Psychological Effects of Digital Overload: Mathematical Exploration of The Viscous Solution Dynamics of Group Dynamics, Using Perron-Ishii's Lemmata

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Abstract: This paper delves into the complex impacts of digital media and social networks on mental health in contemporary society, focusing on the dynamics between opinion leaders and their followers. It examines how opinion leaders' interaction with external negative information affects their supporters' social literacy and information immunity, highlighting the risks of spreading inaccurate information within and outside the group. The study uses viscous solutions of the Eikonal equation (Ishii's Complementary) to model these dynamics, quantifying how information exchange between opinion leaders and followers influences individual and group opinions. Additionally, it references Davis and Martinez's (2021) work on digital resilience in the context of information overload, emphasizing the importance of managing digital information for psychological well-being. The paper aims to provide insights into the convergence and expansion of opinions within group dynamics, demonstrating the intricate relationship between digital influence and mental health.In the second half of the seminar, we will discuss some possible case studies, and simulations, issues, hypotheses and discussions based on the solution method using viscosity and Perron-Ishii's Lemmata will be presented.

Keywords: Digital Environment, Mental Health, Information Immunity, Perron-Ishii's lemmata, Ishii's Lemma, Risk Factors, Digital Resilience, Eikonal Equation, Group Dynamics, Social Literacy, Viscosity Solutions, Aggressive Behavior Patterns, Psychological Well-being

1. Introduction

This paper explores the multifaceted impact of the pervasiveness of the digital environment on the mental health of contemporary society. The impact of the rapid evolution of digital media and social networks on people's mental health is complex, and the quality and quantity of information provided by these platforms can have profound effects on individuals' cognition and behavior. And informative health-aware simulation and hypothesis of group dynamics in digital media environments regarding viscous arguments, opinion extension, and adherence that can occur on group dynamics using an application of Perron-Ishii's Lemmata-based viscosity solution method. The main focus of the project will be on the following topics. In the second half of the seminar, we will discuss some possible case studies, and simulations, issues, hypotheses and discussions based on the solution method using viscosity and Perron-Ishii's Lemmata will be presented.

In particular, we will focus on the dynamics between opinion leaders and their followers and deeply analyze the impact of these relationships on the people around them, especially

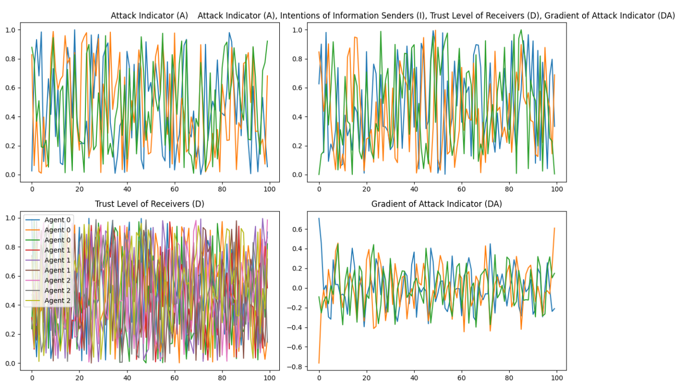

Fig. 1: Attack Indicator (A), Intentions of Information Senders (I), Trust Level of Receivers (D), Gradient of Attack Indicator (DA)

the "uninvolved". Opinion leaders' information immunity and risk factors, self-injurious speech and behavior, and the degree to which they react to external negative information can affect the social literacy and information immunity of their supporters. This suggests that inaccurate information and extreme opinions can be easily spread and influence perceptions and behavior both within and outside the group. The concept of viscous solutions to the Eikonal equation is used to model this dynamics. This equation captures the complexity of the information exchange between opinion leaders and their supporters and quantifies how this affects individual opinions and behavior. The model takes into account variables such as opinion leader influence ($I_i(t)$), risk factors ($R_i(t)$), and supporters' social literacy ($L_i(t)$). The speed of convergence of opinions on group dynamics indicates how opinions move toward agreement within a group. This speed depends on the quality of information and influence of opinion leaders and the efficiency of communication among their supporters; the Eikonal equation allows a numerical analysis of how these factors act on the convergence of opinions within a group. Opinion extension status indicates the extent to which a particular opinion or idea spreads outside the group. This depends on the influence of opinion leaders and the ability of information to spread through the media. Analysis using viscosity solutions makes it possible to investigate in detail how opinions spread and how this affects the perceptions and actions of individuals outside the group. This approach allows for a deeper understanding of the impact of information exchange between opinion leaders and their supporters on individuals inside and outside the group; by utilizing viscous solutions to the Eikonal equation, the impact of complex social interactions and individual behavior patterns on group opinion can be mathematically expressed and, based on this Based on this, it is expected to develop effective communication strategies and intervention techniques.

Opinion leaders' information immunity, risk factors, self-injurious behaviors, and degree of exposure to external negative information may create a risk of reduced social literacy and information immunity among their followers. Furthermore, it has been statistically observed that this dynamic indirectly and subtly affects the cognitive levels of individuals in the vicinity of the endorser. This group dynamic can sometimes lead to misperceptions of "unaffiliated people" and cause aggressive speech and behavior patterns. Also considered are patterns of intentional or unintentional enlightenment by opinion leaders. These interactions can have a pervasive effect, not only within a group, but throughout society, and can pose risks to an individual's informational health.

In this context, an important prior study is Davis and Martinez's (2021) study "Digital Resilience and Psychological Well-Being in the Age of Information Overload." This paper focuses on the impact of information overload in the digital age on an individual's psychological well-being. The study specifically explores the concept of digital resilience and analyzes how the dynamics of information processing in today's society affect an individual's mental health. Davis and Martinez point out that in today's society, where the volume of information in the digital environment is exploding, the quality and quantity of information people encounter can cause mental stress and anxiety. They emphasize the importance of "digital resilience," or the ability of individuals to process and adapt to information effectively, as digital media play an increasing role in our daily lives.

Through an extensive survey and case studies, the study examines the impact of information overload on an individual's psychological well-being. Davis and Martinez found that individuals who are digitally resilient tend to reduce stress and maintain a sense of psychological well-being in information overload situations. They also noted that individuals with low digital resilience are more likely to experience mental stress and anxiety due to the overwhelming flow of information.

Davis and Martinez identified information literacy, critical thinking, and self-regulation skills as factors that enhance digital resilience. These skills help people protect themselves from inaccurate or biased sources of information and effectively discern the information they need. They also conclude that proper management of information in the digital environment is important in supporting psychological well-being.

This research provides an important foundation for individuals to develop and implement strategies to cope with information overload in the digital environment. It emphasizes the importance of digital literacy education in educational institutions and workplaces and suggests concrete ways for individuals to protect themselves from the stress of information overload. The concept of digital resilience also offers a new perspective on promoting mental health and coping with the challenges of the digital age.

Furthermore, following the aforementioned, this study explores how the interaction between opinion leaders and their supporters affects social literacy, information-processing capacity, and responsiveness to external information through mathematical models. In particular, we will analyze the mechanisms by which group dynamics cause aggressive behavior and misperceptions, and assess the indirect effects of these dynamics on the individuals around them. These analyses will provide a basis for assessing risks to informational health within and outside the community and for developing appropriate intervention strategies.

Finally, this paper uses Perron-Ishii's method to obtain viscous solutions to the Eikonal equation and applies them to the informational dynamics model. This allows for a more detailed analysis of the influence of opinion leaders and the speed of convergence of opinions within a group, as well as the behavior of aggression indicators, and a mathematical understanding of the impact of these dynamics in real social situations. In this paper, we use Perron-Ishii's method to obtain a viscous solution to the Eikonal equation and apply this solution to an information dynamics model to gain insight into the convergence speed of opinions and the expansion status of opinions in group dynamics. eikonal equation and information dynamics model, it is formally expressed as

$$|\nabla O(x)| = f(x) \quad \text{in}\,\Omega$$

where $O(x)$represents the state of the opinion and $\nabla O(x)$represents its gradient (rate of change). $f(x)$is a rate function that includes external factors and factors that influence the formation of individual opinions.

