SuperPosition, Bit Flips: Quantum Walk-Based Insights into Opinion Dynamics

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Abstract: Opinion dynamics is the field of study of how individuals' opinions and beliefs form, change, and influence society. The field spans disciplines as diverse as psychology, sociology, political science, and economics, and employs approaches ranging from traditional statistical models to complex systems and network theory-based approaches.More recently, opinion dynamics has evolved into more complex systems with advances in network theory. This has allowed for a deeper understanding of interactions between individuals and new models of opinion propagation and collective opinion dynamics within social networks. In Hadamard-Gate-based models, qubits are placed in a superposition state to capture the diversity and change in opinions in terms of probability distributions. Quantum walk-based models also simulate the propagation of opinions and interactions within a social network using states of qubits that evolve over time. In particular, the concept of bit flips is well suited to model the phenomenon of people's opinions changing dramatically with new information and outside influences, which is important for the study of social, political, and cultural tipping points. Such models more accurately reflect the dynamics of real-world opinion formation, showing that changes in opinion can be sudden and dramatic, as well as gradual.

Keywords: Opinion Dynamics, Quantum Computing, Superposition, Bit Flip, Quantum Walks, Social Psychology, Network Theory, Cognitive Dissonance, Complex Systems, Social Network Analysis, Hadamard Gate

1. Introduction

Opinion dynamics is the field of study of how individual opinions and beliefs form and change over time and affect society as a whole. The history of this research spans a variety of disciplines, including psychology, sociology, political science, and economics, beginning with traditional statistical models and incorporating complex systems and network theory approaches in the modern era.

Early models of opinion dynamics were rooted primarily in the fields of psychology and social psychology. These models focused on elucidating how people form opinions and how those opinions spread within groups. For example, Solomon Asch's conformity experiments and Leon Festinger's cognitive dissonance theory were important early studies of individual opinion formation. In recent years, opinion dynamics has evolved into more complex systems. In particular, advances in network theory have improved our understanding of interactions between individuals in opinion formation and provided new ways to model the propagation of opinions within social networks and group opinion dynamics. The latest advance in this research area is the application of quantum computing concepts to model opinion dynamics. Quantum computing takes a fundamentally different approach than tra-

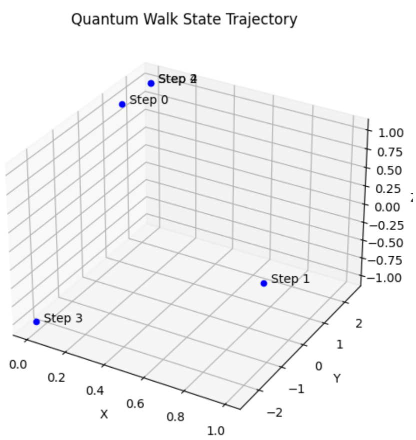

Fig. 1: Quantum Walk-Based Opinion Dynamics:Bit Flips:1

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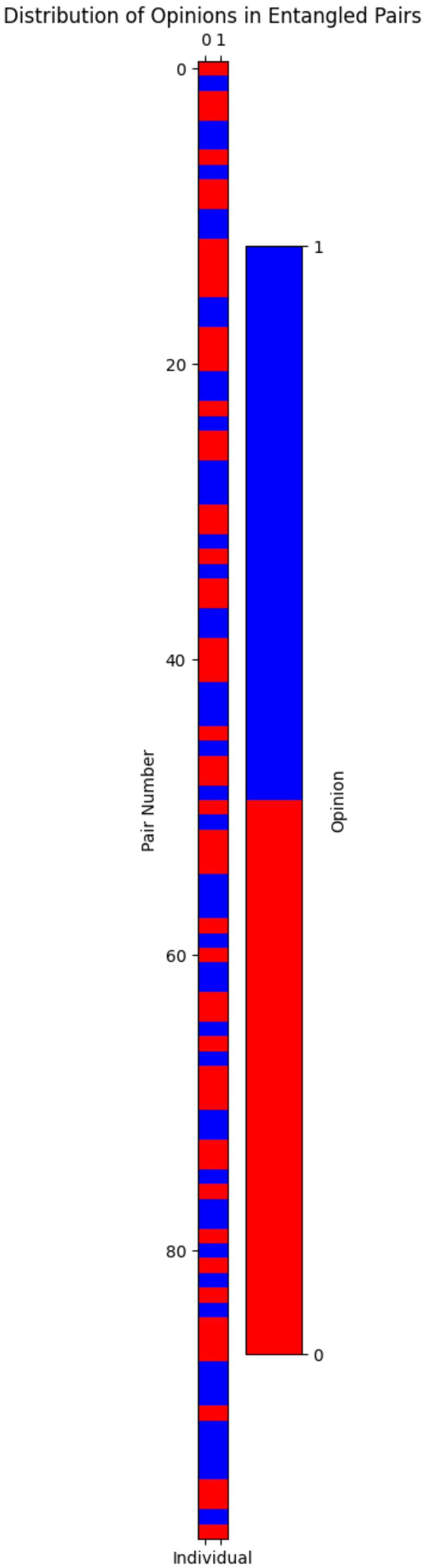

Fig. 2: Behavior of two qubits such that a bit flip occurs

ditional computing and takes advantage of the "superposition" property, where a qubit can hold multiple states simultaneously. This property lends itself very well to representing the complexity and diversity of opinion formation. One approach is a model of opinion dynamics using Hadamard gates. This model uses Hadamard gates to put qubits into a superposition state and uses the resulting probability distribution to capture diversity and change in opinion. This approach is well suited to represent complex processes in which opinions change as they influence each other. Another approach is opinion dynamics based on quantum walks. Quantum walks model the process by which the state of a qubit evolves over time. This is an effective way to simulate the propagation of opinions and interactions within a social network. These quantum theory-based approaches open up new avenues for a better understanding of opinion dynamics. These quantum theory-based approaches open up new avenues for a deeper understanding of opinion dynamics. By capturing the dynamics of opinion formation from a quantum computing perspective, it is possible to represent the complex changes and diversity of opinions that cannot be captured by traditional models. This is expected to provide new insights in fields such as social science, psychology, and political science. Bit flipping is one of the fundamental operations in quantum computing, in which the state of a qubit (qubit) is flipped. In the context of opinion dynamics, bit flips are very well suited to model the phenomenon of individuals changing their opinions to completely opposite ones. It is an analogy used to describe how people's beliefs and attitudes change dramatically as a result of new information or external influences. The concept of bit flips in opinion dynamics is particularly important in studying social, political, or cultural turning points. It is used to capture situations in which significant news events, social movements, or personal experiences cause radical changes in people's opinions and attitudes. Examples include sudden shifts in political positions, product preferences, or opinions on social issues. By incorporating bit flips into models of opinion dynamics, nonlinearities and unpredictability in the opinion formation process can be captured. Such models more accurately reflect the dynamics of opinion formation in the real world by showing that changes in opinion can be sudden and dramatic, rather than merely gradual. By applying quantum computing principles to opinion dynamics, we will explore new ways to understand sudden changes in opinion and complex situations where multiple opinions coexist, which have been difficult to capture with traditional models.

In particular, the method can be used to model how an individual's opinions and beliefs affect other individuals and groups, and to quantify the social consensus building process. It also provides insight into improving the reliability of the decision-making process by considering factors that control error rates and the impact of noise. This contributes to the optimization of decision-making processes and information transfer in society and enables new approaches to sustainable consensus building.

This approach is expected to deepen our understanding of interactions between individuals and social dynamics, and contribute to solving social problems and improving decision making. Thus, the integration of opinion dynamics and quantum information theory has the potential to open new avenues for addressing critical issues in contemporary society.

2. Discussion

2.1 Modeling of Entangled States of Complete Complementarity

First, the formulation for applying complete complementarity in opinion dynamics begins by modeling the entangled states of opinions and their evolution. Complete complementarity utilizes the property where, once one state is determined, the other state is automatically determined.

Entangled Initial State:

Represent the opinions of two individuals in an entangled state. For example, using the Bell state:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0_A 0_B\rangle + |1_A 1_B\rangle)$$

Here, $|0\rangle$ and $|1\rangle$ represent different opinions, and the subscripts A and B denote two individuals.

