

Discussion: Social Dynamics with Spin Glass and Ising Models Using to Filter Bubble Hebbian Learning and Recall Memory

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Abstract: This study presents an ambitious interdisciplinary approach to applying the mathematical framework of quantum field theory, traditionally restricted to the domain of physics, to elucidate digital issues in the social sciences. The principles of remote interaction and proximity interaction are used. By developing a model based on quantum field theory, we mathematically represent the resonance and echo chamber effects of opinions within the filter bubble. The model incorporates non-physical factors known as FP ghosting phenomena, such as misinformation and confirmation bias, to simulate the complexity of social communication. In addition, the model integrates the concept of uncertainty ghosting, similar to the uncertainty principle in the social sciences, to account for information uncertainty and nonlinearities in opinion formation. This approach demonstrates the variability of social opinion and provides a detailed understanding of the dynamics within the filter bubble. The introduction of the spin glass phase provides a new discourse on the energy conservation and memory aspects of arguments within the filter bubble. This is achieved through the combination of the Edwards-Anderson and Hopfield models based on the ferromagnetic Ising model, allowing quantitative discussion of complex phenomena such as social associative memory. Incorporating the Hopfield model allows us to understand the behavior of the system when storing patterns. p patterns 1, 2,... are present at the minima of the Hamiltonian, which facilitates memory recall. This model application shows that when certain patterns are aligned at the minima of the Hamiltonian, the state is low energy and minimal memory storage is possible. In equilibrium, the transition probability $W(S_{jh})$ confirms the increase in the probability $P_t(S)$ of the current state ($-S_j \rightarrow S_j$), consistent with the Boltzmann coefficient of the master equation, thereby stabilizing the left-hand side at zero. This theoretical framework is invaluable in the discussion of long-term memory and energy conservation of fake news diffusion in both the digital and offline space of filter bubbles. Specifically, by extrapolating n to zero, we can theoretically model the selective diffusion of information associated with filter bubbles and obtain accurate results. However, this method requires complex calculations and, as a drawback, may lead to non-intuitive interpretations.

Keywords: Quantum Field Theory, Minima of the Hamiltonian, Social Dynamics, Filter Bubbles, Echo Chambers, Information Dissemination, Hebbian Learning, Ising Model, Spin Glasses, Replica Method, Fake News Proliferation

1. Introduction

This study presents an ambitious interdisciplinary approach to applying the mathematical framework of quantum field theory, traditionally restricted to the domain of physics, to elucidate digital issues in the social sciences. By using the principles of remote interaction and proximity interaction, this innovative approach traces the evolution of opinions and group dynamics among agents and delves deeply into the complexities of social communication and information diffusion.

It introduces the concept of filter bubbles, characterized

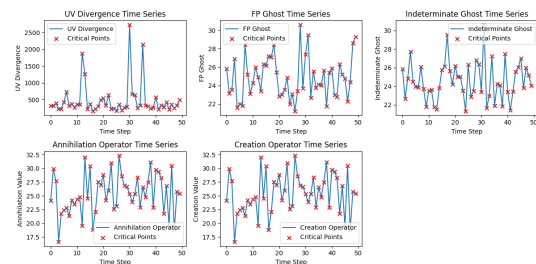


Fig. 1: Identification of critical points during spin glass condition

by the selective diffusion of information and viewpoints, limiting access to a variety of perspectives. By developing a model based on quantum field theory, we mathematically represent the resonance and echo chamber effects of opinions within the filter bubble. The model incorporates non-physical factors known as FP ghosting phenomena, such as misinformation and confirmation bias, to simulate the complexity of social communication. In addition, the model integrates the concept of uncertainty ghosting, similar to the uncertainty principle in the social sciences, to account for information uncertainty and nonlinearities in opinion formation. This approach demonstrates the variability of social opinion and provides a detailed understanding of the dynamics within the filter bubble.

1.1 Spin Glass Phase

The introduction of the spin glass phase provides a new discourse on the energy conservation and memory aspects of arguments within the filter bubble. This is achieved through the combination of the Edwards-Anderson and Hopfield models based on the ferromagnetic Ising model, allowing quantitative discussion of complex phenomena such as social associative memory.

In particular, the application of the replica method in the context of replica symmetry breaking allows for the discussion of coexisting models of diversity and stability. Coexistence models of stability can be discussed. The replica method, which considers multiple copies with different information access patterns, calculates the configurational average of the free energy and proves to be an effective tool in understanding the dynamics of filter bubbles.

This study also delves into the role of the Hebbian rule in models where randomly oriented Ising spins (+1 or -1) attempt to preserve memory aligned in a perfect ferromagnetic state; when J_{ij} is set to $1/N$, the system shows dependence on initial conditions and between different spin states majority state. Incorporating the Hopfield model allows us to understand the behavior of the system when storing patterns. p patterns ξ_1, ξ_2, \dots are present at the minima of the Hamiltonian, which facilitates memory recall. This model application shows that when certain patterns are aligned at the minima of the Hamiltonian, the state is low energy and minimal memory storage is possible. In equilibrium, the transition probability $W(S|h_j)$ confirms the increase in the probability $P_t(S)$ of the current state ($-S_j \rightarrow S_j$), consistent with the Boltzmann coefficient of the master equation, thereby stabilizing the left-hand side at zero.

This theoretical framework is invaluable in the discussion of long-term memory and energy conservation of fake news diffusion in both the digital and offline space of filter bubbles. Specifically, by extrapolating n to zero, we can theoretically model the selective diffusion of information associated

with filter bubbles and obtain accurate results. However, this method requires complex calculations and, as a drawback, may lead to non-intuitive interpretations.

The following is a list of the topics of this discussion, as well as a discussion of the approach. Filter Bubble and Echo Chamber Effect Filter bubbles are a phenomenon in which information on the Internet is filtered based on an individual's behavior and preferences, highlighting only certain viewpoints and information. This creates an "echo chamber effect" in which diverse opinions and information are restricted and an individual's existing beliefs and opinions are reinforced. In this study, we attempt to mathematically represent and understand such social opinion resonance using quantum field theory.

1.2 Application of Quantum Field Theory

Quantum field theory is a theory of physics used to describe the behavior and interaction of particles. The theory uses the concept of fields of quantum states, such as energy and momentum, to mathematically describe the interactions between particles. By applying this theory to the social sciences, specifically the creation of filter bubbles and their social consequences, this research provides a new perspective on the evolution of social opinions and group dynamics.

1.3 FP Ghosting and Uncertainty Ghosting Phenomena

FP ghosting phenomena is a concept introduced to describe non-physical factors such as misinformation and confirmation bias. It is used to represent information distortion or bias within the framework of quantum field theory. The uncertainty ghost phenomenon is also a social science concept analogous to the uncertainty principle in quantum field theory, capturing the uncertainty of information and the non-linearity of opinion formation.

1.4 Spin glass phases and memory models

The spin glass phase refers to the physical state exhibited by a collection of randomly oriented magnetic particles. This state, combined with the Edwards-Anderson model and the Hopfield model, is used to mathematically explain the energy conservation and memorability of opinions in filter bubbles. This allows for quantitative analysis of complex phenomena such as social associative memory.

