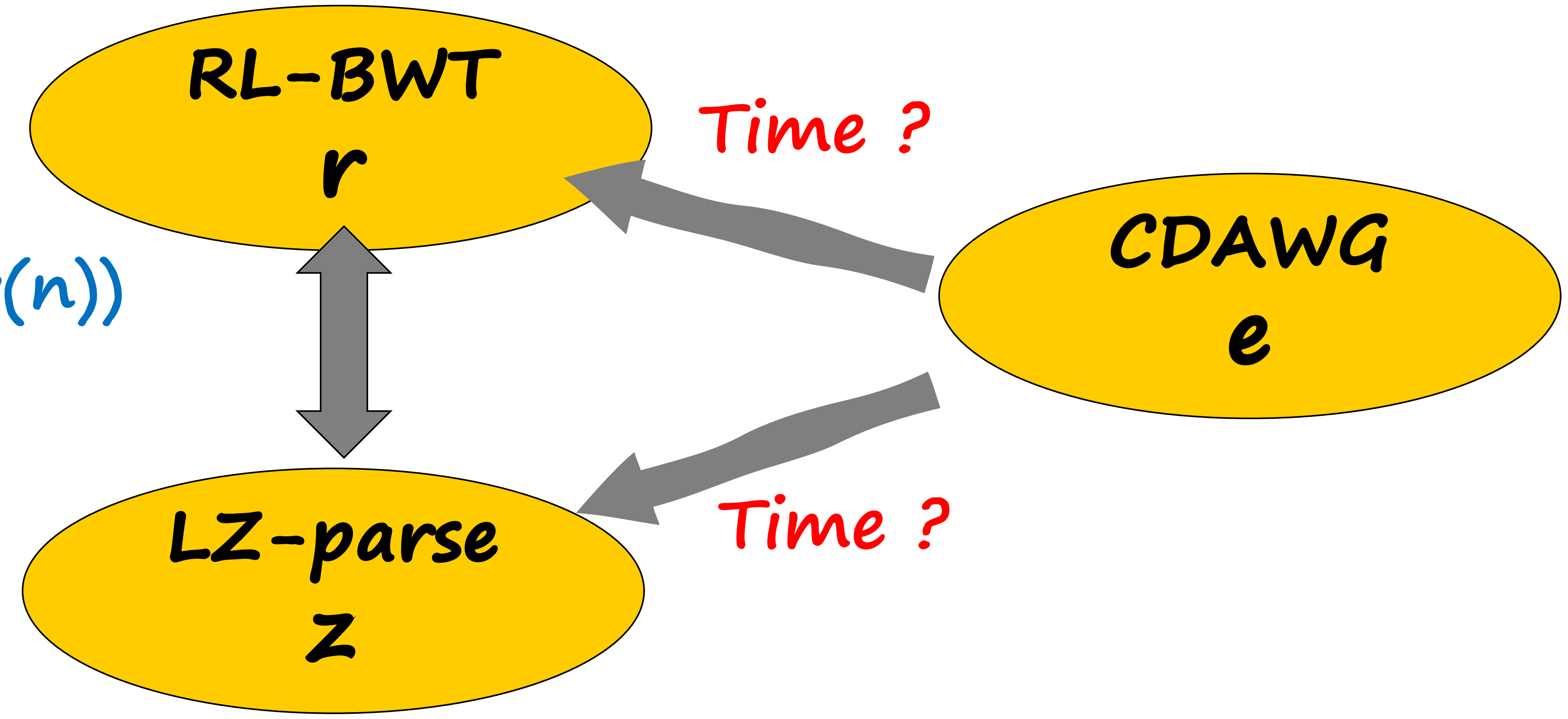
- We focus on the relationship between the indices of the sizes r, z, and e.


Relationship between the sizes
RL-BWT
r
CDAWG
e
LZ-parse
z
$r \le e$
Bellazougui & Cunial CPM2015
$r \le z \log^2 n$
Kempa & Kociumaka, STOC2021
$z \le e$
Bellazougui & Cunial CPM2015

And many others (see survey by Navarro (CSUR, part i, '21)

Brief History

- On the other hand, there are not many results on the sub-linear time and space complexities of conversions . . .