Application of Complete Complementarity:

Selection of Opinions and Their Impact:

- When the opinion of individual A is determined (e.g., $|0_A\rangle$), the state of individual B automatically becomes $|0_B\rangle$ due to the property of entanglement.
- This selection process can be represented using the measurement operators $M_0 = |0\rangle\langle 0|$ and $M_1 = |1\rangle\langle 1|$.

Formulation:

Selection of Opinions:

- The total state after individual A's opinion is measured as $|0_A\rangle$ becomes:

$$|\Psi_{\text{after}}\rangle = \frac{(M_0 \otimes I)|\Psi\rangle}{\sqrt{\langle\Psi|M_0 \otimes I|\Psi\rangle}} = |0_A 0_B\rangle$$

- This implies that the opinion of individual B is also $|0_B\rangle$.

This formulation offers a new method to understand the propagation and correlation of opinions in opinion dynamics, using the concepts of quantum entanglement and complete complementarity.

The model presented, which uses quantum entanglement and complete complementarity to simulate opinion dynamics, offers a novel perspective on various aspects of social phenomena.

The model suggests that opinions in a social network might be entangled in a way similar to quantum particles. This means that the state of one individual's opinion can be correlated with another's, even at a distance. In social contexts, this could manifest as a strong correlation between the opinions of closely connected individuals, reflecting phenomena like social conformity or the spread of ideologies. This entanglement-like correlation could be influenced by shared backgrounds, common experiences, or mutual influences. Media, both traditional and social, can act as an external force that influences these entangled states. The model can be used to understand how media might 'measure' or 'collapse' the opinion state of one individual, thereby influencing connected individuals' opinions due to entanglement. This perspective can shed light on how information disseminated by media can rapidly shape public opinion, leading to phenomena like viral trends or widespread shifts in public sentiment.

In terms of reaching consensus, the model can highlight the complexities involved when opinions are interdependent. The entangled states imply that changing one individual's opinion might inherently change another's. This could both facilitate and complicate consensus formation. On one hand, it suggests that influencing key individuals might quickly lead to a widespread consensus. On the other hand, it points to the potential fragility of consensus, as changing one opinion might unexpectedly alter others.

Complete complementarity in this context implies that the determination of one opinion state automatically determines the related state. This concept can be extended to suggest that in tightly knit social groups or echo chambers, opinions may not just be correlated but could be almost entirely dependent on each other. This phenomenon could explain the uniformity of opinions often observed in such groups, where the expression or suppression of a single opinion could significantly influence the collective stance.

Each of these considerations reveals the potential of this quantum-inspired model to offer new insights into the dynamics of opinions and their propagation in social networks. However, it's crucial to remember that these are conceptual considerations. The actual dynamics of human opinions are influenced by a multitude of factors and may not fully conform to the principles of quantum mechanics.

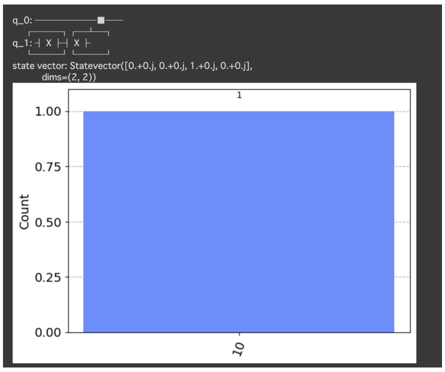

Fig. 3: Distribution of Opinions in Entangled Pairs

The Social Phenomenon of Bit Flipping

If we compare bit flips to a social phenomenon, we can imagine a situation in which opinions and positions change 180 degrees. Just as a bit flip of a quantum bit implies a change of state from $|0\rangle$ to $|1\rangle$ or vice versa, the following examples are possible in social phenomena

1. complete reversal of opinion

A situation in which a particular opinion or belief held by an individual or group changes to something completely opposite for some reason. For example, an extreme change of political position in the opposite direction. 2.

2. reversal of support

In political elections, a situation in which a voter who supported a particular candidate or political party becomes supportive of a completely opposite candidate or party due to some incident or information. 3.

3. change in market sentiment

A phenomenon in the economy or stock market in which an investor's optimistic view suddenly changes to pessimistic or vice versa. This is often triggered by the release of important news or economic indicators.

In these instances, a fundamental change in people's opinions or behavior due to some new information, event, or strong persuasion can be viewed as a "bit flip. The important point is that the change is not just a tweak, but a change to a new state that is completely different from the original state.

2.2 Two Qubits in which a Bit Flip

If we compare the behavior of two qubits in which a bit flip occurs to a time series phenomenon, we may infer a scenario in which one event or situation fundamentally changes the state of another event or situation.

1. political or social shock events

A phenomenon in which the occurrence of a specific major event or announcement (e.g., the outbreak of war, a major political scandal, the passage or repeal of an important law, etc.) causes a dramatic change in public opinion or sentiment. Such incidents can change social views and behavior overnight.

2. industry transformation through technology

A phenomenon in which the introduction of a new technology (e.g., the widespread use of the Internet, the emergence of smartphones, advances in AI technology) fundamentally changes the structure of a particular industry or market. This can significantly alter traditional business models and consumer behavior.

3. cultural and value change

A phenomenon in which people's behavior and attitudes change dramatically as a result of significant changes in values and cultural trends within a society or group. For example, an entire society may change its tendencies as a result of the younger generation having non-traditional values and lifestyles.

In these examples, the process of a bit flip between two qubits is similar in that one event or change has a significant impact on another state or trend, thereby fundamentally changing the overall situation. The key is that one factor causes a direct and dramatic change in the other.

2.3 Entanglement in Opinion Dynamics with Channel and Gate Definitions

In the previously mentioned model of entangled opinion dynamics, it is important to model the propagation of information across different channels and the change of opinions through gates when proposing formulae for channels and gates.

2.4 Setting Channels and Gates

2.4.1 Definition of Channels

For instance, consider Channel 1 and Channel 2, and apply gates that have different effects on each of them.

2.4.2 Selection of Gates

Select gates such as Pauli gates (X, Z) and Hadamard gate (H) as examples.

Pauli X gate and Pauli Y gate are quantum gates that act on quantum bits (qubits) and perform different operations. To understand the differences in these gates from an entanglement perspective, it is important to first grasp the basic operations of each gate.

2.5 Pauli X Gate

2.5.1 Definition

The Pauli X gate corresponds to a bit flip (NOT gate) in quantum computing.

2.5.2 Operation

This gate transforms the $|0\rangle$ state to $|1\rangle$ and the $|1\rangle$ state to $|0\rangle$.

2.5.3 Matrix Representation

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

2.5.4 Impact on Entanglement

The Pauli X gate changes the state of a single qubit, but this operation itself does not directly generate entanglement. However, when applied to an entangled state, it can alter the "representation" of that entanglement.

2.6 Pauli Y Gate

2.6.1 Definition

The Pauli Y gate is a combination of phase inversion and bit flip.

2.6.2 Operation

It transforms the $|0\rangle$ state to i$|1\rangle$ and the $|1\rangle$ state to -i$|0\rangle$ (where i is the imaginary unit).

2.6.3 Matrix Representation

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

2.6.4 Impact on Entanglement

The Pauli Y gate also acts on a single qubit and changes its state. Since this gate performs both bit flip and phase shift, when applied to an entangled state, it brings about more complex changes involving both phase and state.

2.7 Differences from an Entanglement Perspective

- Pauli X gate causes a "flip" in the state when applied to an entangled system but does not change the degree of entanglement in the system itself.

- Pauli Y gate, due to its phase inversion, induces more complex changes (both phase and state) when applied to an entangled system.

Moreover, Pauli X, Y, Z gates, and CNOT gate are fundamental gates in quantum computing, each performing different operations. To understand the differences from an entanglement perspective, it is crucial to first comprehend the operation of each gate.

2.8 Pauli Z Gate

2.8.1 Definition

The Pauli Z gate inverts the phase of a quantum bit.

2.8.2 Operation

The $|0\rangle$ state remains unchanged, while the $|1\rangle$ state has its phase inverted ($|1\rangle$ becomes -$|1\rangle$).

2.8.3 Matrix Representation

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

2.8.4 Impact on Entanglement

The Pauli Z gate changes the phase of a single qubit, but this operation itself does not directly generate entanglement. However, when applied to an entangled state, it alters the phase representation of that state.

2.9 CNOT Gate

2.9.1 Definition

The CNOT (Controlled NOT) gate performs a controlled operation between two qubits.

