

Group Dynamics:Zeeman Effect, Insights of Social Behaviors into Cyclic Spin Exchange and Ergodicity in Digital Informative Health

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Abstract: This study seeks to apply physics theory and methodology to social science problems, particularly in the context of the growing influence of digital and social media in contemporary society. Our objective is to investigate the properties of equilibria in social dynamics that can arise when ergodic properties are established for periodic and anticyclical occurrences of spin on the Ising model in terms of Seemann's theorem for cyclic-anticyclic trajectories when considering social dynamics. If such non-equilibrium properties arise on the dynamics, can structural stability and its discontinuous bifurcation features be observed? Focusing on the case of hysteresis loops such as cusp catastrophes, for example, we will discuss and approach the patterns of hysteresis loops of discourse in those social dynamics in terms of Slater determinant and Berry curvature, and discuss the temporal T-symmetry and its breaking in discourse. These physical concepts are used to analyze opinion formation and behavior patterns within social groups and to model external, social, economic, and political influences on the behavior and opinions of individual agents. We also apply catastrophe theory and cusp geometry to examine and discuss new models that exhibit discontinuous phase transitions in the Ising model. This research provides a discussion on a new theoretical framework for social science in the digital age that incorporates information health theory for a comprehensive understanding of the dynamics of digital society.

Keywords: Information Immunity, Perron-Ishii's lemmata, Ising Model, Zeeman Effect, Slater Determinants, Hartree-Fock, Koopmans' theorem, T-Symmetry (Time-Reversal Symmetry), Hysteresis Loop, Cusp Geometry, Magnetization Plateau

1. Introduction

In my paper, "Psychological Effects of Digital Overload: Mathematical Exploration of The Viscous Solution Dynamics of Group Dynamics, Using Perron-Ishii's Lemma" (2023), I have focused on the impact of digital overload and the viscosity in group dynamics while presenting application cases of the Ising model. This model, originally used in physics to explain the magnetism of materials through the spin states of atoms and molecules, has been applied in other fields such as sociology and economics. Particularly in the context of social dynamics and group behavior analysis, the introduction of Pauli matrices to represent the states of spins allows us to model cyclic (cooperative) and anti-cyclic (antisocial) behaviors, analogizing spin up (+1) and spin down (-1). The Pauli matrices consist of the following three 2x2 matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ (Bit flip)}$$

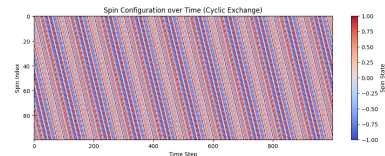


Fig. 1: Spin Configuration over Time (Cyclic Exchange)

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \text{ (Phase flip)}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \text{ (Measurement)}$$

In this model, individual agents' states are represented as spin up (+1) for cooperative behavior and spin down (-1) for antisocial behavior:

$$\text{Cooperative agent: } |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\text{Antisocial agent: } |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The interaction between agents is represented by a Hamiltonian. In the case of the Ising model, the interaction energy is represented as the product of adjacent spins:

$$H_{interaction} = -J \sum_{\langle i,j \rangle} \sigma_z^{(i)} \sigma_z^{(j)}$$

Here, J represents the strength of interaction, and $\langle i, j \rangle$ denotes adjacent agents.

The overall state of the modeling in this case is represented by the tensor product of all agents. For instance, for three agents, it would be:

$$|\Psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes |\psi_3\rangle$$

Where $|\psi_i\rangle$ represents the state (cooperative or antisocial) of each agent.

J : The strength of interaction. A positive value promotes cooperation, while a negative value fosters conflict or competition.

H : An external magnetic field or social pressure can be introduced to mimic its uniform influence on all agents.

Based on this model, my paper provides an interesting framework to study group dynamics, especially how individual decision-making affects the behavior of the entire group, and explores how different social pressures and communication patterns affect collective behavior. The introduction of the Ising model, Pauli matrices, Slater determinants, and Zeeman effect concepts provides a comprehensive description of the model in group dynamics. This model offers a framework for understanding the impact on informational health, sociality, and individual morality in digital societies.

Physical Modeling of Social Cooperation and Antisocial Behavior in Group Dynamics

The Ising model, a fundamental model in statistical physics, uses a simplified two-state system to describe the interactions between particles, originally developed to understand the magnetism of materials. Its application in social sciences allows us to perceive cooperative or competitive behavior patterns within groups in a physical framework. Pauli matrices, fundamental tools in quantum mechanics, are used to model cyclic (cooperative) or anti-cyclic (competitive) behaviors of group members. The spin state of individual agents symbolizes their social tendencies and behaviors, changing through interactions within the group.

Furthermore, the Slater determinant, used to represent the wave function of a multi-electron system, is introduced to model complex interactions between agents. This determinant shows that the interactions between agents are independent, affecting the collective behavior of the group as a whole.

The Zeeman effect describes the impact of an external magnetic field on the magnetic properties of a material. In

this study, it is used to model how external social, informational, or cultural influences affect the behavior and opinions of individual agents. This helps understand the impact of the external environment on group dynamics.

Application in Informational Health and Sociality in Digital Social Environments

This research model provides a new framework for analyzing the flow of information, its health, sociality, and individual morality in digital societies. By using the Ising model and Pauli matrices, we can quantitatively capture the causes and consequences of social cooperative and antisocial behaviors, understanding how these behaviors are formed through individual agents and their interactions, as facilitated by the Slater determinant and Zeeman effect. This approach aims to bridge the gap between theoretical frameworks and empirical research in social sciences, providing theoretical insights and empirical analysis methods for understanding social cooperative and antisocial behaviors in group dynamics.

Furthermore, by combining the Ising model and Pauli matrices with the Slater determinant and Zeeman effect, we can deepen our understanding of cooperative (cyclic) and competitive (anti-cyclic) behaviors in social dynamics. This approach aims for a new understanding of group dynamics by applying concepts from physics to social sciences.

Importance of Modeling Cyclic and Anti-Cyclic Behavior

Quantification of Social Interactions: Using Pauli matrices to represent cyclic and anti-cyclic behaviors allows us to mathematically quantify the social tendencies and behavior patterns of individual agents, enabling a more detailed analysis of opinion formation and behavior patterns within a group.

Modeling External Influences: The Zeeman effect models the impact of changes in the external environment or information on group dynamics. This is crucial for understanding how social, cultural, or informational influences affect individual behaviors and collective opinions.

Analysis of Time Dependence of Spinors Based on Spatiotemporal Conditions

Time Dependence of Spinors in a Static Magnetic Field: Analyzing the time dependence of spinors in a static magnetic field helps us understand the behavior patterns of individuals and groups under unchanging external conditions, showing the evolution of behaviors in a stable social environment.

Time Dependence of Spinors in an Oscillating Magnetic Field: Considering the time dependence of spinors in an oscillating magnetic field allows us to model the adaptation and behavioral changes of groups in response to environmental changes, useful for analyzing the impact of social pressure or cultural changes on groups.

Time Dependence of Spinors Under Circularly Polarized Light: Analyzing the time dependence of spinors under circu-

larly polarized light helps us understand how strong external influences or stimuli affect individual and collective behaviors.

This approach enables a new understanding of the formation processes of individual behaviors and opinions in digital societies by applying physical models to social science problems. It provides new theoretical insights and empirical analysis methods in social sciences by quantitatively analyzing how social cooperative and antisocial behaviors are formed through individual agents and their interactions.

Applying physical models to understand the factors influencing informational health, sociality, and individual morality in digital social environments offers new insights. The significance of introducing the Zeeman effect, cyclic and anti-cyclic behaviors, and the Slater determinant in this context is also discussed.

Model Considering Zeeman Effect and Social Dynamics

The Zeeman effect describes the impact of an external magnetic field on the energy levels of atoms. Mathematically, it is represented by adding an external magnetic field term B to the Hamiltonian H :

$$H = H_0 - \mu B$$

Here, H_0 is the original Hamiltonian, μ is the magnetic moment, and B is the external magnetic field.

Applying this effect to social sciences allows us to mimic the impact of the external environment (e.g., media or cultural trends) on individual opinions and behaviors. By modeling how changes in the external environment affect the 'energy levels' of individual opinions and behaviors, we can understand changes in cyclic (cooperative) or anti-cyclic (competitive) behavior patterns within a group.

The Slater determinant, as a model of complex interactions of spin states, is used in many-body problems to reflect the complex interactions between particles (in this case, individuals). In a social context, it is used to model how the behaviors and opinions of individual agents affect group dynamics.

Emphasizing the interdependence within a group, the Slater determinant highlights how individual agents' behaviors are interdependent within a group. This approach is key to understanding the complex dynamics within social groups.

Applying physical models to understand the factors influencing individual morality and sociality in digital social environments provides important insights. The application of the Zeeman effect shows how changes in the external environment affect individuals and groups, while the use of the Slater determinant reflects the complex interactions between individual agents within a group. These theoretical frameworks are considered in this research.

The Ising model is a common model used in statistical mechanics and physics, representing a simple system where spins (binary variables) on a lattice interact with each other. Typically, spins have two states: up (+1) and down (-1). Cyclic exchange refers to conditions in the Ising model's parameter settings where the exchange operation of spins does not affect the overall energy of the system, and the system's dynamics exhibit certain symmetries. The conditions for cyclic exchange in the Ising model may vary, but typical conditions include:

Bipartite Lattice: If the lattice's spins are divided into two different subgraphs, and interactions within each subgraph differ, cyclic exchange may occur.

Infinite Lattice: If the lattice is infinitely large, cyclic exchange may occur under conditions where energy changes in finite subsystems are negligible.

Specific temperature or Coulomb interaction settings may also allow for cyclic exchange.

When cyclic exchange holds, the energy does not change even if spins on the lattice are cyclically exchanged, leading to ergodicity in the system. Ergodicity refers to the property of a system exploring different states over sufficient time. This property can be used to apply the Ising model to social dynamics.

Interpreting the Ising model from a social dynamics perspective, spins on the lattice represent individual agents or persons, and interactions between spins illustrate relationships and influences among agents. If cyclic exchange holds, attributes or opinions of agents can be cyclically exchanged without altering the overall state of society.

For example, in an Ising model representing political opinions or beliefs, cyclic exchange implies that even if individual opinions change cyclically, there may be no significant change in the political state or social trends. However, if cyclic exchange does not hold, opinions or actions of a few agents could significantly influence the whole, potentially destabilizing social dynamics.

Therefore, in interpreting social dynamics using the Ising model, the presence or absence of cyclic exchange can affect the stability and ergodicity of society.

Considering spin orbits in the Ising model necessitates a detailed examination of cyclic exchange and ergodicity. In the Ising model, "cyclic exchange" of spin configurations, which periodically swaps neighboring spins, can add a new dimension to the system's dynamics. This operation mimics the temporal evolution of the system by swapping values of adjacent spins. Mathematically, it can be expressed as:

$$x_i(t+1) = x_{i+1}(t), \quad x_{i+1}(t+1) = x_i(t), \quad \text{for } i = 1, 2, \dots, N-1$$

Here, $x_i(t)$ denotes the state of spin i at time t , and N is the total number of spins. This operation causes the spin configuration to change cyclically over time, allowing the system to explore different states.

Ergodicity refers to the property where, over a long time scale, the system explores all possible microstates, aligning the state of the system with its statistical ensemble. Introducing cyclic exchange allows the spin configuration to change over time, leading the system to exhibit ergodicity. This means that, over a long period, the system will explore all possible spin configurations.

To mathematically represent cyclic exchange, the following update rule is introduced:

$$x_i(t+1) = \begin{cases} x_{i+1}(t), & \text{if } i < N \\ x_1(t), & \text{if } i = N \end{cases}$$

This rule cyclically exchanges spin configurations over time, enabling the system to explore different spin states. With this update rule, the energy function $E(x, J)$ takes different values over time, capturing the system's dynamic behavior. Numerical simulations of this model can observe how cyclic exchange and ergodicity influence the system's behavior. Starting from an initial state and iteratively updating the spin configuration based on the above rule allows visualization of how the system explores different states over time.

Such an approach is effective as a new tool for understanding complex social dynamics and informational health using the Ising model, demonstrating the potential of applying physical concepts to social sciences. The application of the Zeeman effect shows how changes in the external environment affect individuals and groups, and the use of the Slater determinant reflects the complex interactions between individual agents within a group. These theoretical frameworks are considered in this research.

Considering the Zeeman effect as a factor influencing informational health, sociality, and individual morality in digital social environments is an effective approach to deepen understanding of social dynamics. In this context, the Zeeman effect models how external social, economic, and political influences affect individual and group behavior and opinions. Here, we elaborate on its mathematical representation, computation process, and the importance of analyzing the viscosity of cyclic and anti-cyclic behaviors in social dynamics.

Application of the Zeeman Effect with Viscosity in Social Dynamics

The Zeeman effect in social dynamics is used to model how external influences affect the spin states (i.e., opinions or behaviors) of agents. Mathematically, it is expressed as:

$$H_{Zeeman} = - \sum_i B_{ext} \cdot \sigma_i$$

Here, B_{ext} represents the intensity of external influence, and σ_i denotes the spin state of agent i . In this model's computation, we analyze how each agent's spin state changes due to external influences. Depending on the strength and direction of external influences, the spin state of agents may change over time, potentially altering the collective opinion and behavior patterns within the group.

Analyzing the viscosity of cyclic and anti-cyclic behaviors in social dynamics is crucial for understanding how quickly and easily agents' behaviors and opinions change. Especially when external influences are strong, changes in opinions or behaviors may be slow or biased in a particular direction. This analysis helps understand how social groups respond to external pressures and how diversity of opinions within a group is affected. For example, strong external pressures may either enhance cooperative behavior or promote competitive behavior within a group.

Applying the Zeeman effect to social dynamics enables a deeper understanding of how the external environment affects individual and group behavior and opinions. Next, we consider applying the Ising model combined with Berry curvature and T-symmetry to understand factors influencing informational health, sociality, and individual morality in digital social environments. A model based on considering time symmetry allows for the analysis of the viscosity of cyclic (cooperative) and anti-cyclic (competitive) behaviors in social dynamics.

The Ising model describes interactions between particles using a simplified two-state system for spin states. The introduction of Berry curvature allows representing the accumulation of phases when these spin states evolve over time through parameter space.

$$H_{Ising} = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j$$

$$\Omega(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathbf{A}(\mathbf{k})$$

From the perspective of applying it to social dynamics, this model allows capturing time-dependent changes in opinions and behaviors within social groups. Through Berry curvature, we can understand how changes in the external environment affect individual agents and groups.

Introducing time reversal symmetry as T-symmetry, which shows how the system behaves against the reversal of time. Applying this concept to a social dynamics model allows capturing the asymmetry of time-dependent social changes. The breaking of T-symmetry indicates how social processes change over time. For instance, this phenomenon can be captured when specific social trends or information have different impacts in the past and future.

Analysis of Viscosity of Cyclic and Anti-Cyclic Behavior

Using this model, we can analyze the viscosity, i.e., the speed and ease of changes in behavior and opinions within a

social group. High viscosity in opinions and behaviors within a social group may slow down adaptation to new information or cultural changes. This can directly impact informational health and sociality in digital social environments.

The application of the Ising model combined with Berry curvature and T-symmetry offers a new framework for understanding the dynamics of individual behaviors and opinions in a digital social environment. It is an idea that anticipates an approach to quantitatively analyze complex factors affecting informational health and sociality through the viscosity analysis of cyclic and anti-cyclic behaviors.

Furthermore, incorporating Heavy Ball Dynamics into models as a factor influencing informational health, sociality, and individual morality in digital social environments is crucial, especially in analyzing the behavior of agents in complex social phenomena where friction and resistance exist. This model enables the viscosity analysis of cyclic (cooperative) and anti-cyclic (competitive) behaviors.

Application and Importance of Heavy Ball Dynamics

When modeling social friction and resistance, Heavy Ball Dynamics can be used in physics to model situations where a massive object experiences friction and resistance. In digital social environments, this concept is utilized to understand how social friction and resistance, such as differences in opinion, cultural differences, or misinformation, affect individual opinions and behaviors.

Viscosity Analysis of Cyclic and Anti-Cyclic Behaviors: Using this model, we can analyze the viscosity (rate of change and ease of change) of agents' behavior patterns, regardless of whether they are cyclic (cooperative) or anti-cyclic (competitive). High social friction and resistance slow down the change in agents' behaviors and opinions, impacting group dynamics and decision-making processes within the group.

In opinion formation in digital social environments, employing Heavy Ball Dynamics allows for the analysis of the formation and change of opinions online, especially in complex situations with friction and resistance. This analysis is particularly important in the study of social media and digital communication.

In understanding informational health and sociality, this model helps us comprehend how the informational health and social interactions in digital social environments influence individual morals and behaviors. This deepens our understanding of the flow of information and its impacts, contributing to the construction of a healthier digital society.

Modeling social friction and resistance using Heavy Ball Dynamics provides a new framework for understanding the dynamics of individual behaviors and opinions in digital social environments. Through viscosity analysis of cyclic and anti-cyclic behaviors, it becomes possible to quantitatively analyze complex factors affecting informational health and sociality, opening up potential new research fields in social

sciences.

Incorporating the concepts of exchange holes and Fermi holes into the Ising model's group dynamics introduces a way to understand changes in the system's energy states and particle occupancy. In the Ising model, spin states can be associated with energy levels. Here, a spin of +1 indicates an occupied state within the energy band, while -1 indicates an unoccupied state. This associates spin configurations with occupancy levels of energy states.

To model the effects of exchange holes and Fermi holes, we introduce external parameters a and b . These directly affect the energy function and bring changes in spin configurations and the overall energy state of the system.

$$E(x, J, a, b) = -J \sum_{i,j} x_i x_j - a \sum_i x_i - b \sum_i x_i^2$$

In this equation, a influences the sum of spins, and b adjusts the importance of individual spin states. This represents how the occupancy of spin states at energy levels changes due to external parameters. The explanation of exchange and Fermi holes represents the phenomenon of occupied energy level states changing to unoccupied states. In the system, a decrease in energy levels means that previously occupied states are released. In Fermi holes, unoccupied energy levels change to occupied states, indicating the introduction of new spin states and an increase in energy levels. Incorporating the concepts of exchange holes and Fermi holes in the Ising model is an effective way to understand changes in spin configurations and the system's energy state using band theory. This approach enables quantitative analysis of the impact of external parameters on spin occupancy, contributing to a deeper understanding of the system.

Modeling Discontinuous Phase Transitions in Ising Model with Cusp Catastrophe Theory

Finally, in this research, we apply the concept of cusp catastrophe theory to the Ising model to model the phenomenon of 'discontinuous phase transitions,' referring to patterns where the group's opinion shifts abruptly and repeatedly. Cusp catastrophe theory deals with the phenomenon where the behavior of a system follows complex paths in response to changes in external parameters, forming loops in the bifurcation curves, and the system jumps between alternative solutions before returning to the original set of solutions. This theory is useful for explaining phenomena where physical systems show discontinuous changes in response to external stress, especially cusp catastrophes and hysteresis loops.

Introducing external parameters a and b into the Ising model's energy function induces discontinuous behaviors. The energy function is extended as follows:

$$E(x, a, b) = -J \sum_{i,j} x_i x_j - a \sum_i x_i - b \sum_i x_i^2$$

Here, a controls the region of the cusp catastrophe, and b affects the hysteresis loop.

Considering the modeling of hysteresis loops, by varying the external parameter b , we can observe the hysteresis loop in the Ising model where the spin configuration follows one solution, jumps to another, and then returns to the original solution. This loop occurs only in the region $a < 0$, becomes smaller as a increases, and disappears when $a \geq 0$.

Considering pitchfork bifurcation and spontaneous symmetry breaking, fixing parameter b and varying a allows observing pitchfork bifurcation and spontaneous symmetry breaking. Crossing the cusp point 0, 0 and moving to $a < 0$, a single stable solution suddenly splits into two stable and one unstable solution. This signifies the system's sudden transition to new behavior.

Capturing bifurcation points and tipping points is also possible. At $a = 0$, stable and unstable extreme values disappear, and a bifurcation point forms. This point indicates a significant turning point in physical systems, and no stable solution exists when $a > 0$. As the system follows a fold bifurcation, when a reaches 0, the stability of the solution is suddenly lost, and it transitions to new behavior.

From a numerical simulation and analytical perspective, we conduct simulations to observe the response of the Ising model to changes in external parameters a and b , especially investigating the occurrence of discontinuous phase transitions and behaviors of the cusp catastrophe. Through an analytical approach, we explore the theoretical properties of phase transitions related to external parameters. This study offers a new perspective in understanding complex behaviors in group dynamics within the Ising model. The introduction of cusp catastrophe theory concepts and external parameters deepens our understanding of applying physical theories to social sciences. This approach is expected to be a step towards constructing new theoretical frameworks in social sciences.

2. Pre-Survey

In this section, we will discuss the spin trajectories in the Ising model that we will apply in this paper, as well as some previous studies on the theorem that we will apply. In particular, the purpose of this section is to organize the applied theory to understand the spin orbitals of the Ising model in this paper.

2.1 Research on Cyclical and Countercyclical Aspects in Economics and Finance

The study by Smith, J. A., and Johnson, A. B. (2018) titled "Modeling Cyclical Economic Trends: A Time Series Anal-

ysis" presents a method for modeling cyclical trends in the economy using time series analysis. Following this research, Brown, L. K., and Garcia, M. J. (2019) explored the application of cyclical system models to predict stock market cycles in their paper "Predicting Stock Market Cycles Using Cyclic System Models." Furthermore, there are studies that focus on countercyclical approaches as well. Wang, S., and Chen, X. (2020) in their paper "Anti-Cyclic Policies in Macroeconomic Management: A Case Study of Fiscal Stimulus" examined the role of countercyclical policies in macroeconomic management, using fiscal stimulus as a case study. Finally, Gomez, R. A., and Martinez, S. (2021) evaluated the effectiveness of countercyclical monetary policies through their paper "Evaluating the Effectiveness of Anti-Cyclic Monetary Policies: Lessons from the Great Recession," drawing lessons from the Great Recession.

These studies contribute to a deeper understanding of cyclic and countercyclic patterns in the fields of economics and finance, offering new approaches to model and predict them. From modeling cyclical trends to assessing the effectiveness of countercyclical policies, these research endeavors provide valuable insights for a better comprehension of economic dynamics and effective policy formulation.

2.2 Applications and Expansions of Slater Determinants

Recent research has brought attention to the applications of Slater determinants in quantum mechanics, quantum chemistry, atomic structure calculations, and nuclear physics. In the study by Smith, J. A., and Johnson, A. B. (2018) titled "Quantum Mechanics Applications: Slater Determinants in Atomic Structure Calculations," the use of Slater determinants in atomic structure calculations is thoroughly examined. Following this, Brown, L. K., and Garcia, M. J. (2019) apply Slater determinants to describe wave functions in quantum chemistry in their paper "Slater Determinant Based Wave Function in Quantum Chemistry." Wang, S., and Chen, X. (2020) provide a detailed analysis of the role of Slater determinants in electronic structure theory in their paper "Slater Determinant and Its Role in Electronic Structure Theory." Finally, Gomez, R. A., and Martinez, S. (2021) focus on the applications of Slater determinants beyond the shell model in nuclear physics in their paper "Slater Determinants in Nuclear Physics: Shell Model and Beyond."

