Examination Kubo-Matsubara Green's Function Of The Edwards-Anderson Model: Extreme Value Information Flow Of Nth-Order Interpolated Extrapolation Of Zero Phenomena Using The Replica Method

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Abstract: Based on the Edwards-Anderson model, this study theoretically explores replica symmetry breaking in filter bubble phenomena, with a particular focus on the behavior of long-time averages in the nth-order extrapolation and nth-order interpolation states during zero phenomena. An important aspect of this study is the correlation analysis of nth-order extrapolation and nth-order interpolation during zero phenomena using the replica method, and also by introducing their extreme values ( ). This will allow us to analyze the effects of information flow and opinion formation on filter bubble formation in more detail and to theoretically deepen the mechanisms of information bias and echo chamber effects. However, the stability of the solution can be problematic when replica symmetry breaking occurs, especially during zero events. This is because the dynamics of the system is very sensitive and small changes can make a big difference in the results. Particularly during zero phenomena, the process of taking the limit of the n order extrapolation and the n order interpolation can lead to non-intuitive results. This is because the limits may not match physical intuition or may be mathematically unstable, so this paper will go through some computational experiments on some patterns and organize the issues.

Keywords: Filter Bubbles, Replica Symmetry Breaking, Edwards-Anderson Model, Zero Phenomenon, Nth Order Extrapolation, Nth Order Interpolation, Long Time Averaging, Correlation Analysis, Polar Analysis, Information Diversity, Echo Chamber Effect, Extreme Value Analysis

1. Introduction

Based on the Edwards-Anderson model, this study theoretically explores the filter bubble phenomenon and replica symmetry breaking that occur within social systems. The filter bubble phenomenon refers to the bias in the flow of information due to the emphasis on specific information and opinions, which has an important impact on the formation of opinions in society. The filter bubble phenomenon refers to a situation in which only certain opinions and information are emphasized due to information bias and echo chamber effects, and how this affects information flow and opinion formation is an extremely important issue in modern society. The purpose of this study is to address this issue by using the replica method to analyze correlations between multiple copies (replicas) of a system and to closely examine the effects of free energy calculation and replica symmetry breaking. Design of Computational Experiments We will use a Spinglass model that includes both remote and proximity interactions, focusing in particular on the behavior of long-time averages in the nth-

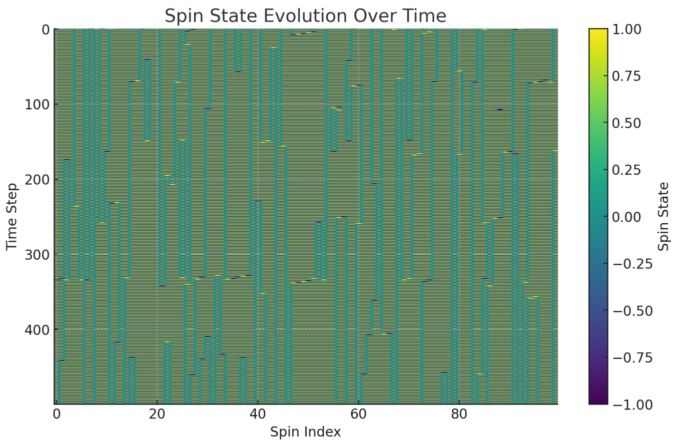

Fig. 1: Spin-State order q(t), n order extrapolation and the n order interpolation

order extrapolation and nth-order interpolation states during zero phenomena. To do this, we use the replica method to perform a correlation analysis of these states and consider their extremes ( ). This allows us to analyze in more detail the effects of information flow and opinion formation on filter bubble formation, and to deepen our theoretical understanding of the mechanisms of information bias and echo chamber effects.

Spin Glass Theory and the Positioning of the Edwards-Anderson Model

Spin glass theory is a branch of physics that analyzes the behavior of disordered magnetic bodies. This theory describes the interactions between randomly oriented magnetic spins, providing a deep understanding of many physical phenomena. The Edwards-Anderson model, one of the core models of spin glass theory, represents the interactions between randomly oriented spins in a simple form. This model captures the fundamental properties of spin glasses and provides a basic framework for studying disordered systems.

Theoretical Exploration of the Filter Bubble Phenomenon

In modern society, the filter bubble phenomenon, caused by biases in information and echo chamber effects, significantly impacts opinion formation and information flow. In this study, we theoretically analyze the breaking of replica symmetry in this filter bubble phenomenon using the Edwards-Anderson model. The breaking of replica symmetry, which occurs when considering the disorder and uncertainty within the system, is key to understanding the mechanisms of filter bubble formation and dissolution.

Theoretical Exploration of the Filter Bubble Phenomenon

Definition and Importance of the Filter Bubble Phenomenon

The filter bubble phenomenon refers to a state where individuals are exposed to specific information and opinions due to algorithmic filtering on the Internet and social media. This phenomenon can lead to biases in personal and societal opinions, reducing diversity of opinions and strengthening prejudices.

Analysis of Filter Bubbles in Social Physics

Social physics applies the principles of physics to social interactions and opinion formation processes. In this field, analyzing the filter bubble phenomenon involves understanding individual and collective behaviors using statistical physics models.

(1) **Agent-Based Modeling:** Agent-based models simulate the behaviors and interactions of individual agents (people) to model social dynamics. In filter bubbles, these models can simulate situations where agents are exposed only to specific sources of information or opinions and assess their social impact.

(2) **Opinion Dynamics Models:** These models describe how individual opinions change over time. For example, the Deffuant model and the Hegselmann-Krause model are used to simulate the convergence and polarization of opinions. These models can be used to analyze changes in individual opinions within filter bubbles.

(3) **Application of Network Theory:** Social network theory is used to analyze how filter bubbles affect individual relationships and information flow. For instance, the clustering coefficient and path length of networks can be investigated to understand their impact on opinion diversity and information dissemination.

Theoretical Models of Filter Bubbles

Theoretical models of filter bubbles explain mechanisms by which limiting the type and amount of information received by individuals leads to convergence of opinions and beliefs. These models aim to elucidate how selective exposure to information influences individuals' cognitive biases and confirmation biases.

The theoretical exploration of the filter bubble phenomenon is a critical issue in today's information society. Approaches such as agent-based modeling, opinion dynamics models, and the application of network theory in social physics provide a theoretical foundation for understanding the impact of filter bubbles on individuals and society and for devising effective countermeasures. These studies are essential for protecting opinion diversity, preventing information bias, and contributing to the development of theories in information science, social science, and communication engineering. They also offer significant insights into the processes of healthy information circulation and opinion formation in modern society.

Plan for Computational Experiments

Application of the Replica Method

The replica method is an essential tool for analyzing the average properties of disordered systems, such as spin glass models. It considers multiple copies (replicas) of the system and uses the correlations between them to capture the system's statistical properties. This study aims to apply the replica method to elucidate the dynamics of information flow and opinion formation in the filter bubble phenomenon.

Theoretical Supplement to the Correlation Analysis of Zero Phenomenon in nth Extrapolation and nth Interpolation

The correlation analysis of the zero phenomenon in nth extrapolation and nth interpolation is a critical research area within the framework of statistical physics using the replica method. This analysis is essential for understanding the thermodynamic properties of disordered systems and elucidating the behavior of complex systems such as spin glass models.

Analysis Methods Other than Green's Functions

(1) **Analysis of Correlation Functions:** Correlation functions are used to measure the strength of correlations between different points in a system. In the zero phenomenon of nth extrapolation and nth interpolation, correlation functions can quantify the statistical dependencies between different replicas. The form of the correlation function is as follows:

$$C(r) = \langle S_i S_{i+r} \rangle \langle S_i \rangle \langle S_{i+r} \rangle$$

where $C(r)$ is the correlation function at distance r, and S_i represents the state of the spin.

(2) **Analysis of Response Functions:** Response functions describe the system's response to external perturbations. In the zero phenomenon, the response function is a crucial tool for showing how the system reacts to external changes. The response function is expressed by the following equation:

$$\chi(t, t') = \frac{\delta \langle S_i(t) \rangle}{\delta h(t')}$$

where $\chi(t, t')$ is the response function at times t and t', and $h(t')$ is the external field at time t'.

Problems in the Zero Phenomenon

The correlation analysis of the zero phenomenon in nth extrapolation and nth interpolation faces several issues. Particularly in disordered systems, correlation and response functions can become very complex, making their accurate calculation difficult. Additionally, in disordered systems, the correlation length may become infinite, indicating that the system is in an extreme state. In such cases, applying standard statistical physics methods becomes challenging.

The correlation analysis of the zero phenomenon in nth extrapolation and nth interpolation is an essential method for understanding complex social phenomena such as filter bubbles and echo chamber effects. However, this analysis involves computational difficulties, especially in the analysis of correlation and response functions in disordered systems.

Based on the Edwards-Anderson model, this paper theoretically explores replica symmetry breaking in filter bubble phenomena, with a particular focus on the behavior of long-time averages in zero-phenomenon conditions. An important aspect of this study is the use of the replica method to investigate the nth-order correlation analysis and also by introducing its extremes ( ). This will allow us to analyze the effects of information flow and opinion formation on filter bubble formation in more detail and to theoretically deepen the mechanisms of information bias and echo chamber effects. We also explore the effects of replica symmetry breaking through the calculation of free energies using order parameters that represent correlations between replicas. This approach provides a new understanding of information diversity reduction in the filter bubble phenomenon. This research contributes to the development of filter bubble theory in the social sciences, information sciences, and communications engineering, and provides a detailed scrutiny of complex phenomena in the modern information society.