This model in the parameter description involves multiple parameters that influence opinion formation and change for individual opinion leaders and group members. These include information immunity, risk factors, social literacy, and responsiveness to external information. These parameters are used to quantitatively assess how they affect the convergence of individual and in-group opinions. The speed of convergence of opinions on group dynamics indicates how fast a group reaches agreement within the group. Using viscosity solutions to the Eikonal equation, we can numerically express how these factors affect opinion convergence within a group. Extension of Scrutiny on Opinion Extension Status indicates the extent to which a particular opinion or idea spreads outside the group. This depends on the influence of opinion leaders and the ability of information to spread through the media. Model analysis using viscosity solutions allows for a more detailed understanding of the process by which opinions spread outside the group and the degree of their influence. This approach allows for a deeper understanding of the processes of convergence and diffusion of opinions in group dynamics using information dynamics models. using viscous solutions of the Eikonal equation, the influence of complex social interactions and individual behavior patterns on group opinions can be mathematically and to develop more effective communication strategies and intervention techniques based on this representation.

2. Discussion:Viscous Dissolution Dynamics in Group Dynamics

To explain the social phenomenon related to the 'viscous dissolution' in group dynamics, it is first necessary to understand what 'viscous dissolution dynamics' means. Viscous dissolution dynamics refer to the phenomenon where opinions, actions, or decisions within a group strongly influence each other, resulting in the entire group leaning towards a single direction or opinion. This means that the uniform behavior of the group supersedes the independent judgment and actions of individual members.

Theoretical Modeling

It is challenging to represent this phenomenon with a formula, but some modeling is possible using theories of social psychology. For example,

$$P(t+1) = P(t) + \alpha \sum_{i=1}^{n} (O_i - P(t))$$

Where,

$P(t)$ represents the state of opinion or behavior of the group at time t.

α is a coefficient indicating the strength of influence within the group, ranging from 0 to 1.

O_i represents the opinion or behavior of individual members within the group.

n is the number of members in the group.

This formula shows how the state of the group $P(t)$ changes over time due to the influence of the members' opinions O_i. Especially, the larger the value of α, the more significant the influence of individual members' opinions on the group's opinion.

Social Phenomena Examples

Examples of social phenomena include organizational culture in companies and groupthink in political movements. In the organizational culture of companies, strong values and beliefs of the organization influence the actions and opinions of individual employees, resulting in a consistent pattern of behavior across the organization. In political movements, shared beliefs and goals within the group can guide the actions of individual participants, creating a strong sense of unity for the entire group.

Impact of Aggressive Opinion Leaders

Center of Opinion Formation

The opinion leader, especially when adopting an aggressive or proactive stance, becomes the center of opinion formation within the group. Their strong personality and beliefs influence other members and determine the direction of the group's opinions and actions.

Imitation and Conformity

Group members tend to imitate the opinions and actions of the opinion leader. Especially when the leader acts confidently, their influence increases, creating conformity pressure within the group.

Enhanced Viscosity

Under the influence of aggressive opinion leaders, the viscosity of opinions and actions within the group is reinforced. This is because group members hesitate to hold different opinions and tend to follow the leader's opinion.

Social Impact

Influence on Public Discourse

Such groups can significantly influence public discussions and opinion formation. Particularly through activities on social media and public forums, they can impact a broad spectrum of society.

Amplification of Extreme Opinions

The presence of aggressive opinion leaders can lead to the amplification of extreme opinions and biases. The homogenization of opinions within the group may suppress neutral or opposing views.

Promotion of Social Division

The activities of these groups can promote social divisions. Differences in opinions between different groups may intensify, narrowing the space for dialogue and compromise.

Case Studies

In actual social phenomena, such dynamics are observed in groups related to political movements or specific social issues. For example, groups that strongly support a particular political stance or actively campaign for specific social issues (such as environmental problems, civil rights, etc.) fall into this category. These groups, especially under charismatic leaders, hold homogenized opinions and exert significant social influence. Understanding these group dynamics is important in comprehending social opinion formation processes and group behavior. It also forms the basis for exploring means to promote healthy social discussions and recognize the importance of diverse opinions and dialogue.

There is an opinion leader and their significantly supportive followers. Concerns arise regarding the potential decrease in the social literacy, information immunity, and resistance to external negative information of the opinion leader's supporters, depending on the opinion leader's own information immunity, risk factors, self-destructive behaviors, and their demand for the influence of external negative information. Moreover, probabilistically, these group dynamics indirectly affect the cognitive level of the supporters, albeit to a small extent, resulting in cases where they repeat patterns of aggressive behavior due to incorrect cognition, either intentionally or unintentionally, as the opinion leader inspires them. These group dynamics, the risk of aggression, and the probability of impairing information health indirectly to the surroundings are considered somewhat mathematically.

For instance, mathematical models used in the past as research cases for phenomena like viral hits are employed to understand the rapid dissemination mechanisms of products and information. When applying this model to construct a mathematical model for the group dynamics of opinion leaders and their supporters, it is essential to quantify the spread, acceptance, and range of influence of information. Below, as an application of the viral hit model, we can propose a mathematical model that represents group dynamics related to opinion leaders.

Let's introduce more concise notations for the formulas you've provided to make them more readable and less cluttered with long keywords. Here's how we can parameterize your equations:

Define: - SL_i for Social Literacy of supporters, - IIS_i for Information Immunity of supporters, - NIE_i for Negative Information Exposure, - ρ for the diffusion rate of influence, - GD_i for Group Dynamics, - OLI_i for OL (Opinion Leader) Influence, - SI_j for Supporter Impact of supporter j, - ψ for the diffusion rate of incorrect cognition, - CR_i for Cognitive Risk, - $Nei(i)$ for the set of neighbors (supporters) of opinion leader i, - $Unr(i)$ for the set of unrelated individuals to opinion leader i, - MI_j for Misinformation Impact of individual j.

Now we rewrite the equations using these notations:

Supporter Impact:

$$SI_i = SL_i \times IIS_i \times (1 - NIE_i)$$

Influence diffusion within the group dynamics:

$$GD_i = \rho \cdot (OLI_i \times \sum_{j \in Nei(i)} SI_j)$$

Risk of incorrect cognition and aggressiveness patterns:

$$CR_i = \psi \cdot \left(GD_i + \sum_{j \in Unr(i)} MI_j \right)$$

With these notations, the equations are significantly cleaner and the key terms are defined separately, improving readability.

This model represents how the influence of an opinion leader propagates to their supporters and indirectly affects unrelated individuals. It also quantifies the risk of incorrect cognition and aggressive patterns through group dynamics, evaluating the potential to impair information health. This mathematical model provides a foundation for designing information intervention strategies and awareness campaigns within communities by understanding the mechanisms of information dissemination and acceptance. It may also be useful in devising strategies for risk management and maintaining the integrity of information.

3. Discussion:Example of model extension as opinion dynamics

Here, this section's discussion on confidence interval definition, range of opinion convergence.

In the context of the Opinion Leader-centered Bounding Confidence Model applied mathematical model, we capture the interaction between the influence of opinion leaders and the confidence intervals of their supporters and demonstrate how misinformation affects them. The model also discusses cases where the complex relationship between information flow and opinion formation in group dynamics is formulated mathematically.

Furthermore, in the Opinion Leader-centered Hegselmann-Krause Model applied mathematical model, we represent the interaction between opinion leaders and their supporters and show how opinions converge. It suggests that the greater the influence of opinion leaders in opinion formation, the more rapidly the opinions of the group converge towards the leader's opinion.

Opinion Leader-centered Bounding Confidence Model Applied Mathematical Model

Parameter Definitions

- OLI_i: Influence indicator of opinion leader i - II_i: Information immunity of opinion leader i - RF_i: Risk factor of opinion leader i - OR_i: External influence of opinion leader i - θ, η: Weight coefficients for risk and external influence

Parameter Definitions

- SC_i: Confidence interval of supporter i - SL_i: Social literacy of supporter i - IIS_i: Information immunity of supporter i - NII_i: Impact of exposure to negative information - ξ: Coefficient of negative information impact

Definition of Supporter's Confidence Interval

$$SC_i = SL_i \cdot (IIS_i - \xi \cdot NII_i)$$

Parameter Definitions

- CS_i: Confidence interval diffusion of i - $Supp(i)$: Set of supporters of opinion leader i

Confidence Interval Diffusion in Group Dynamics

$$CS_i = OLI_i \cdot \sum_{j \in Supp(i)} SC_j$$

Parameter Definitions

- MIV_i: Variability of confidence interval due to misinformation i - σ: Rate of variability of confidence interval due to misinformation - MI_i: Impact of misinformation

Variability of Confidence Interval Due to Misinformation

$$MIV_i = \sigma \cdot (CS_i + MI_i)$$

Discussion

This mathematical model captures the interaction between the influence of opinion leaders and the confidence intervals of their supporters, demonstrating how misinformation affects them. The model also mathematically formulates the complex relationship between information flow and opinion formation in group dynamics.