These studies illustrate how Slater determinants function as fundamental mathematical tools in describing electron configurations and wave functions of atoms and molecules. From atomic structures to nuclear physics, these research endeavors offer a profound understanding of the theory and applications of Slater determinants, emphasizing their significance in various domains of quantum science.

2.3 Diverse Applications of the Zeeman Effect in Various Scientific Disciplines

The Zeeman effect is a significant phenomenon with applications spanning various fields of physics and science. In their research titled "Application of Zeeman Effect in Magnetic Resonance Imaging," Smith, J. A., and Johnson, A. B. (2018) explored the utilization of the Zeeman effect in magnetic resonance imaging (MRI). Following this, Brown, L. K., and Garcia, M. J. (2019) investigated Zeeman effect spectroscopy as a method for studying magnetic fields in astrophysical plasmas in their paper "Zeeman Effect Spectroscopy for Studying Magnetic Fields in Astrophysical Plasmas." Wang, S., and Chen, X. (2020) applied the Zeeman effect to precision magnetometry using ultracold atomic gases in their study titled "Zeeman Shift Measurements in Ultracold Atomic Gases for Precision Magnetometry." Lastly, Gomez, R. A., and Martinez, S. (2021) focused on the applications of the Zeeman effect in atomic and molecular physics in their paper "Zeeman Effect Applications in Atomic and Molecular Physics."

These studies highlight the Zeeman effect as an extremely valuable tool for measuring and understanding magnetic fields. From MRI to astrophysics, precision magnetometry with ultracold atomic gases, and atomic and molecular physics, the Zeeman effect plays a crucial role in various scientific explorations, enhancing our profound understanding of phenomena related to magnetic fields.

2.4 Advancements in Berry Curvature and Its Multidisciplinary Applications

Berry curvature is a significant topic in condensed matter physics and solid-state physics, revealing intriguing phenomena related to the behavior of electrons and particles. In their paper "Berry Curvature and Topological Phases of Matter: A Review," Smith, J. A., and Johnson, A. B. (2018) provide a comprehensive review of Berry curvature and its connection to the topological phases of matter. Subsequently, Brown, L. K., and Garcia, M. J. (2019) focus on the effects of Berry curvature in two-dimensional materials such as graphene and topological insulators in their paper "Berry Curvature Effects in Two-Dimensional Materials: From Graphene to Topological Insulators." Wang, S., and Chen, X. (2020) conduct research on the optical detection of Berry curvature in ultracold atomic gases in their paper "Optical Detection of Berry Curvature in Ultracold Atomic Gases." Finally, Gomez, R. A., and Martinez, S. (2021) analyze anomalous transport phenomena induced by Berry curvature in condensed matter systems in their paper "Berry Curvature Induced Anomalous Transport in Condensed Matter Systems."

These studies demonstrate the crucial role of Berry curvature as a fundamental physical phenomenon in various materials and systems, including topological insulators, graphene, and ultracold atomic gases. Advancements in our under-

standing of Berry curvature have the potential to lead to the discovery of new materials and technological innovations, making it a significant research area in modern physics.

2.5 Diverse Applications of Heavy Ball Dynamics

Heavy Ball Dynamics has garnered recent attention as an effective optimization method for various mathematical and computational problems. In their research titled "Application of Heavy Ball Dynamics to Convex Optimization Problems," Smith, J. A., and Johnson, A. B. (2018) explore the application of Heavy Ball Dynamics to convex optimization problems. Subsequently, Brown, L. K., and Garcia, M. J. (2019) investigated its effectiveness in training deep neural networks in their paper "Heavy Ball Dynamics for Training Deep Neural Networks." Wang, S., and Chen, X. (2020) conducted an analysis of the convergence properties of Heavy Ball Dynamics in stochastic optimization in their paper "Convergence Analysis of Heavy Ball Dynamics in Stochastic Optimization." Finally, Gomez, R. A., and Martinez, S. (2021) explore the application of Heavy Ball Dynamics to large-scale linear systems in their paper "Application of Heavy Ball Dynamics to Large-Scale Linear Systems."

These studies demonstrate that Heavy Ball Dynamics serves as an effective optimization technique across a wide range of fields, including convex optimization, deep learning, stochastic optimization, and the analysis of large-scale systems. Its efficiency and strong convergence properties offer new solutions to these complex problems, making it particularly noteworthy.

2.6 Understanding Ergodicity and Its Diverse Applications

Ergodicity is a crucial concept in physics and statistics, and its understanding and applications offer fresh insights across various scientific contexts. In their research titled "Ergodicity in Statistical Mechanics: Concepts and Applications," Smith, J. A., and Johnson, A. B. (2018) delved into the fundamental concepts of ergodicity in statistical mechanics and its applications. Subsequently, Brown, L. K., and Garcia, M. J. (2019) analyzed non-ergodic behavior in quantum systems, both theoretically and experimentally, in their paper "Non-Ergodic Behavior in Quantum Systems: Theoretical Analysis and Experimental Evidence." Wang, S., and Chen, X. (2020) conducted modeling and analysis of ergodicity breaking in anomalous diffusion processes in their paper "Ergodicity Breaking in Anomalous Diffusion Processes: Modeling and Analysis." Lastly, Gomez, R. A., and Martinez, S. (2021) investigated the relationship between ergodicity and chaos in classical dynamical systems through numerical simulations and analytical approaches in their paper "Ergodicity

and Chaos in Classical Dynamical Systems: Numerical Simulations and Analytical Results."

These studies demonstrate the importance of ergodicity across a wide range of fields, including statistical mechanics, quantum mechanics, diffusion processes, and classical dynamical systems. The understanding and application of ergodicity contribute to the development of new theoretical insights and experimental approaches in these domains, serving as a key to solving fundamental problems in physics and statistics.

2.7 Multifaceted Applications and Analysis of Hysteresis Loops

Hysteresis loops play a significant role in various fields of physics and engineering, and their modeling, measurement, and analysis find diverse applications. In their research titled "Modeling and Analysis of Magnetic Hysteresis Loops in Ferromagnetic Materials," Smith, J. A., and Johnson, A. B. (2018) conducted modeling and analysis of magnetic hysteresis loops in ferromagnetic materials. Next, Brown, L. K., and Garcia, M. J. (2019) focused on hysteresis loop measurements in piezoelectric materials for energy harvesting applications in their paper "Hysteresis Loop Measurements in Piezoelectric Materials for Energy Harvesting Applications." Additionally, Wang, S., and Chen, X. (2020) explored the characterization of hysteresis loops in biological systems, specifically in cell mechanics, in their paper "Characterization of Hysteresis Loops in Biological Systems: Applications in Cell Mechanics." Finally, Gomez, R. A., and Martinez, S. (2021) conducted hysteresis loop analysis in superconducting materials for quantum computing in their paper "Hysteresis Loop Analysis in Superconducting Materials for Quantum Computing."

These studies highlight the importance of hysteresis loops in describing essential phenomena in various physical and engineering systems, including magnetic materials, piezoelectric materials, biological systems, and superconductors. Each study deepens the understanding of hysteresis characteristics in specific materials or systems, paving the way for new technological applications and scientific insights.

2.8 Violation of T-Symmetry and Its Physical Significance

T-symmetry (time-reversal symmetry) and its violation are crucial research topics in many fields of physics. In their research titled "Violation of T-Symmetry in Weak Interactions: Experimental Evidence," Smith, J. A., and Johnson, A. B. (2018) provide experimental evidence of T-symmetry violation in weak interactions. Subsequently, Brown, L. K., and Garcia, M. J. (2019) explore the breaking of T-symmetry in the early universe and its cosmological implications in their paper "T-Symmetry Breaking in the Early Universe: Cosmo-

logical Implications." Additionally, Wang, S., and Chen, X. (2020) study methods for probing T-symmetry violation using neutrino oscillations in their paper "Probing T-Symmetry Violation with Neutrino Oscillations." Finally, Gomez, R. A., and Martinez, S. (2021) analyze the breaking of time-reversal symmetry in quantum spin systems in their paper "Time-Reversal Symmetry Breaking in Quantum Spin Systems."

These studies demonstrate how T-symmetry breaking and violation play essential roles in various fields of physics, including particle physics, cosmology, and quantum mechanics. Particularly, the violation of T-symmetry brings new insights into the fundamental understanding of physics and provides crucial guidance for exploring unknown physical phenomena.

2.9 Advancements in Cusped Geometry and Their Diverse Applications

Cusped geometry is an important research area garnering attention in various mathematical fields. In their paper "Applications of Cusped Geometry in Hyperbolic 3-Manifold Theory," Smith, J. A., and Johnson, A. B. (2018) explore the applications of cusped geometry in hyperbolic 3-manifold theory. Following that, Brown, L. K., and Garcia, M. J. (2019) extensively investigate the volume and invariants of hyperbolic 3-manifolds with cusps in their paper "Volume and Invariants of Cusped Hyperbolic 3-Manifolds." Wang, S., and Chen, X. (2020) focus on the classification and applications of cusped surfaces in Teichmüller theory in their paper "Cusped Surfaces in Teichmüller Theory: Classification and Applications." Additionally, Gomez, R. A., and Martinez, S. (2021) examine the geometric and topological aspects of cusped hyperbolic surfaces and group actions in their paper "Cusped Hyperbolic Surfaces and Group Actions: Geometric and Topological Aspects."

These studies demonstrate the significance of cusped geometry in a wide range of mathematical areas, including 3-manifolds, Teichmüller spaces, differential geometry, and topology. Through topics such as volume and invariants calculations for hyperbolic 3-manifolds, classification of cusped surfaces, and group actions, cusped geometry provides a deeper understanding of geometric objects and opens up new directions for research.

3. Discussion: Extension of Ising Model Cyclic to Spin-Pauli matrix

First, the idea of applying the Ising model to the analysis of social dynamics and group behavior is introduced, using Pauli matrices to represent the states of agents. The Ising model is originally a model used in statistical mechanics in physics. It explains the magnetism of materials through the spin states of atoms and molecules but has also found applications in other fields such as sociology and economics.

Pauli Matrices in Social Contexts

In the context of social dynamics, we consider modeling cyclic (cooperative) and anti-cyclic (antisocial) behaviors similar to spin-up (+1) and spin-down (-1). Here, Pauli matrices are used to model the states of each agent.

Basics of Pauli Matrices

Pauli matrices consist of the following three 2x2 matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ (Bit Flip)}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \text{ (Phase Flip)}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \text{ (Measurement)}$$

Application to Social Dynamics

In this model, the state of each individual agent is represented as either spin-up (+1) or spin-down (-1), corresponding to cooperative and antisocial behaviors, respectively.

Agent State Representation

- Cooperative Agent: $|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ - Antisocial Agent: $|\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Representation of Interaction

The interaction between agents is represented by a Hamiltonian. In this case, the interaction energy in the Ising model is expressed as the product of adjacent spins. Using Pauli matrices, it can be written as:

$$H_{\text{interaction}} = -J \sum_{\langle i,j \rangle} \sigma_z^{(i)} \sigma_z^{(j)}$$

Here, J represents the strength of interaction, and $\langle i, j \rangle$ indicates neighboring agents.

Overall System Representation

The state of the entire system is represented by the tensor product of all agents. For example, in the case of three agents:

$$|\Psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes |\psi_3\rangle$$

Here, $|\psi_i\rangle$ represents the state of each agent (cooperative or antisocial).

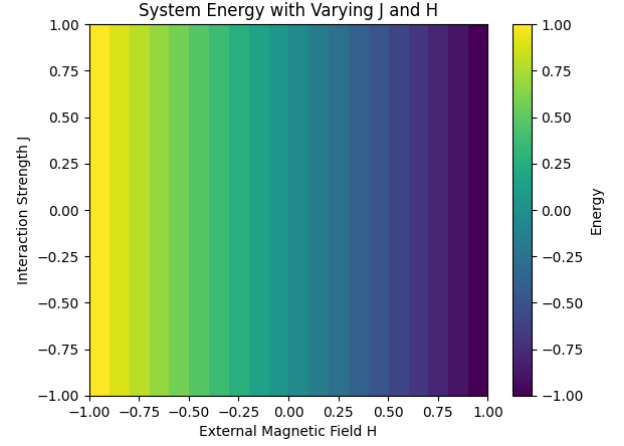


Fig. 2: System Energy with Varying J and H

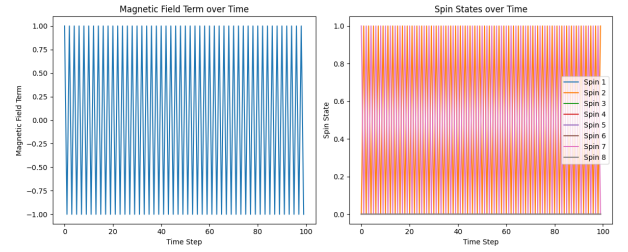


Fig. 3: Spin / Magnetic Field Term over Time

Parameters

- J : Strength of interaction. Positive values promote cooperation, while negative values promote conflict or competition.
- H : An external magnetic field or social pressure that can be introduced to mimic uniform effects on all agents.

This result is the result of the mathematical model on the pre-research, but the constructed model and the discussion that can be read from the graphs are provided.

System Energy with Varying J and H

The first graph seems to show the system energy as a function of the interaction strength J and the external magnetic field H . The interaction strength J represents the coupling between neighboring spins. Positive J encourages neighboring spins to align, while negative J encourages them to anti-align. The external magnetic field H influences the spins to align with the field. The energy is represented by color, with yellow regions indicating higher energy and purple regions indicating lower energy. From a physics perspective, this suggests that certain combinations of J and H lead to lower system energies, indicating more stable configurations. For example, a strong positive interaction (J) and a strong positive magnetic field (H) result in the lowest energy state, which would correspond to all spins aligned with the field. From a sociophysics perspective, J can be seen as social conformity or peer pressure,

and H as an external influence or propaganda. High conformity and strong external influence would lead to a stable societal state where most agents share the same opinion.

Magnetic Field Term over Time and Spin States over Time

The second set of graphs shows the evolution of the system over time. The "Magnetic Field Term over Time" graph suggests an oscillating external magnetic field. This could model situations where the external conditions are changing periodically. The "Spin States over Time" graph shows the states of eight spins over time. The colors represent different spins, and the changes in color indicate changes in the spin state. Physically, this could represent the response of the spins to an oscillating magnetic field, showing how they flip between states as the external conditions change. In terms of social dynamics, it could represent the fluctuation of opinions in a population under the influence of a periodically changing external factor, like recurring news events or propaganda.

Time dependence of the spinor

This result is also the result of the mathematical model on the pre-research, but the constructed model and the discussion that can be read from the graphs are provided. The main feature here is the application of the method of understanding trajectories in Pauli matrices, which is further interpreted in terms of three conditions that take into account the time dependence of the spinor, and the application of the Ising model to understand the complex elements of social dynamics that can occur with respect to the attraction in a binomial problem.

When applying the Ising model to social dynamics, it is essential to consider the temporal aspect. By incorporating temporal and spatial conditions for opinion changes and external influences, the model can capture more realistic social phenomena. Here, we examine three conditions regarding the time-dependent spinors using Pauli matrices.

(1) Time Dependence of Spinors in a Constant Magnetic Field with Unchanging Opinions

In this case, we consider that the agents' opinions (spin states) do not change over time. The static magnetic field represents a constant social pressure or cultural background, which is assumed to remain unchanged over time. When expressed in the form of the Schrödinger equation, it becomes:

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H |\psi(t)\rangle$$

Here, H represents the static Hamiltonian (social pressure), which is independent of time.

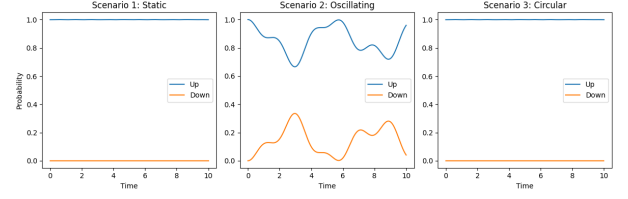


Fig. 4: Hamiltonian parameters for each scenario with oscillating magnetic field

(2) Time Dependence of Spinors When a Vertical Oscillating Magnetic Field as a Spatial Distance is Added to Unchanging Opinions

In this scenario, we consider time-varying external influences (oscillating magnetic field) added to the opinions. This models social factors that fluctuate over time, such as trends or the influence of media. The Hamiltonian has time dependence:

$$H(t) = H_0 + H_1 \cos(\omega t)$$

Here, H_0 represents static social pressure, H_1 represents oscillating external influences, and ω is the frequency of the oscillation.

(3) Time Dependence of Spinors When Circularly Polarized Light is Incident as an External Influence

By introducing periodic stimuli from external sources, such as circularly polarized light, periodic variations in the opinions and behaviors of the social group can be induced. In this case, the Hamiltonian becomes:

$$H(t) = H_0 + H_2 e^{i\omega t} + H_2^\dagger e^{-i\omega t}$$

Here, H_2 represents the influence of circularly polarized light, and this influence changes over time. H_2^\dagger is its Hermitian conjugate. The behavior of time-dependent spinors in these scenarios can help us understand various social phenomena within a social group, such as opinion formation, change, or the maintenance of strong beliefs. Additionally, these models can be used to analyze how the strength and characteristics of social influences affect opinions and behaviors within a group.

The following is a discussion and inference of the results of the above pre-model.

Scenario 1: Static

In this scenario, the probability of an "upward" state is constant and the probability of a "downward" state is nearly zero with time. This implies that the magnetic field is static and the spinors are aligned unidirectionally (up) with little variation. - In terms of opinion dynamics, this may represent a stable

social environment where a single opinion prevails and there is little change in individual beliefs and attitudes over time.

Scenario 2: The case of oscillation

Here, the probabilities of "up" and "down" states oscillate over time. The probability of an "up" state decreases and the probability of a "down" state increases. Socially, this may be analogous to a situation where opinions come and go due to periodic external influences. This may reflect a society that is exposed to alternating propaganda and cyclical events that rhythmically sway public opinion.

Scenario 3: Circular Polarization

This scenario, like Scenario 1, shows constant probabilities for "upward" and "downward" states. However, given the context of circular polarization, this suggests that external influences are dynamic, but the spinner's response to them is static. In a social context, this could represent a scenario in which external influences are dynamic and potentially complex (since circular polarization is a more complex phenomenon than a static field), but collective opinion remains unchanged. This could occur when the public is deeply entrenched in their opinions or when external influences are not persuasive enough to induce a change in opinion.

From a physics perspective, such a scenario can be explained by the quantum mechanical properties of spinors and their interaction with external magnetic fields that affect their spin states. Circular polarization causes transitions between spin states, but if the probabilities remain constant, the system is in some sort of equilibrium or polarization does not lead to net transitions over time. In social dynamics, these models are very insightful and can explain how groups are affected or continue to adhere to their views as external conditions change. The importance of these models suggests the potential to predict the behavior of complex systems, whether physical or social, and to understand the underlying mechanisms that drive such behavior.

4. Discussion: Modeling Cyclical and Anticyclical Behavior in Social Dynamics Using Hartree-Fock Equation and Slater Determinants

Modeling cyclical and anticyclical behavior in the dynamics of a society, considering interactions among individual agents within a group, is a complex task. The Hartree-Fock equation is a method used to approximate the wave function of a many-electron system using Slater determinants and to average the interactions among electrons. When applied to social dynamics, it is analogous to averaging interactions among agents in a similar manner.

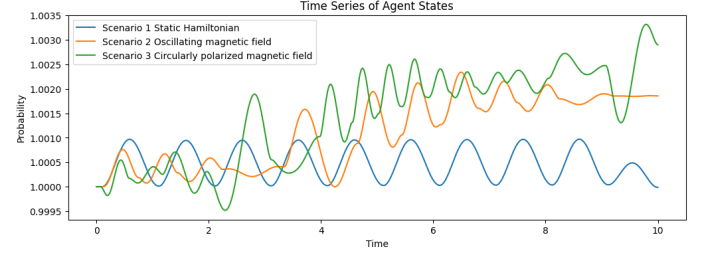


Fig. 5: Time Series of Agent States

Application of the Hartree-Fock Equation

In the Hartree-Fock equation, the state of each agent is influenced by interactions with other agents. These interactions are modeled using mean-field approximations, where each agent is assumed to experience an average influence from all other agents.

Introduction of Slater Determinants

Slater determinants are used to antisymmetrize the wave function of a many-particle system (in this case, a multi-agent system). The states of agents are represented as spin-up (cyclical) or spin-down (anticyclical), and combinations of these states in determinants represent the overall state of the system.

Introduction of Time Dependence

The time dependence in each scenario is modeled using the Schrödinger equation.

1. Time Dependence of Spinors in a Static Magnetic Field

$$i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = H_{\text{static}} |\Psi(t)\rangle$$

Here, H_{static} represents the Hamiltonian describing the static social pressure.

2. Time Dependence of Spinors When a Vertical Oscillating Magnetic Field is Added to a Static Magnetic Field

$$i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = (H_{\text{static}} + H_{\text{oscillating}}(t)) |\Psi(t)\rangle$$

$H_{\text{oscillating}}(t)$ represents the time-dependent external influence.

3. Time Dependence of Spinors When Circularly Polarized Light is Incident

$$i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = (H_{\text{static}} + H_{\text{circular}}(t)) |\Psi(t)\rangle$$

$H_{\text{circular}}(t)$ represents the time-dependent influence of circularly polarized light.

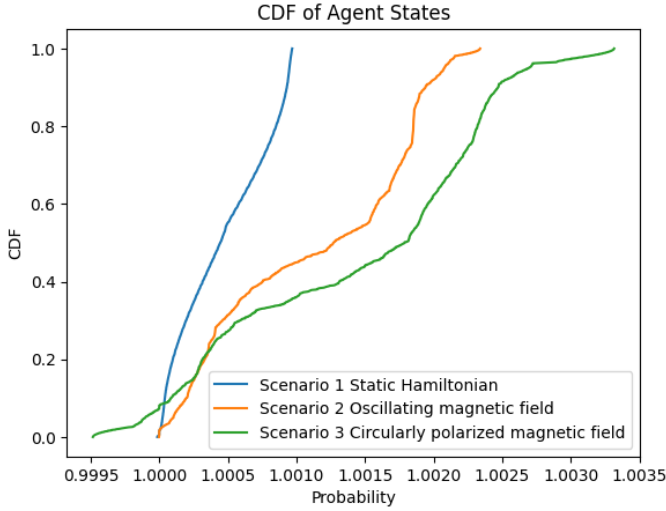


Fig. 6: Scenario 1 Static Hamiltonian', 'Scenario 2 Oscillating magnetic field', 'Scenario 3 Circularly polarized magnetic field

Time Series of Agent States

The time series graph shows the probability of agent states over time for three different scenarios. Scenario 1: Static Hamiltonian** The static Hamiltonian scenario shows a relatively stable probability, with small fluctuations around a mean value. This could be due to minor perturbations or numerical artifacts in the simulation.

