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Note:Harnessing Game Theory for Iterative Optimization to Model Fake News Diffusion Considering Hebbian Learning Insights from Nash Equilibrium

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Abstract: This study utilizes the replica method, a sophisticated concept derived from statistical physics, to simulate the spread of fake news within social networks. By generating multiple replicas of the social network model, we examine the dynamics of fake news diffusion under a variety of conditions and parameters. This methodology facilitates the exploration of the effects of average behavior, variance, and diverse factors on fake news diffusion through extensive computational experiments. This paper further explores the application of a local potential approximation that incorporates a mathematical formulation of local potentials based on pair, triplet, and quadriplet interactions to address both remote and proximity interactions between agents. The impact of nontrivial loops and replica symmetry breaking on the dynamics of fake news diffusion (approached via Hebbian learning) is also highlighted, illustrating the theoretical and computational challenges in accurately modeling such a complex social phenomenon. Visualizations included in this study, such as correlations between replicas, average energy versus interaction strength, and the effect of non-trivial loops on local potentials, provide deep insights into the behavior of the model under a variety of conditions. This comprehensive analysis not only sheds light on the nature of modeling complex systems such as the spread of fake news in social networks, but also highlights the need for advanced methods such as the replica approach and in-depth consideration of the interactions and dynamics in the model. Through this approach, we aim to contribute to a more robust understanding of the fake news phenomenon and provide important insights for developing strategies to increase society's resilience against fake news.

Keywords: Game Theory, Fake News, Factcheck, Replica Method,Local Potential Approximation, Pareto Optimality, Monte Carlo Simulations, Nash Equilibrium, Game Theory, Statistical Physics, Dynamics Analysis

1. Introduction

The research discusses the application of the replica method, a concept borrowed from statistical physics, to model the spread of fake news in social networks. This method involves creating multiple copies of a social network model, referred to as "replicas," to analyze the dynamics of fake news dissemination under various conditions and parameters.

The approach allows for the examination of average behaviors, variances, and the influence of different factors on the spread of fake news by conducting computational experiments across these replicas.

Key steps in applying the replica method include generating replicas, setting initial conditions, simulating dynamics (e.g., using Monte Carlo simulations), conducting statistical analyses, varying parameters, and interpreting the results. This method provides insights into the complexity and vari-

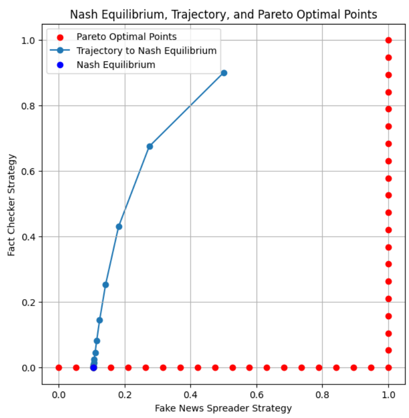

Fig. 1: Nash Equilibrium, Trajectory, and Pareto Optimal Points, Fake News Spreader Strategy

ability of fake news spread, contributing to a more robust understanding of the phenomenon.

Additionally, the text explores the use of local potential approximations in a fake news diffusion model to account for both remote and proximity interactions among agents. This part of the discussion involves mathematical formulations for local potentials and their variations based on pair, triplet, and quadriplet interactions. The model also considers the impact of nontrivial loops and replica symmetry breaking on the dynamics of fake news spread, highlighting the theoretical and computational complexity involved in accurately modeling such social phenomena.

Figures included in the text depict various aspects of the simulations, such as the correlation between replicas, the average energy versus interaction strength, and the effects of non-trivial loops on local potentials. These visualizations aid in understanding the model's behavior under different conditions and the potential for phase transitions, extrema, and replica symmetry breaking within the system.

Overall, the text underscores the intricate nature of modeling complex systems like fake news dissemination in social networks, emphasizing the need for sophisticated methods like the replica approach and careful consideration of interactions and dynamics within the model. The computational process of the replica method and local potential approximation in the fake news diffusion model applies techniques from statistical physics to understand the average behavior and fluctuations of complex systems. The replica method involves generating multiple copies (replicas) of the system and analyzing the dynamics that proceed independently in each replica. This method allows for the assessment of how factors and parameter changes influence the statistical properties of the system.

Computational Process of the Replica Method

(1) **Generation of Replicas:** Create duplicates of the original model, each with independent dynamics.