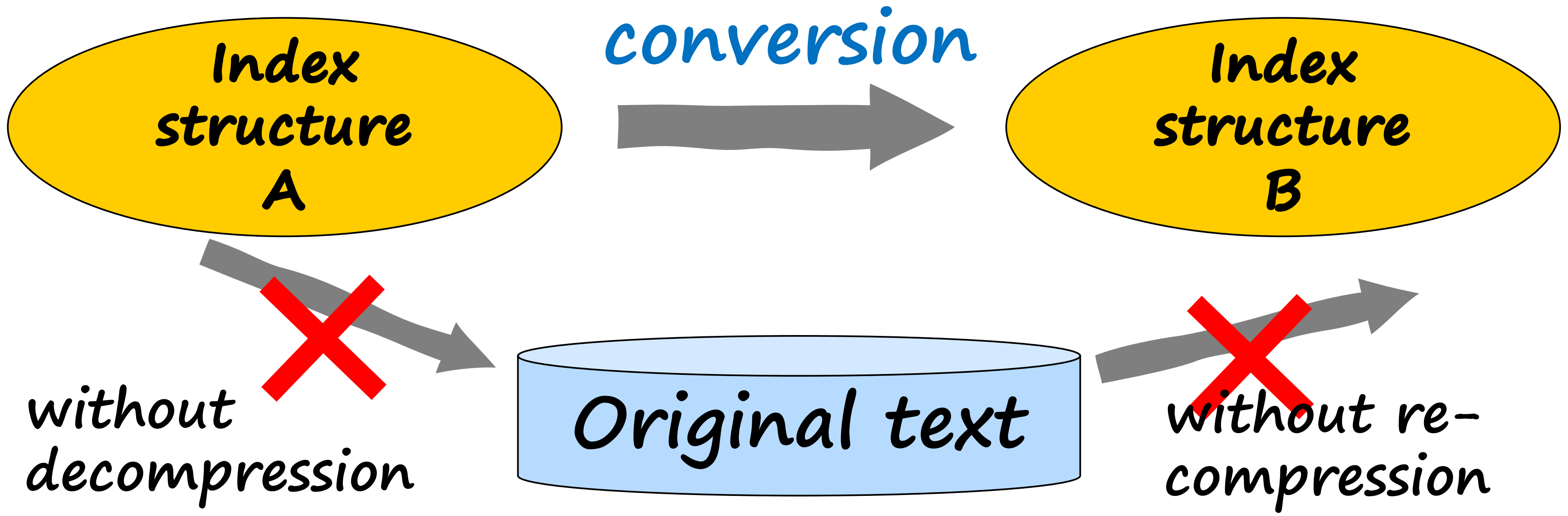


O(z polylog(n)) time

Kempa & Kociumaka, CACM 2022

Our Problem: Conversion problem

- Convert a given compressed index A into another compressed index B without decompression
  - We consider the case that A is the CDAWG of a text T
- Our goal: linear time and space in the combined input and output sizes |A| + |B|



Related works

Sublinear time and space conversion between two indices

- Kempa [SODA'19]
  - Converting an RL-BWT-based index into the irreducible PLCP, CSA, and LZ-parse for a text T of length n in $O(n / \log_\sigma n + r \operatorname{polylog} n)$ time and $O(r)$ space.
- Kempa & Kociumaka [STOC'21, CACM'22]
  - Converting the LZ77-parse of a text T into the RL-BWT for T in $O(z \operatorname{polylog} n)$ time and space.
  - This work solved a long-standing open problem
- Bannai et al. [CPM'13]
  - Converting an SLP of size g into LZ78-parse of size z_{78} in $O(g + z_{78} \log z_{78})$ time and space.
  - Combined with Belazzougui & Cunial [CPM'15], we obtain the conversion from the CDAWG for T into LZ78-parse in $O(e + z_{78} \log z_{78})$ time and space.