Next, we will parameterize and elaborate on the Opinion Leader-centered Hegselmann-Krause Model and an extended version of the Deffuant-Weisbuch Model.

Opinion Leader-centered Hegselmann-Krause Model Applied Mathematical Model

Parameter Definitions

- $O_i(t)$: Opinion of opinion leader i at time t - $N(i)$: Set of supporters influenced by i - ω_{ij}: Weight of influence of supporter j on opinion leader i - μ: Coefficient regulating the convergence speed of overall opinions

Opinion Formation of Opinion Leader

$$O_i(t+1) = O_i(t) + \mu \sum_{j \in N(i)} \omega_{ij}(O_j(t) - O_i(t))$$

Parameter Definitions

- $S_i(t)$: Opinion of supporter i at time t - $M(i)$: Set of other supporters influencing i - ϕ_{ik}: Weight of influence of supporter k on i - ν: Coefficient regulating the convergence speed of opinions among supporters

Opinion Formation of Supporters

$$S_i(t+1) = S_i(t) + \nu \sum_{k \in M(i)} \phi_{ik}(S_k(t) - S_i(t))$$

Parameter Definitions

- $G_i(t)$: State of group dynamics i at time t - ψ_{ij}: Weight of influence of opinion of supporter j on group dynamics i - δ: Coefficient regulating the rate of change in group dynamics

Influence on Group Dynamics

$$G_i(t+1) = G_i(t) + \delta \left(\sum_{j \in N(i)} G_j(t) - G_i(t) \right)$$

Discussion

This model represents the interaction between opinion leaders and their supporters and shows how opinions converge. It suggests that the greater the influence of opinion leaders in opinion formation, the more rapidly the opinions of the group converge towards the leader's opinion.

Extended Version of Deffuant-Weisbuch Model

Parameter Definitions

- $O_{ij}(t)$: Average opinion of agents i and j at time t - μ_{ij}: Convergence rate of influence between i and j - D_{ij}: Trust or distrust of i towards j

Introduction of Trust and Distrust between Agents

$$O_{ij}(t+1) = O_{ij}(t) + \mu_{ij} \cdot (O_j(t) - O_i(t)) \cdot D_{ij}$$

Parameter Definitions

- $C_i(t)$: State of group dynamics i at time t - $N(i)$: Number of supporters for i

Dynamics of Opinion Leaders and Supporters

$$O_i(t+1) = O_i(t) + \frac{1}{N} \sum_{j \in N(i)} (O_j(t) - O_i(t)) \cdot D_{ij}$$

Influence on Group Dynamics

$$C_i(t+1) = C_i(t) + \delta \cdot \left(\sum_{j \in N(i)} C_j(t) - C_i(t) \right) \cdot D_{ij}$$

Section Discussion

In this extended model, we demonstrate how trust and distrust between agents influence opinion convergence. High trust may lead to rapid convergence of opinions among agents, while high distrust may result in slower convergence or polarization. Additionally, the dynamics between opinion leaders and supporters play a significant role in opinion formation within the community, and their influence varies based on the levels of trust and distrust. The impact on group dynamics relates to discussions about the integrity and aggressiveness of information within the community.

4. Discussion:Group dynamics sometimes result in repeated patterns of aggressive speech and behavior as a result of false perceptions to "uninvolved people"

Group dynamics sometimes result in repeating patterns of aggressive behavior due to the incorrect cognition of "unrelated individuals." Furthermore, it is expected that patterns of intentional or unintentional influence by opinion leaders on those dynamics will also be considered.

What cases can be envisaged when assuming these group dynamics, the risk of aggression, and the probability of impairing the information health of indirect surroundings? Let's organize it.

To model the repetition of incorrect cognition and aggressive behavior in group dynamics, and the subsequent impact on the information health of the surroundings, we need to formulate complex dynamics that consider both social influence and individual information processing capabilities. Below, we propose mathematical formulations that include these elements.

Mathematical Model of Misrecognition and Aggressive Dynamics

Diffusion of Misrecognition

$$M_i(t+1) = M_i(t) + \kappa \cdot \sum_{j \in N(i)} \left(M_j(t) - M_i(t) \right) \cdot T_{ij}$$

Here, $M_i(t)$ represents the degree of misrecognition of agent i at time t, T_{ij} is the trustworthiness of agent j affecting i's misrecognition, and κ is a coefficient regulating the speed of misrecognition diffusion.

Dynamics of Aggressiveness

$$A_i(t+1) = A_i(t) + \eta \cdot (M_i(t) \cdot P_i(t) - A_i(t))$$

Here, $A_i(t)$ represents the level of aggressiveness of agent i at time t, $P_i(t)$ is the level of external pressure or stress, and η is a coefficient regulating the responsiveness of aggressiveness.

Impact on Information Health of Surroundings

$$H_i(t+1) = H_i(t) - \theta \cdot \left(A_i(t) + \sum_{j \in N(i)} F_{ij} \right)$$

Here, $H_i(t)$ represents the information health of agent i at time t, F_{ij} represents the negative impact of agent j's aggressiveness on i, and θ is a coefficient for the impact on the surroundings.

Section Discussion

This model illustrates how misrecognition spreads and is associated with aggressiveness. It is expected that higher levels of misrecognition will lead to increased aggressiveness, and this aggressiveness will further reduce the information health of the surrounding environment. This process, especially triggered by opinion leaders, may have repercussions throughout society. Therefore, strategies to mitigate misrecognition and aggressiveness, such as enhancing information literacy education, addressing misinformation, and providing stress management support, are essential. Additionally, this model can be used to predict the impact of specific interventions on group dynamics and evaluate the effectiveness of measures.

The above formulation of group dynamics in the proposed model takes into account the exchange of opinions between opinion leaders and supporters. Furthermore, to calculate the impact of group dynamics on "unrelated individuals" and the information health of the surroundings, we add mathematical models as follows.

External Influence Model on Group Dynamics

Influence on Uninvolved Individuals

$$U_i(t+1) = U_i(t) + \rho \cdot (G_i(t) - U_i(t))$$

Here, $U_i(t)$ represents the opinions of uninvolved individual i at time t, $G_i(t)$ represents the state of group dynamics, and ρ is the coefficient of influence that uninvolved individuals receive from group dynamics.

Relationship between Aggressiveness and Misrecognition

$$C_i(t+1) = C_i(t) + \xi \cdot (A_i(t) \cdot M_i(t) - C_i(t))$$

Here, $C_i(t)$ represents patterns of aggressive behavior at time t, $A_i(t)$ is an indicator of aggressiveness, $M_i(t)$ is the degree of misrecognition, and ξ is the rate coefficient of the impact of aggressiveness and misrecognition on aggressive behavior.

Modeling of Intentional and Unintentional Enlightenment

$$E_i(t+1) = E_i(t) + \zeta \cdot \left(\sum_{j \in N(i)} H_{ij} \cdot O_j(t) \right)$$

Here, $E_i(t)$ represents the state of enlightenment of agent i at time t, H_{ij} represents the weight of enlightenment that opinion leader j's opinion has on i, and ζ is the speed coefficient of the impact of enlightenment.

Section Discussion

This model considers not only the exchange of opinions within the group but also the influence on individuals outside the community. In particular, the influence on uninvolved individuals is crucial in illustrating how dynamics within the community propagate throughout society. The relationship between patterns of aggressive behavior and misrecognition demonstrates how misinformation and bias may enhance aggressiveness. The modeling of intentional and unintentional enlightenment quantifies the extent to which opinion leaders influence opinion formation. Through these models, the risk that group dynamics pose to individual information health can be assessed, and necessary intervention strategies can be developed.

5. Discussion:Viscosity dynamics with respect to the opinion leader's adherence to a particular target and the recipient's adherence to its influence

Regarding aggressiveness, it is believed that opinion leaders exhibit very high stickiness when they have a specific target, and they may repeat patterns even if there are alerts from their surroundings. Additionally, for recipients who are influenced, stickiness to opinion leaders and stickiness resolution become important factors. Let's attempt to construct a model incorporating the concept of stickiness dynamics.