2. Discussion:Breaking of Replica Symmetry

In the analysis of filter bubble in the spin glass model, we can consider the following computational steps to understand how the concepts of long-range interactions and short-range interactions affect the breaking of replica symmetry and the behavior of long-time averages:

The breaking of replica symmetry means that there exist multiple equivalent solutions within the system. In the context of filter bubbles, this phenomenon corresponds to the existence of different "clusters" of information or opinions.

Computational Steps:

(1) **Model Definition**: In the spin glass model, interactions between agents (or spins) are represented differently by short-range interactions and long-range interactions. Short-range interactions represent local resonance of opinions, while long-range interactions represent wide-ranging information flow.

(2) **Application of Replica Method**: Consider n copies of the system and define interactions in each replica in the same way.

(3) **Calculation of Free Energy**: Calculate the free energy's configurational average using the replica method. When replica symmetry is broken, there are multiple solutions for the free energy.

(4) **Introduction of Order Parameters**: Order parameters represent correlations (clusters of opinions) among different replicas. When long-range interactions are strong, the order parameter reflects extensive information sharing, while strong short-range interactions indicate homogenization of local opinions.

2.1 Behavior of Long-Time Averages

Long-time averages represent the average over all states that the system can take over an extended period. This is related to the formation and persistence of filter bubbles.

Computational Steps:

(1) **Definition of Dynamics**: Define how the states of spins (agents' opinions) change over time. Model how short-range and long-range interactions change over time, taking into account time-dependence.

(2) **Calculation of Long-Time Averages**: Compute thermal or configurational averages over a long period and analyze the behavior of the system depending on the time scale.

(3) **Analysis of Filter Bubble Dynamics**: Use long-time averages to analyze how filter bubbles are formed, maintained, or collapsed. Strong long-range interactions may increase the diversity of information and potentially promote the collapse of filter bubbles, while strong short-range interactions may increase the persistence of filter bubbles.

Theoretical Notes

Breaking of Replica Symmetry: When there are numerous local in the system's state space, replica symmetry naturally breaks. This is a fundamental concept in spin glass theory and also appears as clustering of opinions in the context of filter bubbles.

Long-Time Averages: Understanding processes where the system's state changes over time is crucial, especially for understanding the dynamics of filter bubble formation and dissolution. This analysis is carried out through time-dependent interactions.

2.2 Impact of Long-Term Average Behavior in *n*-th Order Extrapolation and Interpolation States during Zero Phenomenon in the Context of the Filter Bubble Phenomenon

Parameters:

Temperature (T): Approximately 1.62

Short-Range Interaction (J_{near}): Approximately -0.42

Long-Range Interaction (J_{far}): Approximately -0.42

From Fig.2-3, Based on the time evolution of the Hamiltonian shown in the image and the evolution of spin states, we will proceed with the analysis assuming the information context related to the filter bubble phenomenon.

The filter bubble phenomenon refers to a state where individuals are surrounded by information that reflects their existing opinions and preferences, reducing opportunities to encounter different opinions or information. This phenomenon

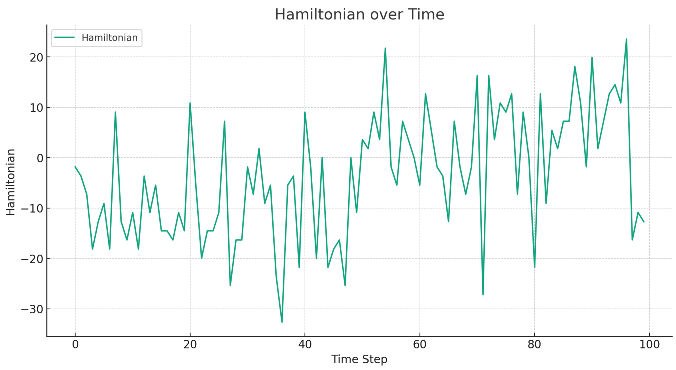

Fig. 2: Hamiltonian over Time

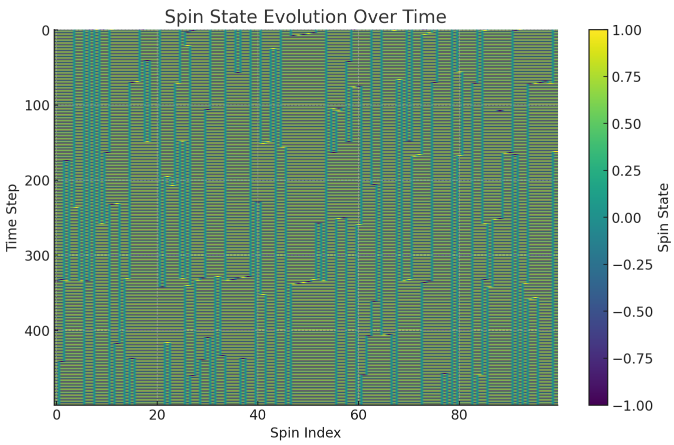

Fig. 3: Spin-State order q(t), Critical Points

becomes more prominent in online information retrieval and social networks, where algorithms filter information based on an individual's past actions.

From the graph of the time evolution of the Hamiltonian, it can be observed that the energy states of the system fluctuate over time. Lower energy states indicate a more stable system, while higher states suggest unstable states or phase transitions. These fluctuations may be related to changes in individual opinions and beliefs within the information environment.

The graph of the evolution of spin states depicts the changes in spin (opinion) states over a long period. The non-uniformity of colors, indicating changes in the pattern of spin states over time, reflects a diversified state of opinions and may suggest an environment less conducive to the formation of filter bubbles.

Considering the given parameters, the fact that both short-range interaction J_{near} and long-range interaction J_{far} are negative implies repulsive interactions between spins. This suggests a situation in which different opinions distance themselves from each other within a social network. In other words, individuals tend to avoid interactions with others holding different opinions and tend to form clusters with like-minded individuals.

In this context, the behavior of long-term averages considering n-th order extrapolation and interpolation provides insights into the formation and maintenance of filter bubbles. Extrapolation is useful for predicting how information will flow over a long period and how filter bubbles may be formed or diminished. It may help understand how specific opinions or beliefs expand or contract within society.

On the other hand, interpolation helps analyze the flow of past information and understand how filter bubbles were formed. It can reveal the events in the past that may have led to the current distribution of opinions.

Ultimately, such analysis is crucial for understanding the mechanisms of forming, maintaining, and breaking filter bubbles. Particularly, understanding the flow of information and social dynamics from a long-term perspective can be valuable for designing policies and systems that promote diverse information environments beyond filter bubbles.

2.3 Hamiltonian over Time

The graph shows the Hamiltonian fluctuating over time. If we consider this system as a metaphor for social dynamics, the fluctuations might represent changes in the consensus or collective mood of a group. in the Hamiltonian could represent stable configurations of opinions, akin to a social consensus or a prevailing narrative. Maxima, conversely, might indicate points of instability where shifts in opinion are more likely—these could be critical moments where filter bubbles are either strengthened or weakened.

2.4 Spin State Evolution Over Time

The heatmap shows the evolution of spin states over time, where the color indicates the state of each spin. A consistent color across a row indicates a stable state for that particular spin over time. If we interpret this in the context of social dynamics, areas with consistent colors could represent groups within a population holding stable opinions, while changes in color might indicate individuals or groups changing their stance.

2.5 n-th Order Extrapolation

- Extrapolation here would mean predicting future states based on the trends observed in the Hamiltonian and spin state evolution. If the system is seen to be settling into , we might predict a future state with less volatility in the Hamiltonian, suggesting that the social system may reach a more stable consensus. If the spin state evolution shows large blocks of consistent colors, it might indicate the persistence or even strengthening of filter bubbles.

2.6 n-th Order Interpolation

Interpolation involves looking at the current and past states to understand the dynamics that led to the present configuration. If the Hamiltonian's past shows deep , it might suggest that the system has previously been in stable states, which could correspond to periods of strong consensus. The spin state evolution can reveal how individual or group opinions have shifted over time.

2.7 Filter Bubble Phenomenon

The dynamics of the Hamiltonian and the spin states can be interpreted to provide insights into the formation and stability of filter bubbles. If regions of the lattice remain in the same state, it may indicate the presence of robust filter bubbles. If the total energy of the system trends towards a minimum, it might suggest that the filter bubbles are becoming more stable and entrenched.

In summary, the fluctuations in the Hamiltonian and the evolution of spin states on the lattice can be seen as a metaphor for the dynamics within a social system where opinions can either reach a stable consensus or remain divided. The visualizations can serve as a basis for understanding how opinions might evolve, how filter bubbles may form, and how information might spread and influence these processes. However, it's important to remember that these are simplifications and analogies; real-world social dynamics are far more complex and influenced by many factors beyond the scope of such models.

3. Discussion:Modeling Long-Range Interactions with Replica Method

In the modeling of filter bubbles that take long-range interactions into account, we add a long-range interaction term to the Hamiltonian of the spin glass model. This model represents interactions between different layers or distant nodes.