1.5 Replica Method and n Extrapolation

The replica method is a technique used in statistical physics to calculate the thermodynamic properties of many-body systems. In this method, multiple copies (replicas) of a system are considered, each taking on a different state to average the

properties of the entire system. In the replica method, extrapolating n to zero is an important step in calculating the free energy of the system. This allows us to theoretically capture the selective diffusion of information and opinion formation process in the filter bubble.

1.6 Hebb Rule and Ising Spin

The Hebb rule is one of the fundamental principles of learning and memory in neuroscience. According to this principle, connections between simultaneously active neurons are supposed to be strengthened. The Ising spin model applies this principle to model memory storage and recall. The Ising spin takes on the state of $+1$ or -1 to represent a particular memory state and to understand how that memory is stored based on the Hebbian rule.

1.7 Hamiltonians and Minimal Memory

The Hamiltonian is a function of physics that represents the total energy of a system. In this study, we use the Hopfield model Hamiltonian to examine how memory is conserved in terms of energy. If certain memory states (patterns 1, 2, and p) have a Hamiltonian minimum, then those states are stable and memory recall is possible.

1.8 The Master Equation and Boltzmann Factor

The master equation is an equation that describes the temporal evolution of a system. Using this equation, the transition probabilities to a particular spin state can be calculated to understand the dynamics of the system. The transition probabilities are weighted by the Boltzmann factor, which captures how the system reaches thermal equilibrium.

Through these theoretical frameworks, this study examines in detail the long-term memorability and energy conservation issues related to the spread of fake news in digital and analog space of filter bubbles. This will allow for a theoretical understanding of how filter bubbles affect the selective diffusion of information and how this affects social opinion formation.

1.9 Application of Quantum Field Theory and Filter Bubble Analysis

The quantum theory of fields enables a new approach to understanding the resonance of opinions and diffusion patterns of information in filter bubbles through the concepts of quantum superposition and entanglement. Information and opinions can be represented as quantum states, and the superposition of different sources and opinions can model the selective diffusion of information and the dynamic evolution of confirmation bias within filter bubbles. This approach allows for quantitative analysis of how fake news is reinforced and preserved over time within specific groups.

Quantitative Analysis of Filter Bubble and Echo Chamber Effects To quantitatively analyze filter bubble and echo chamber effects, we propose a model that combines social network theory and quantum field theory. In this model, the opinions and information states of each individual (agent) are represented by qubits, and their interactions are defined by the quantum field theory. In this way, we can analyze how fake news spreads in social networks and is stored in echo chambers over time.

1.10 Modeling FP Ghosting and Uncertainty Ghosting Phenomena

To model FP ghosting and uncertainty ghosting phenomena, the degree of reliability or certainty of information is expressed as quantum probability amplitudes. This approach allows us to quantitatively assess how fake news and misinformation are perceived and impacted within the filter bubble. We also apply the uncertainty principle to analyze the impact of information uncertainty on opinion formation.

1.11 Application of Spin Glass Phase and Memory Models

By combining the spin glass phase and memory model, we analyze the long-term memorability of information and opinions in filter bubbles. In this model, different opinions and information interact like spin glasses, forming fixed patterns over time. This provides a quantitative understanding of how fake news and certain opinions are stored for long periods of time in the filter bubble.

Application of the replica method and extrapolation of n We apply the replica method to analyze the statistical properties of information diffusion patterns within the filter bubble. n extrapolated to zero allows us to theoretically predict how fake news is selectively diffused and how it affects the social network. The method also provides deep insight into the problem of energy conservation in filter bubbles.

1.12 Integration of the Hebb Rule and Ising

The integration of the Hebb rule and the Ising spin model models the mechanism of storage and conservation of fake news in the filter bubble. The model can simulate the process by which the diffusion of fake news affects the orientation of the Ising spin, resulting in the formation of long-term memories.

These proposed theoretical frameworks and computational processes provide a new analytical approach to the long-term memorability and energy conservation issues related to fake news diffusion in the digital and analog space of filter bubbles. This will enable social scientists and policy makers to better understand the mechanisms of fake news and biased information diffusion and develop effective strategies to address them.

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2. Discussion: Spin Glass Phenomenon

In this thesis, we will further develop theoretical considerations while exploring a hypothetical attempt to deepen the physical interpretation of the filter bubble phenomenon, especially in the context of opinion polarization and division phenomena, similar to those discussed in the previous dissertation on the spin glass phenomenon.

We will theoretically explain the process where the spin glass phenomenon occurs, random systems freeze, and associative memory is maintained, resulting in a fully ferromagnetic state being recorded. This process can be explained based on the Edwards-Anderson model and spin glass theory.

2.1 Edwards-Anderson Model

2.2 Model Setup

$$E = - \sum_{i,j} (J_{ij} S_i S_j) - \sum_i (h_i S_i)$$

Where:

E : Energy of the system

J_{ij} : Interaction constants

S_i : Ising spins with values of ± 1

h_i : Randomly placed magnetic fields

2.3 Process Explanation

- (1) **Random Magnetic Field Configuration:** Initially, random magnetic fields act on the spin system, representing interactions with surrounding spins. These magnetic fields have spatial randomness and remain constant over time.

- (2) **Spin Arrangement:** Spins are arranged on a lattice and interact with neighboring spins. The interaction includes the influence of random magnetic fields.
- (3) **Energy Landscape:** An energy landscape is formed for the spin system, with different landscapes for different spin orientations, depending on the randomly placed magnetic field configuration.
- (4) **Freezing:** When the temperature drops below the transition point T_g , the spin system freezes into minimum energy states within the energy landscape, resulting in the spin glass phenomenon.
- (5) **Ising Model Establishment:** After freezing, the interactions between spins take on the form of a ferromagnetic Ising model, resulting in a fully ferromagnetic state.

2.4 Replica Method for Spin Glass Coordination Average

2.5 Introduction of Replica Method

The replica method is a technique used to calculate the coordination average by introducing n copies (replicas). Each replica has the same bond distribution J_{ij} .

2.6 Model Setup

$$Z_i(T) = \sum_{\{S_i\}} \exp(-\beta E_i)$$

Where:

$Z_i(T)$: Partition function for replica i

β : Inverse temperature

E_i : Energy of replica i

2.7 Calculation Steps

- (1) **Calculation of Helmholtz Free Energy:**

$$F_i(T) = -\beta^{-1} \ln(Z_i(T))$$

- (2) **Calculation of Coordination Average:**

$$F(T) = \frac{1}{n} \sum_i F_i(T)$$

- (3) **Extrapolation to the Limit:**

$$F(T) = \lim_{n \rightarrow 0} F(T; n)$$

3. Discussion: Challenges in Extrapolating n to Zero

There are several theoretical challenges when extrapolating n to zero for calculating the coordination average of spin glass systems:

- (1) **Finite Energy:** When n approaches zero, there is a possibility that the energy becomes finite, which contradicts the requirement for finite energy in physical systems.
- (2) **Behavior of Entropy:** Behavior of entropy may become inappropriate when n approaches zero. The theoretical rigor of extrapolating entropy is not guaranteed.
- (3) **Characteristics of Phase Transitions:** Complex systems like spin glasses may exhibit non-analytical behavior associated with phase transitions. Simply extrapolating n to zero is insufficient to reproduce such behavior accurately.

