Moment Approach for Quantitative Evaluation of Randomness Based on RMT Formula

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Abstract. We develop in this article a quantitative formulation of the randomness-test based on the random matrix theory (RMT-test), in order to compare a subtle difference of randomness between given random sequences. Namely, we compare the moments of the actual eigenvalue distribution to the corresponding theoretical expression that we derive from the formula theoretically derived by the random matrix theory. We employ the moment analysis in order to compare the eigen-value distribution of the cross correlation matrix between pairs of sequences. Using this method, we compare the randomness of five kinds of random data generated by two pseudo-random generators (LCG and MT) and three physical generators. Although the randomness of the individual sequence can be quantified in a precise manner using this method, we found that the measured values of randomness fluctuate significantly. Taking the average over 100 independent samples each, we conclude that the randomness of the random data generated by the five generators are indistinguishable by the proposed method, while the same method can detect the randomness of the derivatives of the sequences, or the initial part of LCG, which are distinctly lower.

Keywords: Randomness measure, RMT-test, Moment analysis, Eigenvalues of cross correlation matrix, Marcenko-Pastur distribution.

1 Introduction

The random matrix theory (RMT, hereafter) [1,2] has a wide variety of application from the nuclear energy levels [3] to the principal component analysis of stock markets [4-8]. Especially, the latter example has attracted much attention of the community of econo-physics in which many researchers have been working to discover possible rules or structures behind the motion of stock prices. As a byproduct of this activity, that we call the RMT-PCA [7,8] to study correlations of

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stock prices, we have realized to apply RMT to measure the degree of randomness of given time series.

In the previous meeting of IDT [9], we proposed the first version of a new algorithm of testing the randomness of marginally random sequences, the qualitative version of the RMT-test, and applied that method on two popular pseudo-random numbers, the Linear Congruential Generator (LCG) [10] and the Mersenne Twister (MT) [11] and showed that both pass the test.

In this article we develop a quantitative formulation of the randomness-test based on the random matrix theory (the quantitative version of the RMT-test), in order to compare a subtle difference of randomness between given random sequences. For this purpose we employ the moment method in order to compare the shape of the eigenvalue distribution of the cross correlation matrix obtained from the data to the theoretically derived eigenvalue distribution from the random matrix theory, which is so-called Marcenko-Pastur distribution [12,13]. We test this new method on the same pseudo-random numbers as before, and three physical random numbers which can be downloaded from the web-site of the Institute of Statistical Mathematics [14].

2 Synopsis of the RMT-Test

The method of the RMT-test is outlined as follows [6,7]. We aim to test the randomness of a long 1-dimensional sequence of numerical data, S.

At the first step, we cut S into N pieces of equal length T by discarding the remainder if the length of S is not divisible by T. Each piece, $S_i = (S_{i,1}, S_{i,2}, \dots, S_{i,T})$, is converted to a normalized vector $x_i = (x_{i,1}, x_{i,2}, \dots, x_{i,T})$ of zero mean and unit variance. Since the original sequence S is random, in general all those vectors are independent. We construct a cross correlation matrix C by taking the inner product of two vectors x_i and x_j,

$$C_{i,j} = \frac{1}{T}\sum_{t=1}^{T} x_{i,t}\, x_{j,t} \tag{1}$$

which is symmetric under the interchange of *i* and *j* by definition. We solve an eigenvalue problem of C

$$C v_k = \lambda_k v_k \quad (k=1,\dots,N) \tag{2}$$

to obtain λ_k , the k-th eigenvalue, and its eigenvector v_k

The distribution of the eigenvalues is compared [4-9] to the theoretical formula derived in the RMT at the limit of $N \to \infty,\ T \to \infty,\ Q = T/N = \text{const}$.

$$P_{RMT}(\lambda) = \frac{Q}{2\pi}\frac{\sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}}{\lambda} \tag{3}$$