Main results

Thm (4.1, 5.1, 5.2): For any integer alphabet Σ, we can convert the CDAWG G of size e for a text T into the following compressed indexing structures for T in O(e) deterministic time and words of space:

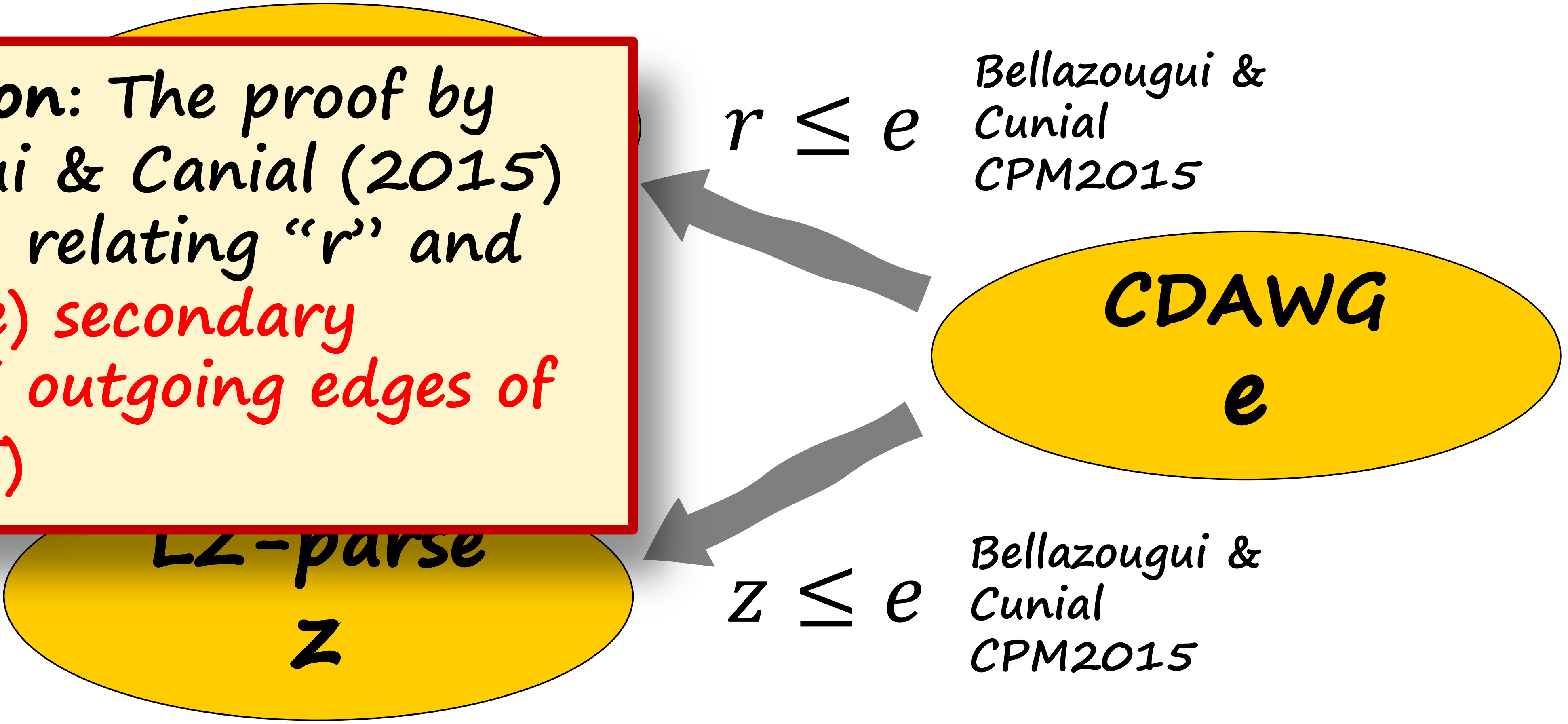
- **The RL-BWT** (run-length BWT) of size r
- **The irreducible PLCP** (permuted LCP) array of size r
- **The quasi-irreducible LPF** (longest previous factor) array of size e (def. Sec. 2 of this paper)
- **The Lex-parse** of size 2r = O(r)
- **The LZ-parse** of size z

G is given in either
- the CDAWG of size e with the read only text of length n,
- the self-index version of CDAWG of size O(e) without a text