The Edwards-Anderson Model

The Edwards-Anderson model considers the probability of preserving a random Ising spin configuration with anisotropic interactions J_{ij} . According to Hebb's rule, different spins have interactions with each other.

Preservation of Associative Memories

In the context of random spin configurations, we aim to preserve a ferromagnetic Ising spin configuration. In this case, since J_{ij} is set to $1/N$, the interactions between spins are, on average, uniform.

Initial Condition Dependency

The value obtained by subtracting the number of down spins may indicate a majority vote state with respect to other spin states. However, depending on the initial random spin configuration, this majority vote state may depend on different values. This demonstrates sensitivity to initial conditions.

Orthogonality and Similarity

When spin configurations exhibit orthogonality, the similarity between different patterns may decrease. Orthogonality means that the inner product of spin configurations is zero, indicating that different patterns are orthogonal to each other. As a result, the similarity between spin configurations may be low, leading to high specificity of stored information.

Coexistence Model of Diversity and Stability

The coexistence model of diversity and stability is an important concept in fields such as ecology and evolutionary biology. It suggests that the coexistence of different species

or individual diversity within a population is essential for the stability of an ecosystem or population.

Introduction of Replica Method

The replica method is a technique used in statistical mechanics and computer simulations to understand the thermodynamic behavior of a system.

Computational Process

The computational process in the coexistence model typically involves the following steps:

- (1) Initialization: Set the initial conditions of individuals within a population with different traits or strategies, introducing diversity.
- (2) Modeling Interactions: Define a model for interactions among individuals, considering different traits or strategies among replicas.
- (3) Replica Generation: Create n copies (replicas) of the same model.
- (4) Simulation of Each Replica: Simulate the evolution of each replica over time, considering interactions among individuals with different traits or strategies. This may involve asynchronous or synchronous updates.
- (5) Averaging Replicas: Average the results from each replica to evaluate the behavior of the coexistence model.
- (6) Extrapolation to $n = 0$: Extrapolate to the limit as n approaches zero to obtain the correct coordination average.

Interpretation of Results

Analyze the simulation results to understand the conditions for the coexistence of diversity and stability and how replica symmetry breaking may influence the system's behavior.

Orthogonality and Similarity in Information Storage

Orthogonality refers to the property where different spin patterns are orthogonal to each other, meaning that their inner product is zero. This implies that different spin configurations are dissimilar and do not overlap.

Similarity, on the other hand, occurs when spin configurations are not orthogonal and have a non-zero inner product. A larger inner product indicates higher similarity between spin configurations.

Pattern Matching for Memory

Pattern matching is the process by which stored spin configurations are compared to an input pattern, and the most similar stored configuration is retrieved. Memory is said to be recalled when the energy is minimized.

Energy Minimization

When memory is recalled through pattern matching, the network's energy is minimized. This occurs because the recalled pattern aligns with the stored pattern, resulting in the minimization of interaction energy and, consequently, the network's energy.

Computational Process for Memory Recall

The process involves calculating the inner product between the stored spin configuration (V_{mem}) and the input pattern (V_{input}).

The configuration (V_{mem}) with the maximum inner product is chosen as the recalled pattern. This selection minimizes the energy.

When the energy is minimized, the system converges to the stored pattern that is most similar to the input pattern.

Application to Digital Filter Bubbles

In the context of digital filter bubbles, we can interpret these theoretical concepts as follows:

- (1) Information Configuration in Filter Bubbles: In the digital environment, user information such as search history, click patterns, and past browsing history forms their information configuration. This can be likened to stored spin configurations.
- (2) Input Patterns: User actions and search queries in the digital environment can be considered as input patterns that create new information configurations.
- (3) Pattern Matching: Within a filter bubble scenario, based on a user's past actions and preferences, specific information configurations are selected, customizing the displayed information. This can be interpreted as pattern matching between input patterns and stored information configurations.
- (4) Energy Minimization: When a user's past actions and preferences align with specific information configurations, the displayed information is optimized, enhancing the user experience. This optimization process can be analogous to energy minimization.
- (5) Digital Filter Bubbles: Digital filter bubbles customize information for users by presenting specific information or perspectives, making it challenging for users to be exposed to and access other information. This is akin to the specificity of stored information in spin glass models.

4. Discussion: Retain Minima Memories

When considering patterns at the Hamiltonian energy minima, specific patterns are recalled, resulting in lower energy states and the ability to retain minima memories. When

analyzing the equilibrium state, one can confirm the increase in probability $P_t(S)$ based on the transition probability $W(S_j|S_j)$ at the current state $(-S_j \rightarrow S_j)$. By applying this to the master equation, it can be observed that it is proportional to the Boltzmann factor. During the equilibrium state, P_t remains constant over time, resulting in the left-hand side being equal to zero. At this point, the concepts of coordination average and thermal average are necessary.

In this model, the storage of memories is related to the energy minima. Below, we provide equations and a computational process explaining the energy minima associated with memory retention.

First, the Hamiltonian H is defined as follows:

$$H = - \sum_i h_i S_i - \sum_{i < j} J_{ij} S_i S_j$$

Here, S_i represents the state of neuron i (+1 or -1), h_i is the bias term for neuron i , and J_{ij} is the coupling weight. Minimizing this Hamiltonian leads to states with minimal energy, which are stored as memories.

The specific calculation process is as follows:

1. To find the energy minima of the Hamiltonian, we take partial derivatives of the Hamiltonian with respect to each S_i and set the result to zero.

$$\frac{\partial H}{\partial S_i} = 0$$

This helps determine the values of each neuron S_i .

2. Verify whether the obtained energy minima are memory patterns. Memory patterns must be states where the given S_i values minimize the Hamiltonian, confirming that H is minimized.

3. Confirm that the energy of memory patterns is minimal. If the energy at the minima states is lower than that of other states, those minima states are considered energetically stable and are stored as memories.

4. When storing multiple memory patterns simultaneously, repeat the above steps for each memory pattern. Confirm that the memory patterns you wish to store are energetically minimal.

Therefore, by finding the energy minima of the Hamiltonian and confirming that the energy is minimal, it becomes possible to store memories. This process represents the fundamental mechanism of memory storage in Hopfield networks.

Additionally, the Edwards-Anderson model, which is a general model for spin glasses, involves the interaction of electron spins in a random and heterogeneous magnetic field. Its behavior is determined by the random arrangement of spins. When associated with the Ising model of strong magnetism, the spin configurations within this model represent memory patterns, and an understanding of how memories are stored through the computational process can be achieved.

In summary, memory storage in the context of the Edwards-Anderson model and Hamiltonian energy minima relies on finding energy minima and confirming their minimal energy state. This process is fundamental to the storage of memories in these models.

5. Discussion: In a Digital Environment and Memory Patterns

In the context of pseudo-thinking about discussions in a digital environment, we hypothesize that discussions resembling filter bubbles create long-term effects, leading to a certain form of temporal and spatial equilibrium. In this case, let's consider how the following master equation can be applied and what calculation process can be envisioned.