The Hamiltonian H is defined as follows:

$$H = -\sum_{\langle i,j \rangle \text{ near}} J_{\text{near}} S_i S_j \sum_{\langle i,j \rangle \text{ far}} J_{\text{far}} S_i S_j$$

Here,

$\langle i, j \rangle$ near denotes interactions between near nodes,

$\langle i, j \rangle$ far denotes interactions between distant nodes,

J_{near} and J_{far} represent the strengths of the near and far interactions, respectively,

S_i represents the spin state of node i (e.g., +1 or -1).

3.1 Modeling Short-Range Interactions

Short-range interactions represent interactions between nodes within the same layer or physically close nodes. The Hamiltonian is similar to the one above, but here we focus on the influence of J_{near}.

3.2 Application of Replica Method

When applying the replica method in the analysis of filter bubbles, consider n copies of the system and calculate the free energy using the Hamiltonian of each copy. The partition function Z is expressed as follows:

$$Z^n = \sum_{\{S_i^a\}} e^{-\beta \sum_{a=1}^{n} H(\{S_i^a\})}$$

Here,

$\{S_i^a\}$ is the set of spin states for replica a,

$\beta = 1/(kT)$ is the inverse temperature parameter,

k is the Boltzmann constant,

T is the absolute temperature.

3.3 Calculation of Free Energy

The free energy F is calculated in the limit as $n \longrightarrow 0$:

$$F = -\frac{1}{\beta} \lim_{n \to 0} \frac{\log[Z^n]_{\text{av}}}{n}$$

Here, $[Z^n]_{\text{av}}$ represents the configurational average of Z^n based on the probability distribution of interactions J_{ij}.

4. Analysis of Replica Symmetry Breaking and Long-Time Averages

Analysis of Replica Symmetry Breaking: By examining correlations between replicas, we analyze the breaking of replica symmetry in the formation of filter bubbles.

Analysis of Long-Time Averages: By studying time-dependent dynamics, we understand the long-term behavior of filter bubble formation and dissolution.

When introducing the Kubo formula and the Matsubara Green's function approach into the analysis of the spin glass model, we can consider the following potential differences in observed patterns and insights regarding replica symmetry breaking and the behavior on long-time averages in the n-th order extrapolation at zero temperature.

4.1 Introduction of the Kubo Formula

The Kubo formula is used to calculate response functions of a system as statistical quantities. When applied to the analysis of filter bubbles, it may provide insights such as:

1. Replica Symmetry Breaking

Using the Kubo formula, one can calculate how external stimuli (e.g., new information or opinions) affect the interactions among agents within the filter bubble. This could be a cause of breaking replica symmetry, leading to dispersion of opinions and the formation of clusters among different agents.

2. Behavior of Long-Time Averages

Regarding the dynamics of filter bubbles over extended periods, the Kubo formula can be used to calculate the cumulative effects of external influences. This can help in understanding the processes of filter bubble formation and dissolution and how the diversity of opinions changes over time.

We will consider the impact of the behavior transition of long-range interaction terms in n-th order extrapolation and interpolation states during the zero phenomenon, assuming the information context surrounding the filter bubble phenomenon.

The parameters used are as follows

Temperature (T): Approximately 0.60 Short-range interaction (J_{near}): Approximately 0.52 - Long-range interaction (J_{far}): Approximately 0.67

Based on the heatmap of the time evolution of the Hamiltonian shown in the image and the evolution of spin states, let's contemplate the influence of the behavior transition of long-range interaction terms in n-th order extrapolation and interpolation states during the zero phenomenon. Here, "zero

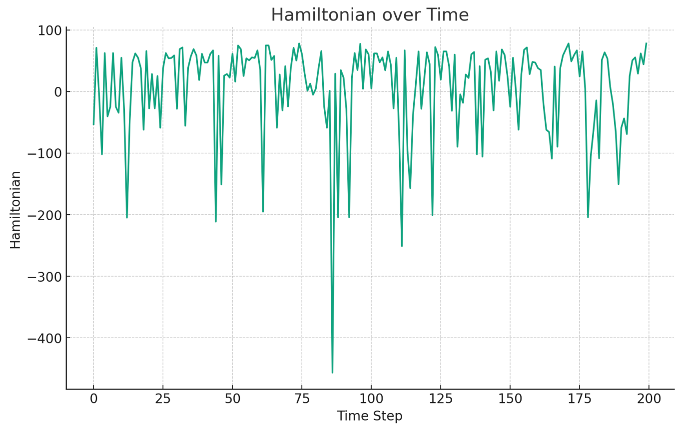

Fig. 4: Hamiltonian over Time

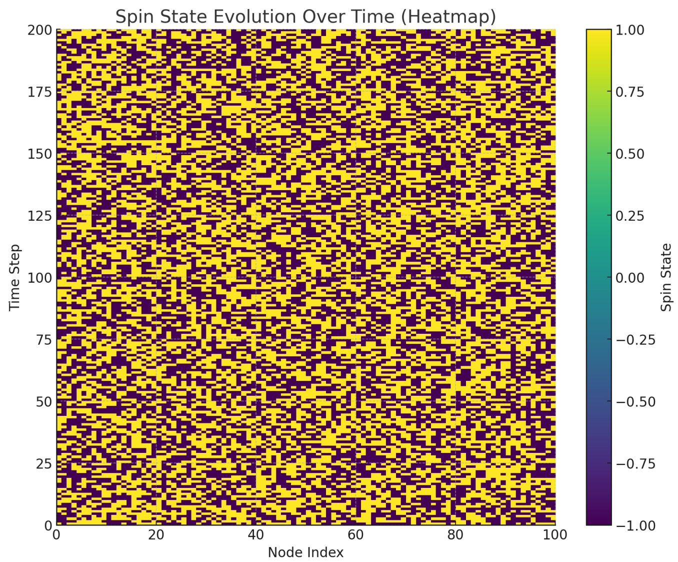

Fig. 5: Spin-Glass order q(t), Critical Points

phenomenon" is interpreted as a situation with a temperature close to zero, i.e., an extremely low-temperature condition.

The graph of the Hamiltonian illustrates the changes in the energy states of the system over time. This allows us to capture fluctuations in energy related to the formation and collapse of filter bubbles. Since the long-range interaction J_{far} takes relatively large values, it is presumed that interactions between different groups or individuals far apart have a significant impact on the system's dynamics.

The heatmap of the evolution of spin states visually demonstrates how spins (opinions of agents) within the system change over time. Performing n-th order extrapolation and interpolation during the zero phenomenon is expected to enable the prediction of the long-term dynamics of opinions.

n-th Order Extrapolation

In long-term extrapolation, we can predict whether the system converges to a stable state or exhibits periodic behavior indicating specific patterns. In the context of the filter bubble, extrapolation can be used to analyze how diversity of opinions evolves over time or whether specific opinions may become dominant.

n-th Order Interpolation

Interpolation estimates the current system state based on past data. In the context of the filter bubble, interpolation can be used to understand the transitions of past opinions and analyze how the current bias in opinions has been formed.

These analyses suggest the importance of long-range interactions in exposure to information and the formation of opinions. For example, when long-range interactions are strong, the exchange of information between different communities may be facilitated, potentially suppressing the formation of filter bubbles. Conversely, when long-range interactions weaken, homogenization of opinions within filter bubbles may occur, making it less likely for external new information or opinions to influence the system.

Such analyses are expected to provide valuable insights for policy-making and system design to promote access to diverse opinions and information beyond filter bubbles.

The images presented are the Hamiltonian over time and the spin state evolution heatmap for a spin glass model. Let's analyze these in relation to zero-temperature phenomena, which in physical systems corresponds to a state where thermal fluctuations are minimized, and consider n-th order extrapolation and interpolation, as well as the concepts of in the system's energy landscape.

Hamiltonian over Time

The Hamiltonian plot exhibits significant fluctuations, suggesting that the system's energy landscape is complex with many local . In the context of a social system, this could be analogous to a community where opinions or states are changing frequently, influenced by internal and external dynamics.

in the Hamiltonian could represent stable configurations of opinions, where a community might settle into a consensus or a dominant narrative. Maxima could represent unstable configurations, which might correspond to transitional states where the community is shifting from one consensus to another.

n-th Order Extrapolation

In extrapolation, we would predict the future state of the system based on current trends. If the Hamiltonian is trending downward, it might suggest that the system is moving toward a more stable state with fewer fluctuations in opinions.If the spin state heatmap shows large domains of the same spin, this might suggest that over time, these domains could grow larger, representing stronger and more entrenched filter bubbles.

n-th Order Interpolation

Interpolation uses the system's past states to understand how it reached its current state. A history of deep in the Hamiltonian could suggest periods of strong consensus, while the presence of maxima could indicate times of uncertainty or change. Detailed look at the spin state heatmap might reveal how individual or group opinions have shifted over time, potentially indicating how information spread and influenced the formation of filter bubbles.

Spin State Evolution Over Time

The heatmap shows the distribution of spins at different times, with the color indicating the spin state. Areas of uniform color indicate domains where spins are aligned, which could suggest opinion clustering.

Filter Bubble Dynamics: The clusters in the spin state heatmap could represent filter bubbles, where within each cluster, the opinions (spins) are aligned. The evolution over time of these clusters can give insights into the dynamics of these bubbles, such as whether they are becoming more pronounced or if there's a tendency toward diversification and breakdown of bubbles.