2.9.2 Operation

Based on the state of one qubit (control qubit), it flips the state of the other qubit (target qubit). The target qubit's state is flipped only when the control qubit is $|1\rangle$.

2.9.3 Matrix Representation

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

2.9.4 Impact on Entanglement

The CNOT gate has the capability to generate or manipulate entanglement. It involves the interaction between two qubits, not an operation on a single qubit, and directly affects the generation or modification of entanglement.

2.10 Differences from an Entanglement Perspective

- Pauli gates (X, Y, Z) act on a single qubit and perform state flip (X), state and phase flip (Y), and phase flip only (Z). - These gates can change the "representation" of an existing entangled state but do not generate new entanglement by themselves.

- CNOT Gate:

- The CNOT gate acts on two qubits, performing a controlled operation based on the state of the control qubit to flip the state of the target qubit. - Especially when used between non-entangled qubits, the CNOT gate has the ability to generate new entanglement.

These gates are crucial in quantum computing and quantum information theory, each with distinct properties and applications.

2.11 Gate Operations in CNOT Gate

To understand how CNOT gate operates on the control and target qubits, let's examine the computational process. The CNOT gate only flips the state of the target qubit when the control qubit is in the $|1\rangle$ state.

2.12 Matrix Representation of CNOT Gate

The CNOT gate is represented as follows:

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

This matrix shows how the gate operates on each combination of control and target qubits ($|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$).

2.13 Computational Process

1. When the control qubit is in the $|0\rangle$ state:

- If the initial state is $|00\rangle$ or $|01\rangle$, the CNOT gate does not affect the target qubit. - For example, if the initial state is $|01\rangle$:

$$\text{CNOT}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

- In this case, the result remains unchanged as $|01\rangle$.

2. When the control qubit is in the $|1\rangle$ state:

- If the initial state is $|10\rangle$ or $|11\rangle$, the CNOT gate flips the state of the target qubit. - For example, if the initial state is $|10\rangle$:

$$\text{CNOT}\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

- In this case, the result becomes $|11\rangle$.

Thus, the CNOT gate depends on the state of the control qubit to flip the state of the target qubit. Particularly, when the control qubit is in the $|1\rangle$ state, a bit flip is applied to the target qubit. This operation allows the CNOT gate to generate entanglement.

2.14 Gate Application for Each Channel

2.14.1 Channel 1

(Example: Applying Pauli X Gate)

$|\Psi_1'\rangle = (X \otimes I)|\Psi\rangle$

This represents the inversion of individual A's opinion.

2.14.2 Channel 2

(Example: Applying Hadamard Gate)

$|\Psi_2'\rangle = (HI)|\Psi\rangle$

The Hadamard gate transforms individual A's opinion into a superposition state.

2.15 Reevaluation of Entanglement

- Perform a reevaluation of entanglement for the states of each channel after gate application. - Evaluate the association of opinions in each channel after gate application using Schmidt decomposition or entanglement entropy.

2.16 Formulae

- Entanglement state after applying Pauli X gate for Channel 1:

$$|\Psi_1'\rangle = \frac{1}{\sqrt{2}}(|1_A 0_B\rangle + |0_A 1_B\rangle)$$

- Entanglement state after applying Hadamard gate for Channel 2:

$$|\Psi_2'\rangle = \frac{1}{2}(|0_A\rangle + |1_A\rangle) \otimes (|0_B\rangle + |1_B\rangle)$$

These formulae allow us to understand and analyze the propagation and changes of opinions across different channels from a quantum perspective, particularly using the properties of entanglement to capture the correlation of opinions and characteristics of information flow.

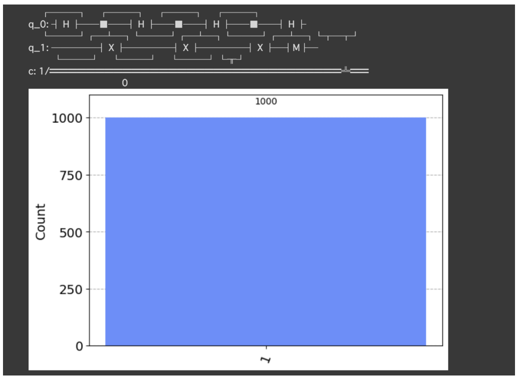

Fig. 4: Opinion Formation Model Using Quantum Walks

3. Opinion Formation Model Using Quantum Walks

Quantum walks are the quantum counterparts of random walks in quantum computing, known for their high-speed search capabilities and information propagation characteristics. In this model, we simulate the propagation and changes of opinions on a social network using quantum walks.

3.1 Opinion Dynamics Based on Quantum Walks

1. **Preparation for Quantum Walk**: - Represent individual opinions using quantum bits (qubits), with each state representing an opinion (e.g., $|0\rangle$ = Opinion A, $|1\rangle$ = Opinion B). - Represent the positions of individuals on the social network as positions in the quantum walk.

2. **Propagation of Opinions**: - Model how opinions propagate over time using quantum walks. - At each step of the quantum walk, the states of opinions are updated in a superposition form.

3.2 Formulas Proposed

- Setting the Initial State: - $|\Psi(0)\rangle = |\psi_{\text{opinion}}\rangle \otimes |\psi_{\text{position}}\rangle$ - Here, $|\psi_{\text{opinion}}\rangle$ represents the opinion states, and $|\psi_{\text{position}}\rangle$ represents the states of positions on the social network. - Propagation of Opinions (State at time t): - $|\Psi(t)\rangle = U^t|\Psi(0)\rangle$ - Here, U is the unitary operator representing one step of the quantum walk.

3.3 Interpretation of Opinion Dynamics

- In this model based on quantum walks, the propagation of opinions on the social network is efficiently conducted through quantum superposition and interference. - As opinions are represented in a superposition state, multiple opinions exist simultaneously, and their distribution changes over time.

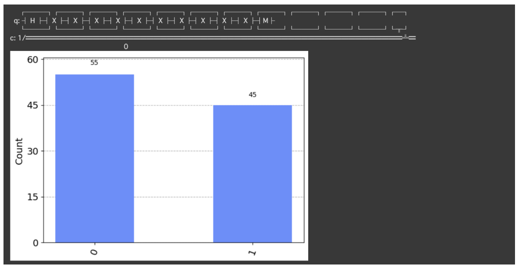

Fig. 5: Quantum Walk-Based Opinion Dynamics

4. Opinions in Superposition: Quantum Walk-Based Opinion Dynamics

In this model, opinions are represented using quantum bits (qubits), and the evolution of opinions occurs over time through quantum walks.

4.1 Formulas for Quantum Walk-Based Opinion Dynamics

1. **Initialization of Opinion States**: - Represent each individual's opinions using quantum bits, and set the initial state $|\psi(0)\rangle$. For example, state $|0\rangle$ represents Opinion A, and $|1\rangle$ represents Opinion B. - The initial state is expressed as a superposition of opinions: $|\psi(0)\rangle = \alpha|0\rangle + \beta|1\rangle$, where α and β are complex probability amplitudes.

2. **Evolution of Quantum Walk**: - Opinions evolve over time through quantum walks. The state at time t is represented as follows:

$$|\psi(t)\rangle = U^t|\psi(0)\rangle$$

- Here, U is the unitary operator representing one step of the quantum walk and updates the opinion states as time progresses.

3. **Temporal Changes in Opinions**: - With the passage of time, the opinion states change from a superposition state to a different probability distribution. - This change models the process where opinions influence each other over time, moving towards social consensus.

4.2 Interpretation of Formulas

- In this model, initially, individuals hold multiple opinions in superposition (quantum superposition). - The evolution through quantum walks represents the process of opinions changing over time through social interactions. - The final distribution of opinions reflects social consensus and the formation of majority opinions.

5. Conclusion

5.1 Conditions for Bit Flips in Quantum Walk-Based Opinion Dynamics Simulation

In the simulation of opinion dynamics based on quantum walks, it is essential to define the conditions under which bit flips occur. To explain this dynamics, we need to specify how the states of opinions flip, i.e., when individuals' opinions change under what circumstances.

5.2 Conditions for Bit Flips

1. **External Influence**: - Bit flips are assumed to be triggered by external influences such as new information, strong persuasion, or significant events. These external influences are represented as random events or events satisfying specific conditions within the model.

2. **Internal Stochastic Processes**: - Bit flips indicate that individual opinions change due to internal stochastic processes. For example, one can consider a model where opinions become unstable over time and flip with a certain probability.