When patterns are at the Hamiltonian energy minima, lower energy states are achieved because specific patterns are recalled, allowing for the possibility of minimal memory retention. When considering the equilibrium state, it is possible to confirm the increase in probability $P_t(S)$ based on the transition probability $W(S_j \rightarrow S_j)$ at the current state $(-S_j \rightarrow S_j)$ through the transition probability $W(S_j \rightarrow S_j)$. When applying this to the master equation, it can be verified that it is proportional to the Boltzmann factor. During the equilibrium state, P_t remains constant over time, resulting in the left-hand side being equal to zero. At this point, the concepts of coordination average and thermal average need to be introduced.

To model discussions or the temporal variations of information, such as filter bubbles, in a digital environment, one can use the master equation. Below, we explain the calculation of memory patterns and energy when patterns are at the Hamiltonian energy minima, as well as the probability distribution in the equilibrium state.

The Master Equation

Master Equation Formulation

The master equation is used to describe the time evolution of probabilistic systems and is expressed as follows:

$$\frac{dP_t(S)}{dt} = \sum [W(S' \rightarrow S)P_t(S') - W(S \rightarrow S')P_t(S)]$$

Here, $P_t(S)$ represents the probability distribution of the system being in state S , and $W(S' \rightarrow S)$ denotes the transition probability from state S' to S . This equation describes the change in the probability distribution $P_t(S)$ with respect to time.

Boltzmann Factor and Equilibrium State

In the equilibrium state, the probability distribution $P_t(S)$ remains constant over time, making the left-hand side time derivative zero. This implies the following equation:

$$\sum [W(S' \rightarrow S)P_t(S') - W(S \rightarrow S')P_t(S)] = 0$$

This equation illustrates that in the equilibrium state, the transition probabilities $W(S' \rightarrow S)$ and the probability distribution $P_t(S)$ are proportional to the Boltzmann factor.

Calculation of Energy at Hamiltonian Minima

When considering the states at Hamiltonian energy minima, specific energy values E_i correspond to spin configurations S_i . This energy E_i is calculated from the Hamiltonian as follows:

$$E_i = \sum_{(i,j)} J_{ij} S_i S_j$$

Here, S_i and S_j represent spin values, and (i, j) denotes pairs of spins with interactions. E_i corresponds to states of minimum energy, which are stored as memories.

Using Methods like the Metropolis Algorithm for Calculation

To calculate the probability distribution $P_t(S)$ and transition probabilities $W(S' \rightarrow S)$ in the equilibrium state, Monte Carlo methods such as the Metropolis algorithm are commonly used. This allows for the determination of probability distributions and energies in the equilibrium state.

Based on the above calculation process, one can model the temporal variations of information and discussions in a digital environment, such as those resembling filter bubbles, and conduct theoretical discussions about information retention and fluctuations.

In a spatially random but temporally invariant scenario, spin patterns emerge. In this situation, the random system freezes, and beyond the transition point T_g , the magnetization curve begins to bend. It can be confirmed that the interactions and strengths of bonds between each spin during this time establish a strongly magnetic Ising model.

To provide a proof for the above, I understand. Below, I will explain the calculation process with specific equations:

1. Setting the Hamiltonian of Spin Configurations:

The Hamiltonian $H(S)$ of spin configurations is expressed as follows:

$$H(S) = - \sum_i \sum_j J_{ij} \cdot S_i \cdot S_j$$

Here, J_{ij} represents the interaction between spin i and spin j , assumed to have random values. These J_{ij} values may exhibit strongly magnetic interactions.

2. Changes in Magnetization beyond the Transition Point T_g :

When the system is at a higher temperature than the transition point T_g , the magnetization susceptibility $\chi(T)$ is usually expressed as:

$$\chi(T) = \frac{\partial M}{\partial H}$$

Here, M represents the magnetization strength, and H is the external magnetic field. To calculate this magnetization, a distribution function $P(S)$ concerning spin configurations is introduced.

3. Introduction of the Distribution Function $P(S)$:

The distribution function $P(S)$ represents the probability distribution of a particular spin configuration S occurring. Based on this probability distribution, the magnetization strength M is expressed as follows:

$$M = \sum_S S \cdot P(S)$$

Here, S denotes a spin configuration, and $P(S)$ indicates the probability of that spin configuration occurring.

4. Minimum Energy State in the Distribution Function $P(S)$:

Since the spin pattern does not change over time, the system converges to a state of minimum energy. The spin configuration S_0 associated with this minimum energy state is found, which corresponds to the minimum value of the Hamiltonian $H(S)$:

$$H(S_0) = \min[H(S)]$$

5. Magnetization Strength M_0 at the Minimum Energy State:

The magnetization strength M_0 at the minimum energy state S_0 is expressed as follows:

$$M_0 = \sum_{S_0} S_0 \cdot P(S_0)$$

Here, $P(S_0)$ represents the probability of the minimum energy state S_0 occurring.

The above constitutes a series of calculation processes to compute the magnetization strength at the minimum energy state based on the Hamiltonian of spin configurations. M_0 at the minimum energy state describes the characteristics of spin glasses and indicates strongly magnetic properties. In a way, these conditions can be related to the thermal average of magnetic moments during the occurrence of filter bubbles, corresponding to spatial averages in the digital space and fulfilling the condition $t_2 \ll t \ll t_1$.

Let's explore these conditions in more detail.

Understood. Below, I will provide a more detailed explanation of the calculation process that satisfies the conditions for the thermal average being related to the distribution of bond strengths J_{ij} .

1. Setting the Hamiltonian of Spin Configurations:

The Hamiltonian $H(S)$ of spin configurations is expressed as follows:

$$H(S) = - \sum_i \sum_j J_{ij} \cdot S_i \cdot S_j$$

Here, J_{ij} represents the interaction between spin i and spin j , assumed to have random values. These J_{ij} values are spatially distributed randomly.

2. Conditions for Calculating the Temporal Average:

To consider the thermal average in relation to the distribution of bond strengths J_{ij} , we consider the conditions for calculating the temporal average, $t_2 \ll t \ll t_1$. Under these conditions, we assume that the system converges to the thermal average after a sufficiently long time t_1 .

3. Introduction of the Distribution Function $P(S)$:

The distribution function $P(S)$ represents the probability distribution of a particular spin configuration S occurring. To calculate the thermal average of a physical quantity A in accordance with this probability distribution, we have the following expression:

$$A = \sum_S A(S) \cdot P(S)$$

Here, $A(S)$ represents the physical quantity associated with a spin configuration S . This equation is used to calculate the temporal average.

4. Calculation of the Temporal Average:

The temporal average $\langle A(t) \rangle$ is calculated by averaging the quantity $A(t)$ over the time range from t_2 to t_1 . Specifically, it is expressed as follows:

$$\langle A(t) \rangle = \frac{1}{t_1 - t_2} \int_{t_2}^{t_1} A(t) dt$$

This integral range assumes that the system converges to the thermal average within the time range $[t_2, t_1]$.

5. Distribution of Bond Strengths J_{ij} :

The distribution of bond strengths J_{ij} corresponds to the probability distribution of random values of J_{ij} . This takes into account the random distribution of interaction strengths within the Hamiltonian of spin configurations.