Algorithms

Backgrounds: Brief History

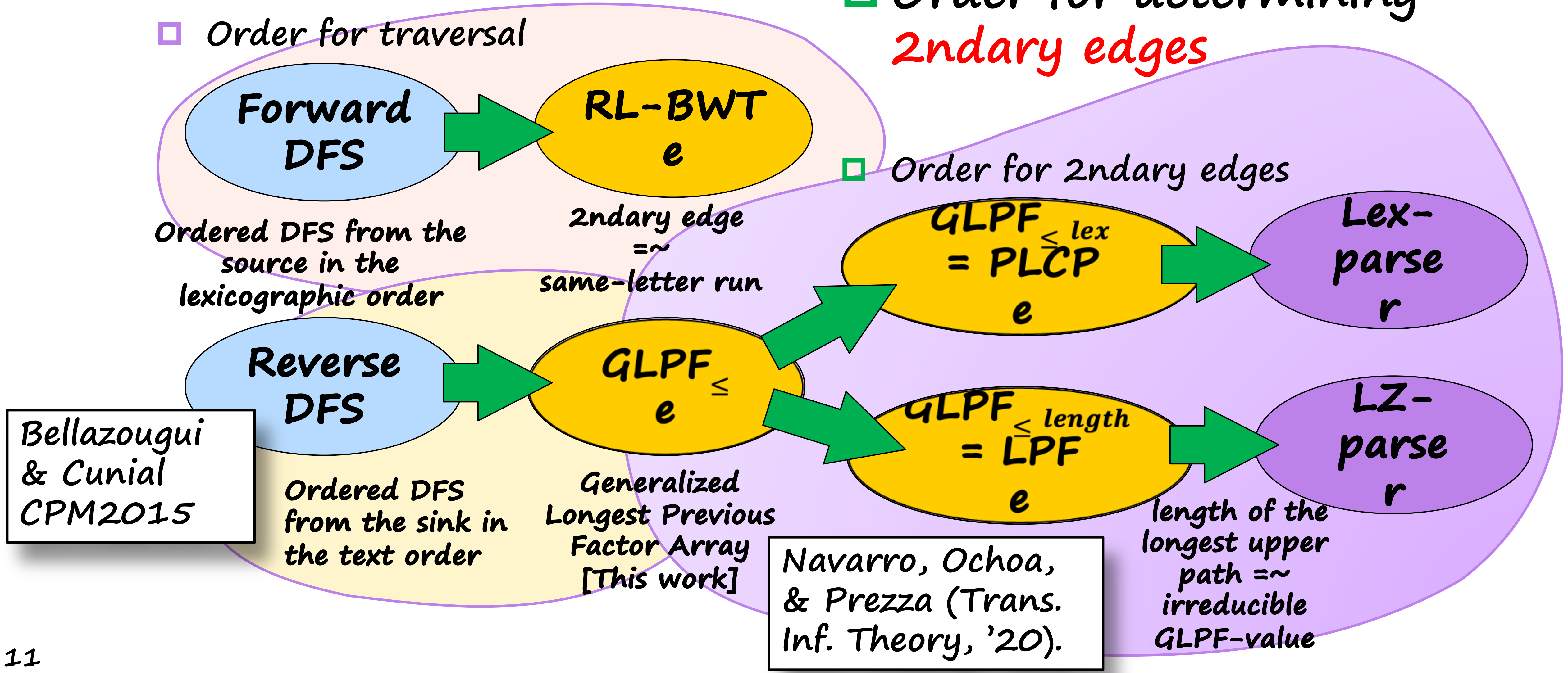
Coming back to the relationship between the sizes …