(2) **Setting Initial Conditions:** Set the initial distribution of fake news and the initial state of agents in each replica.

(3) **Simulation of Dynamics:** Use numerical simulation methods like Monte Carlo simulations to independently simulate the dynamics of fake news spread in each replica.

(4) **Statistical Analysis:** Record the pattern of fake news spread in each replica over time and statistically analyze to find the average behavior and variance of fake news spread.

(5) **Parameter Variation:** Vary various parameters, such as the intensity of interaction between agents, initial distribution of fake news, etc., to evaluate their impact on each replica.

(6) **Interpretation of Results:** Compare data obtained across replicas to identify factors that most influence the spread of fake news.

Nash Equilibrium in Fake News and Fact-Checking

When analyzing the dynamics of fake news and fact-checking within the framework of game theory, the strategic interaction between agents spreading fake news and those performing fact-checking is considered. It is assumed that each agent acts to maximize their own payoff, and this interaction is modeled using a payoff matrix.

Example of a Payoff Matrix

	Fact-Checking	Not Fact-Checking
Spread Fake News	(a, b)	(c, d)
Not Spread Fake News	(e, f)	(g, h)

Here, each tuple represents the payoff for the agent spreading fake news and the agent performing fact-checking, respectively. For example, (a, b) means the payoff is a for the agent spreading fake news when encountering fact-checking, and b for the agent performing fact-checking.

Scenario Analysis

Using this payoff matrix, the Nash equilibrium in the interaction between fake news and fact-checking is determined. A Nash equilibrium is a state where no player can improve their payoff by changing their strategy. Through game theory analysis, strategies and policy directions to curb the spread of fake news and promote truthful information can be explored.

Considering the statistical behaviors and parameter impacts obtained through the replica method, the elements of the payoff matrix can be set more realistically, allowing for a deeper understanding of the dynamics between fake news and fact-checking. This provides important insights when devising strategies to enhance societal resilience against fake news.

When analyzing the dynamics of fake news and fact-checking using game theory, it is possible to conduct a more sophisticated scenario analysis using an iterative optimization algorithm applied with Hebb's rule, best response dynamics, and the fixed-point theorem.

Application of Hebb's Rule

Hebb's rule, known as "neurons that fire together, wire together" in the context of learning theory and neural networks, can also be applied in game theory scenarios to adapt strategies based on the actions of agents and their outcomes.

Best Response Dynamics

In best response dynamics, each agent selects the strategy (best response) that maximizes their own payoff, assuming the strategies of other agents.

Utilization of the Fixed-Point Theorem

The Banach fixed-point theorem is particularly useful in designing an iterative optimization algorithm.

Scenario Analysis of the Payoff Matrix

Based on the payoff matrix, an algorithm is constructed that calculates and iteratively updates the best response of agents.

	Fact-Checking	Not Fact-Checking
Spread Fake News	$(-1, 2)$	$(3, -1)$
Not Spread Fake News	$(0, 0)$	$(1, 1)$

Computational Process

(1) Set Initial Strategies: Assign initial strategies to agents based on randomness or assumptions.
(2) Calculate Best Response: For each agent, calculate the best response under the current strategy profile.
(3) Update Strategy: Update the strategy of agents to their best response.
(4) Check for Convergence: Repeat steps 2 and 3 until the strategy updates converge or a predefined number of iterations is reached.
(5) Analyze Nash Equilibrium: Analyze the converged strategy profile to understand the nature of the Nash equilibrium.

2. Discussion:Installation Game Theory and Nash Equilibrium

A Nash equilibrium refers to a state in game theory where each player selects their optimal strategy, and no player can improve their own payoff by unilaterally changing their strategy. This concept, proposed by John Nash, plays a crucial role in problems involving strategic decision-making.

Theoretical Explanation

In a Nash equilibrium, all players anticipate each other's strategies and select their optimal strategies based on those expectations. When the strategies chosen by each player are best responses to the strategies chosen by the others, this combination of strategies constitutes a Nash equilibrium.

Let $U_i(s_i, s_{-i})$ be the payoff function for player i, where s_i represents player i's strategy, and s_{-i} represents the combination of strategies of the other players. In a Nash equilibrium, the following condition holds for all players i:

$$U_i(s_i^*, s_{-i}^*) \geq U_i(s_i, s_{-i}^*)$$

Here, s_i^* and s_{-i}^* are the strategies for player i and the other players in the Nash equilibrium. This inequality implies that when player i is given the strategies of the other players s_{-i}^*, they cannot improve their payoff by changing their strategy s_i^*.