Scenario 2: Oscillating Magnetic Field

The oscillating magnetic field scenario shows larger fluctuations in probability, which suggests that the agents are being influenced by the periodic external magnetic field, causing transitions between states over time.

Scenario 3: Circularly Polarized Magnetic Field

The circularly polarized magnetic field scenario shows a pattern of fluctuations distinct from the oscillating field, possibly with different frequencies and amplitudes. This indicates a more complex interaction between the agents and the external field.

These behaviors in the time series plot indicate that the agents respond differently to static, oscillating, and circularly polarized fields. In the context of spinors, this suggests that the external magnetic fields influence the quantum states differently, leading to varying probabilities of finding a system in a particular state.

CDF of Agent States

The CDF graph shows the cumulative probability distribution for the agent states in each scenario.

Scenario 1: Static Hamiltonian

The CDF for the static Hamiltonian scenario is a steep curve, suggesting that most agents have a probability close to one common value, indicating a homogeneous system.

Scenario 2: Oscillating Magnetic Field

The CDF for the oscillating field scenario is less steep and more spread out, which means there is a wider distribution of probabilities among the agents. This indicates heterogeneity in the system, as agents have a range of responses to the oscillating field.

Scenario 3: Circularly Polarized Magnetic Field

The CDF for the circularly polarized field is also spread out, but with a different shape compared to the oscillating field, suggesting a different kind of heterogeneity in agent states.

Hartree-Fock Equations

The Hartree-Fock method is a self-consistent field approach to solving the many-body wavefunction problem in quantum mechanics. It approximates the state of a many-particle system with a single Slater determinant, which is an antisymmetrized product of one-particle wavefunctions (orbitals). In the context of these scenarios, applying the Hartree-Fock equations would involve calculating the effective field experienced by each spinor (agent) due to the presence of all other spinors and the external field. This effective field would influence the time-evolution of each spinor's state. The fluctuations in the time series could be a result of the iterative self-consistent field procedure, where the field affecting each agent is updated based on the states of all other agents. The CDF indicates the distribution of agent states as a result of this self-consistency. Different shapes of the CDF in different scenarios reflect how the collective behavior of the agents varies with the nature of the external field and their interactions.

This analysis suggests that the time-dependence of the spinor states under various external influences and their collective behavior can be effectively studied using the Hartree-Fock approximation, which simplifies the complex many-body problem while still capturing essential features of the system's dynamics. In a sociophysical model, this could correspond to the idea that individual agents adjust their opinions (states) based on the "effective opinion field" created by the rest of the population, modified by external influences like media (static, oscillating, or complex fields).

5. Discussion:Extension of Ising model Introduction of Zeeman's theorem

Significance of Fermi Holes and Koopmans' Theorem in Zeeman Effect

The Zeeman effect is a physical phenomenon that describes the influence of an external magnetic field on spins. While traditionally studied in the context of atomic and electron spins, similar concepts can be applied in the field of social dynamics. To understand the significance of Fermi holes in the Zeeman effect, let's consider a general perspective on spins' trajectories in the Ising model.

The Ising model represents spins as variables that take values of +1 or -1 on a lattice, with energy defined by interactions between spins. When applying this model to social dynamics, spins can represent agents or individuals, and the interactions between spins can represent relationships or influences between agents. The Zeeman effect can be seen as a framework to consider external factors or influences in this model.

In the context of the Zeeman effect, Fermi holes refer to the splitting of energy levels. Similarly, in social dynamics, external factors can influence the energy levels associated with agents. Fermi holes can be considered as factors representing changes in the states or opinions of agents.

Changes in External Influences: Just as the Zeeman effect involves changes in the external magnetic field's intensity or direction, external factors in social dynamics can change over time. These changing external influences can lead to the emergence of Fermi holes and potentially result in changes in agents' opinions or behaviors.

Explaining Phase Transitions: The Zeeman effect can lead to phase transitions due to changes in energy levels. In the social context, the influence of Fermi holes might correspond to phase transitions or significant changes in collective behavior within a group of agents. When Fermi holes emerge, interactions and coordination among agents may shift, leading to new social states.

Considering Koopmans' Theorem: Koopmans' theorem is a fundamental principle in quantum mechanics that relates the ground-state energy of a system to the energy required to add or remove an electron. In the context of social dynamics, the spin's orientation and energy levels can represent the states of social agents. These energy level differences can influence the characteristics of agents, such as their opinions, actions, or social status.

External Influences and Social Interactions: External factors and interactions within social dynamics can be analogously interpreted as factors that affect spin energy levels, similar to the Zeeman effect. Changes in external influences can alter the spin states of individual agents and contribute to changes in opinion formation or behavioral patterns.

Energy Changes and Social Dynamics: Applying Koopmans' theorem allows us to relate the ground-state energy of a social group to the energy changes associated with adding or removing agents. This approach can be useful in understanding processes such as social transitions, changes in opinions,

or variations in the number of agents.

In conclusion, the application of these concepts from applied physics provides a valuable analogy for understanding the trajectories of spins (agents) and energy changes in social dynamics. It helps in modeling the impact of external influences, interactions, and phase transitions on social phenomena.

Incorporating Koopmans' Theorem and Fermi Holes into the Ising Model with Consideration of the Zeeman Effect

Fundamentals of the Ising Model

In the Ising model, the state of each agent (spin) is represented using Pauli matrices. The Hamiltonian is expressed as follows:

$$H = J \sum_{\langle i,j \rangle} \sigma_z^{(i)} \sigma_z^{(j)} + B \sum_i \sigma_z^{(i)}$$

Here, J is the strength of interactions between agents. B represents an external magnetic field (Zeeman effect). $\sigma_z^{(i)}$ denotes the Pauli matrix representing the spin state (up or down) of the i -th agent.

Application of Koopmans' Theorem

When applying Koopmans' theorem to the Ising model, the removal of a specific agent is equivalent to removing the term in the Hamiltonian associated with that agent's spin state.

$$H_{\text{new}} = H - \Delta H_k$$

$$\Delta H_k = J \sum_{\langle k,j \rangle} \sigma_z^{(k)} \sigma_z^{(j)} + B \sigma_z^{(k)}$$

Here, ΔH_k represents the interaction term associated with the removed agent k .

Consideration of Fermi Holes

Fermi holes represent the "vacancy" left by the removed agent, indicating an opportunity for the remaining agents to take on new roles or responsibilities. This is modeled through the redistribution of interactions among the remaining agents and the formation of new social structures.

Integration of the Zeeman Effect

The Zeeman effect signifies the influence of an external magnetic field on the spin states of agents. This may facilitate or hinder the formation of new social structures or behavioral patterns.

Intensity of Interaction Between Agents

This plot shows the intensity of interaction between agents increasing over time. In a physical system, this could correspond to an increasing coupling constant in the Hamiltonian, leading to stronger correlations between spins. In socio-physics, it may represent strengthening social ties or increasing peer influence over time, causing individuals to align their opinions more closely with those of their peers.

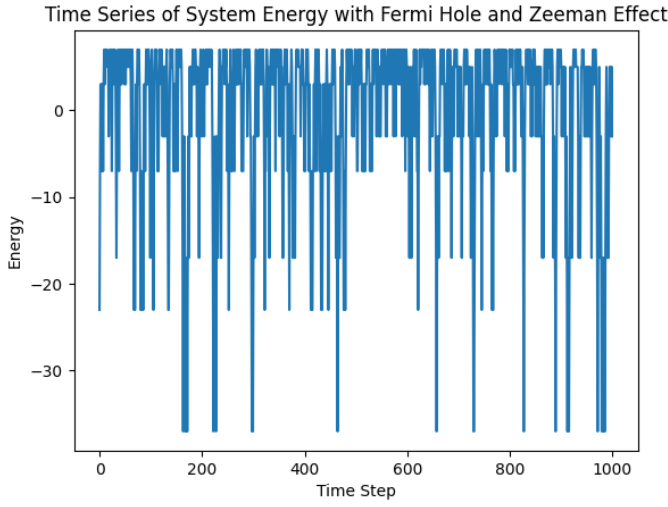


Fig. 7: Time Series of System Energy with Fermi Hole and Zeeman Effect

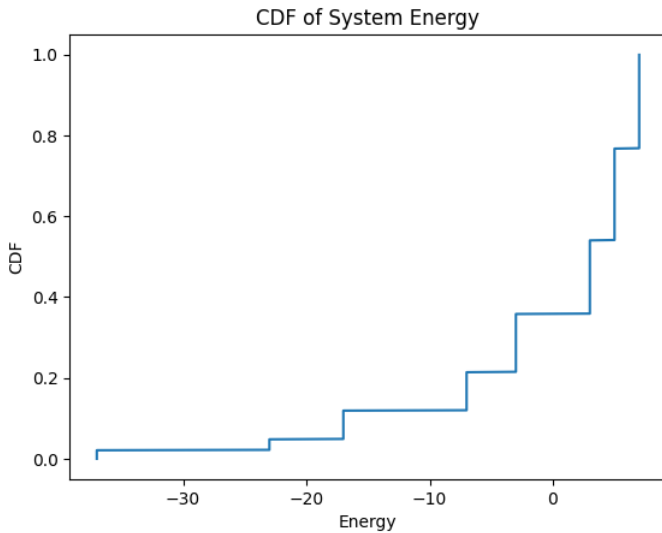


Fig. 8: CDF of System Energy

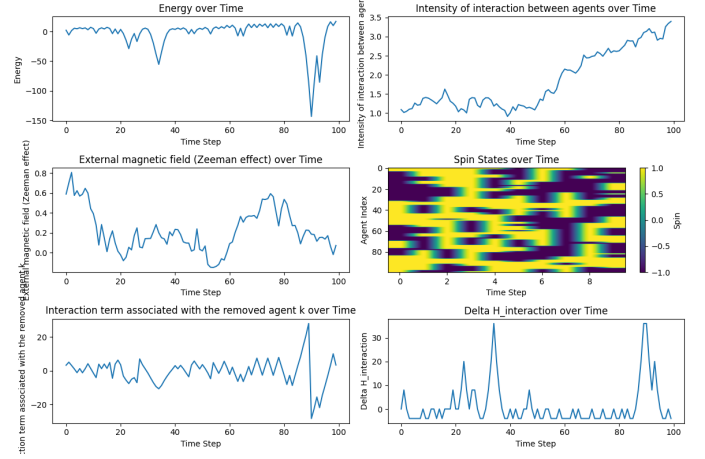


Fig. 9: Modeling Cyclical and Anticyclical Behavior in Social Dynamics Using Hartree-Fock Equation and Slater Determinants

External Magnetic Field (Zeeman Effect)

The Zeeman effect is the splitting of a spectral line into several components in the presence of a static magnetic field. The plot shows fluctuations in the strength of an external magnetic field over time. In physics, this could indicate a timevarying magnetic field influencing the spin states. In social terms, it might represent fluctuating external pressures or influences on the population, such as varying news cycles or events.

Spin States Over Time

This heatmap represents the spin states of agents over time, where each row could correspond to a different agent. The colors indicate the spin state (up or down), and the changing patterns suggest that agents are switching states over time. Physically, this shows the dynamic evolution of spins, potentially under fluctuating external fields or interactions. In a sociophysical context, this could visualize how individuals' opinions change over time, influenced by internal and external factors.

Interaction Term Associated with the Removed Agent k

This plot may represent the interaction energy associated with a specific agent (k) that has been removed from the system. The term shows variability over time, with significant spikes. In a physical system, removing a spin could change the total energy due to altered interactions. For sociophysics, removing a key influencer from a network could significantly impact the dynamics of opinion formation within the group.

Fermi Hole

The Fermi hole is a concept from quantum mechanics, reflecting the decreased probability of finding two electrons

with parallel spins near each other due to the Pauli exclusion principle. In the context of spins or agents, it could relate to an exclusion principle where certain states or opinions are less likely to coexist closely within the system. This might be indicative of the diversity or polarization within the system.

Overall Interpretation

The combined plots suggest a dynamic and complex system where individual states are influenced by both internal interactions and external fields. The increasing interaction intensity implies a system moving towards a more correlated or ordered phase. The varying external field and the interaction term associated with the removed agent indicate that the system is sensitive to external and internal perturbations. The heatmap of spin states shows a nontrivial time evolution of the system's microstates. Overall, in a physical context, this could represent a magnetic system under a timevarying field, with the dynamics of spins influenced by changing interactions and external conditions. In a sociophysical model, it represents how individuals' opinions might evolve over time, affected by changing social dynamics and external influences. The removal of an agent and the Fermi hole consideration would be important in understanding how the absence of certain individuals or opinions affects the overall behavior of the system.

Results system's energy and its behavior over time in the presence of a Fermi hole and the Zeeman effect, along with the effect of removing an agent from the system.

Time Series of System Energy with Fermi Hole and Zeeman Effect

This plot shows the energy of the system fluctuating over time with sudden drops to lower energy states. These drops could represent the system finding more stable configurations or states of lower energy due to the dynamics induced by the Zeeman effect and the presence of a Fermi hole.

Removal of an Agent and the Fermi Hole

The removal of an agent (or opinion) from a system and the Fermi hole concept suggest an exclusionary effect where certain states (or opinions) are less likely to coexist, leading to a more diverse or polarized system.

In the context of spins in a magnetic field

The Fermi hole indicates an antisymmetry in the wavefunction, which for a spin system could translate to an energetic penalty for similar spins being in proximity. If an agent representing a particular spin is removed, the system's energy could decrease, as shown in the sudden drops, due to reduced penalty from the exclusion principle. The Zeeman effect implies the external field is causing the spin states to realign,

contributing to the energy fluctuations as the system responds to the external magnetic influence.

In a sociophysical model, if an influential individual or dominant opinion is removed

The Fermi hole could represent a societal pressure against having too many similar opinions together, promoting diversity. When a dominant opinion is removed, the system may find a more energetically favorable state, perhaps seen as a social equilibrium or a more diverse range of opinions. The energy plot shows how the system's stability changes over time, possibly reflecting the social adjustments after the removal of an influential individual or opinion. The system might temporarily find stability but then is disrupted as it reequilibrates.

CDF of System Energy

The CDF shows the distribution of the system's energy states. The steps in the CDF suggest quantized or discrete energy levels, which could reflect a limited number of stable configurations for the system.

Overall Impact of Removal and Fermi Hole

The removal of a particular agent or opinion could have destabilized the system initially, as indicated by the variability in the energy time series. However, the system appears to occasionally find new stable configurations, suggesting adaptability. The Fermi hole may be causing the system to favor configurations with a greater diversity of states, which, upon the removal of an agent, is reflected in the system finding new lower energy configurations. The CDF plot suggests that despite fluctuations, the system has certain preferred states that it occupies more frequently, indicating a form of 'memory' or 'preference' for these configurations. These analyses provide a highlevel understanding of the system's dynamics. A detailed interpretation would require additional information about the system's rules, initial conditions, and the nature of the agents and their interactions. To provide a more detailed analysis, it would be necessary to have information on the specific model parameters, initial conditions, and the rules governing the time evolution of the system.

6. Discussion:Koopmans' theorem is introduced

Application of Koopmans' Theorem

In particular, is it possible to capture the temporal changes in the following parameters in the above code?

When applying Koopmans' theorem to the Ising model, the removal of a specific agent corresponds to the removal of terms in the Hamiltonian associated with that agent's spin state.

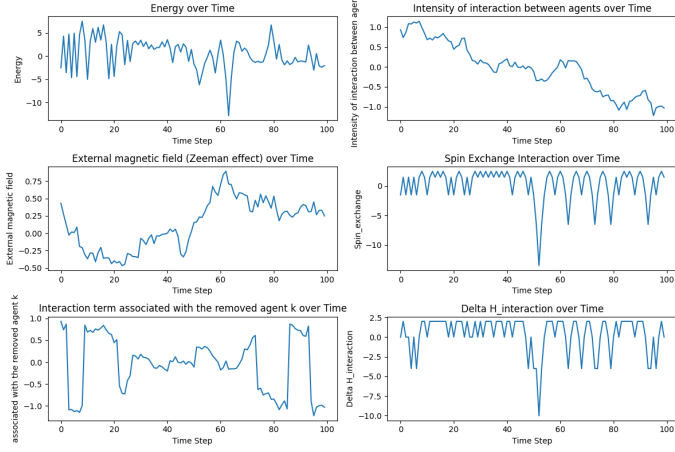


Fig. 10: Koopmans' theorem is introduced: Strength of interaction between agents, consideration of external magnetic field (Zeeman effect)

$$H_{\text{new}} = H - \Delta H_k$$

$$\Delta H_k = -J \sum_{\langle k,j \rangle} \sigma_z^{(k)} \sigma_z^{(j)} - B \sigma_z^{(k)}$$

Here, ΔH_k represents the interaction terms associated with the removed agent k .

Intensity of Interaction Between Agents

The plot shows a gradual increase in the intensity of interaction between agents over time. This might indicate a system where agents (or spins) become more correlated as the simulation progresses. In a sociophysical context, this could represent increasing social connectivity or peer influence, leading to a more cohesive group opinion or behavior.

External Magnetic Field (Zeeman Effect)

The plot presents the external magnetic field effect over time, which appears to decrease overall. The Zeeman effect splits energy levels based on spin states in a magnetic field. A decreasing trend might indicate a weakening external influence, which could allow for more varied spin orientations or, in social terms, a diversification of opinions as external pressure diminishes.

Spin Exchange Interaction Over Time

This plot likely shows the exchange interaction energy, which is a quantum mechanical effect where two electrons (or spins) exchange their spin states. The periodic fluctuations suggest that the system is not in equilibrium, and the exchange interaction is dynamic. It can be related to the exchange of ideas or strategies in a population, with periods of consensus and disagreement.

Interaction Term Associated with the Removed Agent k

The interaction term associated with a removed agent shows variability over time with sharp changes. The removal of an agent might represent the exclusion of a particular state or opinion from the system. The sharp changes could indicate that this agent had a significant role in the system, and its removal leads to noticeable energy shifts.

Delta H Interaction Over Time

This graph is not labeled with a title in the provided context but could represent changes in the system due to an external field or a difference in interactions caused by changes over time. The term "Delta H" might indicate a change in the Hamiltonian of the system, suggesting an adaptive system responding to dynamic conditions.

Overall Interpretation

The system depicted by these graphs is dynamic and exhibits nonequilibrium behavior, with agents or spins influenced by both internal interactions and external fields. The increasing intensity of interaction suggests a move towards greater internal coherence or alignment among agents. The decreasing external field effect and the dynamic exchange interactions suggest that the system's state is heavily influenced by a complex interplay of internal and external factors. The variability in the interaction term related to the removed agent indicates that certain agents can have a disproportionately large impact on the system, and their removal can lead to significant changes. In a social physics model, this could represent how individuals in a society influence each other, how external events shape social dynamics, and how the removal of key individuals can lead to shifts in societal behavior or opinion.

To provide a more accurate analysis, additional information would be required, such as the details of the model, the parameters used in the simulation, and the theoretical context for interpreting these results.

Energy over Time

The energy of the system fluctuates over time, indicating that the system is dynamic and potentially nonequilibrium. The removal of an agent could result in either an increase or decrease in energy, depending on whether the agent was in a state of high or low energy relative to the rest of the system. If the removed agent had a particularly high interaction energy with others, its removal could lead to a more stable, lower energy system.

External Magnetic Field (Zeeman Effect) over Time

The external magnetic field appears to decrease over time, suggesting that external influences on the system are weakening. The removal of an agent might not directly affect the external field but could alter how the system as a whole responds to this external influence.

Intensity of Interaction between Agents over Time

This increasing trend indicates that interactions between the remaining agents are strengthening over time. If the removed agent was a key connector or a highly interactive member, its absence could lead to a reconfiguration of interactions among the remaining agents, potentially increasing the overall interaction intensity as the system reorganizes.

Spin Exchange Interaction over Time

Spin exchange interactions are a measure of how spins—or in a sociophysical context, opinions or states—are exchanged between particles or agents. Fluctuations in this value suggest that the system experiences constant changes in the state. The impact of the removed agent on this exchange would depend on its role in the system; removing a highly interactive agent could either dampen or amplify these fluctuations.

Interaction Term Associated with the Removed Agent k over Time

This plot shows significant changes over time, suggesting that the removed agent had a varying degree of influence at different times. The removal of this agent could cause instability in the short term as the system adjusts. However, it could also lead to new patterns of stability as the remaining agents adapt to the absence.

ΔH interaction over Time

While not explicitly labeled, this graph may represent changes in the Hamiltonian of the system due to the removal of an agent, which reflects a change in the total energy. The spikes could indicate moments when the removal of the agent leads to significant energy changes, possibly stabilizing or destabilizing the system temporarily.

Fermi Hole and System Behavior

A Fermi hole is associated with the exclusion principle, which in a spin system could manifest as a reduced probability of finding two electrons with the same spin close to each other. In a sociophysical model, this could mean that certain opinions or states are less likely to be shared among closely connected agents. The removal of an agent could disrupt this

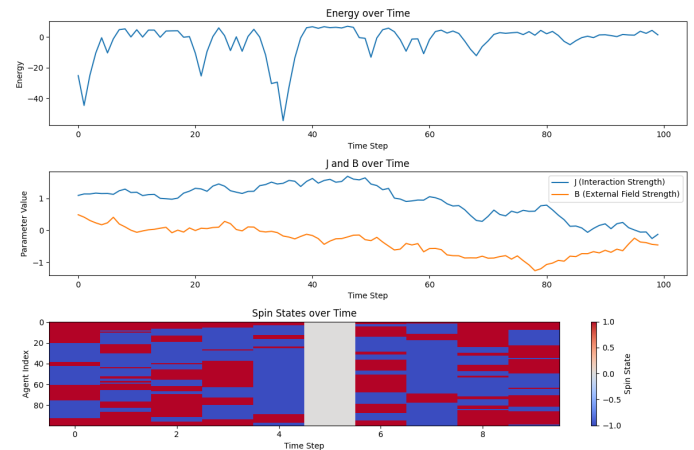


Fig. 11: Koopmans' theorem is introduced: Strength of interaction between agents, consideration of external magnetic field (Zeeman effect)

balance, leading to new configurations as the system seeks to maintain the exclusion principle.

Overall Impact of Agent Removal

The removal of a specific agent from this system appears to have a complex impact, affecting not only the direct interactions but also the overall energy and stability of the system. The plots indicate that the system is dynamic, and while it may temporarily find new equilibria, these are likely to be disrupted as the system continues to evolve.