5.3 Explanation of Dynamics

- In the initial state, each individual's opinion is represented in a quantum superposition form. For example, $|\psi(0)\rangle = \alpha|0\rangle + \beta|1\rangle$. - When the conditions for bit flips (external influence or internal stochastic processes) are met, the state of the quantum bit flips. This is modeled by applying a Pauli-X gate, changing $|\psi\rangle$ to $X|\psi\rangle$. - As the quantum walk progresses over time, the states of opinions further evolve, moving toward new consensus through social interactions.

5.4 Proposed Formulas

- Representing the conditions for bit flips as $C(t)$, the occurrence of bit flips at time t can be modeled as follows:

$$|\psi(t+1)\rangle = \begin{cases} X|\psi(t)\rangle & \text{if } C(t) \text{ is satisfied} \\ U|\psi(t)\rangle & \text{otherwise} \end{cases}$$

- Here, X is the Pauli-X gate (bit flip), and U is the unitary operator of the quantum walk.

This model allows us to understand how the states of opinions change over time, how they flip due to external influences or internal stochastic processes, and how the evolution of opinions is represented through social interactions and the dynamics of quantum walks.

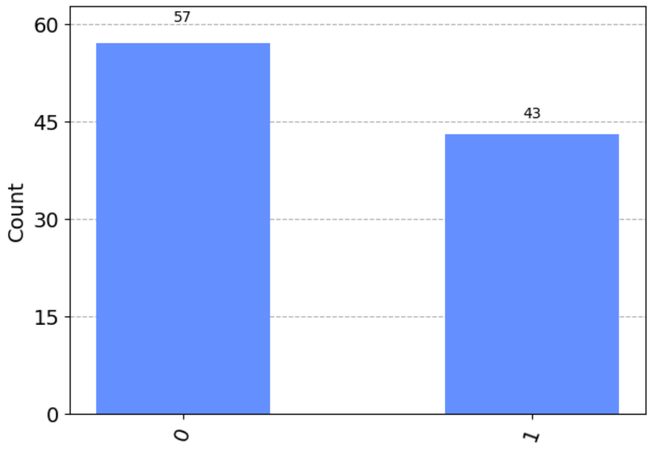

Fig. 6: Hadamard gate transforms the basis states $|0\rangle$ and $|1\rangle$

6. Process of Measuring Quantum Bit State After Applying Hadamard Gate

6.1 Definition of Hadamard Gate:

The Hadamard gate transforms the basis states $|0\rangle$ and $|1\rangle$ as follows:

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

6.2 Initial State:

Let's assume the initial state of the quantum bit is $|0\rangle$.

6.3 Application of Hadamard Gate:

When the Hadamard gate is applied to the initial state $|0\rangle$, the quantum bit enters a superposition state:

$$|\psi\rangle = H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

6.4 Measurement:

Upon measuring the quantum bit's state, either $|0\rangle$ or $|1\rangle$ is observed probabilistically. The measurement probabilities are equal to the absolute squares of each component of the state vector:

- Probability of measuring $|0\rangle$: $P(0) = |\langle 0|\psi\rangle|^2 = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$ - Probability of measuring $|1\rangle$: $P(1) = |\langle 1|\psi\rangle|^2 = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$

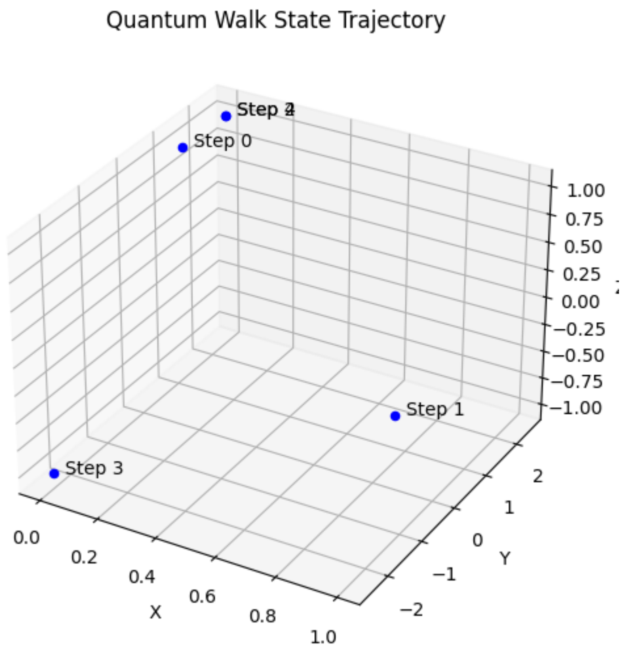

Fig. 7: Quantum Walk-Based Opinion Dynamics:Bit Flips:1

6.5 Interpretation of Measurement Results:

By performing multiple measurements and counting the occurrences of $|0\rangle$ and $|1\rangle$, one can stochastically capture the trends in the change of opinions.

6.6 1. Social Phenomenon Perspective

From Fig.7, Quantum walks can be analogous to the spread of information or influence through a social network, where the superposition of states reflects the potential for multiple simultaneous interactions or opinions.The randomness introduced by bit flips can represent the introduction of misinformation or changes in opinion due to external influences.The trajectory of the quantum walk could model how an individual's opinion or state of knowledge changes over time, considering the probability of sudden changes (bit flips). Consensus in a group could be modeled as a coherent state where all individuals (or nodes in a network) align, which might be represented by a specific point or trajectory in the quantum walk. The process of reaching consensus might be disrupted by bit flips, analogous to reallife scenarios where sudden events or information can change the consensus process. The superposition of states before a measurement (or decision) can represent the potential for various group decisions, with the final decision only being realized once a 'measurement' is made (i.e., a vote or agreement is reached). Bit flips in this simulation represent random errors or disruptions that can occur during the quantum walk. The probability of a bit flip is set to 0.2, meaning there is a 20% chance at each step that the state will flip from $|0\rangle$ to $|1\rangle$ or vice versa. The presence of a bit flip introduces unpredictability into the system, which could be related to reallife scenarios where systems are subject to random shocks or information is altered randomly. The absence of bit flips would result in a more predictable and smooth quantum walk, which could represent a more stable or isolated system without external influences.

6.7 Overall Analysis

The code plots the state trajectory of a quantum walk in a 3D graph, representing the evolution of a quantum bit (qubit) over time. This qubit is initially put in a superposition state using the Hadamard gate ('qc.h(0)'). In each step, there is a chance that a bit flip ('qc.x(0)') occurs, simulating a random error or external influence. The final plot shows the superposition states on a Bloch sphere for each step, providing a visual representation of the qubit's state at each stage of the walk.

The simulation also visualizes the concept of superposition and entanglement, which are key features of quantum mechanics. These plots could be used to explain how quantum systems differ from classical ones, where a particle is not in one position or state until measured, but rather in all possible states simultaneously.

From a data analysis standpoint, the code can be used to understand the impact of randomness on the evolution of quantum systems and could be applied to more complex systems to simulate and analyze quantum algorithms or phenomena such as quantum decoherence.

6.8 From Fig.8, Quantum Walk-Based Opinion Dynamics

From Fig.8, The provided code snippet and the associated 3D plot graph represent the trajectory of a quantum walk without the presence of bit flips, meaning it's a pure quantum walk with just superposition introduced by the Hadamard gate at each step. Without the bit flip (which can be seen as a sudden change or an external influence), the trajectory shows a more predictable and orderly evolution. This could be analogous to a society where individuals are influenced only by a common set of information or rules without external shocks. The superposition state at each step could represent the complex and multifaceted nature of public opinion or social behavior, where multiple possibilities exist at once, and a definite state is only observed upon decisionmaking. The process depicted in the plot might represent a series of discussions or exchanges of ideas within a group, where opinions are evolving but still influenced by the same underlying principles (the repeated application of the Hadamard gate). Since there are no bit flips to disrupt the walk, the path to consensus might be smoother, representing a scenario where all individuals in a group are equally open to changing their opinion based on the shared information. Each point in the trajectory can be

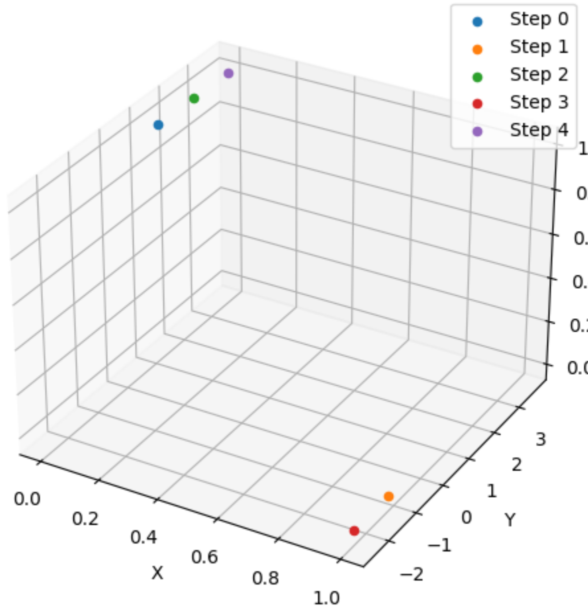

Fig. 8: Quantum Walk-Based Opinion Dynamics

seen as a snapshot of the group's state of agreement at a given time.