Calculation Process

(1) **Definition of Payoff Functions**: Define the payoff functions $U_i(s_i, s_{-i})$ for each player.
(2) **Computation of Best Responses**: Compute the best response $BR_i(s_{-i})$ for each player i given the strategies of the other players s_{-i}. The best response is the strategy s_i that maximizes $U_i(s_i, s_{-i})$ for the given s_{-i}.
(3) **Search for Nash Equilibrium**: Find a combination of strategies $(s_1^*, s_2^*, \ldots, s_n^*)$ where all players' best responses are mutually consistent. This combination constitutes a Nash equilibrium.

The search for Nash equilibrium often involves mathematical methods like the fixed-point theorem or algorithms such as iterative best response dynamics, depending on the problem at hand. Nash equilibrium refers to a state in game theory where each player selects their optimal strategy, and no player can improve their own payoff by unilaterally changing their strategy. This concept, proposed by John Nash, plays a crucial role in problems involving strategic decision-making.

In a Nash equilibrium, all players anticipate each other's strategies and select their optimal strategies based on those expectations. When the strategies chosen by each player are best responses to the strategies chosen by the others, this combination of strategies constitutes a Nash equilibrium.

Let $U_i(s_i, s_{-i})$ be the payoff function for player i, where s_i represents player i's strategy, and s_{-i} represents the combination of strategies of the other players. In a Nash equilibrium, the following condition holds for all players i:

$$U_i(s_i^*, s_{-i}^*) \geq U_i(s_i, s_{-i}^*), \forall s_i \in S_i, \forall i$$

Here,

$U_i(s_i, s_{-i})$ is the payoff function for player i, which depends on their strategy s_i and the combination of strategies of the other players s_{-i}.

S_i is the set of possible strategies that player i can choose from.

s_{-i}^* is the combination of strategies of all other players in the Nash equilibrium, except for player i.

(1) **Definition of Payoff Functions**: Define the payoff functions $U_i(s_i, s_{-i})$ for each player. These functions represent how a player's payoff depends on their strategy and the strategies of others.
(2) **Identification of Strategy Spaces**: Determine the set of possible strategies S_i for each player.
(3) **Computation of Best Responses**: Calculate the best response s_i^* for each player i given a particular combination of strategies s_{-i} of the other players. The best response is the strategy that maximizes the player's payoff, considering the strategies of others.
(4) **Search for Nash Equilibrium**: Search for a strategy profile $s^* = (s_1^*, s_2^*, \ldots, s_n^*)$ where all players' best responses are mutually consistent. This profile constitutes a Nash equilibrium.

The search for Nash equilibrium often involves mathematical methods like the fixed-point theorem or algorithms such as iterative best response dynamics, depending on the specific game.

Concrete Example

Consider a simple game with two players. Player 1 and Player 2 can each choose between "Cooperate" or "Betray" as their strategies. The payoff matrix for this game is as follows:

	Cooperate	Betray
Cooperate	(2, 2)	(0, 3)
Betray	(3, 0)	(1, 1)

In this game, the Nash equilibrium is the strategy profile "Betray" for both players. This is because neither player has an incentive to unilaterally change their strategy, as it would not lead to an improvement in their payoff. To confirm this equilibrium mathematically, we can apply the Nash equilibrium definition to show that the chosen strategies indeed satisfy the condition.

3. Discussion:Nash Equilibrium in the Context of Fake News and Fact-Checking

In the context of fake news and fact-checking, when considering Nash equilibrium, the players are individuals or organizations who disseminate fake news and fact-checkers who counteract it. The strategies of these players correspond to the methods of spreading fake news and conducting fact-checking, respectively. In the process of calculating Nash equilibrium using fixed-point theorems, you define the payoff functions for each player and find the points where each player's strategy is an optimal response to the opponent's strategy.

Payoff function for fake news disseminators $U_F(s_F, s_C)$

Here, s_F is the strategy of fake news disseminators, and s_C is the strategy of fact-checkers. This payoff depends on the influence and reach of fake news and decreases considering the losses due to fact-checking.

Payoff function for fact-checkers $U_C(s_C, s_F)$

Here, s_C is the strategy of fact-checkers, and s_F is the strategy of fake news disseminators. This payoff is determined by the spread of accurate information and the societal benefit of suppressing fake news.