When constructing a model that takes into account stickiness dynamics for opinion leaders' stickiness to specific targets and the stickiness of recipients influenced by them, the following approach can be taken:

Opinion Dynamics Model with Stickiness Dynamics

Model for Opinion Leader's Stickiness to Target

$$P_i(t+1) = P_i(t) + \kappa \cdot (T_i - P_i(t)) - \lambda \cdot \text{AlertLevel}_i(t)$$

Here, $P_i(t)$ represents the stickiness of opinion leader i to the target at time t, T_i is the target stickiness target value, κ is the stickiness adjustment coefficient, λ is the stickiness reduction coefficient based on the surrounding alert level, and $\text{AlertLevel}_i(t)$ is the surrounding alert level.

Stickiness Resolution Model for Recipients to Opinion Leaders

$$D_{ij}(t+1) = D_{ij}(t) + \mu \cdot \left(P_i(t) - D_{ij}(t)\right)$$

Here, $D_{ij}(t)$ represents the stickiness resolution of recipient i to opinion leader j at time t, and μ is the stickiness resolution adjustment coefficient.

Integrated Model of Aggressiveness and Stickiness

$$A_i(t+1) = A_i(t) + \xi \cdot \left(P_i(t) \cdot M_i(t) \cdot I_i(t) \cdot D_{ij}(t) - A_i(t)\right)$$

Here, $A_i(t)$ represents the aggressiveness index at time t, $P_i(t)$ is the stickiness to the target, $M_i(t)$ is the degree of misrecognition, $I_i(t)$ is the level of intention of the information disseminator, $D_{ij}(t)$ is the stickiness resolution of the recipient to opinion leader j, and ξ is the rate coefficient of aggressiveness change.

Section Discussion

This model captures situations where opinion leaders exhibit high stickiness to specific targets and the reactions of recipients influenced by them. When opinion leaders have high stickiness, recipients influenced by them may also exhibit similar stickiness. On the other hand, the stickiness of opinion leaders may decrease when the surrounding alert level is high.

When incorporating the concept of stickiness dynamics into the opinion leader and supporter dynamics model, the following approach can be adopted:

Opinion Dynamics Model Using Stickiness Concepts

Application of Stickiness Partials

Assuming that the opinion O_i of the opinion leader satisfies stickiness partials, it implies that the opinion decreases due to the influence of the surrounding environment and supporters.

$$F_O(x_0, O_i(x_0), D\phi(x_0), D^2\phi(x_0)) \leq 0$$

Here, F_O is a function representing the dynamics of the opinion of the opinion leader, and $D\phi$ and $D^2\phi$ are the gradient and Hessian matrix, respectively, related to the opinion of the opinion leader.

Application of Stickiness Superpartials

Assuming that the opinion S_i of the supporters satisfies stickiness superpartials, it implies that the opinion increases due to the influence of the opinion leader.

$$F_S(x_0, S_i(x_0), D\phi(x_0), D^2\phi(x_0)) \geq 0$$

Here, F_S is a function representing the dynamics of the supporters' opinion.

Integrated Model with Stickiness Superpartials and Partials

To model the interaction between opinion leaders and supporters, the concept of stickiness superpartials and partials can be applied.

$$\text{Interaction}_i(t+1) = \text{Interaction}_i(t) + \rho \cdot (F_O + F_S)$$

Here, $\text{Interaction}_i(t)$ represents the interaction between opinion leader i and supporters at time t, and ρ is the rate coefficient of interaction change.

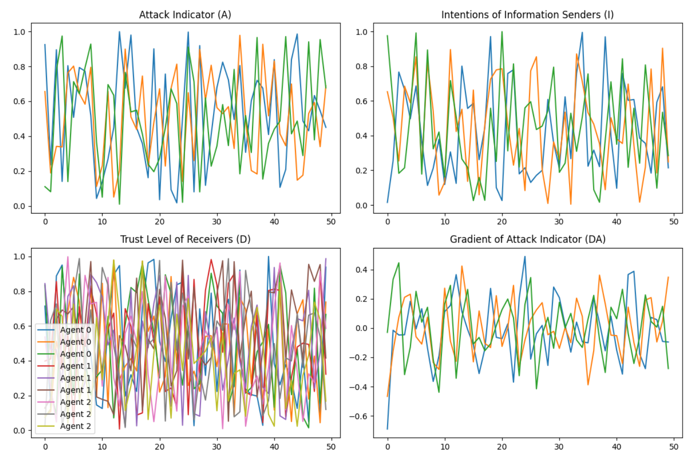

Fig. 2: Attack Indicator (A), Intentions of Information Senders (I), Trust Level of Receivers (D), Gradient of Attack Indicator (DA)

Section Discussion

This model provides a framework to capture a series of dynamics related to opinion leaders, their supporters, and the specific targets of opinion leaders. It allows for the understanding of how the actions of opinion leaders influence their surroundings and how this is reflected in the behavior patterns of the group.

The model considering stickiness of opinion leaders to specific targets and the stickiness of recipients to opinion leaders can help understand the complex aspects of opinion dynamics. A similar approach can be used to model stickiness dynamics in social dynamics, providing insights into the impact of influential individuals on their surroundings.

Attack Indicator (A)

This plot shows the fluctuating levels of an "Attack Indicator" for different agents over time. The variability in the lines suggests that each agent has a unique and changing level of aggression or attack potential throughout the simulation.

Intentions of Information Senders (I)

This plot shows the intentions of information senders, presumably reflecting their propensity to share accurate or misleading information. The lines vary over time, indicating changes in intentions.

Trust Level of Receivers (D)

Here, the trust levels of receivers are plotted. Since trust is a dynamic and subjective attribute, it is natural to see a lot of fluctuations, as depicted. Each agent shows varying levels of trust, which could be influenced by the behavior of the information senders or other factors in the simulation.

Gradient of Attack Indicator (DA)

The gradient of the attack indicator could be interpreted as the rate of change in aggression. The plots show that the aggression levels of agents change at varying rates over time, with some agents experiencing more rapid changes (steeper slopes) than others.

Observations and Insights

Variability, All variables exhibit significant variability, suggesting a complex system where agent states are interdependent and influenced by multiple factors. Correlations, there could be correlations between the attack indicators, intentions, and trust levels. For example, a high attack indicator might correlate with lower trust levels. However, without additional context, it's not possible to draw definitive conclusions.

Patterns and Trends, while there are fluctuations, certain overarching trends might be discernible. For example, if one agent consistently shows higher trust levels despite fluctuations, it could imply an inherent bias or robustness to external influences.

Gradient Interpretation, The gradient plots are particularly useful for understanding the dynamics of the system. Sharp peaks or troughs in the gradient plot suggest moments of rapid change, which could be critical events in the context of the simulation.

Noise vs. Signal

It is essential to distinguish between genuine patterns in the data and noise. Real-world data often contain a mix of both, and the ability to distinguish between the two is crucial for analysis.

Statistical Analysis

Beyond visual inspection, statistical tools could provide more insights. For instance, calculating the correlation coefficients between different agents' behaviors could quantify the relationships observed in the plots.

Model Validation

These visualizations can be used to validate the underlying model. If the behavior of the agents does not align with expected or known patterns, the model's assumptions and parameters may need to be re-evaluated.

Model Complexity

The complexity of the model should be appropriate for the phenomena being simulated. Overly complex models can be as misleading as overly simple ones, as they may fit the noise rather than the signal.

The results are helpful in getting an intuitive understanding of the system's behavior over time, but further analysis would be necessary to draw more specific conclusions about the system dynamics, the interactions between agents, and the potential implications of these interactions.

6. Discussion:Modeling the Relationship between Aggressiveness and Misrecognition

We are considering the application of the concept of Degenerate Elliptic to a model of the relationship between aggressiveness and misrecognition. This concept is related to the classification of partial differential equations (PDEs), where equations meeting certain conditions are referred to as "degenerate elliptic." To apply this concept in the context of opinion dynamics, we propose the following approach:

In essence, we are exploring how to apply the concept of Degenerate Elliptic to a model of the relationship between aggressiveness and misrecognition. This concept is related to the classification of partial differential equations (PDEs), where equations meeting certain conditions are referred to as "degenerate elliptic." To apply this concept in the context of opinion dynamics, we propose the following approach:

Opinion Dynamics Model with Degenerate Elliptic Concept

Definition of Equations

We express the model of opinion dynamics in the form of partial differential equations (PDEs) and design it to satisfy the conditions of Degenerate Elliptic.

$$F(x, O, DO, D^2O) = 0$$

Here, O represents the opinions of opinion leaders, DO represents the gradient of opinions, and D^2O represents the Hessian matrix of opinions (second-order derivatives).