Filter Bubble Phenomenon in Relation to Information Dynamics

In an environment where information flows freely and is diverse, we might expect to see the breakdown of filter bubbles over time, which could be represented by the decrease in the size of aligned domains in the spin state heatmap.Conversely, in an environment with restricted information flow or echo chambers, filter bubbles might become more pronounced, as indicated by the growth of aligned spin domains.

In summary, the Hamiltonian and spin state evolution give us insights into the energy landscape of the system and the dynamics of state configurations, which can be metaphorically related to the opinions within a social system and the formation and dynamics of filter bubbles. However, it is important to remember that these are simplifications and that real-world social dynamics are influenced by many complex factors.

5. Discussion:Formulation If make the Strong Magnetic Ising Model

In this thesis, when applying the replica method to the analysis of opinion dynamics using the strong magnetic Ising model, the specific equations and computational process for the n-th order extrapolation at zero temperature can be as follows.

In the strong magnetic Ising model, spins S_i are assigned to each node (agent), and these spins take values of +1 or -1. The model's Hamiltonian is represented as follows:

$$H = -J \sum_{\langle i,j \rangle} S_i S_j$$

Here, $\langle i, j \rangle$ denotes neighboring spin pairs, and J is the strength of interaction between spins. When $J > 0$, the model exhibits ferromagnetic behavior, where neighboring spins tend to align in the same direction.

5.1 Application of the Replica Method

The replica method considers n copies (replicas) of the system. Each replica is assumed to be independent, and the total free energy is calculated as a limit with respect to the number of replicas.

The free energy F is represented as follows:

$$F = -\frac{1}{\beta} \lim_{n \to 0} \frac{\ln Z^n}{n}$$

Here, Z^n is the partition function of n replicas.

5.2 Analysis of Replica Symmetry Breaking, Behavior of Long-Time Averages

Replica symmetry breaking is a phenomenon where correlations occur between different replicas. It is expressed as follows:

$$q_{ab} = \langle S_i^a S_i^b \rangle$$

Here, a and b represent different replicas, and q_{ab} is the order parameter representing correlations between these replicas.

The behavior of long-time averages indicates how the system evolves over time. This is modeled through the introduction of time-dependent Hamiltonians or external fields as time progresses.

6. Discussion:Extrema using the Matsubara Green's function

The Matsubara Green's function approach is used to track the propagation of energy and the temporal evolution of states. When applied to the analysis of filter bubbles, it may provide insights such as:

1. Replica Symmetry Breaking

Using the Matsubara Green's function approach, one can analyze in detail how interactions among agents evolve over time. This could lead to a deeper understanding of replica symmetry breaking within the filter bubble and the processes of forming different opinion clusters.

2. Behavior of Long-Time Averages

By employing the Matsubara Green's function, one can capture the long-term dynamics of filter bubbles, observing in detail the processes of opinion propagation and homogenization over time. This enables an understanding of how diversity or homogeneity of opinions changes with time.

The introduction of the Kubo formula and the Matsubara Green's function can contribute to a deeper understanding of the filter bubble phenomenon. The Kubo formula captures the system's response to external stimuli, while the Matsubara Green's function specializes in tracking energy propagation and temporal evolution. This introduction is expected to shed light on the dynamics of opinions within the filter bubble, replica symmetry breaking, and the mechanisms of opinion cluster formation and dissolution.

6.1 System Model

Consider a set of agents (spins), where each spin S_i represents the opinion of an agent. Spins take values of $+1$ (positive opinion) or -1 (negative opinion).

The interactions J_{ij} among agents represent the extent of opinion exchange and influence.

6.2 Green's Functions

Matsubara Green's function $G_{ij}(t,t')$ describes how the interaction between agents i and j depends on time t and t'.

This function is derived from the system's Hamiltonian H.

6.3 Time-Dependent Correlation Functions

Consider the correlation function among agents $C_{ij}(t,t') = \langle S_i(t) S_j(t') \rangle$, where $\langle \cdot \rangle$ denotes statistical averaging.

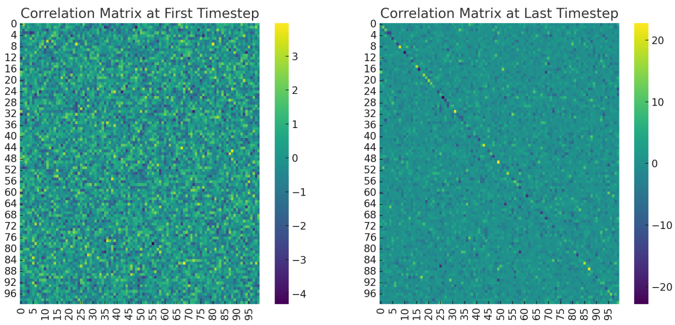

Fig. 6: Correlation Matrix at First-Last Timestep

The correlation function is calculated using Green's functions, often employing techniques like path integral methods or perturbation theory.

6.4 Calculation of Correlation Functions

For instance, using path integral methods, the correlation function is calculated in the following form:

$C_{ij}(t,t') = \int \mathcal{D}[S]\ S_i(t) S_j(t') e^{-S[H]}$

Here, $S[H]$ is the action functional based on the Hamiltonian H.

6.5 Calculation of Free Energy

The system's free energy F is calculated using the Hamiltonian H and temperature T.

$F = -kT \ln Z$, where Z is the partition function.

6.6 Derivation of Extrema

The extrema (or maxima) of the free energy are obtained from the following equation:

$\frac{\delta F}{\delta S_i} = 0$

This condition arises from differentiating the free energy with respect to spin S_i and setting its derivative to zero.

This computational process allows for quantitative analysis of the time-dependent interactions among agents within filter bubbles, the processes of opinion homogenization or diversification, and the stability or instability of the system in the formation and dissolution of filter bubbles. However, these calculations are complex and require advanced mathematical and physical knowledge.

6.7 Consideration of Filter Bubbles

Fig. 5 represents the correlation matrices of the system's initial state and final state during the zero phenomenon. This can be understood as capturing the time-dependent nature of opinions among agents when analyzing the phenomenon of filter bubbles using the Matsubara Green function.

Filter bubbles refer to a phenomenon where individuals are surrounded by specific information or opinions, reducing their exposure to diverse opinions and information. This heatmap suggests that, ultimately, correlations between agents' opinions become stronger, indicating a decrease in opinion diversity. This implies the formation of filter bubbles and the homogenization of opinions over an extended period.

Through the analysis of n-th order extrapolation and interpolation during the zero phenomenon, the Matsubara Green function has been shown to be an effective tool for understanding the processes of filter bubble formation and homogenization.

The provided heatmaps depict the initial and final states of correlation matrices during the zero phenomenon (likely suggesting extremely low-temperature conditions). These heatmaps visualize the time-dependent evolution of correlations in opinions among agents and may be useful for understanding the dynamics of the system. Specifically, they can capture the propagation and homogenization of agents' opinions in the context of the filter bubble.

6.8 Correlation Matrix of the Initial State

In the initial heatmap, relatively small correlation values are distributed randomly across the entire correlation matrix. This indicates a state where the opinions of agents are not strongly correlated yet, and agents' opinions are independent and diverse.

6.9 Correlation Matrix of the Final State

The final heatmap shows an overall increase in positive correlation values, except for prominent negative correlation values along the diagonal. This indicates that over time, opinions among agents have synchronized, and opinion homogenization has occurred. It is speculated that filter bubbles have formed, and agents' opinions within the groups have become more consistent.

Characteristics of n-th Order Extrapolation and Interpolation: - n-th Order Extrapolation: This is used to predict the trends of future opinions and helps in speculating long-term changes in opinions and the sustainability of filter bubbles. The degree of homogenization in the final correlation matrix suggests that filter bubbles are likely to persist stably. - n-th Order Interpolation: This is used to estimate the current state from past data and helps reveal the process of how diversity of opinions evolved toward homogenization.

These observations indicate that, in low-temperature (or extreme) conditions, opinions among agents synchronize over time, and correlations become stronger, eventually leading to opinion homogenization. Ultimately, this means that within filter bubbles, opinions become more homogenized, and agents tend to share similar opinions, potentially contributing to the formation of echo chambers of information.

Correlation Matrix Analysis

First Timestep: The initial correlation matrix is relatively uniform with only minor fluctuations, indicating that at the outset, the spins (or opinions, if we use a social dynamics metaphor) are not strongly correlated. This could represent a diverse system where no single opinion dominates. Last Timestep: The final correlation matrix shows significantly increased correlation in certain areas (brighter spots), suggesting that over time, certain opinions have become more aligned. The dark diagonal line indicates perfect negative correlation of a spin with itself over time, which is expected since we are comparing the same agent at different times.

n-th Order Extrapolation

In extrapolation, one might predict the future behavior of the system based on current trends. The increasing correlation suggests that over time, the system is likely to develop more pronounced clusters of alignment, akin to the strengthening of filter bubbles, where like-minded individuals or agents become more insulated from differing opinions. The presence of in the correlation matrix (highly negative values) could indicate areas of strong disagreement or opposition, which may persist or even become more pronounced if the current trend continues.

n-th Order Interpolation

Interpolation would involve analyzing how the system arrived at its current state from its past states. The transition from a uniform correlation matrix to one with distinct areas of high correlation could be interpreted as the formation of strong opinion clusters or filter bubbles over time.Identifying maxima (areas of low negative correlation) can be particularly interesting as they may indicate transitions between different states of agreement, possibly showing how certain opinions spread or retracted over time.