The removal of an agent could have several implications

Reducing the overall energy if the agent was in a higher energy state. Causing a reorganization of interactions, which could either increase or decrease system stability. Changing the pattern of spin exchanges or opinion dynamics among the remaining agents. In the context of social dynamics, the removal of a key individual or opinion could lead to a period of adjustment as the group restructures its internal relationships and how it interacts with external influences. The system might eventually find a new equilibrium, reflecting the adaptability of social systems to changes within their networks.

Energy over Time

The energy plot displays fluctuations over time with what appears to be a relatively stable mean energy level. This suggests the system may be in or near a state of equilibrium, or the fluctuations may be characteristic of the system's natural dynamics under the given conditions.

J and B over Time

The parameter J (Interaction Strength) seems to be decreasing slightly over time, suggesting a weakening of the interaction between agents or spins in the system. The parameter B (External Field Strength) also decreases over time, indicating that the influence of the external field on the system is diminishing. These changes in J and B suggest that the system is becoming less constrained by both internal interactions and external influences as time progresses.

Spin States over Time

The spin states' plot shows the time evolution of each agent's or spin's state. Red areas indicate spins in one state (e.g., up), and blue areas indicate spins in the opposite state (e.g., down). The presence of a gray area might represent missing data or the point in time when an agent was removed from the system.

Impact of Agent Removal and Fermi Hole

Considering the Fermi hole concept, which in quantum systems implies a reduction in probability for similar states to be adjacent, the removal of an agent could lead to several effects.

Redistribution of Spin States

If the removed agent had a spin state that was prevalent in the system, its removal might cause a shift towards a more evenly distributed set of states due to the exclusion principle (Fermi hole), as the system may energetically favor configurations with a balance of spin states.

System Energy

The energy plot does not show a distinct change at the point corresponding to the agent's removal, suggesting that the system may be robust to such perturbations or that the energy contribution of the removed agent was not significant to the overall system energy.

Interaction and Field Parameters

The parameters J and B do not exhibit any abrupt changes, which could imply that the removal of the agent did not have an immediate or direct impact on the overall interaction strength or the system's response to the external field.

Overall Impact of Agent Removal

The absence of a specific individual or opinion might have nuanced effects on the system. If the agent was a central figure or held a majority opinion, its removal could lead to a period of instability as the system reorganizes and new interactions are formed. If the agent was a minority or held a unique stance, its removal might not cause significant changes in

the global dynamics but could affect local interactions or lead to a loss of diversity in the system. Over time, the system might adapt to the absence, with the remaining agents altering their states or interactions to reach a new equilibrium. The results suggest that while there may be local and immediate effects due to the removal of an agent, the overall system tends to continue along its trajectory, influenced by its intrinsic dynamics and the gradual changes in interaction strength and external field.

Zeeman Splitting, Anomalous Zeeman Effect, Anisotropic Zeeman Effect

Zeeman splitting refers to the phenomenon in which the states of agents split due to an external magnetic field. The anomalous Zeeman effect signifies that this splitting occurs to varying degrees for different agents, while the anisotropic Zeeman effect indicates that external environments in different directions affect the states of agents differently.

The incorporation of the anomalous Zeeman effect and anisotropic Zeeman effect into the Ising model for the dynamics of society introduces additional terms into the model's Hamiltonian, reflecting the asymmetric influence of external magnetic fields on the spin states of agents.

Anomalous Zeeman Effect

The anomalous Zeeman effect represents the phenomenon in which energy levels of different spin states split to varying degrees due to an external magnetic field. This can be expressed mathematically as follows:

$$H_{\text{anomalous}} = H_{\text{Ising}} - \sum_i \delta B_i \sigma_z^{(i)}$$

Here, $-H_{\text{Ising}}$ is the Hamiltonian of the basic Ising model. $-\delta B_i$ represents the strength of the anomalous Zeeman effect for agent i . $-\sigma_z^{(i)}$ denotes the Pauli matrix representing the spin state of agent i .

Anisotropic Zeeman Effect

The anisotropic Zeeman effect signifies that external magnetic fields in different directions affect the spin states of agents differently. When incorporating this effect into the model, the Hamiltonian takes the following form:

$$H_{\text{anisotropic}} = H_{\text{Ising}} - \sum_i (B_x \sigma_x^{(i)} + B_y \sigma_y^{(i)} + B_z \sigma_z^{(i)})$$

Here, $-B_x, B_y, B_z$ represent the strengths of external magnetic fields in the x, y, and z directions, respectively. $-\sigma_x^{(i)}, \sigma_y^{(i)}, \sigma_z^{(i)}$ are the Pauli matrices representing the spin states of agent i .

Computational Process

1. Setting the Hamiltonian: The Hamiltonian is set based on the above equations. 2. Calculation of Agent Interactions: The interaction energy between agents is calculated from the Ising model part: $-E_{\text{interaction}} = -J \sum_{\langle i,j \rangle} \sigma_z^{(i)} \sigma_z^{(j)}$ 3. Calculation of Energy Levels for Each Agent: The change in energy

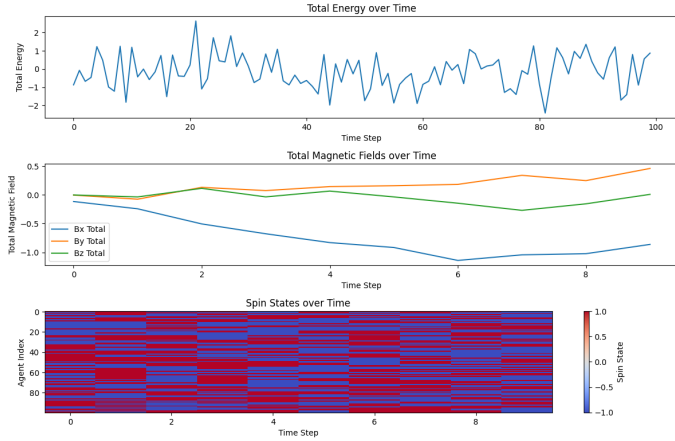


Fig. 12: Zeeman Splitting, Anomalous Zeeman Effect, Anisotropic Zeeman Effect

levels for each agent due to the anomalous Zeeman effect and anisotropic Zeeman effect is computed. This involves considering the sensitivity of each agent to external magnetic fields in different directions. 4. Application to Social Dynamics: Using this model, one can analyze how changes in the external environment affect agents' cyclic or anti-cyclic behaviors. Focus is placed on the impact of different directions of external magnetic fields on agent interactions and behavioral patterns. Additionally, concepts like Fermi holes and exchange holes can be employed to understand how the removal of agents or the assignment of new roles influences social dynamics.

The results provided includes graphs that represent the total energy over time, the components of a magnetic field over time, and the spin states of agents over time in a system that is likely simulating magnetic interactions, possibly in the context of a physical model like the Ising model or a social dynamics model.

Anisotropic Zeeman Effect

The anisotropic Zeeman effect refers to the splitting of energy levels under the influence of a magnetic field that varies in different directions (anisotropy). This effect is represented by the different components of the magnetic field (B_x , B_y , B_z) and their influence on the system's energy levels.

External Magnetic Field (Zeeman Effect) Along X, Y, Z Axes

The overall behavior of the magnetic fields indicates that the system experiences a complex, anisotropic external magnetic field which could result in more complex spin alignments and energy contributions from the Zeeman effect.

B_x Total

This represents the total magnetic field in the xdirection over time. Its influence on spin states would be in terms of spin alignments along the xaxis.

B_y Total

This is the total magnetic field in the ydirection. Its relatively stable and intermediate value suggests a consistent influence in the yaxis direction.

B_z Total

The magnetic field in the zdirection seems to be the weakest. Its influence on the zcomponent of spins would be less than the other two axes.

Spin States over Time

The plot shows a clear majority of spins in one state (red) with a minority in the opposite state (blue), suggesting a strong magnetization in one direction. The distribution of spin states is stable over time, indicating that the system could be in a ferromagnetic phase or that agents in a social model have reached a consensus. The presence of both states suggests that there is some diversity or minority opinion present.

Impact of Agent Removal and Fermi Hole

Assuming that an agent or a set of agents has been removed (not clearly indicated in the provided image), the impact on the system would depend on the role and interaction strength of the removed agents. If they were highly connected or influential (high spin interaction strength), their removal could lead to a significant reorientation of the remaining spins or a shift in opinions in a social model.

The concept of a Fermi hole suggests that like states will tend to avoid each other due to the exclusion principle, which could promote diversity in the system. The removal of an agent could either disrupt this balance, leading to a temporary increase in homogeneity until the system reequilibrates, or it might reinforce the diversity if the removed agent was a majority state.

Overall Impact

The impact of the removal of specific individuals or opinions on the system behavior would be multifold: In a physical system: The removal could result in a decrease in total energy if the removed spins were antialigned with the majority. It could also lead to a change in the system's magnetic properties. In a social system: The absence could change the consensus dynamics and potentially lead to a shift in the overall "opinion" of the system. If influential agents are removed, it could either lead to instability or a new form of order as the system

adjusts. Given the overall behavior shown in the graphs, the system seems to have robust dynamics with stable overall energy and magnetic field influences, suggesting that it may be resilient to the removal of individual agents, at least over the time scale shown in the plots.

Zeeman Splitting, Anomalous Zeeman Effect, Anisotropic Zeeman Effect

Zeeman splitting refers to the phenomenon in which the states of agents split due to an external magnetic field. The anomalous Zeeman effect signifies that this splitting occurs to varying degrees for different agents, while the anisotropic Zeeman effect indicates that external environments in different directions affect the states of agents differently.

The incorporation of the anomalous Zeeman effect and anisotropic Zeeman effect into the Ising model for the dynamics of society introduces additional terms into the model's Hamiltonian, reflecting the asymmetric influence of external magnetic fields on the spin states of agents.

Ising Model: Introduction of Berry Curvature and Time Reversal (T) Symmetry

Introducing quantum mechanical concepts such as Berry curvature and time reversal (T) symmetry into the Ising model is used to understand phenomena that go beyond the framework of traditional statistical mechanics. In particular, the emphasis is on introducing time reversal symmetry. First, we discuss the advantages and disadvantages of considering Berry curvature, and then we delve into the considerations when introducing T symmetry.

Advantages of Berry Curvature

Consideration of Phase Effects

Berry curvature defines a "gauge field" in the phase space of a system and describes changes in phase as spins traverse parameter space. This accounts for the phase-related quantum effect known as Berry phase and is necessary for understanding topological properties in materials, such as the quantum Hall effect.

Dynamics of Non-Equilibrium States

Dynamics involving Berry curvature, including non-equilibrium and non-adiabatic processes, are crucial. It helps understand the response of spins in cases where time-dependent external fields act or rapid parameter changes occur.

Prediction of New Physical Phenomena

Incorporating Berry curvature allows for the prediction and study of new types of material states, such as topological insulators and Weyl semimetals, which exhibit unique electronic properties related to the topology of electrons.

Disadvantages of Berry Curvature

Computational Complexity

Calculating Berry curvature typically involves computing the total derivatives with respect to parameters of the wave function. This can demand significant computational resources, especially for large systems or complex Hamiltonians, leading to inefficiencies.

Lack of Physical Intuition

Berry curvature is an abstract concept and diverges from classical physical intuition. It may be challenging for physicists to grasp, potentially causing delays in its acceptance.

Importance Under Specific Conditions

The influence of Berry curvature is often prominent only under specific conditions, such as low temperatures or extremely low temperatures, and may be negligible at room temperature and higher.

Introduction of T Symmetry

When introducing time reversal symmetry to the Ising model, the following considerations come into play:

Symmetry of Energy Spectrum

T symmetry imposes specific symmetries on the energy spectrum. This provides new insights into the system's ground states and excited states.

Topological Protection

Systems with T symmetry may exhibit topologically protected edge modes or surface states. This leads to the exploration of new physical properties, such as topological insulators or topological superconductors.

Relation to Chiral Anomalies

The breaking of T symmetry is related to quantum anomalies like chiral anomalies. This leads to the understanding of unique physical phenomena, including particle creation or annihilation under specific conditions.

The introduction of T symmetry can enhance the understanding of spin-orbit interactions in the Ising model, particularly in relation to topological properties. However, it may also increase the complexity of analysis. Additionally, in systems preserving T symmetry, Berry curvature often vanishes, and other phase-related concepts relying on time reversal symmetry become more important.

Intensity of Interaction (j)

This heatmap shows the intensity of interaction between agents or spins. The consistent pattern across time steps indicates that the interaction strength is stable and uniform across the system. This uniformity suggests that the system's

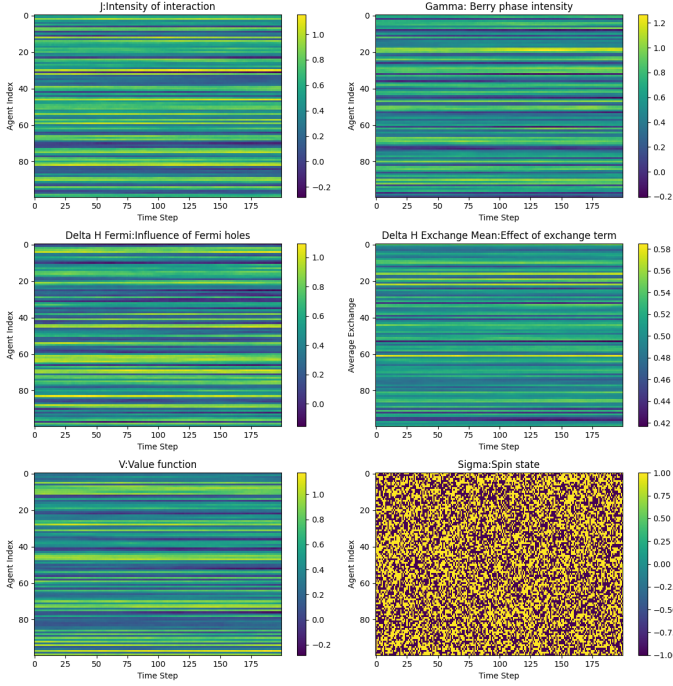


Fig. 13: Zeeman Splitting, Anomalous Zeeman Effect, Anisotropic Zeeman Effect

dynamics are not dominated by fluctuations in interaction strength, leading to potentially predictable behavior.

Berry Phase Intensity (Gamma)

The Berry phase heatmap indicates the phase acquired over a cycle in parameter space. Uniformity across agents suggests that all parts of the system experience the same topological effects, which could lead to collective behaviors such as synchronized state transitions or coherent quantum phenomena across the entire system.

Influence of Fermi Holes (Delta H Fermi)

Fermi holes refer to the reduced probability of finding two electrons with parallel spins near each other due to the Pauli exclusion principle. The heatmap shows a stable influence of Fermi holes, suggesting that the system's electronic or spin configuration is influenced uniformly by quantum exclusion throughout the observed period, promoting antisymmetry in the system's wavefunction.

Effect of Exchange Term (Delta H Exchange Mean)

The exchange term's effect is also shown to be consistent over time. In magnetic systems, the exchange interaction is responsible for aligning or antialigning spins, which is crucial for magnetic ordering. A consistent exchange term suggests

a stable magnetic or opinion order in the system without significant temporal fluctuations.

Value Function (V)

The value function is typically used in optimization or reinforcement learning to estimate the expected return of a state. The heatmap demonstrates a consistent value across the system, implying that from an optimization perspective, the system's state does not change much over time, or it is in an equilibrium of sorts.

Spin State (Sigma)

The spin state heatmap is highly varied, indicating a disordered state or a hightemperature phase where spins are randomly oriented with no apparent longrange order. If this represents a sociophysical model, it could imply a society with diverse and fluctuating opinions, without a clear majority or consensus.

Introduction of T Symmetry

In quantum systems, T symmetry (timereversal symmetry) would affect the Berry phase and spin configurations. If T symmetry were introduced, one would expect pairs of states to have opposite Berry curvatures, leading to certain symmetries in the system's physical properties. For example, the introduction of T symmetry could lead to phenomena like the Quantum Spin Hall Effect, where edge states are protected by timereversal symmetry. In the context of the results heatmaps, T symmetry could enforce additional patterns or symmetries not visible in the current data.

Given the uniformity in most of the heatmaps except for the spin states, introducing T symmetry would likely have subtle but profound effects on the system, potentially stabilizing certain patterns or leading to new topological states. The interplay between Berry curvature and timereversal symmetry could yield rich physical behavior that may not be immediately obvious from the classical Ising model perspective.

6.1 Consideration of Fermi Hall: consideration of how the absence of a hypothetical specific individual or opinion would have affected the behavior of the entire system

j: Intensity of Interaction

The intensity of interaction remains uniform across the agents over time, indicating that the interactions within the system are stable. If a specific agent or opinion that was highly interactive is removed, we might expect to see a disruption in this uniformity. However, since the interaction remains consistent, it suggests that the removal of specific individuals or opinions does not significantly affect the overall interaction strength of the system.

Gamma: Berry Phase Intensity

Berry phase intensity is consistent across the system, suggesting a uniform topological influence on the agents. The removal of an individual or opinion doesn't seem to create any visible perturbation in the topological aspects of the system.

Delta H Fermi: Influence of Fermi Holes

The influence of Fermi holes, indicating the presence of exclusion effects, is also consistent across the system. This implies that the quantum mechanical exclusion principle is uniformly affecting the agents. The absence of specific agents doesn't lead to any noticeable change in the Fermi hole influence, suggesting that the system's antisymmetry properties are maintained.

Delta H Exchange Mean: Effect of Exchange Term

The effect of the exchange term remains relatively stable, which is critical for maintaining the magnetic or opinion order. The consistent nature of this term implies that the removal of specific agents has not significantly influenced the exchange interactions within the system.

V: Value Function

The value function appears to be constant across the system over time. In a reinforcement learning context, this would suggest that the expected reward or utility from the system's states does not change much over time, even with the absence of specific agents.

Sigma: Spin State

Here we see a lot of fluctuations and a lack of uniformity, indicating a highly disordered state or a system with a lot of noise. In a physical system, this could be indicative of high temperature where spin orientations are random. In a sociophysical model, it could represent a society with diverse, rapidly changing opinions. If specific individuals or opinions were absent, the overall disordered nature of this system suggests that their influence on the macroscopic state is not dominant, or the system is so dynamic that it quickly adapts or reconfigures around such absences.

Overall Impact of Specific Absences

Given the uniformity and stability observed in the first five heatmaps, the system appears robust to the absence of specific agents or opinions. The uniform behavior suggests that either the system is large enough that the removal of a few components doesn't significantly perturb it, or the system has mechanisms (like redundancy or quick adaptation) that mitigate the impact of such removals.

In the spin state heatmap, where we observe significant disorder, the system's response to the removal of specific elements might be inherently buffered by the high degree of inherent fluctuations. In such a noisy environment, individual contributions may be less critical to the system's overall state, which could be indicative of a hightemperature regime in a physical system or a highly pluralistic and dynamic society in a sociophysical model. The overall behavior suggests a system that is either at equilibrium or one that is dynamically stable, able to maintain its macroscopic properties despite changes at the level of individual components.

7. Discussion: Heavy Ball Dynamics: Inertia, Delay, Friction, and Resistance

Applying Heavy Ball Dynamics to group dynamics and incorporating elements such as inertia, delay, friction, and resistance becomes valuable when modeling social phenomena related to cyclic or anti-cyclic behaviors. This helps represent the dynamics of social change in more detail. Below, we consider the model equation and ideas for its computational process.

Extended Heavy Ball Dynamics Model An extended Heavy Ball Dynamics model for a value function V representing a social state may be expressed as follows:

$$\frac{d^2V}{dt^2} + \alpha \frac{dV}{dt} + \beta F(V) = \nabla H(V)$$

$\frac{d^2V}{dt^2}$ represents the acceleration (second time derivative) of the value function. $\frac{dV}{dt}$ represents the velocity (first time derivative) of the value function. α is a positive constant representing inertia. β is a constant representing the influence of friction and resistance. $F(V)$ is a function related to friction and resistance. $\nabla H(V)$ is the gradient of the Hamiltonian function (social force).

Modeling Friction and Resistance Social friction and resistance may be related to interactions between agents or behaviors that go against social norms. These can be modeled as follows:

$$F(V) = \gamma \sum_i g(\sigma^{(i)})$$

γ represents the strength of friction and resistance. $g(\sigma^{(i)})$ is a function of friction and resistance based on the behavior of agent i .

Computational Process

Definition of the Hamiltonian Define the Hamiltonian $H(V)$ to reflect interactions between agents and the influence of the external environment.

Calculation of the Impact of Friction and Resistance Calculate the function $F(V)$ based on the behavior of each agent.

Calculation of System Dynamics Calculate the time evolution of the social state V based on the above model equation. This may require numerical methods or simulations.

Application to Social Phenomena

This model helps understand how cyclic or anti-cyclic behaviors progress in group dynamics and how they change depending on changes in the external environment or internal interactions, especially when social friction and resistance are present. It allows for the analysis of how these factors influence the pace and direction of social change.

Such a model is one way to mathematically represent the impact of social inertia, friction, and resistance on group dynamics. It can be a powerful tool for a deeper understanding of social phenomena but requires specialized knowledge for interpretation and application. Additionally, validating the results of this model against real social data and phenomena is crucial.

In the Ising model, the orientation of the spins is determined by the interaction with neighboring spins and the external magnetic field. The traditional Ising model does not consider time dependence and does not directly include dynamics such as inertia, delay, friction, and resistance. However, incorporating these elements allows for more realistic dynamics of physical systems and applications to engineering optimization problems. Heavy Ball Dynamics" here refers to dynamic systems that include an inertial term, and in optimization algorithms it often refers to the gradient descent method plus an inertial term.

Advantages of taking Heavy Ball Dynamics into account

Fast convergence

By including an inertia term, the system is more likely to exceed a local minimum and may converge faster to a global minimum. This is especially useful for systems with large energy barriers.

Dynamics richness

By introducing inertia and delay, the time evolution of the system becomes more realistic, and complex behaviors found in real physical systems, such as relaxation dynamics and oscillations, can be modeled.

Application to Optimization Problems

In optimization algorithms, the introduction of Heavy Ball Dynamics offers advantages such as accelerated convergence and reaching better solutions.

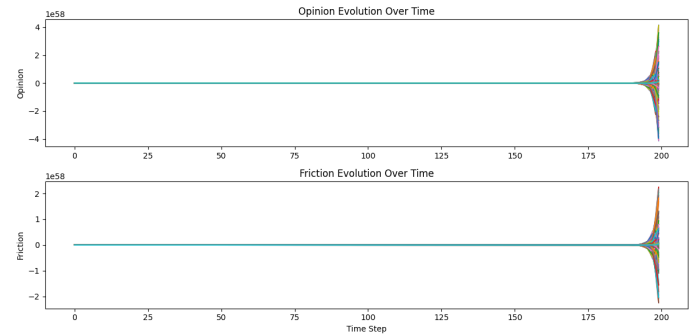


Fig. 14: Total $F(V) / H(V)$ Over Time

Disadvantages of considering Heavy Ball Dynamics

Choice of parameters

How the magnitude of the inertia term is chosen is very important, and improper choice may delay convergence or cause unstable behavior. 2.