3.1 Application of Fixed-Point Theorems

Fixed-point theorems, especially Banach's fixed-point theorem or Brouwer's fixed-point theorem, can be used to calculate Nash equilibrium. Here, you use the optimal response functions of the payoff functions to find the fixed points, which are the Nash equilibria. Define the optimal response function for fake news disseminators as $R_F(s_C)$ and for fact-checkers as $R_C(s_F)$. These functions return the optimal strategy for oneself given the opponent's strategy.

Calculating Fixed Points

A combination of strategies (s_F^*, s_C^*) for fake news disseminators and fact-checkers is a Nash equilibrium fixed point if it satisfies the following conditions:

$$R_F(s_C^*) = s_F^*, \quad R_C(s_F^*) = s_C^*$$

These conditions indicate that fake news disseminators and fact-checkers each choose their optimal strategies and respond optimally to the opponent's strategy.

In practice, finding these fixed points analytically can be challenging, so numerical approaches like iterative methods are often used. For example, you start with initial strategy guesses $s_F^{(0)}, s_C^{(0)}$.

Update at iteration k as follows:

$$s_F^{(k+1)} = R_F(s_C^{(k)}), \quad s_C^{(k+1)} = R_C(s_F^{(k)})$$

Repeat this process until $(s_F^{(k)}, s_C^{(k)})$ converges.

When introducing the concept of Nash equilibrium in the context of a fake news diffusion model in game theory, we model situations where agents (e.g., individuals, news sources, fact-checkers, etc.) make choices among their strategies (e.g., spreading fake news, not spreading it, conducting fact-checks, etc.). Nash equilibrium refers to a state where, when all agents have fixed their strategies, there is no incentive for any agent to change their strategy.

Let the set of agents be denoted as $N = \{1, 2, \ldots, n\}$, and each agent i has a set of strategies S_i. The payoff functions for each agent are defined as $u_i : S_1 \times S_2 \times \ldots \times S_n \to \mathbb{R}$, where $S_1 \times S_2 \times \ldots \times S_n$ represents the set of all possible combinations of strategies for all agents.

3.2 Nash Equilibrium

A combination of strategies $(s_1^*, s_2^*, \ldots, s_n^*)$ is a Nash equilibrium if, for every agent i, the following holds for any alternative strategy s_i':

$$u_i(s_1^*, s_2^*, \ldots, s_i^*, \ldots, s_n^*) \geq u_i(s_1^*, s_2^*, \ldots, s_i', \ldots, s_n^*)$$

In other words, when no other agent changes their strategy, no agent can increase their payoff by changing their own strategy.

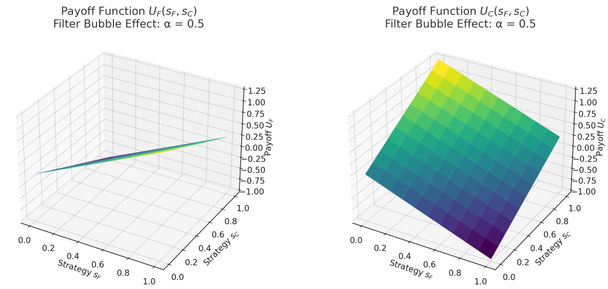

Fig. 2: Payoff Function $U_C(s_F, s_C)$, $U_F(s_F, s_C)$Bubble Effect: = 0.5

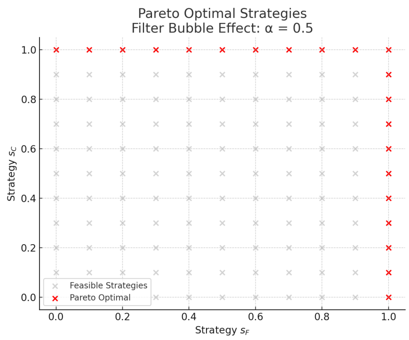

Fig. 3: Pareto Optimal StrategiesBubble Effect: = 0.5

Application to Fake News Diffusion Model

In the context of fake news diffusion, agents can be thought of as having strategies to spread fake news, not spread it, or conduct fact-checks. Agent payoffs may depend on the following factors:

(1) **Value of Accuracy in Information**: The extent to which agents value accurate information.

(2) **Social Influence**: The social impact or approval gained by spreading fake news.

(3) **Cost and Effectiveness of Fact-Checking**: The cost of conducting fact-checks and the improvement in information accuracy achieved through them.