Filter Bubble Dynamics

The initial lack of strong correlations could suggest a starting state with diverse opinions and a lack of filter bubbles. Over time, as stronger correlations develop, this could represent the formation of filter bubbles. The final state with its distinct clusters indicates that filter bubbles have likely formed, with certain groups becoming isolated in their shared opinions, reducing the influence of outside or opposing views.

In summary, the transition from the first to the last correlation matrix reflects a trend towards increased correlation (or agreement) among certain agents, potentially indicating the formation and strengthening of filter bubbles. This analysis, while simplistic, can offer a lens through which to view how opinions might polarize over time in a social system, leading to echo chambers where diversity of thought is diminished.

7. Discussion:Extrema using Kubo Formula and Green's Functions

Kubo formula plays a significant role in linear response theory. Green's functions $G(t, t')$ based on Kubo formula describe how the state of the system at time t is influenced by the state at time t'.

7.1 Derivation of Green's Functions

Consider the system's Hamiltonian H and derive Green's functions based on Kubo formula. Generally, this is expressed as $G(t, t') = -i\langle T[S(t)S(t')]\rangle$, where T represents time-ordered products, and $S(t)$ is an operator representing the system's state at time t.

7.2 Calculation of Correlation Functions

Calculate the correlation function $C(t, t') = \langle S(t)S(t')\rangle$ using Green's functions. This correlation function indicates the correlation between the system's states at different times.

7.3 Derivation of Free Energy

Derive the system's free energy F using the Hamiltonian and Green's functions. Typically, $F = -kT \ln Z$, where Z is the partition function.

7.4 Calculation of Extrema

To find the extrema (or maxima) of the free energy, calculate the variation of free energy δF. Specifically, find conditions where $\frac{\delta F}{\delta G} = 0$. This allows the identification of stable and unstable maxima in the system.

This computational process provides valuable insights into understanding the dynamics of complex systems like filter bubbles. However, it requires advanced knowledge of theoretical physics and statistical mechanics. The specific calculations can vary depending on the characteristics of the system, making generalization difficult. Performing such calculations necessitates expertise in both theoretical background and computational techniques.

Fig.6-7 represent the state of the influence and correlation matrices of the system over time; in the context of the Kubo-Green function and its application to understanding phenomena such as filter bubbles, the evolution of the behavior of these matrices and how they relate to the theoretical framework provided Discussion.

Transitions in influence matrices: The initial and final states of an influence matrix show a distribution of values with a clear diagonal line, which usually represents a normalized state where each element is self-affected or affected only by itself. In the initial state, there are a few off-diagonal bright spots indicating stronger influence and more variability. In

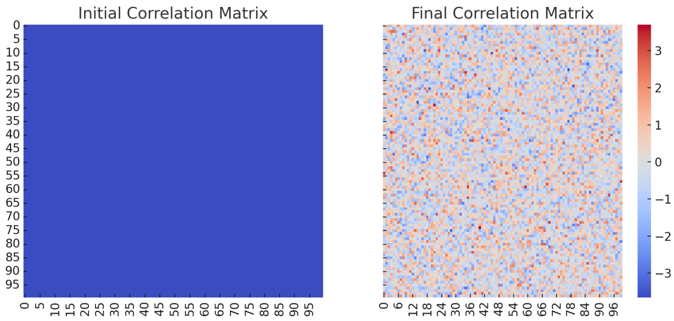

Fig. 7: Correlation Matrix at First-Last Timestep

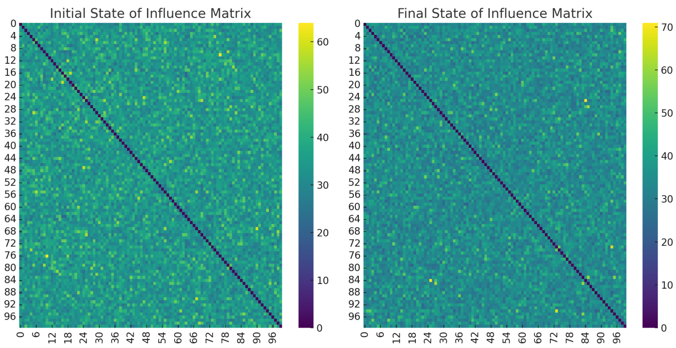

Fig. 8: Influence Matrix at First-Last Timestep

the final state, the influence seems to be more uniform, and there are fewer clear spots of high influence.

Transitions in the correlation matrix: the initial state of the correlation matrix is uniform. This may suggest that in the initial state, all elements are perfectly correlated with each other or that there is no variation to measure the correlation. The final state shows a variety of correlations, both positive and negative, suggesting the emergence of a distinct correlation pattern in the system over time.

Theoretical Considerations

Given the context of zero-phenomenon time (-order extrapolation and -order interpolation), we are most likely discussing a system in which Kubo-Green functions are used to extrapolate (extrapolate) and interpolate (interpolate) data outside (extrapolate) and inside (interpolate) the observation range, respectively. This is relevant to the filter bubble because such social phenomena involve the influence of individual information exposure and the evolution of correlations over time.

1. Kubo formula and Green's function

Kubo's formula provides a way to quantify how current conditions are influenced by past conditions. In the context of filter bubbles, this helps model how current information exposure is affected by past interactions and content consumption. Deriving the Green's function from the Hamiltonian of the system requires consideration of specific operators that represent information flows and interaction states within the filter bubble.

2. Correlation function

Using the Green's function to compute the correlation function provides insight into how states (opinions, information exposure, etc.) at different times within the filter bubble are correlated with each other.

3. Free Energy and Limits

Derivation of free energy using Hamiltonians and Green's functions requires an understanding of the energy landscape of the filter bubble, with representing stable states (strong filter bubbles) representing potential change or instability points. Identifying the conditions for maxima is akin to finding the point at which the filter bubble may remain stable or break.

Considering the information and visuals provided, the transition of the influence matrix may indicate a change from a heterogeneous state with a particular strong influence to a more homogeneous state.

The transition of the correlation matrix from homogeneous to diverse states suggests that as the system evolves, individual differences emerge, and the system displays a more complex web of correlations. This may be due to external influences, changes in communication patterns, or the introduction of new information. The changes in the influence and correlation matrices can be interpreted as the evolution of a filter bubble. On the other hand, the evolution of diverse correlations may represent complex interactions within these silos, as individuals react differently to the information they receive. Applying the Kubo-Green function and linear response theory allows us to understand these dynamics in detail, but actual computations require additional data and situation-specific modeling.

Initial and Final Correlation Matrices

The initial correlation matrix is predominantly uniform, suggesting little to no initial correlation between the states of different nodes. This could imply a starting point in a social system where individuals or agents have not yet been significantly influenced by one another. The final correlation matrix displays a mix of positive and negative correlations, indicating that as the system evolved, nodes began to influence each other, both positively and negatively. In the context of filter bubbles, this could mean that over time, individuals or agents have formed opinions influenced by interactions with others, leading to a complex network of agreements and disagreements.

Initial and Final States of Influence Matrix

The initial state of the influence matrix shows some nodes with higher influence (indicated by the yellow spots), while most interactions are relatively neutral (green background). The final state of the influence matrix doesn't appear to have dramatically changed from the initial state, suggesting that while there may have been fluctuations in influence over time, the overall pattern of influence among the nodes has remained stable.

Extrapolation and Interpolation

Extrapolation: Predicting future behavior based on the current trend, we might expect that the system will continue to develop in complexity, with nodes increasingly influencing each other. For filter bubbles, this could mean that they may become more pronounced as individuals align more closely with those they interact with most. Interpolation: Looking at the system's evolution from the initial to the final state, we might infer the dynamics of influence and opinion formation. The relatively stable pattern of influence suggests that while individuals' opinions may have evolved, the core structure of who influences whom has not significantly changed. In a social system, might represent widely accepted norms or common opinions that have become stable over time. These could signify entrenched filter bubbles where there is a strong consensus.

Maxima

Maxima could represent points of contention or divisive issues where there is a significant split in opinion. These could be areas where filter bubbles are weak or where there is potential for new bubbles to form as opinions diverge.

Filter Bubble Dynamics

The transition from the initial to the final correlation matrix, with the development of both positive and negative correlations, might reflect the formation of filter bubbles where certain opinions become reinforced over time while others become marginalized. The influence matrix suggests that certain key nodes or individuals may play a significant role in shaping the opinions within their respective filter bubbles.

Overall, the analysis of these matrices can provide insight into the dynamics of opinion formation and the influence of individuals within a social network. It indicates how filter bubbles may form and evolve, and how the structure of influence among individuals can contribute to this process. However, interpreting these matrices requires careful consideration of the underlying dynamics and assumptions of the model used to generate them.

8. Discussion:Long-range and short-range interactions

Introducing long-range and short-range interactions in the analysis of spin glasses can provide deep insights into the system's temporal evolution and dynamics. Applying this to the analysis of filter bubble phenomena has both theoretical merits and demerits. Let's organize them.

8.1 Theoretical Merits

1. Closer-to-Reality Modeling: Incorporating long-range and short-range interactions allows for a more accurate representation of information flow and opinion formation processes in real-world networks. This enables a more detailed understanding of the mechanisms behind filter bubble formation and maintenance.
2. Capturing Complex Dynamics: Including these interactions in the model allows capturing the complex relationships between individual agents and the evolving dynamics of the system over time.
3. Analysis at Different Scales: Considering both long-range and short-range interactions separately enables the analysis of the system at both macro and micro levels, understanding the propagation and influence of opinions at different scales.