6. Relationship between Temporal Average and Distribution of Bond Strengths:

If the temporal average $\langle A(t) \rangle$ satisfies the conditions for considering the distribution of bond strengths J_{ij} , it implies that the system converges temporally and that the thermal average converges to the distribution of bond strengths J_{ij} . To meet this condition, it is crucial that the system converges to the thermal average within the time range $[t_2, t_1]$.

The above elaborates on the calculation process that satisfies the conditions for the thermal average being related to the distribution of bond strengths J_{ij} . Calculating the temporal

average of the physical quantity A under these conditions indicates that the system converges temporally, and its thermal average is associated with the distribution of bond strengths J_{ij} .

6. Discussion: Calculation Process for Thermal and Configurational Averages in Spin Glass Phase

In this case, when considering thermal averages in a digital environment, thermal averages are calculated using J_{ij} , while configurational averages are spatially averaged due to the spatial distribution of J_{ij} varying by location. In other words, thermal averages can be hypothesized as time averages of thermal fluctuations that satisfy $t_2 \ll t \ll t_1$ over a long period.

Furthermore, introducing the replica method at this point as an approach to enhance factors such as impersonation and filter bubbles, it is believed that one can calculate the configurational average of the free energy.

Hamiltonian of the Spin Glass Phase

The Hamiltonian $H(S)$ for spin configurations in the spin glass phase is expressed as follows:

$$H(S) = - \sum_i \sum_j J_{ij} S_i S_j$$

Here, J_{ij} represents random values that denote the interaction between spin i and spin j , and it is assumed to have varying spatial distributions by location.

Introduction of Thermal Averages and Configurational Averages

To introduce thermal averages and configurational averages and to consider time averages, the following conditions are considered:

Thermal Average: Considered as the time average of thermal fluctuations that satisfy $t_2 \ll t \ll t_1$. In this case, the time average of a physical quantity A is denoted as $\langle A(t) \rangle$.

Configurational Average: Since J_{ij} has varying spatial distributions by location, configurational averaging involves taking into account different distributions of J_{ij} .

Introduction of the Replica Method

Introduce the replica method to calculate the configurational average of the free energy. The replica method involves considering n copies of the same system, each with different distributions of J_{ij} , and calculating the configurational average of the free energy.

The free energy F is expressed as follows:

$$F = -kT \ln(Z_n)$$

Here, k is the Boltzmann constant, T is the absolute temperature, and Z_n is the partition function for n copies.

Calculation of Configurational Averages

Configurational averaging involves calculating the average of the free energy over different distributions of J_{ij} . This is achieved by taking the average of the free energies of each copy.

$$\langle F \rangle = \frac{1}{n} \sum_i F_i$$

Here, F_i represents the free energy of each copy.

Master Equation

Using the free energy calculated based on the replica method, calculate the time average of a physical quantity A denoted as $\langle A(t) \rangle$. The specific calculation process follows statistical mechanics using the free energy.

This provides a scenario for considering thermal averages and configurational averages in the spin glass phase, and it elaborates on the detailed calculation process using the replica method.

Additionally, by introducing factors such as impersonation and filter bubble diffusion using the replica method, one can calculate the configurational average of the free energy by taking the average over n copies with the same bond distribution J_{ij} and extrapolating to the limit as n approaches 0. This scenario offers a theoretical approach to understanding the diffusion of influencers and filter bubbles.

7. Discussion:Field Quantum Field Memory: Spin Glass Memory Patterns

Let's establish specific equations regarding the comparison of the minimal energy points.

Idea:

Comparing minimal energy points typically focuses on the consistency of energy values and spin configurations. Using the following equations, comparisons can be made:

1. Comparison of Energy:

Compare the energies of minimal energy points E_1 and E_2 . Check if the energies are equal.

$$E_1 = - \sum_{i,j} J_{ij} S_{1i} S_{1j}$$

$$E_2 = - \sum_{i,j} J_{ij} S_{2i} S_{2j}$$

If E_1 and E_2 are equal, they represent the same minimal energy point in terms of energy.

2. Comparison of Spin Configurations:

Also, check if the spin configurations match. Verify if S_{1i} and S_{2i} are identical.

$$S_{1i} = S_{2i}$$

If S_{1i} matches S_{2i} for all i , the spin configurations are consistent.

3. Consistency Determination:

If the energies are equal, and the spin configurations match, minimal energy points E_1 and E_2 represent the same pattern.

In numerical calculations, such comparisons allow for the identification of minimal energy points and the comparison of patterns. Particularly, the consistency of spin configurations is a crucial element.

Furthermore, when considering the Hopfield model to evaluate memory pattern characteristics, the following approach can be considered:

In the Hopfield model, memory patterns are represented as energy minima. To evaluate memory pattern characteristics, the following equations are utilized:

1. Energy Function (Hamiltonian):

The energy function H of the Hopfield model is represented as follows:

$$H(\mathbf{S}) = -\frac{1}{2} \sum_{i,j} J_{ij} S_i S_j - \sum_i \theta_i S_i$$

Here, S_i represents the state of the spins (+1 or -1), J_{ij} is the strength of interactions, and θ_i is the threshold.

2. Energy Minima:

Memory patterns are characterized as energy minima. In other words, for the memory pattern \mathbf{S}^* , the following condition holds:

$$\nabla H(\mathbf{S}^*) = 0$$

Spin configurations that satisfy this condition are memory patterns.

3. Stability of Energy Minima:

For memory patterns to be stable, energy minima must be local minima. This means that all eigenvalues of the Hessian matrix should be positive. The Hessian matrix is represented as follows:

$$H_{ij} = -J_{ij} - \theta_i \delta_{ij}$$

When all eigenvalues of this matrix are positive, memory patterns are stable.

4. Energy Difference:

Calculate the energy difference between memory patterns and other spin configurations. A smaller energy difference indicates greater stability of the memory pattern concerning other spin configurations.

$$E_{\text{diff}} = H(S^*) - H(S)$$

Using these equations, memory pattern characteristics can be evaluated, and aspects like stability and energy properties can be understood. Especially, the stability as an energy minimum is crucial for memory retention.

8. Discussion: Edwards-Anderson Model and Hopfield Model: A Spin Glass Perspective

In this discussion, we theoretically examine the Edwards-Anderson model and the Hopfield model in combination to address the issues of spin glass phenomena and associative memory. Although these two models have different backgrounds, they are both essential for understanding spin systems, so let's clarify their roles.

8.1 Energy Function and Associative Memory

The Hopfield model is a model of neural networks that addresses the problem of associative memory. In this model, an energy function $E(S)$ is defined, and the energy for a spin configuration S (used here to simulate neuron states) is calculated. This energy function is designed to ensure that specific spin configurations (or memory patterns) are the most stable (i.e., have the lowest energy) for the system.

8.2 Spin Glass Freezing

Spin glass phenomena involve the freezing of randomly oriented spins at low temperatures. The Edwards-Anderson model is used to handle this phenomenon by incorporating random interactions between spins. In this model, it is assumed that spins take on random but fixed patterns at temperatures below a certain threshold.