Application of Elliptic Conditions

In the model, we apply specific elliptic conditions to the change in opinions of opinion leaders represented by O.

$$F(x, O, DO, Y) \geq F(x, O, DO, X) \text{ for all symmetric matrices } Y, X$$

This ensures that the equations related to the change in opinions of opinion leaders satisfy the conditions of Degenerate Elliptic.

Modeling Opinion Dynamics as an Elliptic Model

We analyze the dynamics of opinion dynamics using the elliptic model for the change in opinions of opinion leaders.

$$-\Delta O = 0 \text{ where } \Delta \text{ denotes the Laplacian of } O$$

This approach mathematically models the evolution of opinion leaders' opinions over time and helps in understanding how opinion leader opinions are influenced by various factors. Furthermore, this model is valuable for understanding how opinion leader opinions affect the surrounding conditions and factors.

When modeling the relationship between aggressiveness and misrecognition, taking into account the intention of information disseminators and the trust of recipients, the concept of Viscosity Subsolution can be applied to construct a mathematical model as follows:

Modeling Aggressiveness and Misrecognition with Viscosity Subsolution Concept

Definition of Viscosity Subsolutions for Aggressiveness and Misrecognition

In this model, we use the concept of viscosity subsolutions to model the relationship between aggressiveness and misrecognition.

$$F(x_0, A_i(t), DA_i(t), D^2A_i(t)) \leq 0 \quad \forall \phi \geq A_i(t) \text{ neighbor of } x_0$$

Here, $A_i(t)$ represents the aggressiveness index, $DA_i(t)$ represents the gradient of the aggressiveness index (first-order derivative), and $D^2A_i(t)$ represents the Hessian matrix of the aggressiveness index (second-order derivative).

Integration of Information Disseminator's Intention and Recipient's Trust

We model how the intention of information disseminators and the trust of recipients influence the viscosity subsolutions of aggressiveness and misrecognition.

$$F(x_0, I_i(t), DI_i(t), D^2I_i(t)) \leq 0 \quad \forall \phi \geq I_i(t) \text{ neighbor of } x_0$$

$$F(x_0, D_{ij}(t), DD_{ij}(t), D^2D_{ij}(t)) \leq 0 \quad \forall \phi \geq D_{ij}(t) \text{ neighbor of } x_0$$

Here, $I_i(t)$ and $D_{ij}(t)$ represent the intention of information disseminators and the trust of recipients, respectively.

This model mathematically represents the impact of information disseminators' intention and recipients' trust on aggressiveness and misrecognition, and how these elements interact. It is particularly useful for understanding the relationship between opinion leaders and their supporters and how their influence affects communities and external individuals.

When applying the concept of Viscosity Supersolution to model the relationship between aggressiveness and misrecognition, considering the intention of information disseminators and the trust of recipients, a mathematical model can be constructed as follows:

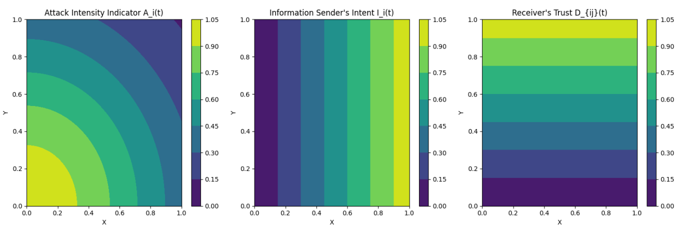

Fig. 3: Attack Intensity Indicator $A_i(t)$, Information Senderś Intent $I_i(t)$, Receiverś Trust $D_{ij}(t)$

Modeling Aggressiveness and Misrecognition with Viscosity Supersolution Concept

Definition of Viscosity Supersolutions for Aggressiveness and Misrecognition

We use the concept of viscosity supersolutions to model the relationship between aggressiveness and misrecognition.

$$F(x_0, A_i(t), DA_i(t), D^2A_i(t)) \geq 0 \quad \forall \phi \leq A_i(t) \text{ neighbor of } x_0$$

Here, $A_i(t)$ represents the aggressiveness index, $DA_i(t)$ represents the gradient of the aggressiveness index (first-order derivative), and $D^2A_i(t)$ represents the Hessian matrix of the aggressiveness index (second-order derivative).

Integration of Information Disseminator's Intention and Recipient's Trust

We model how the intention of information disseminators and the trust of recipients influence the viscosity supersolutions of aggressiveness and misrecognition.

$$F(x_0, I_i(t), DI_i(t), D^2I_i(t)) \geq 0 \quad \forall \phi \leq I_i(t) \text{ neighbor of } x_0$$

$$F(x_0, D_{ij}(t), DD_{ij}(t), D^2D_{ij}(t)) \geq 0 \quad \forall \phi \leq D_{ij}(t) \text{ neighbor of } x_0$$

Here, $I_i(t)$ and $D_{ij}(t)$ represent the intention of information disseminators and the trust of recipients, respectively.

This model mathematically represents the impact of information disseminators' intention and recipients' trust on aggressiveness and misrecognition, and how these elements interact. It is particularly useful for understanding the relationship between opinion leaders and their supporters and how their influence affects communities and external individuals.

series of three 2D contour Results representing different simulation parameters over a twodimensional domain. These parameters are labeled as Attack Intensity Indicator $A_i(t)$, Information Sender's Intent $I_i(t)$, and Receiver's Trust $D_{ij}(t)$.

Attack Intensity Indicator $A_i(t)$

This plot shows the attack intensity indicator as an exponentially decaying function from the origin. The highest values are concentrated at the center (where X and Y are both small), and they fade off toward the edges of the plot. The contour lines are circular and centered, suggesting that the attack intensity is symmetric around the origin.

Information Sender's Intent $I_i(t)$

The intent of information senders is modeled as a linear function of X, independent of Y. The plot has vertical bands of color, each representing a constant value of intent across the Yaxis for a given Xaxis value. The intent increases linearly from left to right, as shown by the color gradient in the contour plot.

Receiver's Trust $D_{ij}(t)$

Receiver's trust is approximated as a linear function of Y, independent of X. The plot has horizontal bands, indicating that trust is uniform along the Xaxis for any given Yaxis value. The trust level increases linearly from bottom to top.

Considerations and Insights

The contour plot of the attack intensity indicator suggests that the risk or severity of an attack decreases as one moves away from the source (origin). The information sender's intent plot indicates that the propensity to convey certain information may be dependent on a particular variable represented by the Xaxis, perhaps signifying a factor like time, influence level, or some other onedimensional metric. The receiver's trust plot suggests that trust is not influenced by the same factor that affects the sender's intent but rather by another factor represented by the Yaxis. The distinct patterns between the three plots indicate that these parameters are influenced by different factors and evolve independently in this simplified model.

Mathematical and Simulation Considerations

The decaying exponential function used for the attack indicator implies that the attack's influence is strongest near the source and diminishes rapidly. The linear functions for intent and trust suggest a direct proportionality with their respective variables, which may not capture more complex or nuanced behaviors often seen in realworld scenarios. While the plots provide a clear visual representation of each parameter, they would need to be coupled with more complex dynamics and interactions for a more realistic simulation. In a more sophisticated model, the interdependencies between these factors would likely be considered, and their representations would not be independent of each other.

Overall, the plots provide an initial visualization of how these parameters might be distributed across a domain. However, for a more detailed and realistic analysis, one would need to incorporate more complex functions and interactions that capture the dependencies and feedback loops between the attack intensity, sender's intent, and receiver's trust.

7. Conclusion:Perron-Ishii's complement, Eikonal Model for Viscosity Solution

Perron-Ishii Method and Viscosity Solutions in Nonlinear PDEs

The Perron-Ishii method is one of the techniques used in constructing viscosity solutions of nonlinear partial differential equations (PDEs), aiding in demonstrating the existence and properties of viscosity supersolutions. It is based on Perron's method and plays a significant role in the analysis of nonlinear PDEs.

Viscosity Supersolution

A viscosity supersolution can be considered as an upper bound solution to a given partial differential equation. According to the definition of a viscosity supersolution, for a function to be a supersolution to a given PDE, it must satisfy the PDE at points where any test function touches it from above.

Perron-Ishii Method

The Perron-Ishii method uses the following steps to demonstrate the existence of solutions:

(1) **Construction of a Family of Supersolutions:** First, construct a family of supersolutions for the considered PDEs. These are functions that satisfy the given PDE.