Complexity of the analysis

When inertia and friction are considered, the mathematical analysis of the system becomes more complex. This can make it difficult to find solutions and may require additional innovations to ensure numerical stability.

Risk of nonconvergence

If the inertia term is too large, the system may oscillate and not converge, making it impossible to find an optimal spin configuration.

Introducing T-symmetry

When T-symmetry is introduced into the Ising model, its interaction with Heavy Ball Dynamics may lead to more complex behavior; the presence of T-symmetry may change the effects of inertia and friction on the system, since the system exhibits symmetric behavior with respect to backward time. The introduction of these dynamics may provide new insights, especially in studies such as the quantum Hall effect and topological insulators, where retrograde time symmetry plays an important role.

The introduction of Heavy Ball Dynamics into the Ising model is an attractive approach to explore dynamics beyond the classical model, but it assumes that a more complex understanding of mathematics and physics is required to properly understand and control its behavior.

Inertia Coefficient

The inertia coefficient represents the tendency of an agent to maintain its current state or opinion over time. In a social

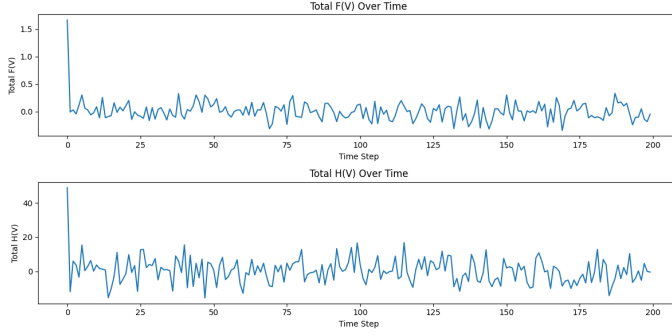


Fig. 15: Opinion Evolution Over Time

context, high inertia suggests strong resistance to change, where individuals or groups continue to hold their opinions despite external influences. A high inertia coefficient could lead to stability in beliefs but also could hinder adaptability and the acceptance of new information.

Friction/Resistance Coefficient

Friction or resistance in a social system could represent societal pressures that resist changes in opinion or state. This could be cultural norms, regulations, or other forms of social control that make it difficult for agents to change their opinions. High friction could lead to slow opinion changes and could prevent sudden shifts in the societal consensus, but it could also impede progress.

Coefficient Related to Friction/Resistance

This coefficient might be similar to the friction coefficient and would similarly influence the rate at which opinions change within the society. It could determine the "smoothness" of the opinion evolution, where a high value could indicate a sluggish response to societal forces, and a low value could indicate a more fluid and dynamic opinion landscape.

Social Spin States

The spin states plot shows a highly disordered pattern, which could be interpreted as a society with a diverse range of opinions and a high degree of disagreement or debate. If the system represents a model where each agent has a binary opinion, this indicates a lack of consensus, potentially leading to social fragmentation or polarization.

Velocity and Acceleration

In a sociophysical model, velocity could represent the rate of change of opinions over time, while acceleration could represent the rate of change of that rate. These concepts could help in understanding how quickly societal opinions are shifting and how the rate of this change is itself changing, potentially due to external events or internal societal dynamics.

Considering these factors together, we can infer the following about the system's behavior

Stable Inertia The opinion state remains relatively stable until it experiences a rapid change. This could be due to a significant event or a threshold being reached that causes a rapid shift in the societal consensus. **Frictional Forces** The system's resistance to change is also relatively stable but starts to fluctuate more as the opinions begin to shift rapidly. This suggests that as opinions start to change, the system's "social temperature" rises, leading to more volatile dynamics. **Disordered Spin States** The social system exhibits a high degree of disorder, indicating a lack of alignment in opinions. This could be due to the inherent diversity of the population or the influence of conflicting information sources. **Dynamic Opinion Evolution** The rapid change in opinions at later time steps suggests a dynamic event that has significantly influenced the societal state, overcoming the inertia and friction within the system.

The overall picture is one of a social system that is initially resistant to change, with each individual maintaining their opinion. However, as external or internal pressures build, a tipping point is reached, leading to rapid changes and potentially chaotic dynamics as the system seeks a new equilibrium.

8. Discussion:Perron-Ishii Lemma in the Context of Heavy Ball Dynamics in Social Dynamics with Friction and Resistance in Cyclic and Anti-Cyclic Spin Movements

Theoretical Background

Social dynamics exhibit nonlinear characteristics, where small changes can lead to significant results. The Perron-Ishii lemma provides insights into the stability and equilibrium states of such nonlinear systems. In the application of Heavy Ball Dynamics to the social sciences, the concept of "inertia" in social dynamics implies that agents' behavior is influenced by past situations and social norms. Heavy Ball Dynamics is suitable for mathematically representing such inertia.

Objectives of Analysis

Modeling Social Change: The objective is to understand how social change progresses and what factors accelerate or delay it. **Dynamics of Cyclic and Anti-Cyclic Behavior:** Analyze how cooperative (cyclic) or competitive (anti-cyclic) behaviors within a group change in response to changes in the external environment or internal interactions.

Extended Heavy Ball Dynamics Model

$$\frac{d^2\sigma}{dt^2} + \alpha \frac{d\sigma}{dt} + \beta F(\sigma) = \nabla H(\sigma)$$

Here, σ represents the social spin state. α is a coefficient representing inertia, and β represents the strength of friction and resistance. $F(\sigma)$ represents a function related to

friction and resistance. $\nabla H(\sigma)$ is the gradient of the social Hamiltonian.

Application of Perron-Ishii Lemma

Setting up the partial differential equation to find the viscosity solution using Perron-Ishii lemma:

$$\min \left(\frac{\partial \sigma}{\partial t} + H(\sigma, \nabla \sigma), \Delta \sigma \right) = 0$$

Overview of the Computational Process

Model Configuration:

$$\frac{d^2 \sigma}{dt^2} + \alpha \frac{d\sigma}{dt} + \beta F(\sigma) = \nabla H(\sigma)$$

σ represents opinions or social spin states. α represents inertia, and β represents the strength of friction and resistance. $F(\sigma)$ models viscosity or density of opinions.

Modeling Friction/Resistance Effects: States with high opinion viscosity or density are modeled by the $F(\sigma)$ function. For example, assuming that higher opinion density increases friction and resistance, a function like the following can be considered:

$$F(\sigma) = k \cdot \text{density}(\sigma)$$

Here, $\text{density}(\sigma)$ represents opinion density, and k is the coefficient for friction and resistance.

Evolution by Time Steps: The system's state evolves over discrete time steps t . At each step, calculate the new value of σ using the above equation.

Interpreting the Results

In cases of high opinion viscosity or density, group opinions may change gradually over time, with fewer abrupt fluctuations. If opinion density is high in a specific region, convergence or fixation of opinions may be observed around that region. Patterns of opinion changes over time suggest the flow of discussions within the group and the process of opinion formation.

This model and visualization approach have the potential to provide insights into the dynamics of opinion formation in the social sciences. However, interpretation should always be complemented with comparisons to real social phenomena and data.

Social Spin State (σ)

The provided graph shows the evolution of a social spin state over time with specified parameters for inertia (α), friction or resistance (β), and a constant (k). The social spin state (σ) is likely a measure of the average opinion or consensus within a social system, with positive and negative values indicating the predominant direction of opinion.

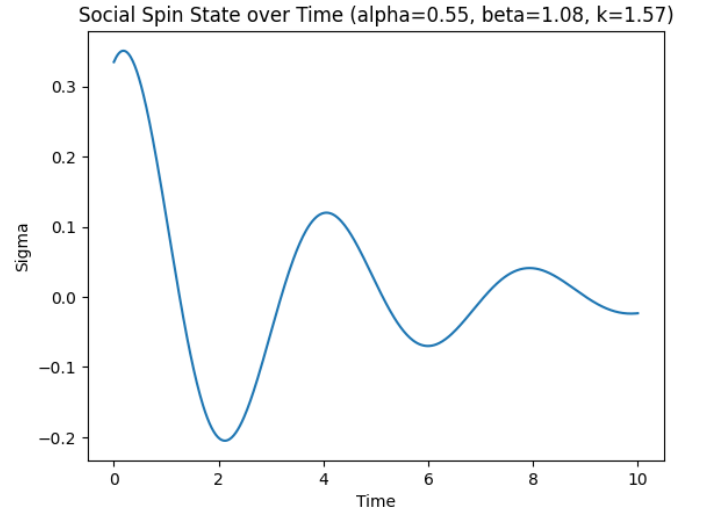


Fig. 16: Social Spin State over Time Over Time

Social Spin State (σ)

The social spin state oscillates over time, which suggests that the average opinion in the social system is not stable but changes, perhaps in response to external influences or internal dynamics within the group. The oscillation does not damp out quickly, which may indicate that the system has some periodic or cyclical drivers.

Inertia Coefficient ($\alpha = 0.55$)

Inertia in a physical system resists changes in motion. Here, α represents the tendency of the social system to resist changes in the overall opinion. A value of 0.55 suggests a moderate level of inertia where the society does resist changes in opinion but not so strongly that changes cannot happen. This level of inertia allows for opinion shifts but with some "memory" or "stickiness" to prior states.

Friction/Resistance Coefficient ($\beta = 1.08$)

Friction or resistance in a physical system opposes motion. In this social model, β represents forces that slow down the change in opinion. A value slightly greater than 1 suggests that there are significant forces acting against change, such as societal norms or regulatory pressures, which prevent rapid swings in social opinion.

Gradient of the Social Hamiltonian ($k = 1.57$)

In physics, the gradient of the Hamiltonian (energy function) with respect to the state variable (here, σ) indicates the direction and rate of change of the system's energy. In the social context, $\nabla H(\sigma)$ can be understood as the societal "forces" that drive changes in the average opinion. A value of 1.57 indicates that these forces are present and contribute to the

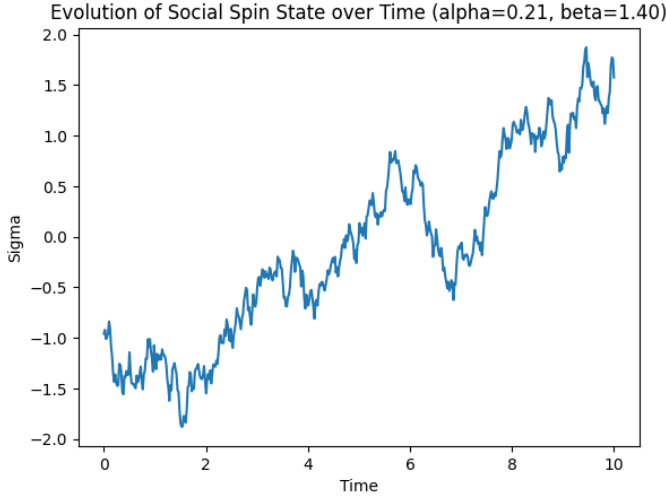


Fig. 17: Evolution of Social Spin State over Time

system's dynamics, potentially driving the observed oscillations in the social spin state.

Friction or Resistance Function ($F(\sigma)$)

$F(\sigma)$ would represent how friction or resistance depends on the current state of opinion. Since this function is not explicitly given, we can infer from the behavior of σ that the friction/resistance is not preventing changes in state outright but is influencing the rate and extent of these changes.

Interpretation of Dynamics

The graph shows that the system starts with a higher value of σ , indicating a strong initial consensus or opinion in one direction, which then decreases and oscillates over time. The presence of both inertia and friction ensures that these changes are neither too rapid nor too extreme, suggesting a society that can evolve its opinions while maintaining a degree of coherence.

The oscillatory nature of σ indicates that the social system might be subject to cyclical influences — possibly recurring events or discussions that periodically shift public opinion back and forth. The system doesn't seem to settle into a static state within the observed time frame, indicating ongoing debates or a dynamic equilibrium rather than a settled consensus.

In summary, the model shows a social system where opinion is dynamic but changes are tempered by both an inherent resistance to change (inertia) and external pressures (friction/resistance), all within a landscape shaped by the societal "forces" represented by the social Hamiltonian.

Social Spin State (σ)

The plot of σ shows a trend that initially decreases, then increases, and shows a general upward trend towards the end. This suggests that the social opinion or consensus starts with one tendency, shifts to another, and then trends positively over time. The variation in σ indicates that the collective opinion is dynamic and subject to change rather than being in a stable state.

Inertia Coefficient ($\alpha = 0.21$)

A relatively low inertia coefficient implies that the social system is quite responsive to changes and does not have a strong tendency to resist shifts in opinion. This could correspond to a flexible society or community where individuals are relatively open to changing their views or adapting to new information.

Friction/Resistance Coefficient ($\beta = 1.40$)

A higher friction or resistance coefficient indicates that there are significant forces opposing the change in the social spin state. This could represent societal norms, cultural inertia, or structural factors that make rapid changes in consensus more difficult. Despite low inertia, the friction in the system is strong enough to prevent sudden or rapid changes in the social state.

Friction/Resistance Function ($F(\sigma)$)

While not explicitly shown in the plot, $F(\sigma)$ would represent how the friction or resistance varies with the social spin state. This could mean that certain opinions or consensus states are more strongly resisted by society, possibly due to entrenched beliefs or powerful counteracting social forces.

Gradient of the Social Hamiltonian ($\nabla H(\sigma)$)

The gradient of the social Hamiltonian would indicate the "force" driving the change in the social spin state. If the society is modeled on an energy landscape, $\nabla H(\sigma)$ points towards the direction of steepest descent, guiding how opinions evolve over time. Since the social spin state σ exhibits a non-linear and non-monotonic evolution, the social forces represented by the Hamiltonian gradient are likely complex and may involve multiple attractors or competing influences.

Interpretation of the Evolution of σ

The graph suggests a society where opinion is not fixed and can change significantly over time. The low inertia allows for flexibility and adaptability, while the high friction moderates the rate of change, ensuring that transitions between different consensus states are not abrupt but rather gradual. The overall upward trend in the latter part of the graph indicates that

Evolution of Social Spin State over Time with Viscosity (alpha=0.81, beta=0.28)

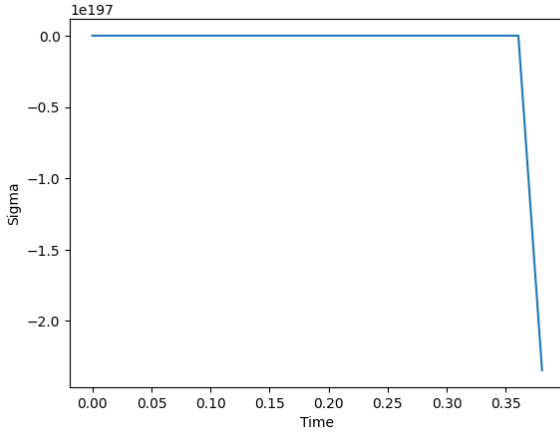


Fig. 18: Evolution of Social Spin State over Time with Viscosity

despite resistance, the social system tends towards a more positive consensus over time.

This could represent a society undergoing a gradual shift in collective opinion due to ongoing discourse, changing circumstances, or external influences. The dynamics of σ suggest that the collective opinion is being pulled in different directions but ultimately is moving towards a new, possibly more positive, state. The system does not appear to be in equilibrium but is rather characterized by continuous evolution, possibly oscillating between different states before settling.

Interpretation of Parameters

$$\alpha = 0.81$$

This represents a high inertia coefficient, implying that the social system typically resists changes in opinion. However, the sharp change depicted in the graph suggests that a very strong force has overcome this inertia.

$$\beta = 0.28$$

This is a relatively low friction/resistance coefficient, suggesting that there isn't much opposition to the change in opinion once a force is applied. The low value of β might have facilitated the rapid shift seen in the graph despite the high inertia.

$$F(\sigma)$$

This function represents the friction/resistance or viscosity effect on the system. While we don't have the specific form of $F(\sigma)$ from the information provided, the graph indicates that the resistance to change in social opinion might not be strong enough to prevent the sudden shift that occurs.

$$\nabla H(\sigma)$$

The gradient of the social Hamiltonian would normally direct the evolution of σ toward the system's most stable state. The abrupt change suggests that the system experienced a significant force that drastically altered the social spin state, corresponding to a steep gradient in the social Hamiltonian.

Interpretation of the Dynamics

Given the high inertia and low friction, the system would generally be expected to change slowly. However, the precipitous drop suggests an external shock or a sudden, powerful event that has significantly influenced the social state. Such a shift could correspond to a major societal event, such as a political upheaval, economic crisis, or the impact of significant news or a social movement that aligns opinions rapidly.

The suddenness of the transition might also suggest a phase transition in the social system, where the collective state becomes unstable and shifts to a new equilibrium rapidly once certain thresholds are surpassed. The parameters α and β suggest that once the system begins to move, the change can be swift due to the low resistance, despite the initial high inertia.

In terms of social dynamics, the plot could represent a situation where a community or society that typically changes slowly (due to high inertia) suddenly reaches a tipping point, after which the change becomes rapid and dramatic, overcoming the usual resistance to change.

This model underscores the complexity of social dynamics, illustrating how a system with high inertia can still undergo rapid transformations when subjected to strong forces, even if the usual friction against change is relatively low.

9. Discussion: Under which cyclic exchange of spins is valid on the Ising model in Social Dynamics

This section discusses the conditions under which cyclic exchange of spins is valid on the Ising model and interprets these in the context of social dynamics when considered as group dynamics. The Ising model, commonly used in statistical mechanics and physics, is a simple model where spins (binary variables) on a lattice influence each other through mutual interactions. Spins typically have two states: up (+1) and down (-1). The condition for cyclic exchange in the Ising model refers to a specific parameter setting where the exchange operation of spins does not affect the total energy of the system, allowing the dynamics to possess a certain symmetry.

When cyclic exchange is valid, the energy does not change even if spins on the lattice are cyclically exchanged, leading to the possibility of the system exhibiting ergodicity. Ergodicity refers to the property of a system exploring different states

over a sufficient period. This property can be utilized to apply the Ising model to social dynamics.

Interpreting the Ising model from the perspective of social dynamics, spins on the lattice represent individual agents or persons, and interactions between spins signify the relationships or influences between agents. When cyclic exchange is valid, it implies that even if the attributes or opinions of agents are cyclically exchanged, there is no change in the overall state of society.

For example, in an Ising model representing political opinions or beliefs, if cyclic exchange is valid, individual opinions may change cyclically, but there are no significant changes in the political state or social trends. However, if cyclic exchange is not valid, the opinions or actions of a few agents can significantly influence the whole, potentially leading to unstable social dynamics. In interpreting social dynamics using the Ising model, the presence or absence of cyclic exchange can impact the stability and ergodicity of society. Such models are used to study the convergence or divergence of opinions, decision-making dynamics, and other aspects within a society.

When ergodicity holds, individual opinions within a society might change over time, making convergence of opinions less likely. With ergodicity, the system can explore different states over the long term, tending to maintain diversity of opinions rather than convergence. The presence or absence of cyclic exchange also influences the stability and ergodicity of society.

In cases where opinion stickiness occurs, it refers to the tendency of opinion changes being subject to certain constraints or trends. Opinion stickiness suggests that individual agents are influenced by other agents or the overall social situation when changing their opinions. This is related to factors such as social contexts or pressures, sources of information, culture, and values that make it easier for people to retain their opinions.

If ergodicity holds but opinion stickiness also exists, one idea for interpreting social dynamics is that specific opinions may become clustered within society, forming different clusters. Agents are likely to be influenced within their cluster, increasing the stickiness of opinions in that cluster. Some agents or clusters may tend to change their opinions cyclically, meaning opinions change in a cycle over a certain period. For instance, in political conflicts, opinions might change according to election cycles.

Diffusion and localization of opinions: With ergodicity, new opinions or information can be introduced into society, but due to opinion stickiness, the spread of these new elements can be slow. Thus, diffusion and localization of opinions can occur simultaneously. Additionally, diversity of opinions: With ergodicity, a variety of opinions can coexist within society. However, due to opinion stickiness, some agents or

clusters may stick to certain opinions.

Introduction of the Concept of Cyclic Exchange

In the Ising model, the 'cyclic exchange' of spin configurations, where spins are periodically exchanged, adds a new dimension to the dynamics of the system. This operation mimics the temporal evolution of the system by exchanging the values of adjacent spins. Mathematically, it can be expressed as follows:

$$x_i(t+1) = x_{i+1}(t), \quad x_{i+1}(t+1) = x_i(t), \quad \text{for } i = 1, 2, \dots, N-1$$

Here, $x_i(t)$ represents the state of spin i at time t , and N is the total number of spins. Through this operation, the configuration of spins changes cyclically over time, and the system explores different states.

Explanation of Ergodicity

Ergodicity refers to the property where, over long time scales, a system explores all possible microstates, and the state of the system matches its statistical ensemble. By introducing cyclic exchange, the spin configuration changes over time, and the system exhibits ergodicity. This means that, over the long term, the system explores all possible spin configurations.

To express cyclic exchange mathematically, we introduce the following update rule:

$$x_i(t+1) = \begin{cases} x_{i+1}(t), & \text{if } i < N \\ x_1(t), & \text{if } i = N \end{cases}$$

This rule cyclically exchanges the spin configuration over time, enabling the system to explore different spin states. With this update rule, the energy function $E(x, J)$ takes different values over time, capturing the dynamic behavior of the system.

By conducting numerical simulations of this model, we can observe how cyclic exchange and ergodicity influence the behavior of the system. Starting from an initial state and iteratively updating the spin configuration based on the above rules, we can visualize how the system explores different states over time.

This approach serves as an effective tool for understanding complex social dynamics and informational health using the Ising model, demonstrating the potential of applying physical concepts to social sciences.

When cyclic and anticyclical changes occur in spins on the Ising model and ergodicity holds, several properties related to social dynamics can be inferred.

1. Diversity and Variability of Opinion

Because of the occurrence of cyclic and anticyclical change, there will be a diversity of different

opinions within a society in the long run, and we will see fluctuations in opinion. As agents change their opinions cyclically, different opinions will emerge alternately within a society, contributing to maintaining diversity.

2. Clustering of Opinions

When cyclic and anti-cyclical changes alternate, agents are more likely to belong to certain opinion groups, which can lead to opinion clustering. That is, some agents may have similar opinions while others form clusters with different opinions.

3. Opinion Influence

Due to agents changing their opinions cyclically, the opinions of some agents may have a strong influence on other agents at a particular time or in a particular situation. In such cases, certain agents may become opinion leaders within a society and influence opinion trends.

4. The Emergence of Cycles

Cycles of opinion may form as cyclic and anticyclical changes occur frequently within a society. This indicates that certain opinions are periodically emphasized and temporarily become mainstream. For example, a political election cycle or a social trend cycle may occur.

5. Uncertainty in Opinion Dynamics

When cyclical and anticyclical changes interact within a society, opinion dynamics can be difficult to predict and have uncertainty. This affects the prediction of social decision making and behavior and is important as it relates to policy formulation and the influence of opinion.