8.3 Establishing the Strongly Magnetic Ising Model

The Ising model is a fundamental model of spin systems, characterized by spins having two possible states (usually +1 or -1). In the strongly magnetic Ising model, all interactions J_{ij} between spins are positive, indicating a tendency for spins to align in the same direction. By using this model to represent the freezing state of spin glasses, we can understand how energy minima are realized.

8.4 Associative Memory and Random Spin Configurations

In the Hopfield model, neuron states (represented here as spins) can be used to represent stored patterns. In the context of spin glass phenomena, the goal is to recall specific patterns from among random spin configurations. In this process, one searches for spin configurations S that minimize the energy function $E(S)$ to reconstruct complete memories based on partial information, emulating the process of recalling full memories based on partial cues.

8.5 Energy Minimization and Associative Memory

In the Hopfield model, the stored patterns ξ_i correspond to energy minima. This means that these patterns are stable states in the network, and ξ_i is likely to be chosen for any S such that $E(\xi_i) < E(S)$.

9. Calculation Procedure

9.1 Setting the Coupling Matrix J_{ij}

Calculate the coupling matrix J_{ij} from the random spin configuration of the spin glass. This matrix represents the strength of interactions between spins and is used to introduce randomness into the Hopfield model.

9.2 Calculating the Energy of Memory Patterns ξ_i

Compute the energy $E(\xi_i)$ for memory patterns ξ_i by substituting them into the energy function $E(S)$. This sets the energy states for memory patterns that serve as criteria for associative memory.

9.3 Exploring Recall Patterns ξ_i^*

To find the memory pattern ξ_i^* that should be recalled from the given partial information S' , apply the dynamics of the Hopfield model. In this process, use asynchronous or synchronous update rules to search for spin configurations S that minimize the energy function $E(S)$. This emulates the process where a neural network recalls the appropriate memory pattern based on given input.

9.4 Confirming Memory Recall

Verify whether the recall process was successful. If successful, the system converges to a specific memory pattern ξ_i^* . This indicates that the network accurately recalled the desired memory. However, if it fails, the system may converge to local minima, signifying erroneous or incomplete memory recall.

Theoretically Explaining the Relationship between Spin Glass Phenomena and Associative Memory

Through the interaction between spin glass phenomena and associative memory, provide a theoretical explanation for the dynamics between random spin configurations in spin glasses and memory recall in the Hopfield model. Show how the interaction between random spin configurations and memory patterns plays a crucial role in physical phenomena (e.g., spin glass freezing) and information processing problems (e.g., filter bubbles).

This approach demonstrates that the combination of the Edwards-Anderson model and the Hopfield model can be useful for theoretically explaining complex phenomena such as spin glass phenomena and associative memory. The dynamics between random spin configurations and memory patterns offer valuable insights into the understanding of physical and information processing systems.

9.5 Time-Dependent Ising Model

Introducing time variation t and considering external opinion influences, calculations using causal Green functions, advanced Green functions, and retarded Green functions are suitable for understanding the dynamic dynamics of the discourse space. Below, we propose equations and a calculation process based on this approach.

Hamiltonian of Time-Dependent Ising Model

The time-dependent Ising model is expressed as follows:

$$H(t) = -J \sum_{\langle i,j \rangle} S_i(t) S_j(t) - H(t) \sum_i S_i(t) \quad (1)$$

Here, $S_i(t)$ represents the state of spins at time t , and $H(t)$ represents the external opinion influence that changes over time.

Definition of Green Functions

9.6 Causal Green Function

Define the causal Green function $G_{ij}^c(t, t')$ as follows:

$$G_{ij}^c(t, t') = -i \langle T[S_i(t) S_j(t')] \rangle \quad (2)$$

Here, $\langle \cdot \rangle$ represents the statistical ensemble average, and T is the time-ordering operator.

9.7 Advanced and Retarded Green Functions

Define the advanced Green function $G_{ij}^a(t, t')$ and the retarded Green function $G_{ij}^r(t, t')$ as follows:

$$G_{ij}^a(t, t') = i \theta(t' - t) \langle [S_i(t), S_j(t')] \rangle \quad (3)$$

$$G_{ij}^r(t, t') = -i \theta(t - t') \langle [S_i(t), S_j(t')] \rangle \quad (4)$$

Here, $\theta(t)$ is the Heaviside function, and $[A, B]$ represents the commutator.

9.8 Calculation of Green Functions

Using the Hamiltonian $H(t)$, calculate the Green functions. This calculation is typically performed using perturbation theory or numerical simulations.

9.9 Analysis of External Opinion Influence

To analyze how external opinion influence $H(t)$ affects the discourse space over time, calculate the response of the discourse space using Green functions. This response is represented by the opinion response function $\chi_{ij}(t, t')$, showing how the discourse space reacts to external opinion influence:

$$\chi_{ij}(t, t') = \frac{\delta \langle S_i(t) \rangle}{\delta H_j(t')} \quad (5)$$

Here, $\chi_{ij}(t, t')$ represents the time-dependent magnetization (or opinion response function), indicating how the discourse space responds to external opinion influence.

10. Consideration of Delayed Effects

Using the retarded Green function $G_{ij}^r(t, t')$, calculate the delayed effects of external opinion influence on the discourse space. These delayed effects indicate that changes in opinion are not immediately reflected and appear with a temporal delay.

11. Calculation of Time-Dependent Correlation Functions

Using the causal Green function $G_{ij}^c(t, t')$, calculate the correlation of opinions at different times. This allows for an understanding of how opinions change and interact over time.

This theoretical framework allows us to understand the dynamic effects of external opinion influence on the discourse space. It is particularly useful for assessing the significance of external influences on the formation of filter bubbles and polarization of opinions. By using Green functions, it becomes possible to analyze in greater detail the temporal behavior of the discourse space and its response to external influences.

12. Conclusion: Associative Memory and Energy in Digital Environments

Applying the Hopfield model pseudo-digittally in a digital environment and considering phenomena like filter bubbles leads to interesting theoretical approaches related to associative memory of information and energy. Below, we present an examination of filter bubbles using this model.

Energy Function and Digital Information

The Hopfield model's energy function, $E(S)$, is associated with the states of digital information. In a digital environment, information has specific states, and corresponding energies are calculated. The energy function can be interpreted as an indicator of information relevance and co-occurrence.

Associative Memory of Digital Information

The Hopfield model can be applied to associative memory of digital information. For example, specific patterns of digital content or information (articles, photos, videos, etc.) are treated as memory patterns ξ_1, ξ_2, ξ_p . These patterns are defined as energy minima, where the energy function $E(\xi_i)$ has its minimum value.

Filter Bubbles and Information Retrieval

In phenomena like filter bubbles, users tend to receive information biased towards specific viewpoints or topics. Using the Hopfield model, users attempt to recall the most relevant information ξ_i^* from the given information S' . This can be perceived as a process of finding information patterns S that minimize the energy function $E(S)$.

Energy Minimization and Filter Bubbles

When filter bubbles are formed, users may converge on specific information patterns. In terms of the Hopfield model, this suggests that users are attracted to information patterns with the lowest energy, exposing them to such information continuously.