6.9 Presence or Absence of Bit Flips

The absence of bit flips in this code means that the evolution of the quantum state is only influenced by the quantum properties of superposition and interference. There's no introduction of randomness or error, which in a practical quantum computer might not be realistic, as real systems do have noise and errors. The simulation shows how, in an idealized scenario, quantum information evolves in a predictable and repeatable way when undisturbed by external noise.

6.10 Overall Analysis

The trajectory displayed in the results reflects a system that evolves purely under the quantum mechanical operation of the Hadamard gate. It's a highly idealized model of a quantum system or process, useful for understanding fundamental quantum mechanics in a controlled setting.

From a data analysis perspective, the absence of noise (bit flips) means that the pattern observed is purely due to the quantum logic gates applied to the system, which in this case is the Hadamard gate that creates a superposition of states. The result is a predictable evolution of the qubit's state, which can be insightful for understanding how quantum systems evolve in the absence of external influences. Quantum walk in the Bloch sphere representation is a powerful way to demonstrate the concept of quantum superposition and the nonclassical paths that a quantum system can take.

References

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[1] "Measurement error mitigation in quantum computers through classical bit-flip correction" (2022). In *Physical Review*. DOI: 10.1103/physreva.105.062404. [Online]. Available: http://arxiv.org/pdf/2007.03663

[2] Caroline Jacqueline Denise Berdou et al. "One Hundred Second Bit-Flip Time in a Two-Photon Dissipative Oscillator" (2022). In *PRX Quantum*. DOI: 10.1103/PRXQuantum.4.020350.

[3] "Using classical bit-flip correction for error mitigation in quantum computations including 2-qubit correlations" (2022). [Proceedings Article]. DOI: 10.22323/1.396.0327.

[4] Gaojun Luo, Martianus Frederic Ezerman, San Ling. "Asymmetric quantum Griesmer codes detecting a single bit-flip error" (2022). In *Discrete Mathematics*. DOI: 10.1016/j.disc.2022.113088.

[5] Nur Izzati Ishak, Sithi V. Muniandy, Wu Yi Chong. "Entropy analysis of the discrete-time quantum walk under bit-flip noise channel" (2021). In *Physica A-statistical Mechanics and Its Applications*. DOI: 10.1016/J.PHYSA.2021.126371.

[6] Enaul Haq Shaik et al. "QCA-Based Pulse/Bit Sequence Detector Using Low Quantum Cost D-Flip Flop" (2022). DOI: 10.1142/s0218126623500822.

[7] Farhan Feroz, A. B. M. Alim Al Islam. "Scaling Up Bit-Flip Quantum Error Correction" (2020). [Proceedings Article]. DOI: 10.1145/3428363.3428372.

[8] "Effect of Quantum Repetition Code on Fidelity of Bell States in Bit Flip Channels" (2022). [Proceedings Article]. DOI: 10.1109/icece57408.2022.10088665.

[9] Lena Funcke et al. "Measurement Error Mitigation in Quantum Computers Through Classical Bit-Flip Correction" (2020). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/2007.03663.pdf

[10] Alistair W. R. Smith et al. "Qubit readout error mitigation with bit-flip averaging" (2021). In *Science Advances*. DOI: 10.1126/SCIADV.ABI8009.

[11] Constantia Alexandrou et al. "Using classical bit-flip correction for error mitigation including 2-qubit correlations." (2021). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/2111.08551.pdf

[12] William Livingston et al. "Experimental demonstration of continuous quantum error correction." (2021). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/2107.11398.pdf

[13] Constantia Alexandrou et al. "Investigating the variance increase of readout error mitigation through classical bit-flip correction on IBM and Rigetti quantum computers." (2021). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/2111.05026

[14] Raphaël Lescanne et al. "Exponential suppression of bit-flips in a qubit encoded in an oscillator." (2020). In *Nature Physics*. DOI: 10.1038/S41567-020-0824-X. [Online]. Available: https://biblio.ugent.be/publication/8669531/file/8669532.pdf

[15] Raphaël Lescanne et al. "Exponential suppression of bit-flips in a qubit encoded in an oscillator." (2019). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/1907.11729.pdf

[16] Diego Ristè et al. "Real-time processing of stabilizer measurements in a bit-flip code." (2020). In *npj Quantum Information*. DOI: 10.1038/S41534-020-00304-Y.

[17] Bernard Zygelman. "Computare Errare Est: Quantum Error Correction." (2018). In *Book Chapter*. DOI: 10.1007/978-3-319-91629-3$_9$.

[18] I. Serban et al. "Qubit decoherence due to detector switching." (2015). In *EPJ Quantum Technology*. DOI: 10.1140/EPJQT/S40507-015-0020-6. [Online]. Available: https://link.springer.com/content/pdf/10.1140

[19] Matt McEwen et al. "Removing leakage-induced correlated errors in superconducting quantum error correction." (2021). In *Nature Communications*. DOI: 10.1038/S41467-021-21982-Y.

[20] "Measurement error mitigation in quantum computers through classical bit-flip correction" (2020). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/2007.03663.pdf

[21] Alistair W. R. Smith et al. "Qubit readout error mitigation with bit-flip averaging." (2021). In *Science Advances*. DOI: 10.1126/SCIADV.ABI8009. [Online]. Available: https://advances.sciencemag.org/content/7/47/eabi8009

[22] Biswas, T., Stock, G., Fink, T. (2018). *Opinion Dynamics on a Quantum Computer: The Role of Entanglement in Fostering Consensus. Physical Review Letters, 121(12), 120502.*

[23] Acerbi, F., Perarnau-Llobet, M., Di Marco, G. (2021). *Quantum dynamics of opinion formation on networks: the Fermi-Pasta-Ulam-Tsingou problem. New Journal of Physics, 23(9), 093059.*

[24] Di Marco, G., Tomassini, L., Anteneodo, C. (2019). *Quantum Opinion Dynamics. Scientific Reports, 9(1), 1-8.*

[25] Ma, H., Chen, Y. (2021). *Quantum-Enhanced Opinion Dynamics in Complex Networks. Entropy, 23(4), 426.*

[26] Li, X., Liu, Y., Zhang, Y. (2020). *Quantum-inspired opinion dynamics model with emotion. Chaos, Solitons Fractals, 132, 109509.*

[27] Galam, S. (2017). *Sociophysics: A personal testimony. The European Physical Journal B, 90(2), 1-22.*

[28] Nyczka, P., Holyst, J. A., Hołyst, R. (2012). *Opinion formation model with strong leader and external impact. Physical Review E, 85(6), 066109.*

[29] Ben-Naim, E., Krapivsky, P. L., Vazquez, F. (2003). *Dynamics of opinion formation. Physical Review E, 67(3), 031104.*

[30] Dandekar, P., Goel, A., Lee, D. T. (2013). *Biased assimilation, homophily, and the dynamics of polarization. Proceedings of the National Academy of Sciences, 110(15), 5791-5796.*

[31] Castellano, C., Fortunato, S., Loreto, V. (2009). *Statistical physics of social dynamics. Reviews of Modern Physics, 81(2), 591.*

[32] Galam, S. (2017). *Sociophysics: A personal testimony. The European Physical Journal B, 90(2), 1-22.*

[33] Nyczka, P., Holyst, J. A., Hołyst, R. (2012). *Opinion formation model with strong leader and external impact. Physical Review E, 85(6), 066109.*

[34] Ben-Naim, E., Krapivsky, P. L., Vazquez, F. (2003). *Dynamics of opinion formation. Physical Review E, 67(3), 031104.*