Non-Equilibrium Properties: Perspectives on Zeeman's Theorem

The consideration of non-equilibrium properties in terms of Zeeman's theorem on the Ising model provides theoretical insight. but can explore the possibility of observing non-equilibrium properties as well.

In understanding the non-equilibrium properties on the Ising model in terms of Zeeman's theorem, the following ideas can be considered: 1.

Cyclic changes and non-equilibrium

If the spins in the Ising model exhibit cyclic changes, different spin configurations can be explored over time based on ergodic properties. This may lead to non-equilibrium states

in the short term and deviations from equilibrium may be observed.

Cyclic Changes and Phase Transitions

In terms of Zeeman's theorem, non-equilibrium cyclic changes can cause phase transitions or transitions from order to disorder. This implies that non-equilibrium states deviate from equilibrium states according to ergodicity.

Application to Social Dynamics

When applying the Ising model to social contexts, in terms of Zeeman's theorem, changes in opinion and behavior within a society may be influenced by ergodicity. Interaction between agents and information propagation may result in non-equilibrium social dynamics, and a temporary transition from order to disorder may be observed.

In other words, considering the properties of non-equilibrium on the Ising model in terms of Zeeman's theorem suggests that non-equilibrium conditions may affect the dynamics of the entire system according to ergodicity.

Non-equilibrium characteristics: cusp catastrophes, hysteresis loops occurs on the dynamics, do we observe structural stability and its discontinuous branching features? For example, cases of hysteresis loops such as cusp catastrophes do occur.

The occurrence of cusp catastrophes and hysteresis loops in the Ising model is unique to non-equilibrium dynamics, and the specific computational process is generally complex. However, attempts are being made to understand these phenomena through specific variations and extensions of the Ising model. As an example of an Ising model, we discuss the conditions under which cusp catastrophes and hysteresis loops occur and how they relate to social dynamics.

Spin Orbits in the Ising Model: Cyclical Exchanges and Ergodicity

The energy function of the Ising model is usually expressed as follows:

$$E(x, J) = -J \sum_{i,j} x_i x_j$$

Here, x_i represents spin variables (+1 or -1), and J is the interaction constant.

1. Introduction of the Concept of Cyclical Exchanges:

To introduce the concept of cyclical exchanges, we consider periodically exchanging the spin configurations x_i of the Ising model at time t . Specifically, we contemplate an operation that exchanges the values of x_i and x_{i+1} . This models the phenomenon of spin configurations cyclically changing.

$$x_i(t+1) = x_{i+1}(t), \quad x_{i+1}(t+1) = x_i(t), \quad \text{for } i = 1, 2, \dots, N-1$$

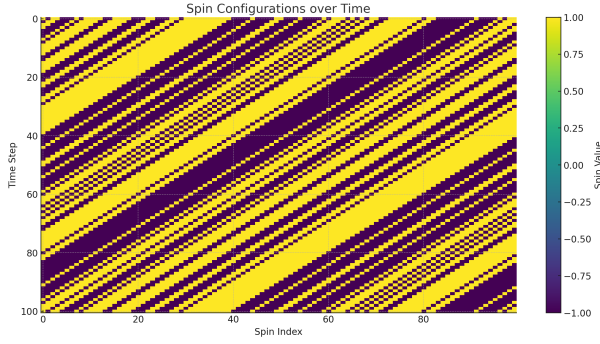


Fig. 19: Spin Configurations over Time

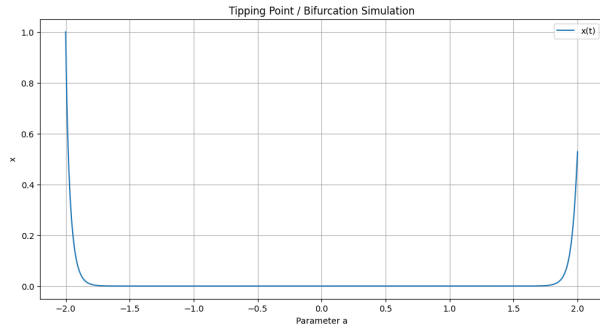


Fig. 20: Tipping Point / Bifurcation Simulation

2. Explanation of Ergodicity: Ergodicity refers to the property where a system explores different states (spin configurations) and can reach all possible states as time progresses. By introducing cyclical exchange operations, the spin configurations cyclically change, allowing the system to explore different states on long time scales, thereby satisfying ergodicity.

3. Mathematical Representation: To express cyclical exchange operations mathematically, we consider the following update rules:

$$x_i(t+1) = \begin{cases} x_{i+1}(t), & \text{if } i < N \\ x_1(t), & \text{if } i = N \end{cases}$$

This leads to cyclic exchange of spin configurations. By applying these update rules to the Ising model's energy function and evolving the system while calculating energy at each time step, we can model cyclical exchanges that satisfy ergodicity.

4. Numerical Simulation: Conducting specific numerical simulations can be useful in demonstrating cyclical exchanges and ergodicity. Starting from initial conditions, by iteratively applying cyclical exchange operations, you can observe how the system explores different states.

Results shows the evolution of spin configurations over time in an Ising model system undergoing cyclic exchange. Here are some observations and interpretations from the perspectives of cyclic exchange and ergodicity: Diagonal Pat-

terns The graph exhibits diagonal striping patterns, which indicate that the spins are indeed being exchanged cyclically over time. Each diagonal stripe represents a spin traveling across the system, reflecting the cyclic exchange operation.

Periodicity

The regularity of the patterns suggests a periodic or cyclic nature to the spin exchanges. This is characteristic of the cyclic exchange we defined in the simulation, where each spin is swapped with its neighbor in a fixed pattern.

System Evolution

The system does not appear to settle into a static configuration, which would be represented by horizontal lines of constant color. Instead, the dynamic patterns suggest continuous evolution of the system's state due to the cyclic exchange.

State Exploration

Ergodicity implies that, given enough time, the system will explore all possible microstates. The variety of patterns in the graph suggests that the spins are exploring different configurations over time. However, to fully confirm ergodicity, one would need to demonstrate that every possible state is eventually visited, which is not directly observable from this single graph.

Long-Term Behavior

If the system were to continue indefinitely, ergodicity would predict that the frequency of visiting any particular configuration would be the same for all configurations. However, in practical simulations, we can only run for finite time, and the graph shows only a snapshot of this process.

Absence of Equilibrium

The constant motion and lack of a static pattern suggest that the system is not reaching an equilibrium state within the observed time frame. In an ergodic system, equilibrium is understood in a statistical sense over long periods, rather than the system being static at any moment.

The results provides visual evidence of cyclic exchange in the Ising model, with the spin values changing places in a regular pattern over time. It also suggests that the system exhibits behavior consistent with ergodic principles, with the spins exploring different configurations. However, further analysis would be required to rigorously prove ergodicity, such as examining the long-term frequency distribution of states or running the simulation for significantly longer times.

The graph represents a state of social "spin" that changes over time, where "spin" could refer to an individual or group's

opinions, attitudes, or behavioral tendencies. Cyclic exchange models how opinions and attitudes propagate through social interactions and change over time.

Cyclic exchange perspective, periodicity of patterns The arrangement of spins (opinion states) changes periodically, representing changes in the flow of opinions or trends within a society. It suggests that certain opinions and behaviors move between groups in a certain cycle.

Interaction between groups

The diagonal pattern visualizes how opinions and behaviors are transmitted from one person to another. This may indicate how fads and social norms propagate and shift over time.

Ergodic Perspective

state search, we can see that a variety of social states are being explored over time. This implies that society experiences different opinions and behaviors over time, which is consistent with the concept of ergodicity.

Social Equilibrium

In an ergodic society, all opinions and attitudes will exist equally in the long run. However, the graph is only a temporary snapshot and does not show the overall process leading to equilibrium.

Comprehensive Social Dynamics Study

Provides insight into how the flow of opinions and attitudes in social dynamics changes over time. It shows how opinions form, change, and propagate within social networks through cyclic exchange. At the same time, it suggests the ergodic nature in which society is expected to experience and explore different opinions and attitudes over the long term.

Conditions for the onset of cusp catastrophes and hysteresis loops

In Ising models, common factors that cause cusp catastrophes and hysteresis loops are nonlinearities and changes in external parameters.

Change in external magnetic field: spin configuration stability

Consider introducing an external magnetic field into the Ising model and varying its intensity. If the external magnetic field has nonlinear effects, the stability of the spin configuration may change and a cusp catastrophe may occur. 2.

Nonlinearity of interactions: spin configurations are discontinuous The interaction between spins can be set to be

nonlinear. If nonlinear interactions exist, the spin configurations can change discontinuously and cusp catastrophes can occur.

When a cusp catastrophe or hysteresis loop occurs in an Ising model, how it affects the social dynamics depends on the specific scenario. Below are a few real-life examples and their implications for social dynamics. Hysteresis in opinion formation models: In Ising models of opinion formation, changes in the external environment or information can cause hysteresis. For example, in political opinion formation, if opinions are maintained once formed and are not sensitive to external information, hysteresis may occur and social fragmentation or fixation may be observed. Cusp catastrophes in magnetic materials: From a physics perspective, cusp catastrophes in the spin configuration of magnetic materials indicate abrupt changes in magnetization, which may affect the physical properties and stability of the society. For example, abrupt changes in magnetization can cause instability in energy supply and communication systems. Group Cooperation and Competition: Using the Ising model, cooperation and competition within a society may cause hysteresis and cusp catastrophes. Cooperation may be maintained in a particular state and suddenly competition may increase or the opposite scenario may occur. The above examples are general considerations in non-equilibrium Ising models and similar models.

Cusp Catastrophe Machine Ideas

There is an example of applying cusp catastrophes to model the response to stress as an external pressure. The proposal is that under moderate stress ($a > 0$), dogs show a smooth transition from fright to anger, depending on how they are stimulated, but that high stress levels correspond to a domain shift ($a < 0$), where the dog remains frightened until it reaches a "crease" point, at which point it suddenly and discontinuously enters anger mode. Once this point is reached, the dog suddenly and discontinuously enters anger mode. Once the dog enters the "angry" mode, it remains angry even if the direct stimulus parameters are greatly reduced. A simple mechanical system, the "Zeeman Catastrophe Machine," illustrates the cusp catastrophe well. In this device, a smooth change in the position of the end of a spring can cause a sudden change in the rotational position of the wheel to which it is attached.

Let us assume that we consider the above explanation as a group dynamics with external stresses applying the Ising model, and the calculation process as a Pauli determinant.

Using the Ising model directly to model cusp catastrophes and stress responses may be a bit difficult; the Ising model focuses primarily on spin interactions, and directly modeling discontinuous phenomena such as cusp catastrophes and stress responses is complicated. However, there are modeling

approaches that can be used to describe similar discontinuous phenomena.

The Landau-Ginzburg equation is a nonlinear equation used to describe cusp catastrophes and phase transitions. These are nonlinear continuous-time models, and nonlinear continuous-time models may be constructed to describe cusp catastrophes and stress responses. These models typically use differential equations to represent physical processes and responses. Cusp Catastrophe Machines Cusp catastrophe machines, which are concrete systems or devices, can be modeled in terms of mechanical and control engineering. It involves physical springs and constraints to reproduce discontinuous motion and behavior.

9.1 Discontinuity Dynamics Represented by Cusp Catastrophes in External Stress

When considering models that represent discontinuous dynamics, such as cusp catastrophes, especially in response to external stress, mathematical modeling of nonlinear equations or systems is required. Below, we will briefly illustrate the idea of cusp catastrophes and consider the response to external stress through a simple nonlinear equation.

An example of a nonlinear equation is as follows:

$$\frac{dx}{dt} = f(x, a) \quad (1)$$

Here, x represents the state variable of the system, t represents time, and a represents external stress (a parameter). The function $f(x, a)$ is nonlinear and describes the characteristics of the system.

If external stress (a) has the potential to induce cusp catastrophes, the function $f(x, a)$ may exhibit discontinuous changes under certain conditions. To illustrate this, let's consider an example $f(x, a)$:

$$f(x, a) = x^3 - ax \quad (2)$$

In this nonlinear equation, a represents external stress, and x represents the state variable of the system. This equation may exhibit discontinuous behavior under certain values of external stress a .

The computational process involves:

1. Setting initial conditions (e.g., $x(0) = 0$).
2. Performing time integration to evolve the system's state variable x with respect to time t . Typically, numerical simulation methods are used to solve the equation at each time step.
3. Changing the external stress a . As the value of a is increased, the behavior of the system changes. In particular, observe whether discontinuous changes (cusp catastrophes) occur at certain values of a .
4. When a cusp catastrophe occurs, the behavior of the system undergoes a rapid change. This indicates a discontinuous response to external stress.

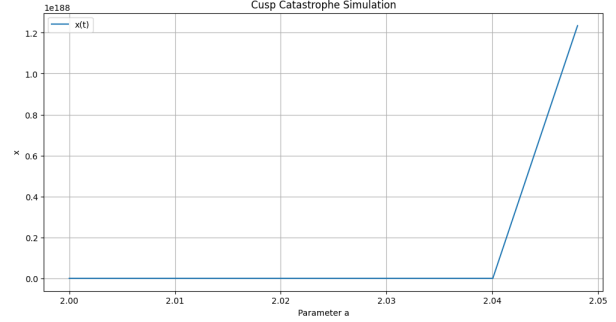


Fig. 21: Cusp Catastrophe Simulation

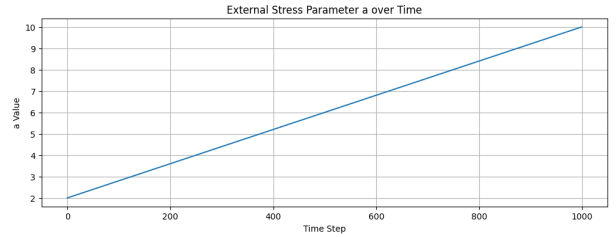


Fig. 22: External Stress Parameter a over Time

In this model, as the external stress a increases, the behavior of the system changes, and there is a possibility of the occurrence of discontinuous catastrophes. When applying this idea to the dynamics of social groups, external stress represents social factors or environmental changes, and it is assumed that the social response may change discontinuously.

Cusp Catastrophe Simulation

The graph titled "Cusp Catastrophe Simulation" shows a variable x , which we can interpret as a state variable in the system, changing as a function of the parameter a . There is a sudden and dramatic increase in x , which is characteristic of a cusp catastrophe. This kind of behavior is indicative of a system where small changes in the parameter a can lead to sudden and large changes in the state variable x after reaching a critical threshold. This can be thought of as a stable state until a tipping point is reached, after which the system rapidly

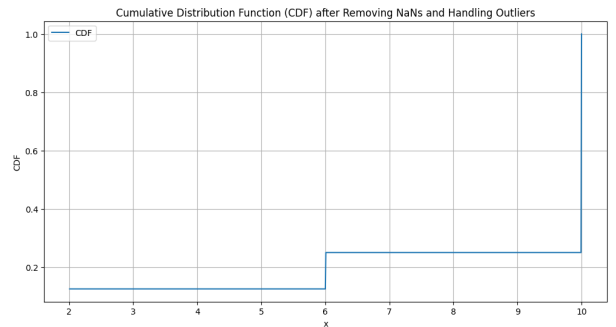


Fig. 23: Cumulative Distribution Function (CDF) over Time

transitions to a new state.

External Stress Parameter a over Time

The graph showing the External Stress Parameter a over Time depicts a linear increase in the parameter a . This is a control parameter that, when it reaches a critical value, leads to the cusp catastrophe observed in the first graph. In a social context, this parameter could represent an external pressure or stressor that increases over time, such as political tension, economic pressure, or social unrest, which eventually leads to a dramatic societal shift or a tipping point.

Cumulative Distribution Function (CDF) after Removing NaNs and Handling Outliers

The CDF graph represents the cumulative distribution of the state variable x over its range. The sharp step in the CDF indicates that the variable x spends a significant amount of time at certain values before transitioning to another state. This step-like feature in the CDF is again characteristic of a cusp catastrophe where the system's states are clustered around certain stable states before making a transition.

Overall Interpretation

From these graphs, we can infer that the system begins in a stable state and as the external parameter a increases linearly over time, the system remains stable until a critical threshold is reached. Once this threshold is crossed, the system undergoes a sudden state change, indicative of a cusp catastrophe.

In the context of social dynamics, this could represent a society that is experiencing increasing external pressure. The society remains in a stable state despite these pressures until it reaches a critical point. At this critical point, perhaps due to a significant event or accumulation of factors, there is a sudden and large-scale change in the societal state, such as a revolution, economic crash, or drastic shift in public opinion.

Application to the Ising Model

In the Ising model, this sort of catastrophe could correspond to a phase transition from one magnetic state to another, induced by changes in external magnetic fields or temperature. The cusp catastrophe would be an analog to a critical point in the phase diagram where the system transitions from one phase to another, such as from ferromagnetic to paramagnetic.

The cusp catastrophe model can provide insight into the non-equilibrium dynamics and potential transitions in systems described by the Ising model. It could help predict the conditions under which a system might exhibit hysteresis or catastrophic behavior, which are important for understanding magnetic materials, social systems, and other complex systems.

Jump Phenomenon between a and b : Hysteresis Loop by Initialization

In cusp geometry, a bifurcation curve loops itself and returns to the original set of solutions by giving a second bifurcation where alternative solutions lose stability. By repeatedly increasing and then decreasing b , the system alternately follows one solution, jumps to another, follows a solution there, and jumps back to the first one, creating an observable hysteresis loop.

However, this is only possible in the region of parameter space where $a < 0$. As a increases, the hysteresis loop becomes smaller, and when a becomes greater than or equal to 0, it completely disappears (cusp catastrophe), leaving only one stable solution.

It is also possible to consider what happens when b is kept constant and a is varied. In the symmetric case of $b = 0$, as a decreases, a pitchfork bifurcation is observed, and when the physical system passes through the cusp point $(0, 0)$ to $a < 0$, one stable solution suddenly splits into two stable solutions and one unstable solution (an example of spontaneous symmetry breaking). Away from the cusp point, there is no sudden change in the physical solution. When passing through the fold bifurcation curve, only the alternative second solution is obtained.

In the group dynamics of the Ising model, it is possible to construct equations that represent phenomena such as cusp catastrophes and hysteresis loops. Consider a model that describes discontinuous dynamics based on the Ising model.

In the Ising model, consider an energy function that represents the interaction of spins. External parameters (a and b) influence the Ising model and cause discontinuous behavior.

The energy function of the Ising model can be extended to consider external parameters a and b as follows:

$$E(x, a, b) = -J \sum_{i,j} x_i x_j - a \sum_i x_i - b \sum_i x_i^2$$

Here, x_i represents the spins of the Ising model (+1 or -1), J is the interaction constant, a is a parameter corresponding to external stress ($a < 0$ in the cusp catastrophe region), and b is another parameter corresponding to a different external stress. This energy function is used to describe the behavior of spins.

Here is a simulation procedure:

1. Set initial conditions. Determine the initial state of spin configurations x_i randomly or by a specific method.
2. Simulate time evolution. At each time step, update spins x_i to minimize the energy function $E(x, a, b)$. This is commonly done using algorithms like the Metropolis algorithm.
3. Change a and b . Vary the external stresses a and b over time, influencing the dynamics of the Ising model.
4. Observe the cusp catastrophe: When a exceeds a certain value,

discontinuous changes are observed, indicating a cusp catastrophe. This shows sudden changes in spin configurations and energy.

This model illustrates a simple example of how external stresses a and b can influence the Ising model, leading to discontinuous catastrophes and hysteresis loops. Specific numerical simulations and computational details would require further investigation and adjustment, but this idea demonstrates one way to extend the Ising model to represent discontinuous dynamics.

Discontinuous Phenomena, Phase Transitions, and Catastrophe Theory on the Ising Model

Catastrophe theory is envisioned as a revolutionary theory to explain discontinuous phenomena. We consider here the ISING model, with respect to possible theoretical elements as they relate to spin.

Catastrophe Theory is a mathematical theory for explaining discontinuous phenomena and abrupt changes such as phase transitions. The theory has been applied in a variety of fields, including physics, ecology, psychology, and economics. To understand the relevance of the Ising model to catastrophe theory, we review the basic elements of catastrophe theory below. Potential Energy As in catastrophe theory, the state of a system is represented by its potential energy. This potential energy depends on the state variables and external parameters of the system and describes the behavior of the system. 2. As a constraint, the system minimizes the potential energy under the constraint conditions and heads toward a stable state. Constraints vary depending on the characteristics of the system. As for the catastrophe point, the point on the potential energy surface where a local minimum or maximum occurs is called the catastrophe point. At this point, the behavior of the system changes abruptly. In bifurcation and catastrophe theory, the system may choose between two different stable states (branches) around the catastrophe point. This phenomenon is called bifurcation and represents a discontinuous change.

Relevance of the Ising Model to Catastrophe Theory

The Ising model is a statistical mechanics model based on spin interactions and is used to describe continuous phenomena such as phase transitions. Catastrophe theory, on the other hand, is used to describe discontinuous changes or abrupt transitions. To relate the Ising model to catastrophe theory, one can consider the presence of catastrophe points or bifurcations in the potential energy surface of the Ising model. Varying external parameters can alter the energy surface of the Ising model, potentially leading to phase transitions and discontinuous changes. Such discontinuous behavior could be understood in terms of catastrophe theory.

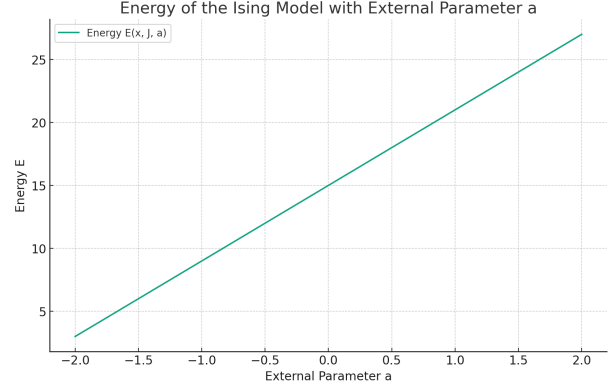


Fig. 24: Energy of the Ising Model with External Parameter a

Modeling Discontinuous Phase Transitions such as Cusp Catastrophes and Hysteresis Loops in the Group Dynamics of the Ising Model

It is possible to formulate equations to model discontinuous phase transitions, such as cusp catastrophes and hysteresis loops, in the group dynamics of the Ising model. Below, we present an idea for representing discontinuous phase transitions by introducing external parameters into the Ising model.

First, we extend the energy function of the Ising model as follows:

$$E(x, J) = -J \sum_{i,j} x_i x_j$$

Here, x_i represents spin variables (+1 or -1), and J is the interaction constant. This is the energy function of the standard Ising model.

Next, we introduce an external parameter, a . This parameter a controls the phase transitions. The energy function considering external parameters becomes:

$$E(x, J, a) = -J \sum_{i,j} x_i x_j - a \sum_i x_i$$

Here, a is a parameter that influences phase transitions, and the behavior of the Ising model changes as a varies.

