

# Conference on Algebraic Representation Theory

December 2–5, 2016

organized by

Susumu Ariki (Osaka University)

Hebing Rui (Harbin Institute of Technology, Shenzhen Graduate School)

Toshiaki Shoji (Tongji University )

Harbin Institute of Technology, Shenzhen Graduate  
School

## Speakers:

Susumu Ariki(Osaka Univ)

Junbin Dong (Academy of Mathematics and System Science)

Zhaobin Fan, (Harbin Engineering University)

Naoki Fujita (Tokyo Institute of Technology)

Ryo Fujita (Kyoto University)

Tatsuyuki Hikita (RIMS, Kyoto University)

Ryosuke Kodera (Kyoto University)

Shiyuan Liu (Tongji University)

Kengo Miyamoto (Osaka University)

Satoshi Naito (Tokyo Institute of Technology)

Liangang Peng (Sichuan University)

Fan Qin (Shanghai Jiaotong University)

Linliang Song (Harbin Institute of Technology, Shenzhen Graduate School)

Toshiaki Shoji (Tongji University)

Toshiyuki Tanisaki (Osaka City University)

Jinkui Wan (Beijing Institute of Technology)

Kentaro Wada (Shinshu University)

## Lecture Room:

Talks will be given in Building A, Room 413 (2 December–4 December, 2016) and 414 (5 December, 2016).

## Schedule of the talks:

Friday, 2 December, 2016	
9:00–9:10	Opening
9:10–10:00	S. Naito: <i>Pieri-Chevalley type formula for equivariant <math>K</math>-theory of semi-infinite flag manifolds</i>
10:00–10:20	Tea Break
10:20–11:10	Jinkui Wan: <i>Modular representations and branching rules for affine and cyclotomic Yokonuma-Hecke algebras</i>
11:10–14:00	Lunch Break
14:00–14:50	T. Hikita: <i>On canonical bases in equivariant <math>K</math>-theory of conical symplectic resolutions</i>
14:50–15:10	Tea Break
15:10–16:00	J. Dong: <i>Irreducibility of certain subquotients of Spherical Principle Series Representations of Reductive Groups with Frobenius Maps</i>
16:00–16:20	Tea Break
16:20–17:10	N. Fujita: <i>Newton-Okounkov polytopes of Schubert varieties and perfect bases with positive properties</i>

<b>Saturday, 3 December, 2016</b>	
9:00–9:50	T. Tanisaki: <i>The Drinfeld pairing and the braid group action for a quantized enveloping algebra</i>
9:50–10:10	Tea Break
10:10–11:00	F. Qin: <i>quantum cluster algebras, triangular bases and dual canonical bases</i>
11:00–12:00	Lunch Break
12:00–	Excursion

<b>Sunday, 4 December, 2016</b>	
9:10–10:00	S. Ariki: <i>Representation type for block algebras of Hecke algebras</i>
10:00–10:20	Tea Break
10:20–11:10	S. Liu: <i>Fermionic form for Double Kostka polynomials</i>
11:10–14:00	Lunch Break
14:00–14:50	Z. Fan: <i>Affine flag varieties and quantum symmetric pairs</i>
14:50–15:10	Tea Break
15:10–16:00	R. Kodera: <i>Geometric construction of spherical cyclotomic rational Cherednik algebras</i>
16:00–16:20	Tea Break
16:20–17:10	K. Miyamoto: <i>Stable AR components containing non-periodic Heller lattices of the Kronecker algebra over a complete d.v.r.</i>
17:30	Dinner

<b>Monday, 5 December, 2016</b>	
9:10–10:00	L. Peng: <i>Modified Ringel-Hall algebras and Drinfeld double</i>
10:00–10:20	Tea Break
10:20–11:10	Ryo Fujita: <i>Arakawa-Suzuki functor on a deformed BGG category of <math>\mathfrak{gl}_m</math></i>
11:10–14:00	Lunch Break
14:00–14:50	Kentaro Wada: <i>Finite dimensional simple modules of deformed current Lie algebras</i>
14:50–15:10	Tea Break
15:10–16:00	L. Song: <i>Isomorphisms between simple modules of degenerate cyclotomic Hecke algebras.</i>
16:00–16:20	Tea Break
16:20–17:10	T. Shoji: <i>Kostka functions associated to complex reflection groups and Finkelberg's conjecture</i>
17:30	Dinner