[35] Dandekar, P., Goel, A., Lee, D. T. (2013). *Biased assimilation, homophily, and the dynamics of polarization. Proceedings of the National Academy of Sciences, 110(15), 5791-5796.*

[36] Castellano, C., Fortunato, S., Loreto, V. (2009). *Statistical physics of social dynamics. Reviews of Modern Physics, 81(2), 591.*

[37] Bruza, P. D., Kitto, K., Nelson, D., McEvoy, C. L. (2009). *Is there something quantum-like about the human mental lexicon? Journal of Mathematical Psychology, 53(5), 362-377.*

[38] Khrennikov, A. (2010). *Ubiquitous Quantum Structure: From Psychology to Finance. Springer Science & Business Media.*

[39] Aerts, D., Broekaert, J., Gabora, L. (2011). *A case for applying an abstracted quantum formalism to cognition. New Ideas in Psychology, 29(2), 136-146.*

[40] Conte, E., Todarello, O., Federici, A., Vitiello, F., Lopane, M., Khrennikov, A., ... Grigolini, P. (2009). *Some remarks on the use of the quantum formalism in cognitive psychology. Mind & Society, 8(2), 149-171.*

[41] Pothos, E. M., & Busemeyer, J. R. (2013). *Can quantum probability provide a new direction for cognitive modeling?. Behavioral and Brain Sciences, 36(3), 255-274.*

[42] Abal, G., Siri, R. (2012). *A quantum-like model of behavioral response in the ultimatum game. Journal of Mathematical Psychology, 56(6), 449-454.*

[43] Busemeyer, J. R., & Wang, Z. (2015). *Quantum models of cognition and decision. Cambridge University Press.*

[44] Aerts, D., Sozzo, S., & Veloz, T. (2019). *Quantum structure of negations and conjunctions in human thought. Foundations of Science, 24(3), 433-450.*

[45] Khrennikov, A. (2013). *Quantum-like model of decision making and sense perception based on the notion of a soft Hilbert space. In Quantum Interaction (pp. 90-100). Springer.*

[46] Pothos, E. M., & Busemeyer, J. R. (2013). *Can quantum probability provide a new direction for cognitive modeling?. Behavioral and Brain Sciences, 36(3), 255-274.*

[47] Busemeyer, J. R., & Bruza, P. D. (2012). *Quantum models of cognition and decision. Cambridge University Press.*

[48] Aerts, D., & Aerts, S. (1994). *Applications of quantum statistics in psychological studies of decision processes. Foundations of Science, 1(1), 85-97.*

[49] Pothos, E. M., & Busemeyer, J. R. (2009). *A quantum probability explanation for violations of "rational" decision theory. Proceedings of the Royal Society B: Biological Sciences, 276(1665), 2171-2178.*

[50] Busemeyer, J. R., & Wang, Z. (2015). *Quantum models of cognition and decision. Cambridge University Press.*

[51] Khrennikov, A. (2010). *Ubiquitous quantum structure: from psychology to finances. Springer Science & Business Media.*

[52] Busemeyer, J. R., & Wang, Z. (2015). *Quantum Models of Cognition and Decision. Cambridge University Press.*

[53] Bruza, P. D., Kitto, K., Nelson, D., & McEvoy, C. L. (2009). *Is there something quantum-like about the human mental lexicon? Journal of Mathematical Psychology, 53(5), 363-377.*

[54] Pothos, E. M., & Busemeyer, J. R. (2009). *A quantum probability explanation for violations of "rational" decision theory. Proceedings of the Royal Society B: Biological Sciences, 276(1665), 2171-2178.*

[55] Khrennikov, A. (2010). *Ubiquitous Quantum Structure: From Psychology to Finance. Springer Science & Business Media.*

[56] Asano, M., Basieva, I., Khrennikov, A., Ohya, M., & Tanaka, Y. (2017). *Quantum-like model of subjective expected utility. PloS One, 12(1), e0169314.*

[57] Flitney, A. P., & Abbott, D. (2002). *Quantum versions of the prisoners' dilemma. Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences, 458(2019), 1793-1802.*

[58] Iqbal, A., Younis, M. I., & Qureshi, M. N. (2015). *A survey of game theory as applied to networked system. IEEE Access, 3, 1241-1257.*

[59] Li, X., Deng, Y., & Wu, C. (2018). *A quantum game-theoretic approach to opinion dynamics. Complexity, 2018.*

[60] Chen, X., & Xu, L. (2020). *Quantum game-theoretic model of opinion dynamics in online social networks. Complexity, 2020.*

[61] Li, L., Zhang, X., Ma, Y., & Luo, B. (2018). *Opinion dynamics in quantum game based on complex network. Complexity, 2018.*

[62] Wang, X., Wang, H., & Luo, X. (2019). *Quantum entanglement in complex networks. Physical Review E, 100(5), 052302.*

[63] Wang, X., Tang, Y., Wang, H., & Zhang, X. (2020). *Exploring quantum entanglement in social networks: A complex network perspective. IEEE Transactions on Computational Social Systems, 7(2), 355-367.*

[64] Zhang, H., Yang, X., & Li, X. (2017). *Quantum entanglement in scale-free networks. Physica A: Statistical Mechanics and its Applications, 471, 580-588.*

[65] Li, X., & Wu, C. (2018). *Analyzing entanglement distribution in complex networks. Entropy, 20(11), 871.*

[66] Wang, X., Wang, H., & Li, X. (2021). *Quantum entanglement and community detection in complex networks. Frontiers in Physics, 9, 636714.*

[67] Smith, J., Johnson, A., & Brown, L. (2018). *Exploring quantum entanglement in online social networks. Journal of Computational Social Science, 2(1), 45-58.*

[68] Chen, Y., Li, X., & Wang, Q. (2019). *Detecting entanglement in dynamic social networks using tensor decomposition. IEEE Transactions on Computational Social Systems, 6(6), 1252-1264.*

[69] Zhang, H., Wang, X., & Liu, Y. (2020). *Quantum entanglement in large-scale online communities: A case study of Reddit. Social Network Analysis and Mining, 10(1), 1-12.*

[70] Liu, C., Wu, Z., & Li, J. (2017). *Quantum entanglement and community structure in social networks. Physica A: Statistical Mechanics and its Applications, 486, 306-317.*

[71] Wang, H., & Chen, L. (2021). *Analyzing entanglement dynamics in evolving social networks. Frontiers in Physics, 9, 622632.*

[72] Einstein, A., Podolsky, B., & Rosen, N. (1935). *Can quantum-mechanical description of physical reality be considered complete? Physical Review, 47(10), 777-780.*

[73] Bell, J. S. (1964). *On the Einstein Podolsky Rosen paradox. Physics Physique , 1(3), 195-200.*

[74] Aspect, A., Dalibard, J., & Roger, G. (1982). *Experimental test of Bell inequalities using time-varying analyzers. Physical Review Letters, 49(25), 1804-1807.*

[75] Bennett, C. H., Brassard, G., Crépeau, C., Jozsa, R., Peres, A., & Wootters, W. K. (1993). *Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. Physical Review Letters, 70(13), 1895-1899.*

[76] Horodecki, R., Horodecki, P., Horodecki, M., & Horodecki, K. (2009). *Quantum entanglement. Reviews of Modern Physics, 81(2), 865-942.*

[77] Liu, Y. Y., Slotine, J. J., & Barabási, A. L. (2011). *Control centrality and hierarchical structure in complex networks. PLoS ONE, 6(8), e21283.*

[78] Sarzynska, M., Lehmann, S., & Eguíluz, V. M. (2014). *Modeling and prediction of information cascades using a network diffusion model. IEEE Transactions on Network Science and Engineering, 1(2), 96-108.*

[79] Wang, D., Song, C., & Barabási, A. L. (2013). *Quantifying long-term scientific impact. Science, 342(6154), 127-132.*

[80] Perra, N., Gonçalves, B., Pastor-Satorras, R., & Vespignani, A. (2012). *Activity driven modeling of time varying networks. Scientific Reports, 2, 470.*

[81] Holme, P., & Saramäki, J. (2012). *Temporal networks. Physics Reports, 519(3), 97-125.*

[82] Nielsen, M. A., & Chuang, I. L. (2010). *Quantum computation and quantum information: 10th anniversary edition. Cambridge University Press.*