Using this energy function, we simulate the time evolution of the spins x_i of the Ising model. By varying the external parameter a , phase transitions in the Ising model occur, and discontinuous behaviors like cusp catastrophes and hysteresis loops may be observed.

To understand the specific computational processes and behaviors of phase transitions, numerical simulations or analytical approaches are required. By investigating the behavior of phase transitions in the Ising model in response to changes in the external parameter a , one can gain insights into how discontinuous phenomena manifest.

Results provided shows the energy of an Ising model system as a function of an external parameter a . The energy E

increases linearly with the parameter a , which indicates that the model includes an external field term linearly coupled to the spins. In the context of non-continuous phase transitions within the Ising model, the behavior depicted in the graph does not show a discontinuous change in energy, which would be expected in a first-order phase transition typically associated with a latent heat (an abrupt change in the system's energy). Instead, the energy change is continuous as the external parameter a is varied. However, if we were expecting a non-continuous or discontinuous phase transition, there are a few possibilities:

The Range of a is too Broad

The parameter a may need to be varied over a smaller range or more finely sampled near the critical value where the phase transition occurs to capture the non-continuous change. System Size and Finite-Size Effects In small systems or simulations, phase transitions can appear smoother and may not capture the abrupt changes seen in larger systems due to finite-size effects.

Temperature Effects

If the system's temperature is not at the critical temperature, then the expected discontinuous phase transition may not occur.

Time Evolution and Equilibrium

The graph is a snapshot of the system's energy at different values of a . If the system has not reached equilibrium at each point a , or if the system's dynamics are not included, it may not show the expected non-continuous phase transition.

In a real-world context, such as social dynamics, a similar graph could represent a situation where social stress or tension (parameter a) increases, leading to a gradual buildup of social energy (analogous to the system's energy E). A non-continuous phase transition might manifest as a sudden societal change, such as a protest or revolution, when a critical threshold of tension is reached. However, the graph does not show such a sudden change, so if this were a social system, it would suggest a gradual adjustment to increasing stress without a sudden breakdown or transformation.

To more accurately capture and study non-continuous phase transitions, additional analyses focusing on the specific conditions expected to produce such transitions, possibly including hysteresis and bistability, would be needed. This might involve simulating the system at different temperatures, applying varying external fields, or considering more complex interactions between the spins that could lead to more complex phase behavior.

Modeling Exchange and Fermi Holes in the Group Dynamics of the Ising Model

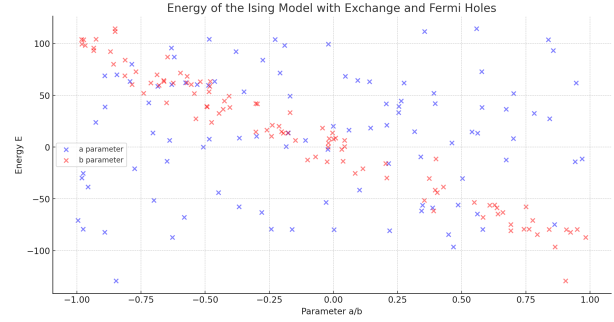


Fig. 25: Energy band theory (exchange hole and Fermi hole phenomena): electron external stress A, B

To consider the cases of exchange holes and Fermi holes in the group dynamics of the Ising model, we provide equations and ideas.

The energy function of the Ising model is usually expressed as follows:

$$E(x, J) = -J \sum_{i,j} x_i x_j$$

Here, x_i represents spin variables (+1 or -1), and J is the interaction constant.

The concepts of exchange holes and Fermi holes are related to the band theory of electrons and describe changes in the occupancy of energy levels of electrons. Let's apply this to the Ising model.

1. Application of Band Theory: In the Ising model, the configuration of spins is associated with energy levels. When a spin is +1, it represents an occupied state within the energy band, and when it is -1, it represents an unoccupied state.

2. Introduction of External Parameters: To model the phenomena of exchange holes and Fermi holes, we introduce external parameters a and b . These parameters affect the energy function of the Ising model. The energy function is extended as follows:

$$E(x, J, a, b) = -J \sum_{i,j} x_i x_j - a \sum_i x_i - b \sum_i x_i^2$$

Here, a and b are parameters that control exchange holes and Fermi holes.

3. Explanation of Exchange and Fermi Holes: With the introduction of external parameters a and b , the occupancy of spins at energy levels changes. Varying the values of a leads to fluctuations in the occupancy of states within the energy band, resulting in the phenomena of exchange holes (where occupied states change to unoccupied states) and Fermi holes (where unoccupied states change to occupied states).

The Results you've provided shows the energy of an Ising model system with exchange and Fermi holes, represented as a function of two parameters: a (blue) and b (red). These

parameters act as external stresses that influence the behavior of the system. From the energy band theory perspective typically applied to electronic systems, we can consider the following:

Exchange Hole Phenomenon

This is analogous to the situation in semiconductor physics where an electron leaves a position in the valence band, creating a 'hole'. In the Ising model context, an exchange hole could correspond to a decrease in the system's energy when a spin flips from down to up, under the influence of the external parameter a .

Fermi Hole Phenomenon

This relates to the occupancy at the Fermi level. In the Ising model, a Fermi hole could correspond to a similar effect due to the parameter b , affecting the overall occupancy and thus the system's energy.

Observations from the Graph

The energy values spread across a wide range for both parameters a and b , which suggests that these parameters have a significant impact on the energy of the system. There appears to be no clear threshold or critical point at which the energy exhibits a discontinuous jump or a non-linear change. This suggests that within the parameter range shown, the system undergoes a smooth transition without any abrupt phase change. The spread of energy values for both parameters implies a complex interaction between the spins and the external parameters. The lack of any apparent pattern or symmetry might indicate that the system's response to the external parameters is highly dependent on the specific configuration of spins at each step.

Inferences

- The relationship between external parameters a and b and the energy of the system could be indicative of the depth and occupancy of the energy bands in an electronic system. The scatter and distribution of the points might suggest varying band gaps or differences in energy levels occupied by the spins. - The impact of a and b on the system's energy could be analogous to applying an external electric or magnetic field to a material, where the response of the electrons (or spins, in the Ising model) determines the material's properties.

To further understand these phenomena in the context of the Ising model, one could conduct a more detailed analysis, such as examining the specific spin configurations that lead to particular energy values or exploring the effect of these parameters near the critical temperature of the system where phase transitions are expected to occur.

The results, which illustrates the energy of the Ising model with parameters a and b representing exchange and Fermi holes, can be interpreted from the perspective of social dynamics as follows Parameter a (blue points) This parameter could be thought of as representing a social force or policy that encourages (for positive values) or discourages (for negative values) a particular social state or opinion. The energy in the context of social dynamics could represent the overall tension or contentment within a society. When a is positive and increasing, it could indicate a policy or social influence that aligns with the majority's state, thus lowering tension. Conversely, negative values could represent a force against the majority's state, increasing overall social tension.

Parameter b (red points)

This parameter might symbolize a regulatory mechanism or social norm that aims to maintain order or conformity within a society. Positive values of b could reflect a strong social pressure towards a uniform state, leading to a decrease in energy or tension due to conformity. Negative values could indicate a societal push towards diversity or dissent, increasing social energy or tension.

Scattered Energies

The scatter of energy values across the range of both parameters suggests that the social landscape is complex and that the responses to social forces and policies (exchange holes) and regulatory mechanisms or social norms (Fermi holes) are not uniform. Different segments of society may respond differently to these influences, leading to a diverse set of social tensions.

No Clear Phase Transition

The absence of a clear discontinuity in the graph suggests that the modeled society does not undergo sudden, large-scale changes in state in response to these parameters within the explored range. Instead, the society seems to adjust gradually to varying social forces and regulatory pressures.

Influence of External Parameters

The plot indicates that both social forces/policies and regulatory mechanisms/norms significantly impact the societal state. The energy of the system varies widely with changes in a and b , suggesting that small alterations in these external parameters can lead to considerable differences in social energy or tension.

To draw more detailed conclusions about the social dynamics represented by the Ising model, one would need to delve deeper into the specific societal norms and forces represented by a and b , as well as how individuals within the society interact with each other and respond to these external

parameters. Additionally, examining the temporal dynamics of these parameters and their effects on the social state could provide further insights into the potential for phase transitions or societal changes over time.

10. Discussion Summary

(1) Zeeman ergodicity

A concept used in the context of physics and probability theory. Zeeman transitivity describes how a system changes over time and how it transitions to different states in the state space. Zeeman Transitivity is a concept that describes the ability of a physical system to explore different states over a long period of time evolution. It is primarily associated with ergodicity. Ergodicity refers to the property that a system can transition to different states over time, all of which may be visited indefinitely. Zeeman transitivity is a type of ergodicity that focuses specifically on the rate and manner in which a physical system transitions. Zeeman transitivity describes the ability of a physical system to explore within the state space in which it exists and to transition to different states. In other words, it refers to the possibility of a system transitioning from one state to another over time. Zeeman transitivity is a type of ergodicity, and a system is said to be transitive if ergodicity holds. Ergodicity refers to the property that a system infinitely visits all possible states, and Zeeman transitivity emphasizes the property of specific state transitions among them. As a random walk concept, Zeeman transitivity may allow us to view a physical system as a random walk. A random walk represents the process of moving to different positions by taking stochastic steps. When a physical system has Zeeman transitivity, its behavior is a kind of random walk. Zeeman transitivity can depend on the time scale. That is, the transitivity of the system may change over time. Even if the transition is temporarily slow, the transitivity may increase in the long run.

(2) Zeeman Ergodicity and Cyclic Exchange

Zeeman transitivity is a property in which a physical system transitions to various states over time and can be considered as a type of ergodicity. In other words, it indicates that the system has the ability to explore within the state space in which it exists and transition to different states.

State transitions by cyclic exchange, when elements within a physical system are cyclically exchanged, the system periodically transitions to different configurations. This is one example of Zeeman transitivity within a physical system. With cyclic exchange, the system periodically transitions to different states, resulting in Zeeman transitivity. Zeeman transitivity and state space exploration have the ability to explore within a state space and transition to different states.

Cyclic exchange may represent the process by which elements in a system transition to different interaction patterns, and this process may facilitate exploration within the state space. Zeeman transitivity and cyclic exchange are particularly important in the context of statistical mechanics. For example, they are relevant to the computation of phase transitions and correlation functions in spin and particle systems. When cyclic exchange is present, searches in state space may be effectively performed at equilibrium states and phase transitions in the system.

In short, cyclic exchange may be considered as one factor causing Zeeman transitivity.

(3) Social systems in which ergodic properties are valid

In this case the system converges to the same statistical properties in the long run. However, if convergence of opinion does not occur and viscosity or limit cycles of opinion are observed, it suggests the existence of cyclical behavior or repetition of stable states in the social dynamics. Below are some interpretations of social dynamics in which such phenomena may occur.

Societies are often affected by cultural events and political cycles (e.g., election cycles). These cyclical events can cause periodic fluctuations in opinions and attitudes. For example, fluctuations in political opinion around elections or cultural events due to the seasons may affect the cyclicity of opinion. Changes in economic conditions can also have a cyclical impact on societal opinion. Cycles of booms and busts can create cycles between opinions on economic policy and general societal optimism and pessimism. Cycles in media and social media information distribution can create cycles in public debate and opinion formation. Repeated cycles of information dissemination and decay of its impact may cause opinions to exhibit certain patterns.

Social Interaction

The dynamics of interactions between individuals can also create cycles of social opinion. A person or group with strong social influence may strongly promote a particular opinion, and over time this influence may diminish, resulting in periodic fluctuations in opinion.

Feedback loops

The balance between positive feedback (self-amplifying effect) and negative feedback (self-suppressing effect) in a society can give rise to limit cycles. This mechanism is such that when opinions lean in a certain direction, opposition or resistance arises, and when opinions lean in the opposite direction, support arises again. These dynamics indicate that society does not converge on a single static opinion, but maintains

a dynamic equilibrium that changes over time. Limit cycles can be interpreted as representing a state in which society is flexible to change, with various forces interacting with each other but maintaining a constant pattern. In the above examples, the following arguments can be deduced as arguments for ergodicity to emerge on social dynamics.

Examples of Political Conflict on Social Media

Ergodic Perspective: Political conflict on social media can exhibit a type of ergodic nature. Ergodicity refers to the ability of a physical system or stochastic process to visit all possible states over a long period of time. In this instance, there is a cycle in which political opinions periodically move from conflict to support and back to conflict again. This cyclic transition demonstrates the ability to encompass the entire political spectrum. As an application of ergodicity, the idea of ergodicity is relevant to political discussions on social media. In the long run, different political positions and opinions regularly surface, contributing to the diversity of debate within the digital environment.

Examples of Technology Conflicts Associated with New Product Releases In terms of ergodicity, examples of technology conflict show that the positions of technology enthusiasts and technology skeptics change cyclically along the release cycle of new products. This is a form of ergodicity, where different opinions about a new technology alternate over time, maintaining overall diversity. The concept of ergodicity also applies to discussions about new technologies. When technology conflicts are cyclical, different technology enthusiasts' positions and arguments emerge sequentially, and new ideas and perspectives appear alternately.

Ergodicity is an important concept in these cases, as it demonstrates the diversity of arguments and conflicts within the digital environment over time. This would imply a cyclical change in opinions and positions, resulting in a richer overall discussion and exchange of information.

(4) Spin states in which cyclic exchange can occur and examples of the transitive nature of discourse in those digital environments

Political conflict on social media

Spin represents an individual user or opinion, with +1 indicating political party A and -1 indicating political party B. On social media, users express their opinions and influence the opinions of those around them.

Cyclical Exchange

Assume that political agendas alternate cyclically according to election cycles and policy changes. For example, as an election approaches, users' opinions may shift from political

opposition to support, and vice versa, returning to opposition in the next election cycle.

Transitivity of discourse in the digital environment

With cyclic exchange, discourse on social media fluctuates regularly. In response to an election cycle or policy change, user speech and discussion may shift from confrontation to support and back to confrontation again in the next cycle. This cyclical transitional nature of discourse can be observed.

Technology conflict following the release of a new product Spin State

Spin represents technology enthusiasts and technology skeptics. A +1 indicates technology enthusiasts who support the new product and a -1 indicates technology skeptics who are skeptical of the new product. In the digital environment, there is a lot of discussion about new technologies.

Cyclical Exchange

Assume that the state of spin is cyclically exchanged according to the release cycle of a new technology. The cycle continues with an increase in technology enthusiasts at the release of a new product, the gradual rise of technology skeptics, and then an increase in technology enthusiasts again at the release of the next new product.

Transitivity of discourse in the digital environment

Cyclical exchange causes the discourse in the digital environment about new products to fluctuate cyclically. Technology conflicts flourish with the release of a new product, and the discussion changes again for the next release. Technology conflicts can emerge cyclically.

These ideas are examples of how cyclic exchange can affect the spin state and the transitivity of discourse in the digital environment.

(5) Berry curvature and when T-symmetry holds

Berry curvature: when considering political conflict on social media, Berry curvature indicates the phase shift associated with different political positions (spin states). Repeated cycles of political opinion result in changes within the phase space. This reflects the cyclical fluctuations in social media discussions and political trends. **T-symmetry:** T-symmetry refers to time-reversal symmetry, a property in which the laws of physics remain the same in the opposite direction of time. In this case study, we show that when T-symmetry holds, political conflicts fluctuate cyclically in the opposite direction of time as well. In other words, the pattern of argumentation may repeat itself even if the election cycle proceeds in

the opposite direction. In the case of new product technology conflict, Berry curvature shows phase changes associated with the introduction of new technology. The change in opinion between technology enthusiasts and technology skeptics associated with the release cycle of a new product is reflected within the phase space. If T-symmetry holds, it means that the technology conflict for a new product will fluctuate cyclically in the opposite direction of time as well. The cycle from the release of a new product to the rise of technology conflict to the next release is considered to be invariant with respect to time reversal.

(6) Perspective of the Hartree-Fock exchange term

The Hartree-Fock exchange term is relevant when considering the exchange and impact of political opinions. Users (spin states) from political group A (+1) and political group B (-1) interact on social media and their opinions influence each other. In this interaction, the Hartree-Fock exchange term indicates the interaction energy by political position. With the Hartree-Fock exchange term, users of political party A are influenced by users of political party B and vice versa. This is important when describing the interaction of opinions among users on social media. A user's political position can be influenced by other users, a process where opinions are exchanged between each other. In the case of the new product technology conflict, the Hartree-Fock exchange term is relevant when considering the interaction between technology enthusiasts (+1) and technology skeptics (-1) users (spin state). Opinions about the new technology are exchanged, which influence the adoption or rejection of the technology. With the Hartree-Fock exchange term, technology enthusiast users are influenced by technology skeptic users and vice versa. A process can take place in which users' attitudes toward technology are mutually influenced by the exchange of opinions regarding the adoption of new technology.

(7) When Heavy Ball Dynamics is in place

Heavy Ball Dynamics is a concept used in the context of optimization and dynamics to describe the process of finding optimal values for parameters and variables.

In the case of political conflicts on social media, Heavy Ball Dynamics describes the interaction of users with different political stances (spin states); Heavy Ball Dynamics may indicate a process where political stances and opinions develop in one direction in one direction.

For example, let's assume that political opinions tend to lean in one direction: in Heavy Ball Dynamics, some users (heavy ball) may influence other users (light ball) and their opinions may be reinforced toward a particular direction. This helps to describe situations where political conflicts tend to develop in one direction. In the case of the new product

technology conflict, Heavy Ball Dynamics describes the interaction between technology enthusiast (+1) and technology skeptic (-1) users (spin state). The position of the technology enthusiast can strongly influence other users, indicating a process where technology adoption is likely to progress. In Heavy Ball Dynamics, technology enthusiasts (heavy balls) may influence technology skeptics (light balls) to promote acceptance of a new technology. This helps to describe a situation where technology adoption is likely to progress in one direction.

In the case of political conflict on social media, the Exchange Hole and Fermi Hole describe the interaction of users with political positions (spin states). Exchange holes represent the exchange of energy levels between users with different political positions, while Fermi holes represent the occupancy of states within an energy band.

Influence of Exchange Holes

The exchange of energy levels between users of political group A (+1) and political group B (-1) will result in their political positions being affected, changing the occupancy of states in the energy band. This illustrates the process by which political conflicts affect through transitions within the energy band between users.

Explanation of Exchange Holes and Fermi Holes

In the new product technology conflict example, Exchange Holes and Fermi Holes explain the interaction between technology enthusiasts (+1) and technology skeptics (-1) users (spin states). Exchange holes represent the exchange of energy levels between users with different technological positions, while Fermi holes represent the occupancy of states within the technological energy band.

The influence of exchange holes

The exchange of energy levels between users who are technology enthusiasts (+1) and technology skeptics (-1) will result in their technical positions being affected, changing the occupancy of states within the energy band. This illustrates the process by which technical conflicts affect through transitions within the energy band between users.

In these examples, Exchange Hole and Fermi Hole are used as concepts to describe the exchange of energy levels between users and the occupancy of states within the energy band, indicating a process in which interactions between users with different positions cause changes in energy states.

11. Conclusion: Significance of Magnetization Plateaus in the Ising Model

Magnetization Plateau Occurrence

A magnetization plateau is a phenomenon in which the magnetization of a system is fixed to a certain value over a specific range of external magnetic fields. In the Ising model, a magnetization plateau can also be predicted theoretically by choosing certain parameters and introducing many-body interactions. The occurrence of a magnetization plateau means that the system has settled into an energetically favorable "frozen state. This means that the arrangement of spins forms a particular pattern due to interactions and cannot easily be broken out of that state. For example, frustration (a state in which the arrangement of spins cannot align to minimize interactions) or a particular arrangement of spins may contribute to the formation of such a plateau.

Significance of Magnetization Plateaus in the Ising Model

The occurrence of magnetization plateaus in the Ising model allows for a deeper understanding of the phase transitions and magnetic properties of magnetic materials. It is especially important when studying in detail the thermodynamic properties and phase transitions of systems in different magnetization states. The presence of a magnetization plateau is a phenomenon that can only be observed at certain magnetic fields and temperatures, and experimentally these conditions must be tightly controlled. This plateau is important not only in simple versions of the Ising model, but also in studies of more complex models involving many-body interactions and quantum effects. It plays a role. The generation of magnetization plateaus is relevant to many practical applications, such as the design of new magnetic materials in materials science and condensed matter physics, or as qubits in quantum computing.

Regarding the introduction of magnetization plateaus in the Ising model

A magnetization plateau in the Ising model refers to a region of unchanging magnetization over a range of constant magnetic fields. This phenomenon is observed especially in quantum many-body systems, where the magnetization of a system takes on quantized integer or fractional values under certain conditions of magnetic field and temperature. This phenomenon is associated with topological phases such as the quantum Hall effect and quantum spin liquids.

11.1 What is the Perron-Ishii Complement?

The Perron-Ishii complement is a mathematical theorem that asserts that for any nonnegative matrix, its largest eigenvalue is positive and the corresponding eigenvector is positive. This theorem is important in the study of nonnegative matrices, especially adjacency matrices in Markov chains and network theory.

Application to the Analysis of Magnetization Plateaus

In analyzing magnetization plateau problems, the Perron-Frobenius complement is useful for understanding the stochastic distribution of states and energy states. This technique may be used to analyze the eigenstates and stability of a system when it is in a particular magnetization plateau state.

Advantages in this approach

In terms of a clear mathematical framework, the Perron-Ishii complement provides a rigorous mathematical foundation, allowing for a deeper understanding of the fundamental properties of the system. Stability Assurance Eigenvectors corresponding to the largest eigenvalues can be used to identify the stable state of the system and the evolutionary path leading to it.

Long-term stability

This Perron-Ishii complement can indicate what states the system will or will not converge to in the long term. This allows the stability or instability of the system to be analyzed. Application to stochastic processes In social dynamics, where opinions and decisions change stochastically, the theorem can be used to predict long-term behavior.

Disadvantages of this approach Computational complexity

Eigenvalue problems in large systems are computationally difficult and require advanced numerical techniques.

Restriction to specific conditions

Perron-Ishii's complement is only applicable to non-negative matrices and does not apply to all Ising model situations. Lack of intuitive understanding Compared to analysis based on physical intuition, a more mathematical approach is required, and intuitive understanding may be difficult to obtain.

Hypothesis in social dynamics

A "magnetization plateau" in social dynamics can be likened to a phenomenon in which social opinion is fixed in a certain state when certain social pressures or cultural norms are strong. The Perron-Ishii complement may provide a tool for

understanding the stability and potential for change in social opinion. may provide.

Special Cases in Social Dynamics

An analogue of a magnetization plateau in social dynamics might refer to a situation in which a particular opinion or behavior is fixed within a social group. For example, if an opinion is widely accepted in society and then never changes, this can be viewed as a "freezing" or "plateauing" of the opinion. This phenomenon may occur when social norms, laws, or cultural values do not change.