# Abstracts

## **Representation type for block algebras of Hecke algebras**

Susumu Ariki  
Osaka University

Our main result is to tell the representation type for each block algebra of Hecke algebras of classical type (except for type  $D$  in characteristic 2). The result follows from the case of type  $B$  with equal or unequal parameters, and the proof requires recent results on cyclotomic quiver Hecke algebras.

## **Irreducibility of certain subquotients of Spherical Principle Series Representations of Reductive Groups with Frobenius Maps**

Junbin Dong  
Academy of Mathematics and System Science

For infinite reductive groups with Frobenius maps, we show that certain subquotients of abstract representations of the groups induced from 1-dimensional representations of Borel subgroups are irreducible.

## **Affine flag varieties and quantum symmetric pairs**

Zhaobin Fan  
Harbin Engineering University

The quantum groups of finite and affine type A admit geometric realizations in terms of partial flag varieties of finite and affine type A. Recently, the quantum group associated to partial flag varieties of finite type B/C is shown to be a coideal subalgebra of the quantum group of finite type A. In this talk, I will present the structures of Schur algebras and Lusztig algebras associated to (four variants of) partial flag varieties of affine type C. The quantum groups arising from Lusztig algebras and Schur algebras via stabilization procedures are shown to be (idempotent) coideal subalgebras of quantum groups of affine  $\mathfrak{sl}$  and  $\mathfrak{gl}$  types, respectively. In this way, we provide geometric realizations of eight quantum symmetric pairs of affine types. We construct monomial and canonical bases of all these quantum (Schur, Lusztig, and coideal) algebras. For the idempotent coideal algebras of affine  $\mathfrak{sl}$  type, we establish the positivity properties of the canonical basis with respect to multiplication, comultiplication and a bilinear pairing. In particular, we obtain a new and geometric construction of the idempotent quantum affine  $\mathfrak{gl}$  and its canonical basis. This is a joint work with C.Lai, Y. Li, L. Luo and W.Wang.

## **Arakawa-Suzuki functor on a deformed BGG category of $\mathfrak{gl}_m$**

Ryo Fujita

Kyoto University

Arakawa-Suzuki functor is a functor which associates a module over the general linear Lie algebra  $\mathfrak{gl}_m$  with a module over the degenerate affine Hecke algebra  $H_n$  of  $GL_n$ . In this talk, we consider the functor on the deformed BGG category of  $\mathfrak{gl}_m$  which is introduced by Soergel and show that it gives a fully embedding of a block of the deformed BGG category into the module category of a certain completion of  $H_n$ . We apply a general theory of tilting modules in affine highest weight category.

## **Newton-Okounkov polytopes of Schubert varieties and**

# perfect bases with positive properties

Naoki Fujita

Tokyo Institute of Technology

A Newton-Okounkov polytope is a polytope constructed from a polarized variety with a valuation on its function field. Kaveh (resp. the speaker and Naito) proved that a Littlemann string polytope (resp., a Nakashima-Zelevinsky polyhedral realization) is identical to the Newton-Okounkov polytope of a Schubert variety with respect to a specific valuation, which is defined algebraically to be a highest term valuation. By the way, there are valuations which arise naturally from geometric data of a polarized variety, more precisely, some sequences of its subvarieties. Lazarsfeld-Mustata, Küronya-Lozovanu-Maclean, and others were focused on this kind of valuation. In this talk, we see that the valuation used by Kaveh (resp., by the speaker and Naito) and the one coming from a sequence of specific subvarieties of the Schubert variety are identical on the dual basis of a perfect basis with some positive properties. The existence of such a perfect basis follows from a categorification of associated Newton-Okounkov polytopes coincide through an explicit affine transformation. This is a joint work with H. Oya.