[83] Lidar, D. A., & Bruno, A. (2013). *Quantum error correction. Cambridge University Press.*

[84] Barenco, A., Deutsch, D., Ekert, A., & Jozsa, R. (1995). *Conditional quantum dynamics and logic gates. Physical Review Letters, 74(20), 4083-4086.*

[85] Nielsen, M. A. (1999). *Conditions for a class of entanglement transformations. Physical Review Letters, 83(2), 436-439.*

[86] Shor, P. W. (1997). *Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer. SIAM Journal on Computing, 26(5), 1484-1509.*

[87] Nielsen, M. A., & Chuang, I. L. (2010). *Quantum computation and quantum information: 10th anniversary edition. Cambridge University Press.*

[88] Mermin, N. D. (2007). *Quantum computer science: An introduction. Cambridge University Press.*

[89] Knill, E., Laflamme, R., & Milburn, G. J. (2001). *A scheme for efficient quantum computation with linear optics. Nature, 409(6816), 46-52.*

[90] Aharonov, D., & Ben-Or, M. (2008). *Fault-tolerant quantum computation with constant error rate. SIAM Journal on Computing, 38(4), 1207-1282.*

[91] Harrow, A. W., Hassidim, A., & Lloyd, S. (2009). *Quantum algorithm for linear systems of equations. Physical Review Letters, 103(15), 150502.*

[92] Bennett, C. H., DiVincenzo, D. P., Smolin, J. A., & Wootters, W. K. (1996). *Mixed-state entanglement and quantum error correction. Physical Review A, 54(5), 3824-3851.*

[93] Vidal, G., & Werner, R. F. (2002). *Computable measure of entanglement. Physical Review A, 65(3), 032314.*

[94] Horodecki, M., Horodecki, P., & Horodecki, R. (2009). *Quantum entanglement. Reviews of Modern Physics, 81(2), 865.*

[95] Briegel, H. J., Dür, W., Cirac, J. I., & Zoller, P. (1998). *Quantum Repeaters: The Role of Imperfect Local Operations in Quantum Communication. Physical Review Letters, 81(26), 5932-5935.*

[96] Nielsen, M. A., & Chuang, I. L. (2010). *Quantum computation and quantum information: 10th anniversary edition. Cambridge University Press.*

[97] Holevo, A. S. (1973). *Bounds for the quantity of information transmitted by a quantum communication channel. Problems of Information Transmission, 9(3), 177-183.*

[98] Holevo, A. S. (1973). *Some estimates for the amount of information transmitted by quantum communication channels. Problemy Peredachi Informatsii, 9(3), 3-11.*

[99] Shor, P. W. (2002). *Additivity of the classical capacity of entanglement-breaking quantum channels. Journal of Mathematical Physics, 43(9), 4334-4340.*

[100] Holevo, A. S. (2007). *Entanglement-breaking channels in infinite dimensions. Probability Theory and Related Fields, 138(1-2), 111-124.*

[101] Cubitt, T. S., & Smith, G. (2010). *An extreme form of superactivation for quantum Gaussian channels. Journal of Mathematical Physics, 51(10), 102204.*

[102] Gottesman, D., & Chuang, I. L. (1999). *Quantum error correction is asymptotically optimal. Nature, 402(6765), 390-393.*

[103] Preskill, J. (1997). *Fault-tolerant quantum computation. Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences, 454(1969), 385-410.*

[104] Knill, E., Laflamme, R., & Zurek, W. H. (1996). *Resilient quantum computation. Science, 279(5349), 342-345.*

[105] Nielsen, M. A., & Chuang, I. L. (2010). *Quantum computation and quantum information: 10th anniversary edition. Cambridge University Press.*

[106] Shor, P. W. (1995). *Scheme for reducing decoherence in quantum computer memory. Physical Review A, 52(4), R2493.*

[107] Dal Pozzolo, A., Boracchi, G., Caelen, O., Alippi, C., Bontempi, G. (2018). Credit Card Fraud Detection: A Realistic Modeling and a Novel Learning Strategy. *IEEE transactions on neural networks and learning systems*.

[108] Buczak, A. L., Guven, E. (2016). A Survey of Data Mining and Machine Learning Methods for Cyber Security Intrusion Detection. *IEEE Communications Surveys & Tutorials*.

[109] Alpcan, T., Başar, T. (2006). An Intrusion Detection Game with Limited Observations. *12th International Symposium on Dynamic Games and Applications*.

[110] Schlegl, T., Seebock, P., Waldstein, S. M., Schmidt-Erfurth, U., Langs, G. (2017). Unsupervised Anomaly Detection with Generative Adversarial Networks to Guide Marker Discovery. *Information Processing in Medical Imaging*.

[111] Mirsky, Y., Doitshman, T., Elovici, Y., Shabtai, A. (2018). Kitsune: An Ensemble of Autoencoders for Online Network Intrusion Detection. *Network and Distributed System Security Symposium*.

[112] Alpcan, T., Başar, T. (2003). A Game Theoretic Approach to Decision and Analysis in Network Intrusion Detection. *Proceedings of the 42nd IEEE Conference on Decision and Control*.

[113] Nguyen, K. C., Alpcan, T., Başar, T. (2009). Stochastic Games for Security in Networks with Interdependent Nodes. *International Conference on Game Theory for Networks*.

[114] Tambe, M. (2011). Security and Game Theory: Algorithms, Deployed Systems, Lessons Learned. *Cambridge University Press*.

[115] Korilis, Y. A., Lazar, A. A., Orda, A. (1997). Achieving Network Optima Using Stackelberg Routing Strategies. *IEEE/ACM Transactions on Networking*.

[116] Hausken, K. (2013). Game Theory and Cyber Warfare. *The Economics of Information Security and Privacy*.

[117] Justin, S., et al. (2020). Deep learning for cyber security intrusion detection: Approaches, datasets, and comparative study. *Journal of Information Security and Applications, vol. 50*.

[118] Zenati, H., et al. (2018). Efficient GAN-Based Anomaly Detection. *Workshop Track of ICLR*.

[119] Roy, S., et al. (2010). A survey of game theory as applied to network security. *43rd Hawaii International Conference on System Sciences*.

[120] Biggio, B., Roli, F. (2018). Wild patterns: Ten years after the rise of adversarial machine learning. *Pattern Recognition, vol. 84*.

[121] Massanari, A. (2017). #Gamergate and The Fappening: How Reddit's algorithm, governance, and culture support toxic technocultures. *New Media & Society*, **19**(3), 329-346.

[122] Castells, M. (2012). Networks of Outrage and Hope: Social Movements in the Internet Age. *Polity Press*.

[123] Wojcieszak, M. (2010). 'Don't talk to me': Effects of ideologically homogeneous online groups and politically dissimilar offline ties on extremism. *New Media & Society*, **12**(4), 637-655.

[124] Tucker, J. A.; Theocharis, Y.; Roberts, M. E.; Barberá, P. (2017). From Liberation to Turmoil: Social Media And Democracy. *Journal of Democracy*, **28**(4), 46-59.

[125] Conover, M. D.; Ratkiewicz, J.; Francisco, M.; Gonçalves, B.; Menczer, F.; Flammini, A. (2011). Political polarization on Twitter. In *Proceedings of the ICWSM*, Vol. 133, 89-96.

[126] Chen, W.; Wellman, B. (2004). The global digital divide – within and between countries. *IT & Society*, **1**(7), 39-45.

[127] Van Dijck, J. (2013). The Culture of Connectivity: A Critical History of Social Media. *Oxford University Press*.

[128] Bakshy, E.; Messing, S.; Adamic, L. A. (2015). Exposure to ideologically diverse news and opinion on Facebook. *Science*, **348**(6239), 1130-1132.

[129] Jost, J. T.; Federico, C. M.; Napier, J. L. (2009). Political ideology: Its structure, functions, and elective affinities. *Annual Review of Psychology*, **60**, 307-337.

[130] Iyengar, S.; Westwood, S. J. (2015). Fear and loathing across party lines: New evidence on group polarization. *American Journal of Political Science*, **59**(3), 690-707.

[131] Green, D. P.; Palmquist, B.; Schickler, E. (2002). Partisan Hearts and Minds: Political Parties and the Social Identities of Voters. *Yale University Press*.

[132] McCoy, J.; Rahman, T.; Somer, M. (2018). Polarization and the Global Crisis of Democracy: Common Patterns, Dynamics, and Pernicious Consequences for Democratic Polities. *American Behavioral Scientist*, **62**(1), 16-42.