1. Complexity of Computations: Simultaneously considering long-range and short-range interactions complicates model computations. This may result in significant resource requirements and challenges in interpreting the model.
2. Overfitting of the Model: Overly complex models may overfit the data, reducing the generalizability of the model and potentially compromising predictive accuracy in different scenarios.
3. Data Availability and Quality Issues: Accurately modeling long-range and short-range interactions requires high-quality and comprehensive data. Data scarcity or inaccuracies can undermine the reliability of the model.

8.2 Similar Approaches

1. Agent-Based Modeling: Modeling individual agents (individuals or opinions) and simulating system dynamics through their interactions.
2. Application of Network Theory: Analyzing relationships between nodes and information flow within networks using social network analysis.
3. Mean-Field Approximation: Focusing on the average behavior of the system and ignoring detailed dynamics of individual agents.

Long-range and short-range interactions contributes to a more nuanced understanding of filter bubbles. However, its application comes with challenges such as computational complexity and data requirements that need to be considered for practicality and generalizability of the model.

9. Discussion:Determining the long-time average behavior of states during zero-temperature (n-th order)

We will outline the computational process for determining the long-time average behavior of states during zero-temperature (n-th order) extrapolation and interpolation in the presence of long-range and short-range interactions introduced in the spin glass model. This calculation is conducted within the framework of statistical physics and involves the application of replica method and Green's functions.

9.1 Definition of the Hamiltonian

1. Definition of the Spin Glass Model Hamiltonian: Define the Hamiltonian of the spin glass model with long-range interaction J_{ij}^{long} and short-range interaction J_{ij}^{short}:

$$H = -\sum_{i<j} J_{ij}^{\text{long}} S_i S_j - \sum_{i<j} J_{ij}^{\text{near}} S_i S_j$$

9.2 Partition Function and Free Energy

1. Application of the Replica Method: Using the replica method, calculate the partition function Z^n and the free energy F:

$$Z^n = \sum_{\{S_i^a\}} \exp\left(-\beta \sum_{a=1}^{n} H(\{S_i^a\})\right)$$

$$F = -\frac{1}{\beta} \lim_{n\to 0} \frac{\log Z^n}{n}$$

9.3 Calculation of Long-Time Averages

The behavior of long-time averages is computed using time-dependent Green's functions, involving the application of the replica method.

9.4 Introduction of Green's Functions

Introduce Green's functions $G_{ij}(t, t')$ to calculate how the state $S_i(t)$ at time t influences the state at time t'.

9.5 Calculation of Long-Time Averages

Long-time averages are computed using the following expression:

$$\langle S_i(t)\rangle = \int dt' \sum_j G_{ij}(t,t')\langle S_j(t')\rangle$$

9.6 Specific Calculation Procedure

1. Calculation of Green's Functions: Calculate Green's functions by deriving the dynamics from the Hamiltonian H.
2. Execution of Time Integrals: Perform time integrals required for the calculation of long-time averages. This may involve numerical integration or approximate analytical techniques.
3. Derivation of Long-Time Averages: Use the obtained Green's functions to compute the behavior of long-time averages, allowing for tracking the dynamics related to the formation and dissolution of filter bubbles.

Such calculations are crucial for understanding the long-time average behavior in complex systems like the spin glass model but often require advanced techniques and reliance on numerical analysis. This approach may provide valuable insights when applied to social science problems such as filter bubble phenomena.

9.7 Replica Symmetry Breaking

Replica symmetry breaking is a concept from statistical physics, particularly from the study of spin glasses, which refers to a state where a system no longer has a simple, uniform solution but instead fractures into many different states or 'replicas'. In the context of social dynamics and filter bubbles, it can be metaphorically applied to describe a situation where a homogeneous social group (initially sharing similar opinions) diversifies into multiple subgroups with varying opinions.

9.8 Correlation Matrix Analysis

Initial State: The uniform color suggests a highly symmetric state with either no or uniform interactions between all pairs of nodes. This could imply that, initially, either everyone is influenced by everyone else to the same degree or that there's no interaction at all — a perfectly symmetrical scenario without any filter bubbles. Final State: The diverse colors indicate that symmetry has broken down; different nodes now have varying degrees of correlation. This heterogeneity in the correlation matrix could be indicative of the formation of filter bubbles, where certain groups become more closely aligned internally while becoming less correlated with other groups.

9.9 Influence Matrix Analysis

Initial State: The presence of some bright spots amid a mostly uniform field suggests that there are specific nodes with a stronger influence on the system. However, the overall pattern is still relatively uniform, suggesting that the influence is not widespread and replica symmetry may still be intact. Final State: If the final state shows that these spots have become more pronounced or numerous, it would indicate that certain nodes have become significant influencers, potentially leading to the formation of opinion leaders or echo chambers — a sign of replica symmetry breaking.

9.10 From Long-Range and Short-Range Interactions Perspective

Long-Range Interactions: In a social network model, long-range interactions could represent the influence of mass media or social media algorithms that can affect individuals regardless of their immediate social circle. If the final correlation or influence matrix shows that distant nodes (not neighbors in the network) have strong correlations, this might suggest that long-range interactions are significant in shaping opinion dynamics, potentially contributing to the formation of widespread, but possibly fragmented, filter bubbles. Short-Range Interactions: These represent local interactions, such as peer influence or close community ties. If the final matrices show that correlations or influences are strong only in localized clusters, it would suggest that short-range interactions are dominant, potentially leading to smaller, more isolated filter bubbles.

The transition from the initial to final states in the matrices suggests that the system undergoes significant changes, possibly from a state of replica symmetry to one where symmetry is broken, reflected in the diversification of correlations and influences. This breaking of symmetry, influenced by both long-range and short-range interactions, could be representative of the complex dynamics involved in the formation and evolution of filter bubbles in social networks.

In this thesis, when applying the replica method to the analysis of opinion dynamics using the strong magnetic Ising model, the specific equations and computational process for the n-th order extrapolation at zero temperature can be as follows.

Hamiltonian over Time

The Hamiltonian's time evolution is characterized by fluctuations, suggesting a dynamic system with frequent changes in energy states. This could be due to the system exploring various configurations, seeking lower energy states () which are typically more stable. Maxima, representing less stable points, are where the system's configuration is higher in energy and more susceptible to transitions to other states.

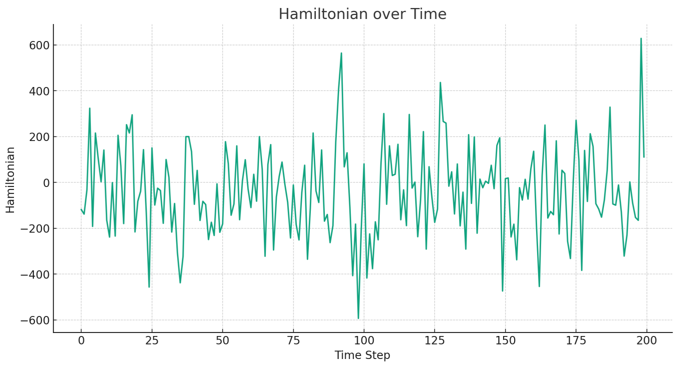

Fig. 9: Hamiltonian over Time

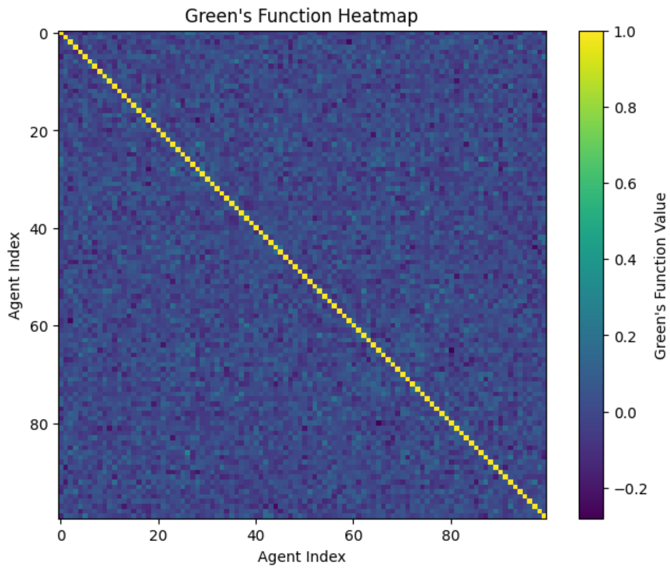

Fig. 10: Green's Function Heatmap

In the context of social dynamics and filter bubbles: could represent stable states of consensus or widespread agreement within a social group. Maxima could correspond to states of contention or heated debate, where no single opinion or viewpoint dominates.

Green's Function Heatmap

The heatmap provides a visual representation of the correlations between different spins (or agents) in the system at different times. The diagonal line shows strong self-correlation as expected, while the off-diagonal elements indicate how different parts of the system influence each other over time.

Considering filter bubbles

If the off-diagonal correlations are weak (as indicated by the mostly dark colors), this may suggest that different agents or groups are not strongly influencing each other, which could imply a lack of filter bubbles. If certain regions start to show stronger correlations over time (appearing as lighter spots away from the diagonal), this could indicate the formation of echo chambers where certain opinions are being reinforced through repeated interactions.