(2) **Definition of an Upper Bound Function:** Then, define a new function by taking the upper bound (the least upper bound) of this family of supersolutions. This new function is a candidate for being a viscosity solution of the PDE.

(3) **Verification as a Viscosity Supersolution:** Finally, show that the upper bound function is a viscosity supersolution of the PDE. This is typically done by proving that the upper bound function has the necessary properties under certain conditions.

Perron's method is a technique used in classical potential theory, and Ishii applied it in the context of viscosity solutions of PDEs. This method constructs a viscosity solution by taking an upper bound of functions that satisfy the boundary conditions, thus providing a powerful means of demonstrating the existence of solutions.

This method is particularly important in finding solutions to nonlinear and non-convex problems. However, it is abstract, and many technical details are involved in constructing the solution in practice. When applying the Perron-Ishii

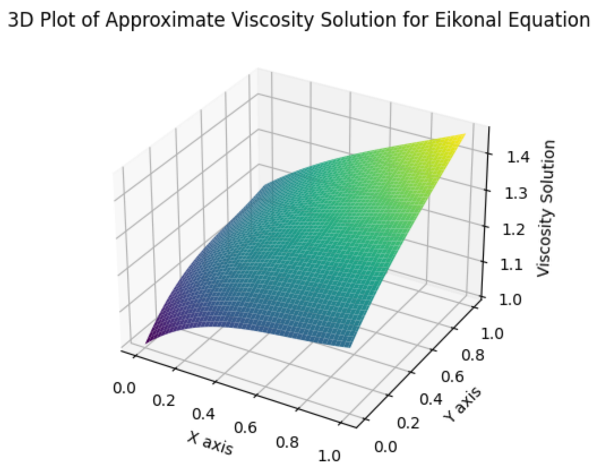

Fig. 4: Approximate Viscosity Solution for Eikonal Equation

method to each specific PDE, fine-tuning according to the characteristics of that equation is necessary.

To illustrate the Perron-Ishii method using formulas, consider the following nonlinear PDE:

$$F(x, u, Du, D^2u) = 0$$

Here, F is a nonlinear operator, u is the unknown function, Du represents the gradient (first-order derivatives) of u, and D^2u denotes the Hessian matrix (second-order derivatives) of u.

By defining a function ‘f(x, y)‘ based on a simplified combination of three approximated factors: an attack indicator ‘A‘, the intentions of information senders ‘I‘, and the trust level of receivers ‘D‘. In the specific case of the Eikonal equation, the function ‘f(x, y)‘ would typically represent the squared speed function that the gradient norm of the solution should match.

Analysis of the 3D Plot

The 3D plot displays the viscosity solution ‘Z‘ as a surface over the domain. The surface appears smooth, which suggests that the combined effect of the three factors ‘A‘, ‘I‘, and ‘D‘ leads to a smooth change in the viscosity solution over the domain. The plot uses the ‘viridis‘ colormap, which typically maps low values to purple and high values to yellow, indicating that the highest values of the viscosity solution are found where the function ‘f‘ is largest.

Considerations for Interpretation

Center of Activity: The peak of the solution is near the origin, indicating the highest 'activity' in terms of the combination of the three factors is there. Linear Influence: The linear increase of ‘I‘ and ‘D‘ with ‘X‘ and ‘Y‘ respectively, suggests that the viscosity solution would also exhibit a linear increase in these directions, but this is modulated by the exponential decay of ‘A‘. Physical Interpretation: In the context of information dynamics, this plot could represent the overall 'state' or 'potential' in a social system where the attack indicator, intentions, and trust levels interact.

Mathematical Rigor

While this plot provides a good visualization, it is essential to note that in a realworld scenario, determining the viscosity solution to a PDE like the Eikonal equation would require rigorous numerical methods that ensure convergence to the true solution. The square root operation used here is a simplification and may not always provide an accurate viscosity solution for more complex or different forms of the Eikonal equation.

In summary, the plot seems to show a wellbehaved approximate solution to an Eikonallike equation, capturing the combined effects of the defined variables. However, for a rigorous analysis, one would need to ensure that the numerical methods used to approximate the solution are valid and that the solution satisfies the necessary conditions for viscosity solutions, which typically involve comparison principles and consistency checks.

Step 1: Construction of a Set of Supersolutions

First, consider a set of supersolutions.

$$S = \{v \in USC(\Omega) : F(x, v, Dv, D^2v) \geq 0 \text{ in } \Omega\}$$

Here, $USC(\Omega)$ denotes the set of upper semicontinuous functions, and Ω is the domain of the functions. S is the set of all possible supersolutions.

Step 2: Definition of an Upper Bound Function

Next, define the upper bound function $\bar{u}$ from the set of supersolutions.

$$\bar{u}(x) = \sup\{v(x) : v \in S\}$$

Step 3: Verification as a Viscosity Supersolution

Finally, it is necessary to verify that the upper bound function $\bar{u}$ is indeed a viscosity supersolution. For verification, use a test function ϕ and demonstrate the following condition.

$$F(x_0, \bar{u}(x_0), D\phi(x_0), D^2\phi(x_0)) \geq 0$$

The test function ϕ is a C^2 function that “touches” $\bar{u}$ and x_0 is the point of contact. If this condition is satisfied, then $\bar{u}$ is a viscosity supersolution.

Notes

> To demonstrate that the upper bound function $\bar{u}$ is actually a viscosity supersolution of the PDE, additional technical conditions must be satisfied. For example, F might need to meet certain continuity or monotonicity conditions.
>
> The Perron-Ishii method depends on how boundary conditions are handled. Proper treatment of boundary conditions ensures the existence and uniqueness of solutions.

This is a mathematical overview of the Perron-Ishii method, but in practice, it requires very advanced analytical techniques. Depending on each PDE, the conditions and verification procedures may vary.

Eikonal Equation and Viscosity Vanishing Method

The Eikonal equation is a nonlinear first-order PDE that appears in geometric optics and Hamilton-Jacobi theory. The equation is as follows:

$$|\nabla u(x)| = f(x)$$

Here, u is the function to be found, ∇u is the gradient (vector of first-order spatial derivatives) of u, and $f(x)$ is a given non-negative function. The Eikonal equation typically appears in contexts such as the propagation of light waves or the shortest path problem.

The viscosity vanishing method is an approach for finding viscosity solutions of nonlinear PDEs, especially Hamilton-Jacobi equations. This method involves adding a small viscous term (a second-order differential term) to regularize the original equation and then considering the limit as this viscous term approaches zero to obtain the viscosity solution of the original equation.

Applying the viscosity vanishing method to the Eikonal equation, consider the following regularized equation:

$$|\nabla u_\epsilon(x)| = f(x), \quad u_\epsilon(x) - \epsilon \Delta u_\epsilon(x) = 0$$

Here, Δu_ϵ is the Laplacian (sum of all second-order spatial derivatives) of u_ϵ, and $\epsilon > 0$ is a small positive parameter representing the viscous term. This regularized equation is expected to be more amenable to finding a viscosity solution, and as ϵ approaches zero, it is expected to converge to the viscosity solution of the original Eikonal equation.

Ultimately, as ϵ approaches zero, we obtain the following viscosity solution:

$$|\nabla u(x)| = f(x)$$

Here, u is the viscosity solution of the Eikonal equation.

This viscosity vanishing method has become a powerful tool in the analysis of nonlinear PDEs, such as Hamilton-Jacobi equations. Viscosity solutions provide an appropriate concept of solution even when the equation does not have smooth solutions or when shock waves or discontinuous solutions appear.

Shock Wave Problem in the Eikonal Equation

The problem of shock waves in the Eikonal equation is generally modeled by the following equation:

$$|\nabla u(x)| = 1 \quad \text{in} \quad \Omega$$

Here, $u(x)$ represents the arrival time of the wavefront, and Ω is the domain (usually the entire space or a subset of it). Shock waves can occur when waves propagate through heterogeneous media with different propagation speeds or through media where the speed varies with distance from the source.

To solve this problem using the viscosity vanishing method, first regularize the Eikonal equation by adding a small viscous term as follows:

$$|\nabla u_\epsilon(x)| = 1, \quad u_\epsilon(x) - \epsilon \Delta u_\epsilon(x) = 0$$

Here, ϵ is a small positive parameter controlling the strength of the viscous term. Due to the presence of the viscous term, this equation is expected to have a smooth solution.