Competing Information Patterns

In a digital environment, various information patterns may compete. The Hopfield model can be used to select the information pattern with the lowest energy from among competing patterns, determining the information provided to the user. Minimizing the energy function may assist in resolving competition.

In summary, the scenario of n extrapolating to 0 provides theoretical insights into associative memory of information and energy concerning phenomena like filter bubbles in digital environments. Theoretical considerations of information retrieval processes and competition resolution through energy

function minimization may contribute to a deeper understanding of the mechanisms behind filter bubbles.

We will discuss the utility and drawbacks of the n extrapolation scenario in the context of filter bubbles in digital and analog spaces and the long-term memory and heat preservation discussions related to fake news diffusion.

13. Conclusion: Utility and Drawbacks of n Extrapolation Scenario

Utility

- (1) **Theoretical Modeling:** The scenario of n extrapolating to 0 is useful for theoretical modeling of filter bubbles and information diffusion. It provides a detailed understanding of selective diffusion of information and the selection process.
- (2) **Gaining Insights:** This approach allows for gaining deep insights into how specific information is chosen and diffused. It aids in understanding the mechanisms and impacts of filter bubbles.
- (3) **Proposing Countermeasures:** Based on computational results, countermeasures against filter bubbles or methods to control selective information diffusion can be proposed, potentially mitigating biased information diffusion.

Drawbacks

- (1) **Simplification:** The scenario of n extrapolating to 0 simplifies the complexity of real-world filter bubbles. Real-world information diffusion is influenced by many factors, and simple models may not account for all of them.
- (2) **Difficulty in Empirical Validation:** Empirical data and evidence are required for n extrapolation to 0. However, real-world information about filter bubbles is limited, making model validation challenging.
- (3) **Excessive Assumptions:** This scenario incorporates many assumptions, and they may not align with the actual situation. Results can be significantly affected by these assumptions.

In conclusion, the n extrapolation scenario provides theoretical insights but may oversimplify the complexity of real-world filter bubbles. Recognizing the limitations of the model and integrating it with real data is crucial for comprehensive discussions.

Analyzing the Phase Transition of the Strong Magnetic Ising Model

Let's consider ideas for analyzing the phase transition of the Ising model for strong magnetic materials. The strong mag-

netic Ising model is a simple model with spins having two states, +1 or -1, but it exhibits interesting properties related to phase transitions.

Ideas

(1) Hamiltonian of the Ising Model:

The Hamiltonian of the strong magnetic Ising model is defined as follows:

$$H = -J \sum_{\langle i,j \rangle} S_i S_j - H \sum_i S_i$$

Here, S_i represents the spin values (+1 or -1), $\langle i, j \rangle$ denotes pairs of adjacent spins, J is the exchange interaction constant, and H is the external magnetic field.

(2) Characteristics of Phase Transition:

The strong magnetic Ising model exhibits a phase transition at the critical temperature T_c . At this transition point, physical quantities such as magnetization and susceptibility undergo rapid changes. Exponents called critical exponents describe the characteristics of this change.

(3) Analysis of the Transition Point:

To analyze the phase transition point T_c , the following steps are performed:

Numerical simulations (e.g., Monte Carlo methods) are used to find the critical temperature T_c .

Physical quantities like magnetization and susceptibility are calculated in the vicinity of T_c to determine the critical exponents. The critical exponents are expressed as follows:

$$M \propto (T - T_c)^\beta, \quad \chi \propto |T - T_c|^{-\gamma}$$

Here, M is magnetization, χ is susceptibility, and β and γ are critical exponents.

Size scaling methods are used to consider finite size effects and obtain the values of critical exponents.

(4) Evaluation of Analytical Results:

The calculated critical temperature and critical exponents are compared with experimental results and other theories to assess the validity of the model. It is verified whether theoretical results match experimental observations.

The phase transition of the strong magnetic Ising model is a classic topic in statistical mechanics, and various analysis methods are available. By performing numerical simulations and using size scaling techniques, critical temperature and critical exponents can be calculated, providing insights into the phase transition of matter.

14. Conclusion

Calculation of Coordination Average in Spin Glass

In the context of spin glass, the calculation of the coordination average plays a crucial role in understanding its properties. This average involves considering thermal fluctuations and spatial distributions of bond interactions J_{ij} .

In the spin glass phase, the thermal average is obtained from J_{ij} , while the coordination average involves averaging over different spatial distributions of J_{ij} . The thermal average corresponds to a time average over a long time between t_2 and t_1 , where $t_2 \ll t \ll t_1$. This distinction is important for characterizing spin glass behavior.

Introduction to the Replica Method

The replica method is a powerful technique for calculating the coordination average in spin glass systems. It involves introducing n identical copies or replicas, each with the same joint distribution J_{ij} .

Setting Up the Partition Function

For each replica i ($i = 1, 2, \dots, n$), we define a partition function $Z_i(T)$, where T represents temperature. The partition function is given by:

$$Z_i(T) = \sum_{\{S_i\}} \exp(-\beta E_i)$$

Here, β is the inverse temperature ($1/T$), and E_i represents the energy of the i -th replica.

Calculation of Helmholtz Free Energy

The Helmholtz free energy $F_i(T)$ for each replica is calculated based on the partition function:

$$F_i(T) = -\beta^{-1} \ln(Z_i(T))$$

Calculation of the Coordination Average

A crucial step in the replica method is to average the free energies of the n replicas to obtain the adjusted mean, denoted as $F(T)$:

$$F(T) = \frac{1}{n} \sum_{i=1}^n F_i(T)$$

Extrapolation to the Limit

To obtain an accurate coordination average, it is necessary to extrapolate the value of n to zero:

$$F(T) = \lim_{n \rightarrow 0} F(T; n)$$

Challenges in Extrapolating n to Zero

While the replica method is a valuable tool for calculating coordination averages, there are theoretical challenges associated with extrapolating n to zero in spin glass systems.

Finite Energy

As n approaches zero, there is a concern that the energy may become finite. This violates the fundamental physical requirement that energy should remain finite in a physical system, posing a problem.

Entropy Behavior

Entropy is another critical factor in coordination average calculations. However, as n approaches zero, the behavior of entropy may become inappropriate. Ensuring the theoretical rigor of extrapolating entropy becomes challenging.

Characteristics of Phase Transitions

Systems like spin glasses may exhibit non-analytic behavior associated with phase transitions. Simply extrapolating n to zero may not accurately reproduce this behavior at phase transition points, necessitating additional methods and numerical approaches.

Given these theoretical challenges, accurately extrapolating n to zero is problematic, particularly for complex systems like spin glasses. It requires careful consideration and potential modifications to ensure accurate coordination average calculations.

Challenges in Understanding Filter Bubbles

Filter bubbles, which occur in digital environments, present several challenges in their understanding and analysis. These challenges include:

Algorithmic Opacity

One significant challenge is the algorithmic opacity of content recommendation systems. The algorithms used by platforms to personalize content often lack transparency, making it difficult to discern the criteria used for content selection. This opacity hinders a clear understanding of why certain information is presented to users while other content is filtered out.