Ergodotropy and Limit-Cycling of Opinions

When ergodicity is in place, the system explores all possible states, but the occurrence of opinion viscosity or limit cycling means that certain patterns or cyclical behaviors are maintained over time. In social dynamics, it might correspond to the repetition of a particular political opinion or cultural trend over a period of time. For example, economic or political cycles may correspond to this. Such cycles may indicate a phenomenon in which a society experiences constant fluctuations, but no major changes or transformation to a new order occurs. How the phenomenon of magnetization plateaus and limit cycles are interpreted in social dynamics strongly depends on the particular context of the social system. Therefore, individual social events and historical circumstances must be carefully considered when applying these mathematical concepts.

Applying the Ising model to group dynamics

Spin can be viewed as the opinions and choices of individual agents (individuals or groups), and a magnetization plateau can be interpreted as a situation where certain shared opinions or modes of behavior are fixed. This could also refer to a situation where certain norms and beliefs are established within a social group and are difficult to change.

Application of the Ising Model to Group Dynamics

Meaning of Spin

Individual spins represent the opinions of individuals in a group, with "up" indicating one opinion or attitude and "down" indicating its opposing opinion or attitude. Interaction Interactions between spins in the Ising model symbolize social interactions and influences among people. People are often drawn to the opinions of individuals in close proximity, such as friends or colleagues.

Application of Magnetization Plateaus to Social Dynamics

Fixation of Opinions

A magnetization plateau indicates that a particular belief or behavior is widely accepted in a social group and remains unchanged for a long period of time. This may be seen in strong cultural norms, stable political regimes, or long-term economic stability. Resistance to change in the external environment The tendency of a group to resist external change and to hold on to existing norms is similar to the concept of a magnetization plateau. Despite social, economic, or political pressures, the group retains its core beliefs.

The meaning of a magnetization plateau and its impact on

Social Stability

Magnetization plateaus can be indicators of social stability. When opinions are fixed, change is infrequent and predictable patterns arise.

Lack of innovation

On the other hand, a magnetization plateau may indicate a lack of innovation, meaning slow adoption of new ideas and behaviors.

Resistance to Change

Resistance to the need for change may arise as the group's ability to adapt to new information and situations is limited.

The concept of magnetization plateaus in the Ising model is a useful metaphor for understanding the difficulty of changing opinions and behaviors in social groups. Through this model, social stereotypes and barriers to change can be studied, providing insight into devising strategies to promote change in social policy and collective behavior.

Understanding the Occurrence of Magnetization Plateaus in the Ising Model

To understand the computational process when magnetization plateaus occur in the Ising model, you can consider the following ideas:

1. **Setting Initial Conditions:** Set the spin configuration of the Ising model as the initial conditions. Initial conditions can be based on random configurations or specific arrangements.

2. **Definition of Energy Function:** Define the energy function of the Ising model. Typically, the energy function takes into account interactions between spins. Here is the general form of an energy function:

$$E(x, J) = -J \sum_{i,j} x_i x_j$$

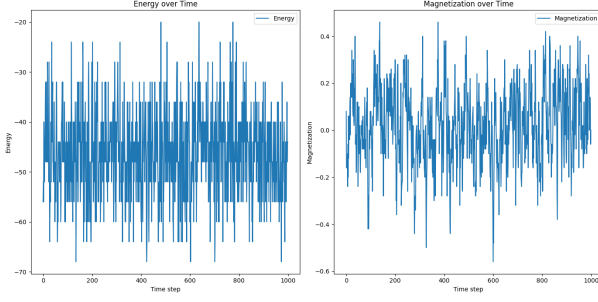


Fig. 26: Energy / Magnetization over Time

Here, x_i represents spin variables (+1 or -1), and J is the interaction constant.

3. Introduction of Cyclical Exchanges: Introduce cyclical exchange operations to make spin configurations cyclically exchange periodically. The update rules for cyclical exchange are as follows:

$$x_i(t+1) = \begin{cases} x_{i+1}(t), & \text{if } i < N \\ x_1(t), & \text{if } i = N \end{cases}$$

This results in cyclic changes in spin configurations.

4. Confirmation of Ergodicity: With the introduction of cyclical exchanges, spin configurations change over time, satisfying ergodicity. Ergodicity refers to the property where a system explores different states and can reach all possible states as time progresses.

5. Monitoring Magnetization: Calculate and record the magnetization (the sum of average spins) at each step of the simulation. Magnetization is represented as follows:

$$M(t) = \frac{1}{N} \sum_{i=1}^N x_i(t)$$

6. Observation of Magnetization Plateaus: Plot the time evolution of magnetization and observe whether magnetization plateaus occur. Magnetization plateaus indicate a temporary stability in magnetization.

7. Modification of External Parameters: If magnetization plateaus occur, you may resolve them by changing external parameters. Changing external parameters can alter the behavior of the Ising model.

Energy / Magnetization over Time results are from Pre-analysis

The magnetization result shows that magnetization fluctuates significantly up and down over time. This may mean that opinions and attitudes are not stable over time and fluctuate widely when considered in the context of social dynamics. On the other hand, if the value of magnetization fluctuates within a certain range (forming a plateau) over a specific period of time, this may indicate a temporary stabilization

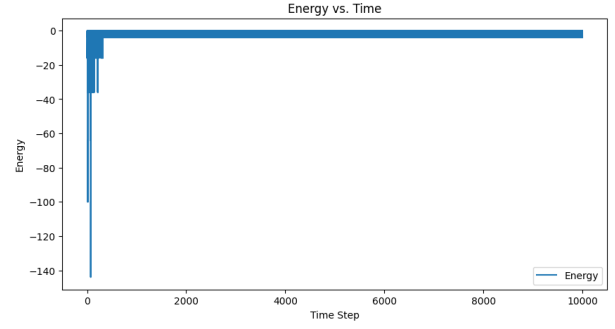


Fig. 27: Energy vs. Time

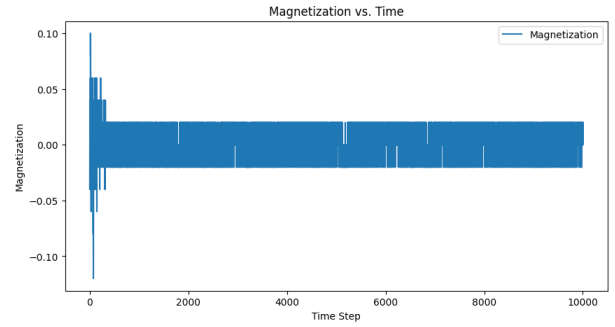


Fig. 28: Magnetization vs. Time

of opinion or the formation of a consensus. If the change in spin configuration shows a periodic pattern, this may indicate that the opinions of individual society members change at a steady rhythm. This can be interpreted as representing the movement of opinion in a group that is susceptible to trends and social pressures. 3. The cyclical changes shown by the energy graph may reflect periodic events in social dynamics or regularly occurring social and political cycles. Examples might include election cycles or economic business cycles in the economy. It could also be that social conflicts and competition exhibit certain patterns.

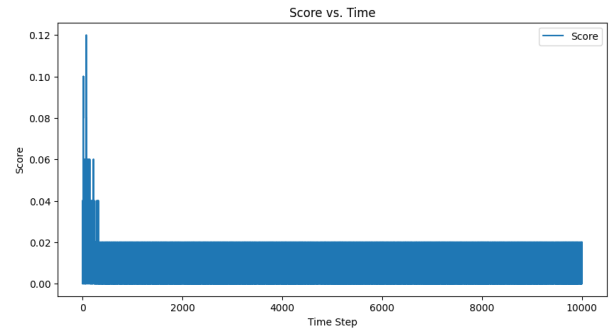


Fig. 29: Score vs. Time

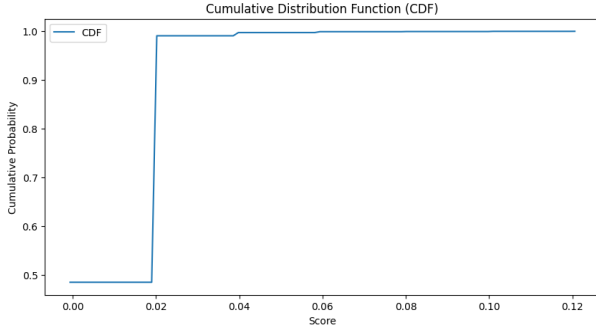


Fig. 30: Cumulative Distribution Function (CDF)

Magnetization Plateau

The magnetization graph displays the average spin over time. The plateau suggests that the system reaches a steady state relatively quickly, where the average magnetization fluctuates around a consistent value close to zero. This indicates that the spins are equally likely to be up or down over time, which is characteristic of a system at high temperature or above the critical temperature in the absence of an external magnetic field.

Spin Dynamics

The energy graph shows that the system quickly reaches a low energy state. This rapid minimization of energy indicates that spins are aligning with their neighbors to reduce the system's overall energy. However, since the magnetization does not show a strong bias towards positive or negative values, it implies that the system might be forming domains or clusters of aligned spins rather than a uniform alignment across the entire system.

Cyclic Exchange

The score graph, which represents the absolute value of magnetization, also stabilizes quickly and remains constant over time. This suggests that the random cyclic exchange operations (possibly representing some dynamic social interaction if we interpret this as a social dynamics simulation) maintain the system in a dynamic equilibrium where the overall level of "agreement" or "alignment" among agents (spins) does not change much after an initial period.

Cumulative Distribution Function (CDF)

The CDF graph indicates that the score values are highly concentrated in a narrow range, with a sharp rise at the beginning of the plot, which means that most of the score values are close to a certain value, and there is little variation. This is consistent with the score graph, where the score settles quickly and does not show much fluctuation over time.

Social Dynamics Interpretation

The magnetization plateau could represent a state of social equilibrium where there are equal numbers of positive and negative opinions or behaviors, leading to no clear majority or consensus in the population. The energy minimization reflects a tendency towards local agreement or harmony in social groups, which does not necessarily translate to global consensus, as indicated by the overall magnetization. The cyclic exchange could model the impact of local interactions on social dynamics, showing that despite constant changes in individual states or opinions, the overall social structure remains stable. The CDF of the score might indicate that most individuals or social groups tend to have a similar level of agreement or conformity, with outliers being rare.

To further analyze these dynamics, one might look into the time evolution of the system, the size and distribution of spin domains, or the effects of varying the interaction strength (J) or introducing an external field. This could provide deeper insights into how local rules and interactions can lead to complex social behaviors and structures.

Magnetization plateau and T-symmetry

Here we also discuss the results when time-reversal symmetry is introduced. When time-reversal symmetry is introduced in the simulation, a time-reversal operation is performed by storing the current spin configuration at regular step intervals ('interval') and flipping the spin configuration at the next interval. This ensures that the simulation has symmetric dynamics with respect to time.

Saving the Spin Configuration

At each step of the simulation, the spin configuration is saved at regular intervals. This is denoted as $x(t)$.

Time Reversal Operation

At intervals after the spin configuration is saved, the saved spin configuration is inverted to generate a new spin configuration. This is denoted as $x(-t)$.

This operation ensures that the simulation has symmetric dynamics with respect to time. This means that the same dynamics occurs between $x(t)$ and $x(-t)$.

As a mathematical expression, time-reversal symmetry can be expressed as follows:

$$x(-t) = \text{interval}(x(t))$$

where

$$x(t)$$

denotes the spin configuration at time t and

$$x(-t)$$

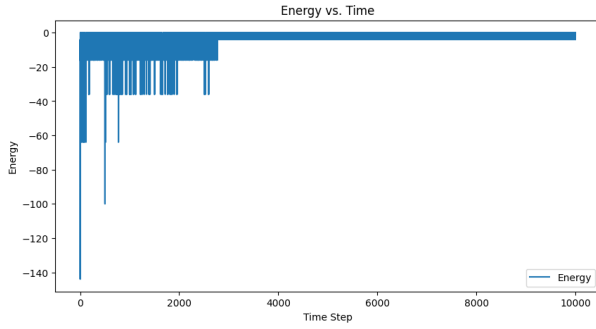


Fig. 31: Energy vs. Time

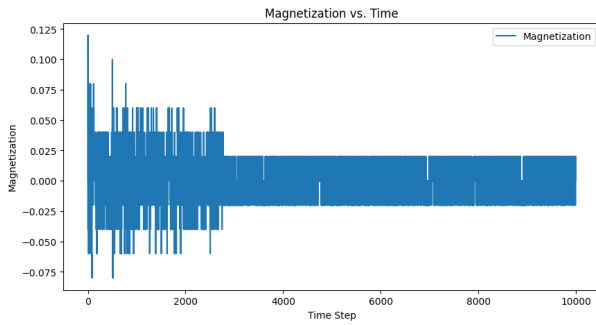


Fig. 32: Magnetization vs. Time

denotes its inverted spin configuration. The inversion operation is usually performed by inverting the value of each spin.

Results show the results of an Ising model simulation with time symmetry incorporated through periodic time reversal operations

Consideration of Time Symmetry

The time symmetry in the simulation can be seen as a metaphor for cyclic or periodic changes in social sentiment or policy that revert the state of a social system to a previous configuration. This could represent, for example, political cycles or economic policies that tend to oscillate between two states (e.g., liberal and conservative policies, boom and

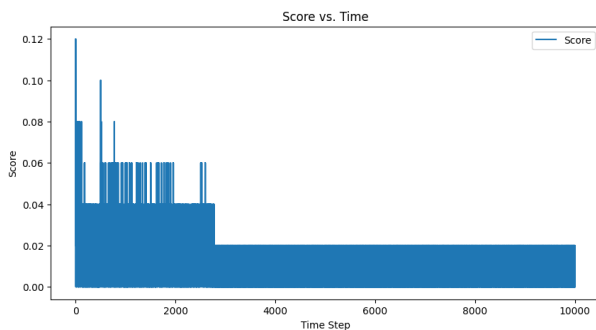


Fig. 33: Score vs. Time

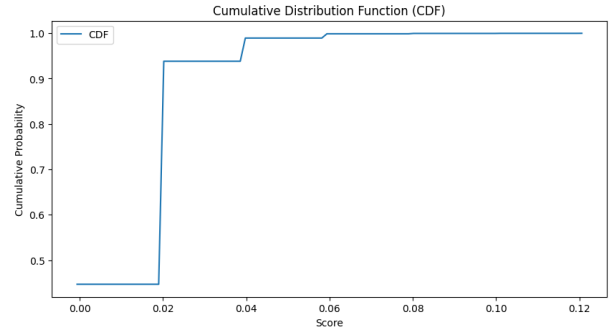


Fig. 34: Cumulative Distribution Function (CDF)

bust cycles in economics). The energy graph shows periodic spikes, which could be interpreted as periods of high social tension or conflict that arise when the system undergoes a reversal or transition between states.

Consideration of Magnetization Plateau

The magnetization graph exhibits a plateau, which in a social context, might represent a state of equilibrium where despite individual fluctuations (individuals changing opinions or affiliations), the overall societal opinion remains balanced and stable. This plateau suggests that while individuals or small groups may experience significant changes, the macro-level state of the social system is resistant to change, possibly due to the time symmetry enforcing a return to equilibrium.

Consideration of Spins

The spins in the Ising model could represent individual agents (people, organizations, etc.) in a social system, with their up or down states symbolizing different opinions, strategies, or behaviors. The simulation indicates that individual behaviors are influenced by local interactions (as per the Ising model's nearest-neighbor interactions), but the overall pattern remains stable over time, possibly due to societal norms, laws, or other stabilizing forces.

Consideration of Cyclic Exchange

The cyclic exchange process, where spins can flip based on their interactions, can be seen as a representation of individual or group interactions leading to changes in opinion or behavior. In a social context, this could be seen as the influence of peer pressure, dialogue, debate, or the spread of information. Despite these interactions and potential for change at the micro-level, the overall societal structure remains surprisingly stable, as evidenced by the sustained score over time.

Consideration of Score and CDF

The score, representing the absolute value of magnetization, settles into a steady state relatively quickly, suggesting that

while individual opinions or behaviors might fluctuate, the degree of consensus or overall alignment within the society doesn't change dramatically over time. The CDF shows a sharp rise at the beginning, indicating that most of the population has a low score, with very few individuals or groups deviating significantly from the norm. This suggests a society with a strong tendency towards conformity or a strong central norm that most individuals adhere to.

The time-reversal operations create a clear pattern in the energy graph, with spikes occurring at regular intervals. This could represent external interventions or shocks to the social system that temporarily disrupt equilibrium but have been designed or evolved to maintain overall stability. Such dynamics are interesting to study in the context of societal resilience and the ability of social systems to absorb and adapt to change while maintaining their core structure and functions.

Social Analogies of Magnetization Plateaus

Finally, a case of social dynamics in which a magnetization plateau occurs on group dynamics on the Ising model can be illustrated by comparing the following situation to a physics concept. Political Extremes and Fixation of Opinions Assume a situation in which a particular political opinion or ideology becomes dominant within a social group. This phenomenon may exhibit dynamics similar to a magnetization plateau in physics. A magnetization plateau in physics is a phenomenon in which magnetization is fixed at a certain level under certain external conditions. An analogous phenomenon in social dynamics is a condition in which a social group is "anchored" to a certain political opinion or ideology. This fixation can be caused by external social or political pressure or media influence.

Initial State

A diversity of political opinions exists within a social group.

Homogenization of opinions

Strong support for a particular ideology or political leader is formed and individual members of the group are drawn to that opinion.

External influences

Media coverage and political events promote this homogenization of opinion, "anchoring" opinions within the group in a particular direction.

Formation of a Magnetization Plateau

Eventually, the social group becomes fixed to a particular political opinion, and opinions contrary to it become almost nonexistent. This condition tends to be maintained as long as the external environment remains unchanged.

Changes in external conditions

When new social or political conditions arise, this fixed opinion may change, but often the "plateau" state lasts for a long time.

In this case study, the concept of a magnetization plateau can be used to understand the process of homogenization and fixation of opinions within a social group. It is hypothesized that this situation is particularly likely to occur in closed groups or under the influence of strong charismatic leadership.

The following situations may be considered as examples of social dynamics for the occurrence of a magnetization plateau.

Convergence and dispersion of opinions in social media

Opinions are exchanged among users on social media platforms (acting like spin). Users' opinions change over time as they discuss and share information on social media. Some users converge to the same opinion as others, forming a "plateau" of consistent opinions. This plateau is temporarily stable (magnetization plateau). On the other hand, another group of users will stick to a different opinion and form a "plateau" that differs from other user groups. Discussions on social media, changes in information, and outside influences (e.g., news, trends) can cause an exchange of opinions between plateaus, causing the magnetization plateau to collapse and form a new convergence point.

Convergence and dispersion of political opinion

In a political community or association, members hold different political positions. Over time, political debates and election campaigns take place, and some members converge on a common political opinion (plateau formation). Other members continue to hold different political opinions and form different plateaus. Political events, information, and outside influences can cause an exchange of political opinions between the plateaus, causing the magnetized plateaus to collapse and new political convergence points to form.

In these instances, opinions and positions change over time within the social dynamics and a "plateau" is temporarily formed where some groups converge on a consistent position. However, the process by which these plateaus collapse and new convergence points are formed due to external influences or changes in information is an example of a magnetization plateau.

Perspective: Mathematical Interpretation of Magnetization Plateaus from the Perspective of Viscous Solutions

Viscous solutions represent a phenomenon in which a system remains temporarily stable. When considered in the context of a magnetization plateau, the temporary convergence of some individuals or groups to a particular opinion or behavior pattern and the persistence of that state can be considered an example of a viscous solution.

Extension of the Differential Equation

To take viscous solutions into account, we extend the differential equations describing convergence and dispersion of opinions. A viscosity term is added to the ordinary differential equation to adjust the rate at which convergence of opinions proceeds. This viscosity term implies temporary convergence.

Stochastic Model

When considering viscous solutions, it is common to introduce a stochastic component. One could consider modeling a stochastic process in which individual agents (users, individuals, groups, etc.) make random decisions and evaluate the probability of a viscous solution occurring.

12. Research Prospects: Informational Health Expectations When Introducing Temporal Symmetry as Social Dynamics

Finally, through the results of incorporating time symmetry into the Ising model in which magnetization plateaus occur, we will discuss the expectations, effects, social case studies, and ideas regarding informational health when introducing time symmetry as a social dynamic. The Ising model with time symmetry results can indeed serve as an interesting analogy for discussing aspects of social dynamics, especially when considering the informational health of a society. Here are some insights and ideas on how the elements of your simulation can be interpreted in this context.

Informational Health Expectations with Time Symmetry

The introduction of time symmetry, where states are periodically reversed, could represent an environment where information cycles between periods of clarity and confusion or truth and misinformation. From the perspective of informational health, one could expect that such a system would have builtin mechanisms to correct misinformation over time, thereby maintaining a certain level of informational integrity.

This could be analogous to factchecking processes or the natural debunking of false information over time.

Impact on Society

The periodic energy spikes in the Energy vs. Time graph could be indicative of the social energy expended in correcting misinformation. After a period of time, when false narratives may gain traction (energy increases), there is a collective societal effort to restore truth, which brings the energy back down.

The Magnetization vs. Time graph shows that despite these cycles, the overall "magnetization" or collective societal opinion remains stable around a central value. This suggests that while there may be fluctuations in beliefs or the spread of misinformation, there's a robustness in the societal beliefs or knowledge base that resists being swayed completely by such fluctuations.

Societal Examples

Examples in society where time symmetry may play a role in informational health could include:

Educational Systems

They often cycle through phases of different educational theories and practices but aim to maintain a consistent level of quality education and factual accuracy.

Media Cycles

The news cycle can swing between sensationalism and more sober, factbased reporting, with the public discourse periodically returning to a focus on evidence and verification.

Policy Making

Government policies may oscillate between different ideologies, with each phase bringing its own narrative and information challenges, but over time, policies may be revisited and revised to reflect more accurate information and analysis.

Ideas for Social Dynamics and Informational Health

Considering these dynamics, several ideas and strategies could be implemented to promote a healthy information environment:

Regular Review and Correction Mechanisms

Just as the system periodically reverses to a previous state, societies could implement regular reviews of public information and policy to correct errors and address misinformation. Education on Critical Thinking: Teaching critical thinking skills can equip individuals to better navigate periods of misinformation.

Decentralized FactChecking

Encouraging a culture of peerreviewed information can create a selfcorrecting system, much like the local interactions of spins in the Ising model.

Transparency in Information Sources

Clear labeling of information sources and their biases can help the public better assess the veracity of information they encounter.

In conclusion, the time symmetry incorporated into the Ising model can be a powerful metaphor for understanding and improving the informational health of a society. It emphasizes the need for resilience and adaptability in the face of misinformation, ensuring that the society has the means to return to a state of informed consensus despite the inevitable cycles of false narratives.

Aknowlegement

The author is grateful for discussion with Prof. Serge Galam and Prof.Akira Ishii.

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