(1) **Formulation of the Regularized Eikonal Equation with Viscous Term:** Set up the above equation.

(2) **Selection of Numerical Approximation Method:** Choose a method for numerically solving this regularized equation. Common methods include finite difference, finite volume, or finite element methods.

(3) **Discretization:** Discretize the equation based on the chosen numerical approximation method. For instance, using the finite difference method, an approximation like the following is made:

$$\frac{u_\epsilon(x+h) - 2u_\epsilon(x) + u_\epsilon(x-h)}{h^2} \approx \Delta u_\epsilon(x)$$

Here, h is the grid step size.

(4) **Iterative Computation:** Perform iterative computations to solve the discretized equation. This can be achieved, for example, using methods like Newton's method or fixed-point

8. Computational Processes for Viscosity Solutions of the Eikonal Equation Using Perron-Ishii Method

Setting Up the Eikonal Equation

The Eikonal equation is given in the form:

$$|\nabla O(x)| = f(x) \quad \text{in}\,\Omega$$

Here, $O(x)$ represents the function to be solved, $f(x)$ is the given speed function, and $\nabla O(x)$ denotes the gradient of O.

Setting Up the Lower and Upper Bound Functions

In the Perron-Ishii method, lower bound (subsolution) and upper bound (supersolution) functions are used to construct the viscosity solution. For the Eikonal equation, rational lower bound function U and upper bound function V are set as:

$$|\nabla U(x)| \leq f(x), \quad |\nabla V(x)| \geq f(x) \quad \text{in}\,\Omega$$

Constructing the Solution Using Perron's Method

The viscosity solution $O(x)$ of the Eikonal equation is constructed using the lower and upper bound functions. The solution is defined as the supremum of all functions between U and V:

$$O(x) = \sup\{W(x) : U(x) \leq W(x) \leq V(x),\, W \text{ is a subsolution}\}$$

Applying Ishii's Lemma

Ishii's Lemma is used to confirm that the constructed function is a viscosity solution of the Eikonal equation. This lemma ensures that a function is a viscosity solution if it is both a viscosity subsolution and a viscosity supersolution.

Verifying the Uniqueness of the Solution

The uniqueness of the viscosity solution of the Eikonal equation is verified using an appropriate comparison principle.

Through this computational process, the viscosity solution of the Eikonal equation is obtained. However, depending on the characteristics of the Eikonal equation and the specific problem setting, this process may be modified. It is also important to note that this is a complex process requiring mathematical insight and expertise.

Viscosity Solutions for Aggression and Misrecognition Models with Trust Integration Using Perron-Ishii Method

Setting Up the Eikonal Equation

Set up the Eikonal equation representing the aggression and misrecognition model with trust integration. For example, the equation for the aggression index A can be:

$$|\nabla A(x)| = f(x)$$

Here, $f(x)$ is a function influencing the aggression index, dependent on factors like the stickiness of opinion leaders, misrecognition, and the intent level of the information disseminator.

Setting Up the Lower and Upper Bound Functions

Define rational lower bound function U and upper bound function V for the aggression index A.

Constructing the Solution Using Perron's Method

Construct the viscosity solution $A(x)$ of the Eikonal equation using the lower bound function U and the upper bound function V. This is defined as the supremum of all functions between U and V.

Applying Ishii's Lemma

Use Ishii's Lemma to confirm that the constructed function is a viscosity solution of the Eikonal equation.

Verifying the Uniqueness of the Solution

Verify the uniqueness of the viscosity solution of the Eikonal equation using an appropriate comparison principle.

This process yields the viscosity solution of the Eikonal equation for the aggression and misrecognition model with trust integration. However, depending on the specific form of the function $f(x)$ and the details of the model, this computational process may be subject to changes. Also, this process requires mathematical expertise, so detailed analysis and calculations may need the advice of mathematics experts.

Viscosity Solutions for Eikonal Equation with Randomness in Parameters

Setting Up the Eikonal Equation

Set up the Eikonal equation:

$$|\nabla O(x)| = f(x) \quad \text{in}\,\Omega$$

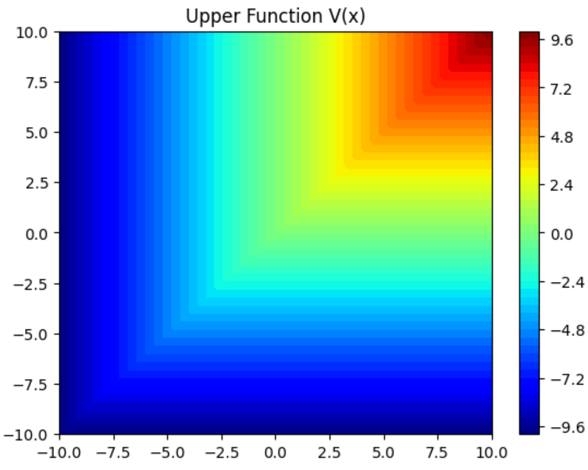

Fig. 5: Upper Function V(x)

Here, $f(x)$ is the given speed function, and parameters are generated using randomness.

Setting Up the Lower and Upper Bound Functions

Set parameters using randomness for the lower bound function $U(x)$ and the upper bound function $V(x)$. Ensure that the lower bound function is always smaller, and the upper bound function is always larger than $f(x)$:

$$|\nabla U(x)| \leq f(x) + \epsilon_1(x), \quad |\nabla V(x)| \geq f(x) - \epsilon_2(x)$$

Here, $\epsilon_1(x)$ and $\epsilon_2(x)$ represent small deviations due to randomness.

Constructing the Solution Using Perron's Method

Construct the viscosity solution $O(x)$ of the Eikonal equation as the supremum of all functions between $U(x)$ and $V(x)$:

$$O(x) = \sup\{W(x) : U(x) \leq W(x) \leq V(x),\ W \text{ is a subsolution}\}$$

Applying Ishii's Lemma

Use Ishii's Lemma to confirm that the constructed function is a viscosity solution of the Eikonal equation.

Ishii's Lemma is a result in the theory of viscosity solutions

Particularly for second-order partial differential equations. It provides a way to handle the comparison between a test function and a viscosity solution at points where the test function touches the viscosity solution from above or below. This is crucial in establishing uniqueness and stability properties of viscosity solutions.

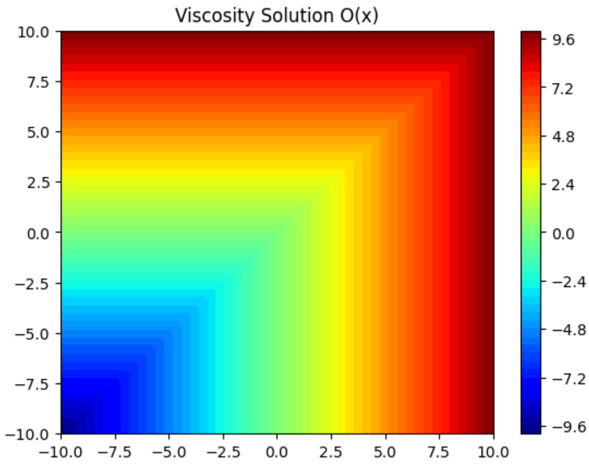

Fig. 6: Viscosity Solution O(x)

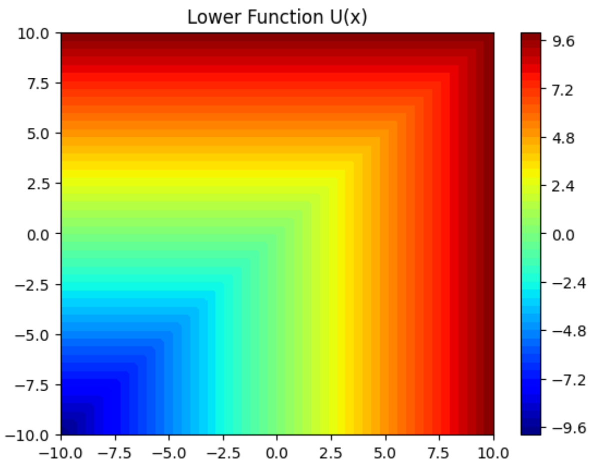

Fig. 7: Lower Function U(x)