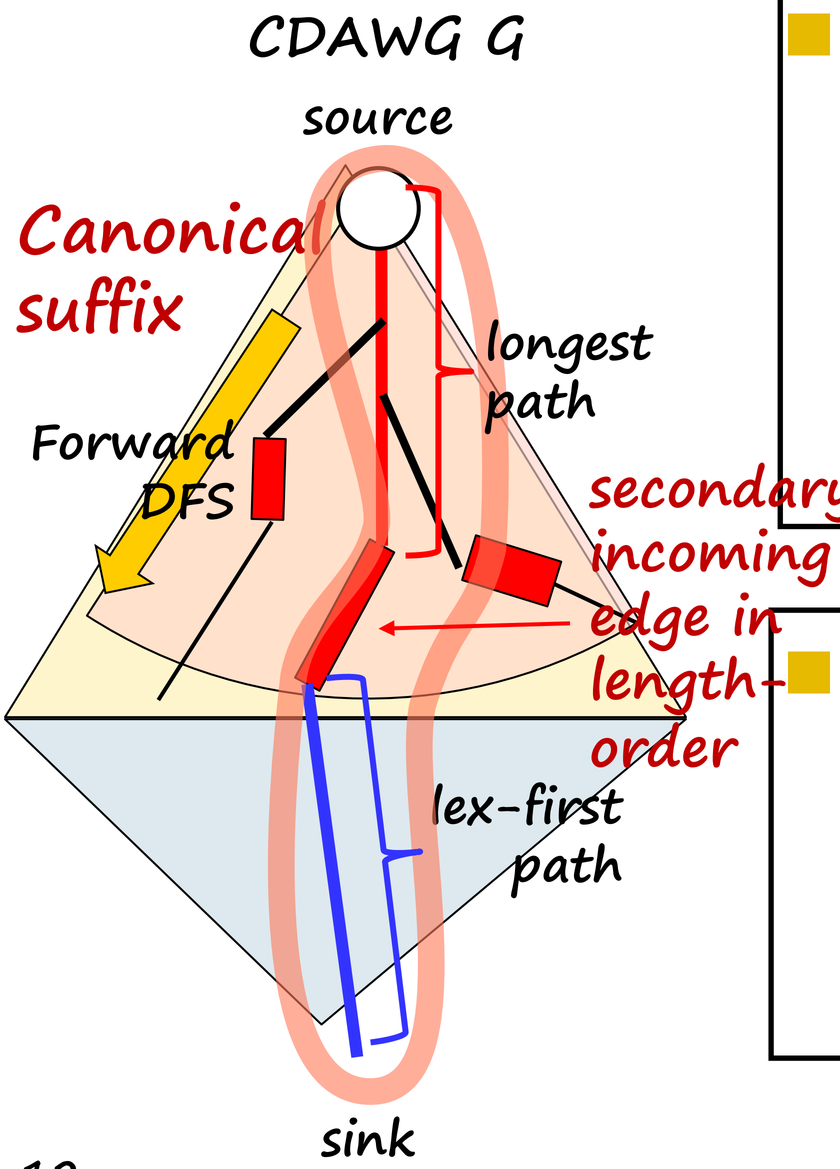
Observation: The proof by Bellazougui & Canial (2015) is done by relating "r" and "z" to O(e) secondary incoming/ outgoing edges of CDAWG(T)

Our approach

- We use two orders of paths
  - One for traversal of CDAWG
  - Order for determining 2ndary edges


Order for traversal
Forward DFS
RL-BWT e
Ordered DFS from the source in the lexicographic order
2ndary edge =~ same-letter run
Reverse DFS
GLPF$_\le$ e
Bellazougui & Cunial CPM2015
Ordered DFS from the sink in the text order
Generalized Longest Previous Factor Array [This work]
Order for 2ndary edges
GLPF$_{\le lex}$ = PLCP e
Lex-parse r
GLPF$_{\le length}$ = LPF e
LZ-parse r
length of the longest upper path =~ irreducible GLPF-value
Navarro, Ochoa, & Prezza (Trans. Inf. Theory, '20).