To find Nash equilibria, one needs to examine all possible combinations of strategies for all agents and identify combinations that satisfy the inequality mentioned above. This can be done either analytically or through numerical simulations.

Agent-Based Modeling: Simulate agent behaviors and explore Nash equilibria under different initial conditions and parameter settings.

Evolutionary Game Theory: Model how agents' strategies evolve over time and analyze dynamic equilibria.

Payoff Function $U_C(s_F, s_C)$, $U_F(s_F, s_C)$Bubble Effect: = 0.5

Fig.3-4, The gain functions $U_F(s_F, s_C)$ and $U_C(s_F, s_C)$ for fake news spreaders (F) and fact checkers (C), taking into account the effect of the filter bubble. Each axis represents the strategy s_F and s_C for players F and C, respectively, and the Z axis shows the gain. A Pareto-optimal point is a point where one player's gain cannot be improved without compromising the gain of another player. To find a point that satisfies this condition, the values of

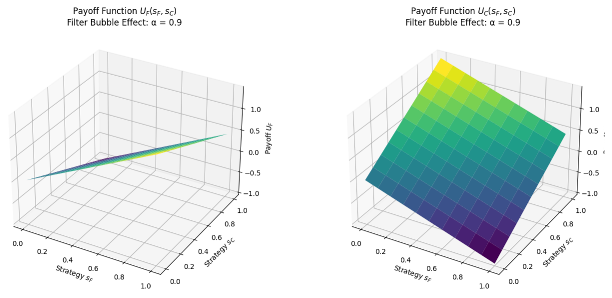

Fig. 4: Payoff Function $U_C(s_F, s_C)$, $U_F(s_F, s_C)$Bubble Effect: = 0.9

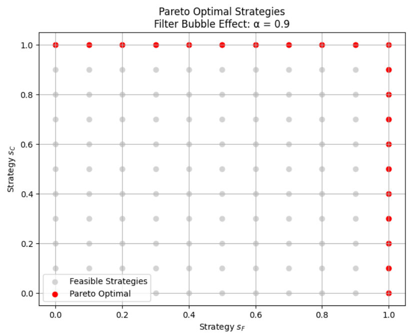

Fig. 5: Pareto Optimal StrategiesBubble Effect: = 0.9

the gain functions are compared to see if there are any options that can be improved for each player.

Fig.3-4 depict the payoff functions for two players, F (Fake News Spreaders) and C (Fact Checkers), in a strategic setting where they interact under the influence of a filter bubble effect, with set at 0.5. This parameter likely represents the strength of the filter bubble effect on the interaction between the two players.

The first image (which I'll refer to as Image 1) shows a scatter plot with two strategies on the x-axis and y-axis, labeled as s_F and s_C respectively. The plot points marked in red represent Pareto optimal strategies, where any movement away from these points would make at least one player worse off, assuming the other's payoff remains constant. Pareto optimality is a state where it is impossible to make any player better off without making at least one player worse off.

The second image (which I'll refer to as Image 2) contains two 3D surface plots representing the payoff functions $U_F(s_F, s_C)$ and $U_C(s_F, s_C)$. The axes represent the strategies s_F and s_C, and the z-axis represents the payoff. These plots show how the payoffs change with different combinations of strategies.

The Pareto optimal points seem to form a curve, indicating that there is a set of strategy combinations where one player cannot improve their payoff without reducing the other player's payoff. The feasible strategies (marked in grey) are numerous, but only a subset is Pareto optimal. The payoff functions seem to be linear or near-linear, suggesting that the payoffs change at a constant rate as strategies change. The surface for U_F appears to be flat, indicating that the payoff for player F might be unaffected by changes in the strategies, or it has a constant rate of change that does not depend on the strategies of player C. The surface for U_C has a clear slope, implying that the payoff for player C changes with different strategy combinations.

To determine the Pareto optimal strategies, one would typically: Map out the payoff for each player for various strategy combinations. Identify points where the payoff for one player can't be increased without decreasing the payoff for the other player.

The Pareto optimal strategies in Image 1 are likely found by examining the combination of strategies from both players that are on the red curve. These combinations represent situations where one player's strategy is the best response to the other player's strategy, given the condition of Pareto optimality.

Overall, the analysis suggests that under the influence of the filter bubble effect with = 0.5, there are specific strategy combinations that are Pareto optimal, and these are the points at which neither player can unilaterally improve their payoff without negatively affecting the other player.