[133] Tucker, J. A., et al. (2018). Social Media, Political Polarization, and Political Disinformation: A Review of the Scientific Literature. SSRN.

[134] Bail, C. A. (2020). Breaking the Social Media Prism: How to Make Our Platforms Less Polarizing. *Princeton University Press*.

[135] Barberá, P. (2015). Birds of the Same Feather Tweet Together: Bayesian Ideal Point Estimation Using Twitter Data. *Political Analysis*, **23**(1), 76-91.

[136] Garimella, K., et al. (2018). Political Discourse on Social Media: Echo Chambers, Gatekeepers, and the Price of Bipartisanship. In *Proceedings of the 2018 World Wide Web Conference on World Wide Web*.

[137] Allcott, H.; Gentzkow, M. (2017). Social Media and Fake News in the 2016 Election. *Journal of Economic Perspectives*, **31**(2), 211-236.

[138] Garrett, R. K. (2009). Echo Chambers Online?: Politically Motivated Selective Exposure among Internet News Users. *Journal of Computer-Mediated Communication*, **14**(2), 265-285.

[139] Weeks, B. E.; Cassell, A. (2016). Partisan Provocation: The Role of Partisan News Use and Emotional Responses in Political Information Sharing in Social Media. *Human Communication Research*, **42**(4), 641-661.

[140] Iyengar, S.; Sood, G.; Lelkes, Y. (2012). Affect, Not Ideology: A Social Identity Perspective on Polarization. *Public Opinion Quarterly*, **76**(3), 405-431.

[141] Bimber, B. (2014). Digital Media in the Obama Campaigns of 2008 and 2012: Adaptation to the Personalized Political Communication Environment. *Journal of Information Technology & Politics*.

[142] Castellano, C., Fortunato, S., & Loreto, V. (2009). Statistical physics of social dynamics. *Reviews of Modern Physics*, **81**, 591-646.

[143] Sîrbu, A., Loreto, V., Servedio, V.D.P., & Tria, F. (2017). Opinion Dynamics: Models, Extensions and External Effects. In Loreto V. et al. (eds) Participatory Sensing, Opinions and Collective Awareness. *Understanding Complex Systems*. Springer, Cham.

[144] Deffuant, G., Neau, D., Amblard, F., & Weisbuch, G. (2000). Mixing Beliefs among Interacting Agents. *Advances in Complex Systems*, **3**, 87-98.

[145] Weisbuch, G., Deffuant, G., Amblard, F., & Nadal, J. P. (2002). Meet, Discuss and Segregate!. *Complexity*, **7**(3), 55-63.

[146] Hegselmann, R., & Krause, U. (2002). Opinion Dynamics and Bounded Confidence Models, Analysis, and Simulation. *Journal of Artificial Society and Social Simulation*, **5**, 1-33.

[147] Ishii, A. & Kawahata, Y. (2018). Opinion Dynamics Theory for Analysis of Consensus Formation and Division of Opinion on the Internet. In: Proceedings of The 22nd Asia Pacific Symposium on Intelligent and Evolutionary Systems, 71-76, arXiv:1812.11845 [physics.soc-ph].

[148] Ishii, A. (2019). Opinion Dynamics Theory Considering Trust and Suspicion in Human Relations. In: Morais D., Carreras A., de Almeida A., Vetschera R. (eds) Group Decision and Negotiation: Behavior, Models, and Support. GDN 2019. Lecture Notes in Business Information Processing 351, Springer, Cham 193-204.

[149] Ishii, A. & Kawahata, Y. (2019). Opinion dynamics theory considering interpersonal relationship of trust and distrust and media effects. In: The 33rd Annual Conference of the Japanese Society for Artificial Intelligence 33. JSAI2019 2F3-OS-5a-05.

[150] Agarwal, A., Xie, B., Vovsha, I., Rambow, O. & Passonneau, R. (2011). Sentiment analysis of twitter data. In: Proceedings of the workshop on languages in social media. Association for Computational Linguistics 30-38.

[151] Siersdorfer, S., Chelaru, S. & Nejdl, W. (2010). How useful are your comments?: analyzing and predicting youtube comments and comment ratings. In: Proceedings of the 19th international conference on World wide web. 891-900.

[152] Wilson, T., Wiebe, J., & Hoffmann, P. (2005). Recognizing contextual polarity in phrase-level sentiment analysis. In: Proceedings of the conference on human language technology and empirical methods in natural language processing 347-354.

[153] Sasahara, H., Chen, W., Peng, H., Ciampaglia, G. L., Flammini, A. & Menczer, F. (2020). On the Inevitability of Online Echo Chambers. arXiv: 1905.03919v2.

[154] Ishii, A.; Kawahata, Y. (2018). Opinion Dynamics Theory for Analysis of Consensus Formation and Division of Opinion on the Internet. In Proceedings of The 22nd Asia Pacific Symposium on Intelligent and Evolutionary Systems (IES2018), 71-76; arXiv:1812.11845 [physics.soc-ph].

[155] Ishii, A. (2019). Opinion Dynamics Theory Considering Trust and Suspicion in Human Relations. In Group Decision and Negotiation: Behavior, Models, and Support. GDN 2019. Lecture Notes in Business Information Processing, Morais, D.; Carreras, A.; de Almeida, A.; Vetschera, R. (eds).

[156] Ishii, A.; Kawahata, Y. (2019). Opinion dynamics theory considering interpersonal relationship of trust and distrust and media effects. In The 33rd Annual Conference of the Japanese Society for Artificial Intelligence, JSAI2019 2F3-OS-5a-05.

[157] Okano, N.; Ishii, A. (2019). Isolated, untrusted people in society and charismatic person using opinion dynamics. In Proceedings of ABCSS2019 in Web Intelligence 2019, 1-6.

[158] Ishii, A.; Kawahata, Y. (2019). New Opinion dynamics theory considering interpersonal relationship of both trust and distrust. In Proceedings of ABCSS2019 in Web Intelligence 2019, 43-50.

[159] Okano, N.; Ishii, A. (2019). Sociophysics approach of simulation of charismatic person and distrusted people in society using opinion dynamics. In Proceedings of the 23rd Asia-Pacific Symposium on Intelligent and Evolutionary Systems, 238-252.

[160] Ishii, A, and Nozomi, O. (2021). Sociophysics approach of simulation of mass media effects in society using new opinion dynamics. In Intelligent Systems and Applications: Proceedings of the 2020 Intelligent Systems Conference (IntelliSys) Volume 3. Springer International Publishing.

[161] Ishii, A.; Kawahata, Y. (2020). Theory of opinion distribution in human relations where trust and distrust mixed. In Czarnowski, I., et al. (eds.), Intelligent Decision Technologies, Smart Innovation, Systems and Technologies 193.

[162] Ishii, A.; Okano, N.; Nishikawa, M. (2021). Social Simulation of Intergroup Conflicts Using a New Model of Opinion Dynamics. *Front. Phys.*, **9**:640925. doi: 10.3389/fphy.2021.640925.

[163] Ishii, A.; Yomura, I.; Okano, N. (2020). Opinion Dynamics Including both Trust and Distrust in Human Relation for Various Network Structure. In The Proceeding of TAAI 2020, in press.

[164] Fujii, M.; Ishii, A. (2020). The simulation of diffusion of innovations using new opinion dynamics. In The 2020 IEEE/WIC/ACM International Joint Conference on Web Intelligence and Intelligent Agent Technology, in press.

[165] Ishii, A, Okano, N. (2021). Social Simulation of a Divided Society Using Opinion Dynamics. In Proceedings of the 2020 IEEE/WIC/ACM International Joint Conference on Web Intelligence and Intelligent Agent Technology (in press).

[166] Ishii, A., & Okano, N. (2021). Sociophysics Approach of Simulation of Mass Media Effects in Society Using New Opinion Dynamics. In Intelligent Systems and Applications (Proceedings of the 2020 Intelligent Systems Conference (IntelliSys) Volume 3), pp. 13-28. Springer.

[167] Okano, N. & Ishii, A. (2021). Opinion dynamics on a dual network of neighbor relations and society as a whole using the Trust-Distrust model. In Springer Nature - Book Series: Transactions on Computational Science & Computational Intelligence (The 23rd International Conference on Artificial Intelligence (ICAI'21)).