## On canonical bases in equivariant $K$ -theory of conical symplectic resolutions

Tatsuyuki Hikita

RIMS, Kyoto University

Lusztig defined certain involution and inner product on the equivariant  $K$ -theory of Slodowy varieties and conjectured the existence of certain canonical bases characterized by the involution and the inner product. This conjecture was proved by Bezrukavnikov-Mirkovic by constructing certain exotic  $t$ -structures on the derived category of coherent sheaves. In this talk, I will explain a purely  $K$ -theoretic reformulation of Lusztig's involution which should make sense for more general conical symplectic resolutions which have Hamil-



tonian torus actions with finitely many fixed points.

## **Geometric construction of spherical cyclotomic rational Cherednik algebras**

Ryosuke Kodera

Kyoto University

Recently Braverman-Finkelberg-Nakajima gave a mathematically rigorous definition of Coulomb branches of 3d  $N=4$  supersymmetric gauge theories. They introduced a family of commutative algebras together with their noncommutative deformations as convolution algebras. This commutative algebra gives the coordinate ring of the Coulomb branch by definition. We call the noncommutative deformation the quantized Coulomb branch.

In this talk, we consider the quantized Coulomb branches associated with quiver gauge theories of Jordan type. We prove that they are isomorphic to the spherical parts of cyclotomic rational Cherednik algebras. In other words, we obtain a geometric construction of spherical cyclotomic rational Cherednik algebras. This is a joint work with Hiraku Nakajima.

## **Stable AR components containing non-periodic Heller lattices of the Kronecker algebra over a complete d.v.r.**

Kengo Miyamoto

Osaka University

In the representation theory, to determine the shape of Auslander–Reiten quiver is a classical problem. However, in the case of the Krull dimension 1, there are few examples since it is difficult to compute almost split sequences much more than the case of algebras over a field. Thus, we want more examples of AR quivers of orders over a complete d.v.r. Recently, if we focus on the special lattices called "Heller lattices", then there are possibilities that one can compute AR components containing such lattices. In this talk, we will

determine the shape of stable AR components containing non-periodic Heller lattices of the Kronecker algebra over a complete discrete valuation ring.

## **Pieri-Chevalley type formula for equivariant $K$ -theory of semi-infinite flag manifolds**

Satoshi Naito

Tokyo Institute of Technology

In this talk, we give a Pieri-Chevalley type formula for a variant of equivariant (with respect to the Iwahori subgroup)  $K$ -theory of semi-infinite flag manifolds. This formula describes the product (in the  $K$ -theory) of the structure sheaf of a semi-infinite Schubert variety with a line bundle (associated to a dominant weight) on the semi-infinite flag manifold, in terms of semi-infinite Lakshmibai-Seshadri (LS) paths; the main ingredient in our proof is the combinatorial version of standard monomial theory for semi-infinite LS paths. We note that our result can be regarded as a semi-infinite analog of a result for the torus equivariant  $K$ -theory of ordinary finite-dimensional flag manifolds, which is due to Pittie-Ram.

This talk is based on a joint work with S. Kato and D. Sagaki.

## **Modified Ringel-Hall algebras and Drinfeld double**

Liangang Peng

Sichuan University

In this talk, we will discuss the Ringel-Hall algebras of a hereditary abelian category. From the category of the  $\mathbb{Z}/2$ -graded complexes over a hereditary abelian category, we consider a certain quotient algebra and some localization of its Ringel-Hall algebra, which is called the modified Ringel-Hall algebra. When this hereditary abelian category is finitary, in particular the category of representations of a quiver and the category of coherent sheaves on a weighted projective line over a finite field, we prove that the modified Ringel-Hall algebra

is isomorphic to the Drinfeld Double of the extended Ringel-Hall algebra of the hereditary abelian category. This is a joint work with Ming Lu.