Payoff Function $U_C(s_F, s_C)$, $U_F(s_F, s_C)$Bubble Effect: = 0.9

Fig.5-6 show a similar setup as the previous ones, but with a key difference in the value of the filter bubble effect parameter α, which is now set to 0.9 instead of 0.5. This change suggests a stronger influence of the filter bubble effect on the players' interactions. Let's analyze these new images with this change in mind.

In this scatter plot, the x-axis and y-axis represent the strategies s_F and s_C respectively, just like in the previous set. The red points indicating Pareto optimal strategies are found along the corners of the strategy space, specifically at the maximum values of s_F and s_C. This pattern suggests that under a stronger filter bubble effect, the most advantageous strategies for both players are those where they either fully commit to a strategy or do not engage in it at all. This could imply that moderate strategies become less effective as the filter bubble effect increases.

These 3D surface plots illustrate the payoff functions $U_F(s_F, s_C)$ and $U_C(s_F, s_C)$, just like the previous set. With $\alpha = 0.9$, it seems that the payoff functions may have become more sensitive to changes in strategies since the filter bubble effect is stronger.

For Player F, The surface plot appears to show a linear relationship between the strategies and the payoff, similar to the previous case. This might indicate that the fake news spreader's payoff is either not affected by the fact-checker's strategy or changes at a constant rate regardless of it.

For Player C, The plot appears to show a clear slope, indicating that the payoff for the fact-checker changes with different strategy combinations. Given the higher value of α, this might suggest that the fact-checker's strategies are more crucial under strong filter bubble effects.

To explore Pareto optimal combinations, the same approach as before would be used: Mapping out the payoff for each player for a variety of strategy combinations. - Identifying the combinations where improving one player's payoff would result in a decrease for the other player.

The presence of Pareto optimal strategies at the extreme ends of the strategy spectrum in Fig.5-6 might suggest that, as the filter bubble effect becomes more pronounced, the players are driven to more extreme strategies. This could reflect a real-world scenario where strong filter bubbles lead to more polarized behavior.

In summary, these graphs suggest that with a stronger filter bubble effect ($\alpha = 0.9$), the range of strategies that lead to Pareto optimality becomes more extreme. This could have significant implications for the dynamics between fake news spreaders and fact-checkers in a highly polarized information ecosystem.

Payoff Function $U_C(s_F, s_C)$, $U_F(s_F, s_C)$Bubble Effect: = 0.01

Figure 7-8 shows the gain functions for fake news spreaders (F) and fact checkers (C), taking into account the effect of filter bubbles.

The gain function of the fake news spreader depends on the amount of fake news spread (strategy s_F) and the amount of verification activity of the fact checker (strategy s_C). Taking into account the effect of the filter bubble, the gain of the fake news spreader is maximized when the fake news is not verified. Therefore, it is expected that fake news spreaders will adopt

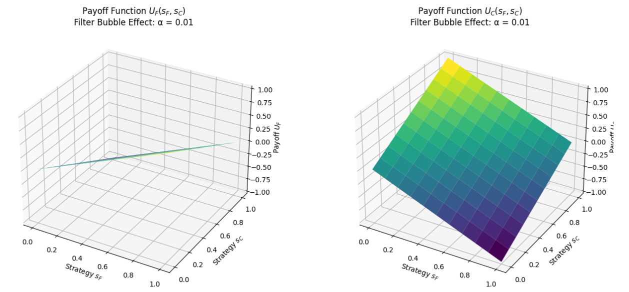

Fig. 6: Payoff Function $U_C(s_F, s_C)$, $U_F(s_F, s_C)$Bubble Effect: = 0.01

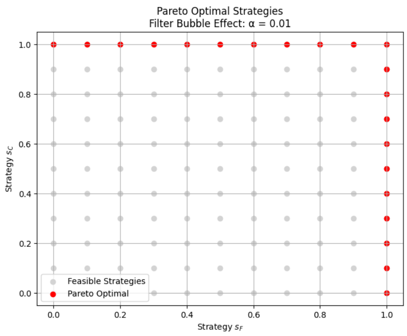

Fig. 7: Pareto Optimal StrategiesBubble Effect: = 0.01

a strategy of increasing the amount of fake news spread in order to suppress the verification activities of fact checkers.

