Impact Objectives

- Understand the diverse forms and dynamics of the natural world from the perspective of pluripotent cells - a network of unstable solutions
- Contribute to society from the perspective of 'mathematics of the commons' driven by both qualitative mathematical models and quantitative data analysis

Mathematics of the commons

Mathematician **Professor Emeritus Yasumasa Nishiura's** research involves the universality of mathematics in nature and Mathematics of the Commons, both of which could help solve a range of global problems



How did you come to be involved in the field of mathematics research?

In my first year at university I

encountered some mathematics that defy intuition, including pathological examples (such as Peano curves) which fill up two-dimensional spaces and functions that are continuous but not differentiable everywhere. I particularly remember being moved by the limits of our perception along with the mysteriousness of infinite operations. The simple question of where the special predictive power that the mathematics have come from was one of the things that ultimately led me in the direction of my current research.

Can you share what some of the main goals of your work are?

The simple question of how protean, unfettered and flexible patterns in the natural world come to be is the starting point.

My aim is to reproduce robust space-time structures that change moment-by-moment and possess the beauty of being unified with the micro, and then to do this using mathematical models and extract the mathematical mechanisms that drive them. Mathematical models are particular to each phenomenon, but extracted mathematical

mechanisms have an extremely wide-ranging universality. Self-replicating patterns, selfrepairing dynamics, collision dynamics, etc., are classic examples of dissipative, complex dynamics that appear in biosystems, chemical reaction systems, fluid systems, etc. The processes and structures that carry their substance are completely different, but they are systems that are extremely tough against perturbation from the outside world. That is because they have techniques for manipulating instability in a very flexible manner. In order to explicitly show those, we need to derive moderately reduced mathematical models and pioneer mathematics that can describe that. I want to establish a framework in which a stable system is one where suppressed unstable forces are controlled and selectively released according to the perturbation.

From your perspective, what is the ultimate impact of your studies?

There is a huge ripple effect from having concretely shown through dissipative, complex dynamics that the entire system is controlled by unstable things that cannot be put into experiments or simulations and by relationships that do not depend on substance. As such, the perspective that pluripotent cells or networks of unstable solutions whose peak is an extremely highly unstable solution gives us a guide for understanding large-scale, complex systems.

Things such as signs that they are reaching a catastrophe, known as a 'tipping point', are also an example of this perspective. The important method that is the counterpart to this qualitative way of seeing things is the quantitative technique, and it has become possible to actually use it due to the development of observation networks and data assimilation. Techniques combining these suggest solutions to many specific problems.

What do you believe needs to be the focus of future research?

One of my central philosophies is that of 'Mathematics of the Commons'. In simple terms, this philosophy does not represent a specific field or subject in mathematics, but instead refers to a group of problems which mathematical science can make great contributions to solving.

These problems are generally societal and as such are rooted in the 21st century, although the central tenets stretch back for centuries. The Mathematics of the Commons refers to the shared property of mankind, all of the activities of human beings, and their mutual interactions. Most of the urgent issues we face today fall in this category. It is essential that we proceed in parallel to qualitative mathematical model and quantitative data analysis in order to solve these.

Unstable control networks

Researchers are working on the philosophical idea of Mathematics of the Commons - a viewpoint that emphasises the notion that mathematics can make a positive change to societies around the world

rofessor Emeritus Yasumasa Nishiura is a mathematician who has dedicated his career to understanding more about the profound impact mathematics has on the world around us. He worked as Research Director at the Alliance for Breakthrough between Mathematics and Sciences (ABMS) cells, and the dynamics involved in their in Japan between 2007 and 2016 where he was supporting research activities in mathematical science that highlights their potential for solving societal problems. 'My notion is that maths is a means of looking at the world in a way that is prescient, is at the centre of organisation and this influences the research I conduct,' he emphasises.

Nishiura is working with a team of researchers based at the Advanced Institute for Materials Research (AIMR), Tohoku University in Japan where they are studying self-organisation patterns that naturally

Large Disc

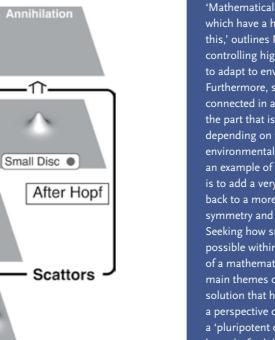
Before Hopf

manifest without design but have rhythm in space and time, such as polymers, convection, slime moulds and chemical reactions, to help learn more about pattern dynamics. 'Spatially localised particle patterns are especially suggestive of living self-reproduction and collisions is thoughtprovoking,' comments Nishiura. 'Particle solutions spontaneously and repeatedly selfreplicate over time, and the heterogeneity of the environment (reflecting diversity of the external world) triggers to create a variety of functions such as successive pulsing.' He explains that both of these processes are the outward manifestations of what he refers to as the hidden instability within the system.