## **quantum cluster algebras, triangular bases and dual canonical bases**

Fan Qin

Shanghai Jiaotong University

We introduce the common triangular bases of quantum cluster algebras arising from quantum groups. We show that these bases agree with the dual canonical bases for symmetric Cartan type, which implies the categorification conjectures by Fomin-Zelevinsky and Hernandez-Leclerc. We discuss recent progress and conjectures in symmetrizable Cartan type.

## **Fermionic form for Double Kostka polynomials**

Liu Shiyuan

Tongji University

The  $X = M$  conjecture asserts that the  $1D$  sum and the fermionic formula coincide up to some constant power. In the case of type  $A$ , both the  $1D$  sum and the fermionic formula are closely related to Kostka polynomials. Double Kostka polynomials  $K_{\lambda, \mu}(t)$ , indexed by two double partitions  $\lambda, \mu$ , are polynomials in  $t$  introduced as a generalization of Kostka polynomials. In this talk, we consider  $K_{\lambda, \mu}(t)$  in the special case where  $\mu = (-, \mu'')$ . We formulate a  $1D$  sum and a fermionic formula for  $K_{\lambda, \mu}(t)$ , as a generalization of the case of ordinary Kostka polynomials. Then we prove an analogue of the  $X = M$  conjecture.

## **Kostka functions associated to complex reflection groups and Finkelberg's conjecture**

Toshiaki Shoji

Tongji University

Kostka functions associated to complex reflection groups (or  $r$ -Kostka functions, in short) are an analogue of the original Kostka polynomials, which are indexed by pairs of  $r$ -partitions. It is known that the original Kostka polynomials have an interpretation in terms of Lusztig's partition function. Finkelberg conjectured, in his talk in the conference in Shanghai last December, that a similar formula holds also for  $r$ -Kostka functions. In this talk, we show that Finkelberg's conjecture holds under a mild restriction. We also discuss about a multi-variable version of  $r$ -Kostka functions.

## **Isomorphisms between simple modules of degenerate cyclotomic Hecke algebras**

Linliang Song

Harbin Institute of Technology, Shenzhen Graduate School

In this talk, we classify maximal vectors of certain tensor modules in super parabolic category  $\mathcal{O}$  for general linear Lie superalgebras. Via this result and Brundan-Losev-Webster's results on tensor product categorification together with Brundan-Kleshchev's results on Schur- Weyl duality for higher levels, we give explicit isomorphisms between simple modules of degenerate cyclotomic Hecke algebras defined via various cellular bases. A special case of our result gives a generalized Mullineux involution in degenerate case. This is a joint work with Hebing Rui.

## **The Drinfeld pairing and the braid group action for a quantized enveloping algebra**

Toshiyuki Tanisaki

Osaka City University

I will present some results on the Drinfeld pairing of a quantized enveloping algebra. Especially, I will give new proofs of the invariance of the Drinfeld pairing under the action of the braid group.

# Finite dimensional simple modules of deformed current Lie algebras

Kentaro Wada  
Shinshu University

A deformed current Lie algebra was introduced in the representation theory of cyclotomic  $q$ -Schur algebras at  $q=1$ , and it is a filtered deformation of the current Lie algebra. In this talk, we study representations of deformed current Lie algebras. In particular, we will give a classification of finite dimensional simple modules which are parametrized by the set of tuples of "monic polynomials" with additional data.

# Modular representations and branching rules for affine and cyclotomic Yokonuma-Hecke algebras

Jinkui Wan  
Beijing Institute of Technology

We give an equivalence between a module category of the affine Yokonuma-Hecke algebra (associated with the group  $\mathbb{Z}/r\mathbb{Z}$ ) and its suitable counterpart for a direct sum of tensor products of affine Hecke algebras of type  $A$ . We then develop several applications of this result. In particular, the simple modules of the affine Yokonuma-Hecke algebra and of its associated cyclotomic algebra are classified over an algebraically closed field of characteristic  $p$  when  $p$  does not divide  $r$ . The modular branching rules for these algebras are obtained, and they are further identified with crystal graphs of integrable modules for quantum affine algebras.