Confirmation Bias

Filter bubbles can reinforce users' existing beliefs and preferences, leading to confirmation bias. Users may be exposed primarily to content that aligns with their views, limiting their

exposure to diverse perspectives. This bias can be challenging to quantify and analyze effectively.

Echo Chambers

Filter bubbles can create echo chambers where users interact primarily with like-minded individuals and content. This segregation of information and opinions can hinder constructive dialogue and compromise, making it challenging to identify the extent of echo chamber effects in digital spaces.

Limited User Awareness

Many users are unaware of the existence of filter bubbles or the extent to which their online experiences are personalized. This lack of awareness can make it difficult to gauge the impact of filter bubbles on individual behavior and perceptions.

Benefits of Understanding Filter Bubbles

Despite these challenges, gaining a deeper understanding of filter bubbles in digital environments offers several valuable insights and benefits:

Improved Personalization

Understanding the mechanisms of filter bubbles can lead to improved content personalization. By identifying and addressing biases in recommendation algorithms, platforms can provide users with a more diverse range of information and perspectives, enhancing the quality of user experiences.

Mitigating Polarization

Insights into filter bubbles can help mitigate polarization by introducing mechanisms that expose users to alternative viewpoints. This can contribute to a more balanced information ecosystem and reduce extreme ideological divisions.

Media Literacy Enhancement

Awareness of filter bubbles can encourage media literacy efforts. Educating users about the presence and potential effects of personalized content can empower them to critically evaluate information sources and make informed decisions.

Policy Development

A nuanced understanding of filter bubbles is essential for policymakers. It can inform the development of regulations and guidelines aimed at promoting transparency and fairness in content recommendation systems, thereby protecting democratic values and diverse discourse.

In conclusion, while the challenges in understanding filter bubbles are substantial, the potential benefits of gaining deeper insights into their dynamics are equally significant. By

addressing these challenges and leveraging the knowledge obtained, we can work towards a more informed, balanced, and inclusive digital information landscape.

15. Summary

The context in which the scenario of extrapolating n to 0 to obtain accurate results is considered involves the introduction of the replica method to model phenomena related to selective diffusion of information, such as filter bubbles and the spread of misinformation. In this scenario, the replica method is used to theoretically investigate the diffusion of information and the process of selection.

Hamiltonian Definition

- (1) To model the selective diffusion of information, a Hamiltonian is defined. This Hamiltonian represents interactions between different pieces of information or users. The general form of the Hamiltonian is as follows:

$$H = \sum_i \sum_j J_{ij} S_i S_j$$

Here, J_{ij} represents the interaction between information elements i and j , and S_i and S_j represent the states of information elements i and j .

Application of the Replica Method

- (1) The replica method involves considering n copies with different patterns of information access to calculate the coordination average of the free energy. Each copy has a different pattern of information access but shares the same Hamiltonian.

Calculation of Free Energy

- (1) The free energy F_n for each copy is calculated. The free energy is expressed as follows:

$$F = -kT \ln(Z_n)$$

Here, k is the Boltzmann constant, T is the temperature, and Z_n is the partition function for n copies.

Calculation of Coordination Averages

- (1) Coordination averages are obtained by taking the average of the free energies among different copies.

$$\langle F \rangle = \frac{1}{n} \sum F_n$$

Here, F_n represents the free energy of each copy.

Extrapolation to $n = 0$

- (1) Finally, accurate coordination averages are obtained by extrapolating n to 0. This extrapolation is a method to obtain the average energy related to different patterns of information access.

Addressing Equilibrium Concerns

- (1) Addressing the point about breaking the equilibrium state is important. To handle the scenario where n extrapolates to 0 properly, it provides theoretical supplements on maintaining equilibrium and its countermeasures.

Maintaining Equilibrium

- (1) In the replica method, when n extrapolates to 0, there is a potential risk of breaking the equilibrium state of the system. To address this issue, methods to maintain a pseudo-equilibrium state are needed.
- (2) Common strategies include the application of numerical computation methods such as Monte Carlo methods or molecular dynamics methods to prevent the system from deviating from equilibrium. These methods adjust the calculations to bring the system closer to equilibrium.

Verification of Equilibrium

- (1) In the replica method, it is crucial to confirm whether the equilibrium state is maintained when n extrapolates to 0. This can be achieved by using physical indicators such as ergodicity and time-reversal symmetry to verify the equilibrium state.

Proposal of Calculation Process and General Formula

- (1) Specific calculation processes and general formulas for maintaining equilibrium depend on the nature of the system or problem in question. Depending on the properties of the system or model, appropriate calculation methods should be chosen to prevent the disruption of equilibrium and construct the corresponding general formulas.
- (2) Common precautions include verifying the conservation of energy during the calculation process and selecting appropriate sampling methods. Also, constraints on transition probabilities and time scales can be introduced to maintain equilibrium.

Relevance to Fact-Checking

- (1) Measures to maintain equilibrium are also relevant to fact-checking. Fact-checking aims to provide accurate information by preventing the disruption of equilibrium.

Therefore, strategies for maintaining equilibrium can enhance the reliability of the fact-checking process.

Advantages and Disadvantages of Information Hygiene Scenario

The scenario mentioned above - how does it benefit the hygiene of the information space? Please also provide disadvantages. The scenario of extrapolating n to 0 to obtain accurate results has theoretical disadvantages as well.

Advantages:

1. **Improved Reliability:** In fact-checking and information verification, the ability to obtain accurate results enhances reliability. It serves as an effective means to prevent the spread of false or misleading information.
2. **Enhanced Data Quality:** Gathering and analyzing data based on accurate information improves data quality. This allows for more informed decision-making and policy formulation.
3. **Information Discrimination:** Utilizing methods to acquire information without disrupting the equilibrium state enhances the ability to distinguish useful information from less reliable sources. This enables efficient information filtering.

Disadvantages:

1. **Computational Costs:** There is a potential increase in computational costs to maintain the equilibrium state. It may require advanced computational resources and time.
2. **Information Delay:** Maintaining the equilibrium state might introduce delays in information provision or access. Urgent information may be constrained.
3. **Limitations on Information:** Constraints imposed to maintain the equilibrium state may lead to bias or limitations in accessing specific information. It could be challenging to comprehensively gather a wide range of information.
4. **Limited Applicability:** It may not be applicable to all information environments or contexts. It could be restricted to specific problems or information sources.

Disadvantages of Extrapolating n to 0 for Accurate Results:

1. **Breakdown of Equilibrium:** If it's difficult to maintain the equilibrium state, there's a risk of the system deviating from equilibrium. In such cases, there are concerns about the reliability of the computed results.

2. **Scope of Applicability:** The difficulty in maintaining the equilibrium state might limit its applicability to certain problems or systems. Therefore, this approach may not be suitable for all information.
3. **Resource Requirements:** Additional computational resources and time may be required to maintain the equilibrium state, potentially increasing operational costs.
4. **Access Constraints:** Access constraints to maintain the equilibrium state could lead to an inability to access some information.

In summary, the advantages and disadvantages of maintaining accurate information and equilibrium state interact with each other. When aiming to improve information quality and reliability, it's essential to consider the merits of collecting and verifying information without disrupting the equilibrium state and carefully assess factors like cost and applicability.

Aknowlegement

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