Nishiura explains that unstable solutions are important because the dynamic patterns

are all intermediate organisms similar to life. 'In other words, they are structures that cannot be maintained without metabolism.' he clarifies. Because of that, interaction with the outside world is always essential. If the surrounding environment changes, then the response to that becomes an issue. 'Patterns that can be taken from a given environment are in a tentative stable state, and if the environment changes, it is inevitable that they will change to another stable state,' says Nishiura. In the process of change, there will always be an 'unstable' state. 'The unfettered and flexible adaptation of life can also be said to adeptly be using this unstable state,' he points out. This means that highly unstable states that give rise to a variety of outputs are probably effective at surviving, in the sense that they have a lot of leeway, which allows them to respond to different environmental changes.

'Mathematically speaking, saddle solutions, which have a high codimension, address this,' outlines Nishiura. 'Paradoxically, controlling high instability allows us to adapt to environmental changes. Furthermore, such unstable states are connected in a network, and it is believed the part that is to be used will change depending on the size and type of environmental change.' Nishiura offers an example of how a smart way to do this is to add a very huge perturbation to go back to a more primitive state with high symmetry and gradually adapt from there. Seeking how such a mechanism is made possible within the wider solution structure of a mathematical model is one of the main themes of the research underway. 'A solution that has high instability from such a perspective can probably be described as a 'pluripotent cell' of a dynamic pattern as brought forth by mathematical models,' he



Network of separators at collision process: Peanut, large disk and small disk are unstable states that sort out the behaviours of orbits after collision. Scattors represent the set of separators.

Peanut

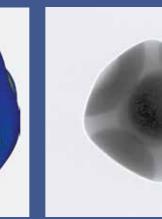
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An example that Nishiura offers to help understand how containing instability is important for adaptation in accordance with any change in circumstances is through the comparison of fighter planes and passenger planes. He says that unlike passenger planes which demand stable flight at high cruising altitudes, fighter planes would get shot down unless they could instantly change direction and speed. For that reason, their fuselages are designed to immediately take in unstable states. 'Naturally, piloting is more difficult, but the ability to follow changes is markedly higher,' notes Nishiura. 'Systems that adapt to environments can promptly expose certain types of instability that have been suppressed behind the scenes, allowing them to transition to new

To explain about spatially-localised patterns, Nishiura uses the example of polymer systems (such as nanoparticles) of extremely varied 3D shapes that occur as a subtle balance between intrinsic and extrinsic forces. 'Their sizes are small, generally between 100 nm and 300 nm, and multiple polymers are involved,' he says. 'They have a variety of shapes that look like viruses or cell groups which can change from simple shapes following environmental changes in solvent or temperature, in addition to changes in physical parameters such as chain length.' This means that an understanding about the relationship between polymers inside and the solvent around such systems can

be achieved. As a result, the forms of local patterns (shape of cells) and the mutual separation at the micro scale inside them (internal structure of the cells) becomes varied. 'The existence of pluripotent cells as the organisation centre that gives rise to a variety of shapes is suggested and is in the process of being unravelled,' confirms Nishiura. 'This allows us to create reduced mathematical models and depends on our ability to globally compute a depiction where we roll down the energy landscape that determines the shapes.'

Ultimately, Nishiura's research interests are not strictly bound by mathematical science. His philosophical approach to his studies and responsibilities means that his approach often extends beyond purely mathematical considerations. Indeed, the bulk of his motivations could be said to be influenced by the idea that his findings will filter through into other areas of scientific enquiry. 'I would like to construct a new worldview based on an understanding of qualitative structural instability and quantification of uncertainty in reduced mathematical models.' he highlights. 'I would like to do this together with the experienced mathematical scientists involved in the PRESTO and CREST mathematics programmes (ABMS) implemented by JST between 2007-2016. Through collaboration and the development of common goals, Nishiura hopes to use the research findings to deliver tangible solutions to societal problems.



Numerically obtained 3D cubic morphology for copolymer particle (left) prior to experiments, which is realised in experiments shown in TEM image (right). The diameter is about 250 nm.

Project Insights

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