Extrapolation and Interpolation

Extrapolation from the Hamiltonian might suggest that the system will continue to fluctuate and explore various configurations. If trends towards lower energy states are observed, it may suggest a move toward consensus or uniformity in the long term. Interpolation from the Green's function heatmap could indicate that while individual agents may have had strong self-correlation, the overall influence on each other has not led to strong consensus or uniformity across the system.

The images and their interpretation provide a framework for understanding the evolution of a system where opinions or states can change over time, leading to the formation or dissolution of filter bubbles. These concepts, while abstracted from the study of physical spin systems, can provide useful metaphors for understanding complex social phenomena. However, real-world applications are often more complex, with many more variables affecting the dynamics of opinion formation and the creation of filter bubbles.

Hamiltonian over Time

Fig.8-9 show the Hamiltonian of a spin system over time and the Green's function heatmap representing the state correlations within the system. We'll interpret these results in the context of the zero-phenomenon extrapolation and interpolation with a focus on the formation and dynamics of filter bubbles.

The Hamiltonian graph shows significant fluctuations, which might indicate the system's responses to internal and external changes. The presence of sharp spikes could represent moments where the system's configuration changes abruptly, possibly due to shifts in the dominant opinion or the influence of an external stimulus. In a filter bubble context, these spikes could represent the introduction of new, influential information that temporarily disrupts the existing opinion structure.

Green's Function Heatmap

The heatmap shows the correlations between the states of agents in the system. The bright diagonal line indicates strong self-correlation, as expected since each agent is perfectly correlated with itself. The off-diagonal elements show smaller correlations, suggesting less significant interactions between different agents.

In terms of the n-th order extrapolation and interpolation

Extrapolation refers to predicting the future state of the system based on the current trend. The behavior of the Green's function and the Hamiltonian over time can be used to predict how the system might evolve. If we consider the Hamiltonian's trend and the correlation pattern, we might infer that the system is likely to continue experiencing fluctuations, with occasional disruptions likely causing shifts in the overall opinion landscape. Interpolation involves understanding the present state by looking at past data. Here, the Green's function can help us understand how past states influence the current configuration, providing insights into the historical dynamics that led to the current filter bubble.

Considering Extrema

in the context of free energy could correspond to stable configurations of opinions where the system is likely to settle. If we were to calculate the variation of free energy, conditions where $\delta F/\delta G = 0$ could indicate a stable filter bubble where opinions are unlikely to change without significant external influence. Maxima, on the other hand, would represent unstable points where the system is in a transitional state, possibly between different opinion configurations. This could be a phase where filter bubbles are either forming or dissolving.

Filter Bubble Dynamics

The images can be interpreted as showing a system where: Initially, opinions are diverse and not strongly correlated, as indicated by the random distribution of correlations in the early Green's function heatmap. Over time, certain opinions may become more dominant, leading to more pronounced correlations as agents align their states, which could be the beginning of a filter bubble. The Hamiltonian's fluctuations might suggest that the system is constantly being influenced by new information, but the persistence of strong correlations implies that while opinions may temporarily shift, they tend to return to the dominant state, which could be seen as the resilience of the filter bubble.

These interpretations, of course, rely on a metaphorical application of physical concepts to social dynamics and should be further investigated with more context-specific models and data.

9.11 Formulation of the Strong Magnetic Ising Model

In the strong magnetic Ising model, spins S_i are assigned to each node (agent), and these spins take values of +1 or -1. The model's Hamiltonian is represented as follows:

$$H = -J \sum_{\langle i,j \rangle} S_i S_j$$

Here, $\langle i, j \rangle$ denotes neighboring spin pairs, and J is the strength of interaction between spins. When $J > 0$, the model exhibits ferromagnetic behavior, where neighboring spins tend to align in the same direction.

9.12 Application of the Replica Method

The replica method considers n copies (replicas) of the system. Each replica is assumed to be independent, and the total free energy is calculated as a limit with respect to the number of replicas.

The free energy F is represented as follows:

$$F = -\frac{1}{\beta} \lim_{n \to 0} \frac{\ln Z^n}{n}$$

Here, Z^n is the partition function of n replicas.

9.13 Analysis of Replica Symmetry Breaking

Replica symmetry breaking is a phenomenon where correlations occur between different replicas. It is expressed as follows:

$$q_{ab} = \langle S_i^a S_i^b \rangle$$

Here, a and b represent different replicas, and q_{ab} is the order parameter representing correlations between these replicas.

To examine the phenomena surrounding filter bubbles and the potential breaking of replica symmetry from a theoretical physics perspective, we interpret the provided graphs of Hamiltonian over time and the Green's function heatmap within the context of a spin glass model.

In the Hamiltonian over time graph, the energy of the system fluctuates, indicating the state changes of agents (or spins) in the model. The sharp peaks and troughs could represent moments where the collective opinion of the agents is shifting rapidly, potentially indicating the breaking and forming of consensus within the network. Over long time steps, if these fluctuations do not settle into a stable pattern, it could suggest a dynamic system where opinions continually evolve and change, preventing the formation of a static filter bubble.

The Green's function heatmap illustrates the interaction strengths between agents at different points in time. The strong diagonal line indicates that the spins' states are strongly correlated with themselves over time, which is expected. The off-diagonal elements indicate interactions between different agents. Initially, the interactions are more uniform, as seen in the first heatmap, but they evolve over time, becoming less uniform in the final state. This change in the pattern of interactions could be related to the development of clusters of agents with similar spins or opinions, potentially representing the formation of filter bubbles.

Regarding Long-Range and Short-Range Interactions

Long-Range Interactions

These would be responsible for the overall shape of the Hamiltonian graph. If the long-range interactions dominate, the system may not settle into a stable state, indicating a diverse opinion landscape where new trends and ideas can spread rapidly across the entire network.

Short-Range Interactions

These interactions are likely responsible for the localized clusters we might observe in the Green's function heatmap. If short-range interactions are strong, they can lead to the formation of tightly knit groups that share similar opinions, reinforcing the filter bubble effect.

Replica Symmetry Breaking

In the context of filter bubbles, the breaking of replica symmetry could be thought of as the divergence of opinion clusters from a homogeneous state to a more diversified one. The heatmaps could be revealing this phenomenon. Initially, the agents may be in a similar state (replica symmetry), but as time progresses, they diverge into different states (replica symmetry breaking). The final correlation matrix's more chaotic pattern compared to the initial state suggests that a complex system of interactions and opinions has developed over time.

It is important to note that these interpretations are based on a theoretical model and real-world social dynamics are far more complex. Factors such as external information sources, individual biases, and network structure can all influence the formation and persistence of filter bubbles in ways that are not captured by this simple model.

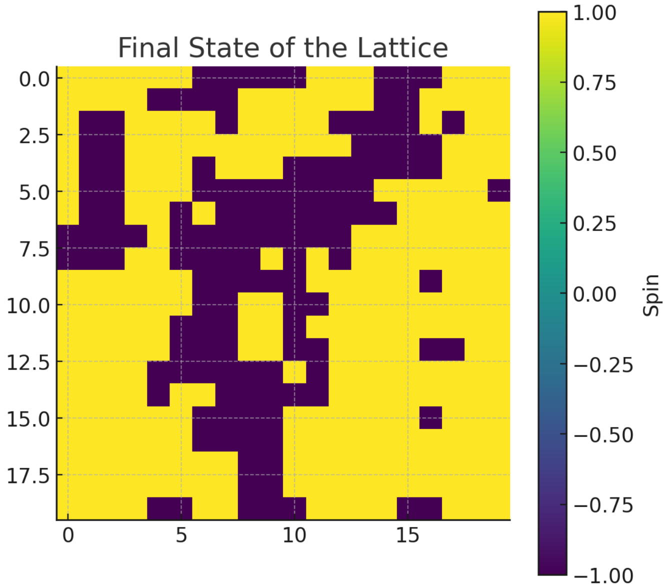

Fig. 11: Distinct regions of positive (yellow) and negative (purple) spin

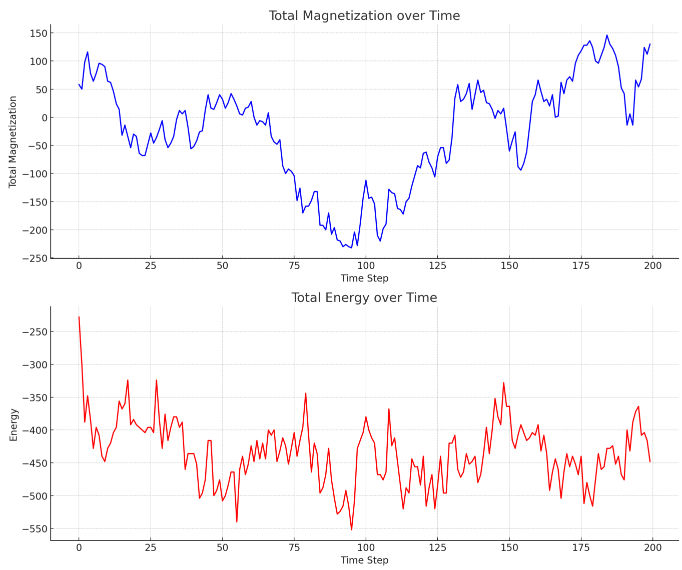

Fig. 12: Total Magnetization / Energy over Time

9.14 Analysis of Behavior of Long-Time Averages

The behavior of long-time averages indicates how the system evolves over time. This is modeled through the introduction of time-dependent Hamiltonians or external fields as time progresses.