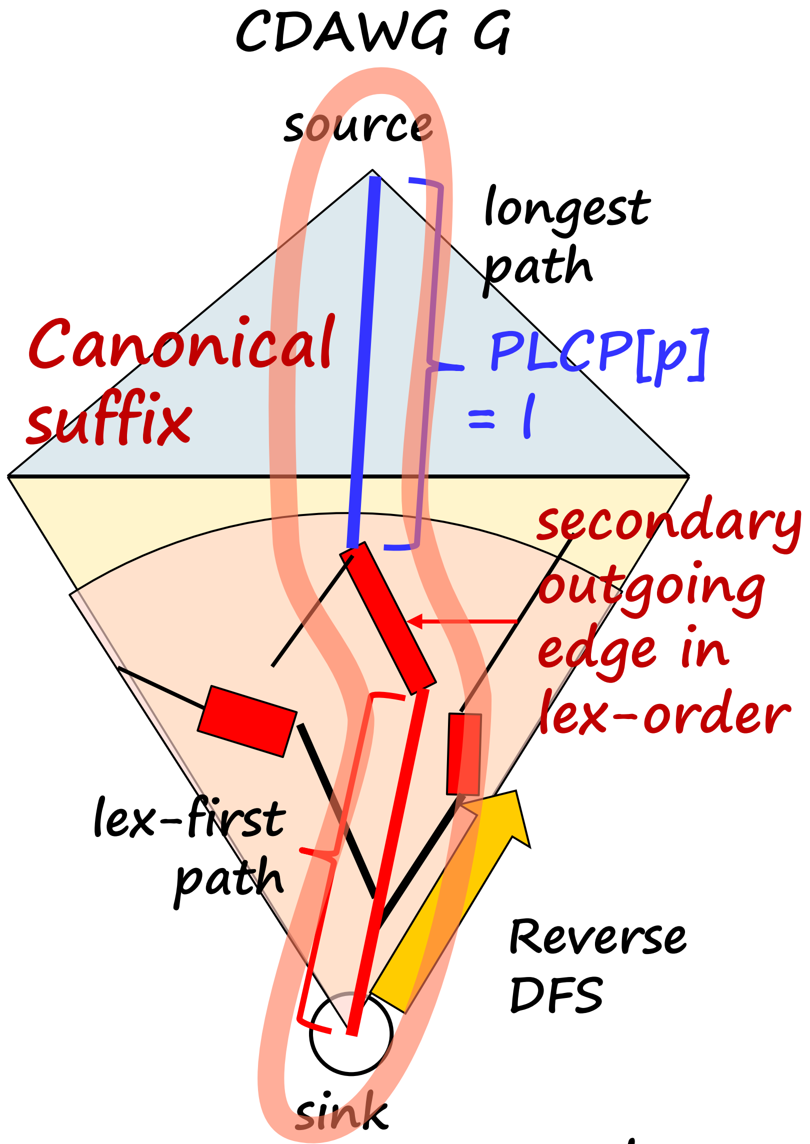
Sec4. Computing RL-BWT in O(e) time&space



- **Observation A1**: O(e) secondary incoming edges of CDAWG(T) correspond to subintervals of the same-letter runs of the BWT under the length-order. (this is because such a search path defines a non-left-maximal factor in T)

- **Observation A2**: O(e) incoming edges of CDAWG(T) can be enumerated in the lexicographic order of its "canonical suffix" by the forward DFS from the source.

Sec5. Computing PLCP in O(e) time&space



- **Observation A1**: O(e) secondary outgoing edge of CDAWG(T) determines the value PLCP[p] = l by the length l of the longest path from the source to the corresponding branching node under the length-order

- **Observation A2**: O(e) secondary outgoing edges can be enumerated in the text order of its "canonical suffix" by the reverse DFS from the sink.

We can extend the above result from PLCP to PLPF by employing the definition of 2ndary outgoing edges in length-order

Conclusions

- Conversion problem from the CDAWG into other compressed indices for highly-repetitive texts
- O(e) time and space conversion from either the CDAWG of a text T or its self-index into the following structures:
    - RL-BWT, (quasi-) irreducible PLCP and LPF arrays, Lex-parse, and LZ-parse for T.
    - Effective version of the result by Belazzougui & Cunial (CPM'15) that r <= e and z <= e to actual conversion.
- Techniques:
    - Characterization of the "irreducible values" by secondary edges.
    - Forward/reverse DFS under the lexicographic/text order
- Future Work:
    - Sub-linear time and space conversion from RL-BWT and LZ-parse into CDAWG.

Thank you!