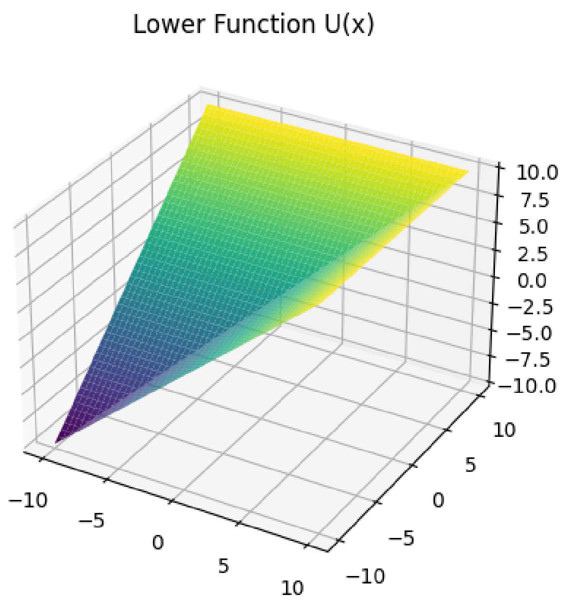

Fig. 8: Lower Function U(x)

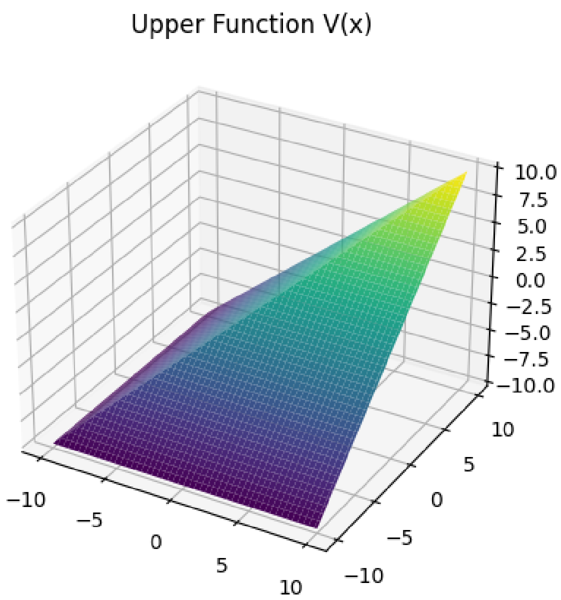

Fig. 9: Upper Function V(x)

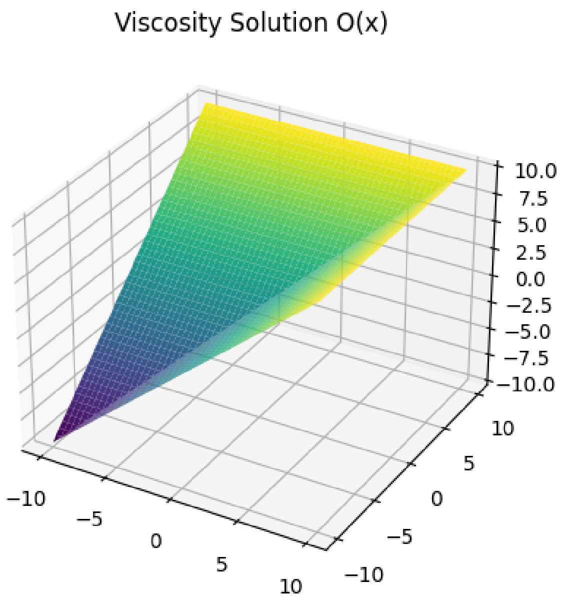

Fig. 10: Viscosity Solution O(x)

Upper Function V(x)

The heatmap for 'Upper Function V(x)' shows a gradient of colors from blue to red, where blue represents lower values and red represents higher values. The function seems to increase linearly from the bottom left corner to the top right corner. If we consider this in the context of a PDE, 'V(x)' could represent an upper bound or a supersolution to the PDE.

ower Function U(x)

Similarly, the heatmap for 'Lower Function U(x)' shows a gradient from blue to red, increasing from the top left corner to the bottom right corner. In PDE terms, 'U(x)' could represent a lower bound or a subsolution to the PDE.

Viscosity Solution O(x)

The heatmap for 'Viscosity Solution O(x)' seems to represent the maximum of the two functions 'U(x)' and 'V(x)' at each point, as suggested by the provided code snippet. This is a common approach in viscosity solution theory to combine subsolutions and supersolutions to form a candidate viscosity solution. The colors suggest that 'O(x)' takes on the higher values from either 'U(x)' or 'V(x)' at each point.

Considerations Based on Heatmaps

The heatmaps show a clear continuous change in values, indicating that the functions are at least continuous with respect to both spatial dimensions. The transition from blue to red in the 'Viscosity Solution O(x)' heatmap indicates that the viscosity solution is indeed capturing the maximum values between the 'Lower Function U(x)' and 'Upper Function V(x)', as expected from the theory of viscosity solutions. The smoothness of the color transitions suggests that the gradients of these functions would also be continuous, which is important for the Eikonal equation that requires the gradient norm to be equal to a given speed function.

Subsolution Verification

Mathematically, we would need to verify that for any smooth test function that touches 'O(x)' from above, the Eikonal condition is satisfied from above.

Supersolution Verification

Similarly, for any test function that touches 'O(x)' from below, the Eikonal condition should be satisfied from below.

Uniqueness

The visualizations alone do not confirm the uniqueness of the solution. This would require a comparison principle argument, showing that if there were two distinct viscosity solutions, they would necessarily be equal.

Stability and Convergence

In numerical analysis, stability and convergence are critical. The visualizations suggest a stable solution as there are no apparent anomalies like oscillations or discontinuities, which are common numerical issues.

In summary, the visualizations align well with what one would expect for upper and lower bounds and a viscosity solution in the context of the Eikonal equation. However, rigorous mathematical checks are necessary to confirm that $O(x)$ is indeed a viscosity solution. The heatmaps provide an intuitive understanding of the functions' behavior across the domain but do not substitute for the necessary mathematical proofs.

Verifying the Uniqueness of the Solution

Verify the solution's uniqueness using an appropriate comparison principle. This principle is used to confirm that different solution candidates satisfying the same conditions are identical.

Through this computational process, the viscosity solution of the Eikonal equation can be determined. However, using randomness can result in some variability in the results, so selecting an appropriate range and precision of randomness is important. Additionally, this process is theoretical, and actual calculations may require numerical analysis expertise. In considering the application of viscosity solutions of the Eikonal equation using the Perron-Ishii method in opinion dynamics, the following cases and scenarios might be inferred:

Case Study 1: Information Spread on Social Media

Problem Setting

The spread of information on social media is shaped by the opinion dynamics among users. This involves modeling the process through which information about a particular topic spreads via exchanges of opinions and influences among users.

Application of the Eikonal Equation Using Perron-Ishii Method

In this scenario, the Eikonal equation represents the speed and direction of opinion diffusion. The speed of information spread $f(x)$ varies depending on factors like the influence of opinion leaders, media exposure, or the urgency of the topic.

Utilization of the Viscosity Solution

The viscosity solution demonstrates how specific information spreads among certain user groups and is eventually accepted as a general opinion. This analysis helps understand how particular information might exert social influence.

Case Study 2: Impact and Mitigation of Fake News

Problem Setting

Exploring how fake news and misinformation spread in online communities and how to minimize their impact.

Application of the Eikonal Equation Using Perron-Ishii Method

The Eikonal equation is used to quantify the spread rate and patterns of fake news. Here, $f(x)$ depends on the persuasiveness of the fake news and the emotional reactions associated with the topic.

Utilization of the Viscosity Solution

The viscosity solution shows how certain communities respond to fake news and how they counteract it. This information is useful in developing strategies to prevent the spread of fake news.

Case Study 3: Convergence of Social Opinions

Problem Setting

Understanding the convergence or divergence of opinions on major social issues (e.g., political events or public health crises).

Application of the Eikonal Equation Using Perron-Ishii Method

The Eikonal equation represents the speed of formation and change of opinions. The convergence speed of opinions $f(x)$ is influenced by factors such as media coverage, expert opinions, and political leadership.

Utilization of the Viscosity Solution

The viscosity solution illustrates how opinions on specific social issues are formed and change over time. This helps understand patterns of how social opinions either converge or diverge.

In these case studies, the concept of viscosity solutions of the Eikonal equation using the Perron-Ishii method enables us to quantitatively capture the complex interactions and outcomes in opinion dynamics. This allows for a better understanding of social phenomena and aids in developing more effective communication strategies and policy-making.

Aknowlegement

The author is grateful for discussion with Prof. Serge Galam and Prof.Akira Ishii.

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