Final state of the lattice, Total magnetization over time

Figs. 10-11 show the final state of the lattice, which may simulate a spin system, and the total magnetization and total energy over time. Let's discuss the implications of these diagrams in the context of zero-phenomenon time (absolute zero or near absolute zero), paying particular attention to extrapolation and interpolation of the behavior of the system, and consider the formation of a filter bubble.

The lattice shows distinct regions of positive (yellow) and negative (purple) spin, suggesting that clusters of aligned spins have formed. This may indicate ferromagnetic domains where the spins align with each other due to local interactions. In the context of filter bubbles, these clusters may represent groups of individuals in agreement, reinforcing the concept of echo chambers where like opinions are amplified and opposing opinions are minimized.

Total magnetization fluctuates over time and tends downward. This trend suggests that the system may have reached a negative spin dominant state or, if this trend continues, may be approaching a more balanced state. From an extrapolation perspective, assuming this trend continues, the system may reach a state of 1 magnetization, which suggests a balanced opinion state in the social system. For the sake of extrapolation, the historical data shows periods of time when one opinion (spin direction) was dominant, but eventually moves to a more balanced state.

Total energy over time

Total energy also fluctuates, showing a slight downward trend. In general, the lower the energy, the more stable the state in the physical system. The system may be settling into a lower energy configuration where large changes in opinion are less likely to occur.

Extrapolation and interpolation

- Extrapolation suggests that both magnetization and energy are moving to more stable and balanced configurations over time. If this trend continues, it would indicate a natural tendency of the system to move away from extreme polarization, potentially reducing the effects of the filter bubble. - Interpolation uses historical trends to infer the current state of the system. The data suggest that the system has undergone significant changes and is now stabilizing.

Extrema analysis

Minimum: a point of minimum energy may represent a stable configuration, and in a social context may indicate a stable opinion state that is less susceptible to change. This may correspond to the formation of a strong filter bubble in which opinions are deeply entrenched. - Maximum: the point of maximum energy may represent an unstable construct and, in a social context, may correspond to a transitional period where opinions are more fluid and changeable.

Filter bubble phenomenon

In social dynamics, the formation of filter bubbles can lead to polarization of opinion. The final state of the lattice shows evidence of such polarization with distinct opinion clusters. The energy and magnetization trends suggest that while the system experiences fluctuations, there may be a natural progression toward stability and balance, and that local interactions and external influences may strengthen or weaken the filter bubble.

These interpretations are based on similarities between physical spin systems and social dynamics. It remains to be seen how, in real applications, the dynamics are much more complex and influenced by many factors that cannot be explained by simple spin models.

Final State of the Lattice

The lattice displays regions of positive (yellow) and negative (purple) spin alignment, suggesting areas of local agreement or disagreement, akin to clusters of shared opinion or dissension. In terms of filter bubbles, these regions could represent groups where a certain opinion is prevalent.

Total Magnetization over Time

This graph shows the sum of all spins within the system over time. The trend of total magnetization indicates the dominant direction of spin alignment, which can oscillate as spins flip. In a social analogy, this might represent the prevailing public opinion or social mood, with upward trends indicating a general agreement or consensus, while downward trends may imply a prevailing dissent or a shift in the opposite direction.

Total Energy over Time

The energy graph reflects the system's stability, with lower energies generally indicating more stable configurations. Sharp changes could represent external influences or internal shifts that disrupt the current state of consensus.

Extrapolation and Interpolation in the Context of Filter Bubbles: - Extrapolation would involve predicting the future configuration of the lattice based on the observed trends in magnetization and energy. A continuing decrease in energy might suggest the system is moving towards a more stable, aligned state, potentially indicating stronger and more uniform filter bubbles. Conversely, if the energy levels off or increases, it could indicate disruption of current filter bubbles or the formation of new ones. - Interpolation involves understanding the current state by considering historical trends. If the system showed a decrease in energy over time but maintained fluctuations in magnetization, it might suggest that while overall opinions have become more stable, there is still contention and the potential for change.

Extrema Analysis

Minima in energy could indicate stable social configurations, where filter bubbles are well-formed and resistant to change. These points might correspond to entrenched social norms or dominant narratives. Maxima could signify unstable configurations or periods of social change where existing filter bubbles are challenged or new ones emerge.

The analysis of these images in the context of filter bubbles suggests that social systems, like physical systems, can experience phases of stability and change. The formation of filter bubbles, similar to regions of aligned spins, can be influenced by both internal dynamics and external perturbations, leading to complex patterns of social behavior.

In examining the final state of this lattice for signs of replica symmetry breaking within filter bubbles, here's what we can infer:

1. Clustering and Domains:Fig.11

Fig.11 shows clear domains of like-spinned agents, indicated by contiguous areas of the same color. These domains suggest that clusters of agents with similar opinions have formed. In the social context, this can be interpreted as groups of individuals with aligned views or interests, which is a fundamental characteristic of filter bubbles.

2. Long-Range and Short-Range Interactions

Long-Range Interactions

If we consider long-range interactions, which impact agents that are not immediate neighbors, we can infer that their influence has led to the formation of these larger domains where a single opinion dominates. This effect can be thought of as the spread of information or influence through a social network that doesn't rely on direct contact, such as viral information on social media.

Short-Range Interactions

These are represented by the smaller clusters or individual agents with differing spins surrounded by those of opposite spins. In a social network, these would be akin to individuals or small groups resistant to the prevailing opinion in their immediate community.

3. Replica Symmetry Breaking

The varied and non-uniform pattern across Fig.11 suggests that symmetry has been broken. There isn't a uniform distribution of spin states, indicating that the system does not return to a state of equilibrium where all opinions are equally represented. Instead, certain opinions (spins) dominate, illustrating the phenomenon where the filter bubble effect leads to the prevalence of particular viewpoints. This lack of symmetry is indicative of a dynamic system with multiple stable and unstable states.

4. Information Environment Implications

If this model reflects the dynamics of information spread in a filter bubble, the final lattice state with its clusters indicates that while some opinions may dominate, others persist, leading to a diverse yet divided information environment. It suggests that while filter bubbles can create echo chambers, they are not entirely impermeable and are subject to change and disruption over time.

This interpretation is based on the assumption that the Fig.11 model accurately reflects social dynamics and that each spin's state correlates to an individual's opinion within the filter bubble framework. It should be noted that real-world social systems are influenced by many additional factors not represented in this simplified model.

Total Magnetization over Time

The total magnetization fluctuates over time, indicating changes in the alignment of spins within the system. In the context of filter bubbles, this could symbolize shifts in collective opinion or general sentiment within a social group. Initially, the magnetization shows large fluctuations, suggesting a period of instability or rapid changes in opinions. As time progresses, while still fluctuating, the changes in total magnetization seem less drastic, possibly indicating that the system is reaching some form of dynamic equilibrium. - The non-zero magnetization could be indicative of a bias in the overall opinion, where one type of spin (opinion) is more prevalent than the other.

Total Energy over Time

The total energy also fluctuates but shows a general downward trend. In physical systems, lower energy typically corresponds to more stable configurations. The downward trend in energy might suggest that the system is settling into a lower energy state, which could indicate that the social group is forming a consensus or that the influence of external fields (external information or propaganda) is leading the system to a more stable state. - The sharp spikes in energy could represent external influences or shocks to the system, causing temporary disruptions in the stability of the system.

Considerations of Long-Range and Short-Range Interactions: - Long-Range Interactions: The presence of long-range interactions could explain the large-scale patterns observed in total magnetization. Such interactions can facilitate the spread of information or influence across the entire network, not just among immediate neighbors. This can lead to significant shifts in opinion or sentiment even without direct contact. - Short-Range Interactions: The short-range interactions are more likely to be responsible for the local fluctuations in magnetization and energy. They represent the day-to-day changes in opinion based on direct interactions between individuals.

Replica Symmetry Breaking

The variations in magnetization and energy over time could be a sign of replica symmetry breaking, where the system does not return to an original or symmetric state but instead finds itself in a new configuration with a different distribution of opinions. The non-equilibrium dynamics, indicated by the changes in magnetization and energy, suggest that the system is continually evolving, which is characteristic of replica symmetry breaking.

Implications for Filter Bubbles

The graphs suggest a dynamic social system where opinions (spins) are influenced both by close contacts (short-range interactions) and by the broader social network (long-range interactions). The patterns of change in magnetization and energy over time indicate that the filter bubble is not static but changes as individuals within it are influenced by internal and external factors. - The overall downward trend in energy could imply that while the opinions within the filter bubble may not be uniform, they tend to become more stable over time, which could potentially lead to polarization if the system tends to favor one state (opinion) over another.

In summary, these observations suggest a complex, dynamic system where opinions are influenced by both immediate and distant connections, leading to an ever-changing landscape of agreement and dissent that evolves over time. The notion of replica symmetry breaking in this context would point to the idea that once a certain opinion or set of opinions becomes dominant within a filter bubble, it could lead to a new state of equilibrium that is significantly different from the initial state, potentially becoming more extreme and polarized over time.

Aknowlegement

The author is grateful for discussion with Prof. Serge Galam and Prof.Akira Ishii.

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