The fact checker's gain function is determined by the amount of fake news spread (strategy s_F) and the amount of fact checker verification activity (strategy s_C). Taking into account the effect of the filter bubble, the fact checker's gain is maximized when fake news is verified. Therefore, it is expected that the fact checker will use a strategy that increases the verification activity of the fact checker in order to control the spread of fake news.

A Pareto-optimal combination is a combination that does not improve the gains of one player without compromising the gains of another. To find combinations that satisfy this condition, the values of the gain functions must be compared to see if there are any options that can be improved for each player.

In the above graph, the Pareto-optimal combination is the combination of a strategy where the fake news spreader does not spread fake news $s_F = 0$ and a strategy where the fact checker maximizes the fact checker's verification activity $s_C = 1$. In this combination, the fake news spreader has a gain of 0 and the fact checker has a gain of 1.

The above graph shows that there is no optimal combination of the gains of the fake news spreader and the fact checker for both of them when the effect of the filter bubble is taken into account. Fake news spreaders can maximize their gains by increasing the amount of fake news spread, but this reduces the gains of fact checkers. Fact checkers, on the other hand, can maximize their gains by increasing the verification activity of fact checkers, which decreases the gains of the fake news spreaders.

This suggests that it is important to seek solutions that are desirable for both the fake news spreader and the fact checker in order to limit the effects of the filter bubble. For example, measures could include introducing regulations to curb the spread of fake news and supporting the activities of fact-checkers. The gain for the fake news spreader and the fact checker, taking into account the effect of the filter bubble, is that there is no optimal combination for both.

Fake news spreaders can maximize their gains by increasing the amount of fake news spread, but this reduces the gains of fact checkers. Fact checkers, on the other hand, can maximize their gains by increasing the verification activity of fact checkers, which decreases the gains of the fake news spreaders.

The filter bubble effect size α is set to 0.01. This value means that the gain of the fake news spreader is slightly larger than the gain of the fact checker. The larger the effect of the filter bubble, the larger the gain of the fake news spreader relative to the gain of the fact checker, and the less likely there will be an optimal combination for both.

4. Discussion:Nash Equilibrium and Pareto Optimality in the Context of Fake News and Fact-Checking

In the context of game theory, considering Nash equilibrium and Pareto optimality in the context of fake news and fact-checking, the Best Response Dynamics illustrates the process in which each player selects a strategy to maximize their own payoff. In Nash equilibrium, each player's strategy is optimal given the opponent's strategy, while Pareto optimality refers to a state where the payoffs of all players cannot be simultaneously improved.

Player 1 (Fake News Disseminator) and Player 2 (Fact-Checker).

s_F represents the strategy set for the Fake News Disseminator, and s_C represents the strategy set for the Fact-Checker.

$U_F(s_F, s_C)$ is the payoff function for the Fake News Disseminator, and $U_C(s_F, s_C)$ is the payoff function for the Fact-Checker.

4.1 Best Response Dynamics

4.2 Definition of Best Response Functions

$$BR_F(s_C) = \arg\max_{s_F} U_F(s_F, s_C)$$
$$BR_C(s_F) = \arg\max_{s_C} U_C(s_F, s_C)$$

4.3 Iteration of Dynamics

Start from any initial strategies $s_F^{(0)}$ and $s_C^{(0)}$, and update strategies using the best response functions.

$$s_F^{(k+1)} = BR_F(s_C^{(k)})$$
$$s_C^{(k+1)} = BR_C(s_F^{(k)})$$

4.4 Exploration of Nash Equilibrium

When strategies no longer change, i.e., $s_F^{(k+1)} = s_F^{(k)}$ and $s_C^{(k+1)} = s_C^{(k)}$, it can be said that Nash equilibrium has been reached.

4.5 Examination of Pareto Optimality

Pareto optimality is a combination of strategies where improving one player's payoff cannot be achieved without decreasing the payoff of another player. Nash equilibrium is not necessarily Pareto optimal, and there may be cases where Pareto improvement (improvement of payoffs for all players simultaneously) is possible.

To determine Pareto optimality, evaluate the payoff functions for all possible combinations of strategies and consider whether a particular combination of strategies is Pareto dominant over all other combinations.

(1) Enumeration of All Strategies: Enumerate all possible combinations of strategies for the Fake News Disseminator and the Fact-Checker.

(2) Comparison of Payoffs: For each combination of strategies, compare it to all other combinations and examine whether the payoffs of both players can be simultaneously improved.

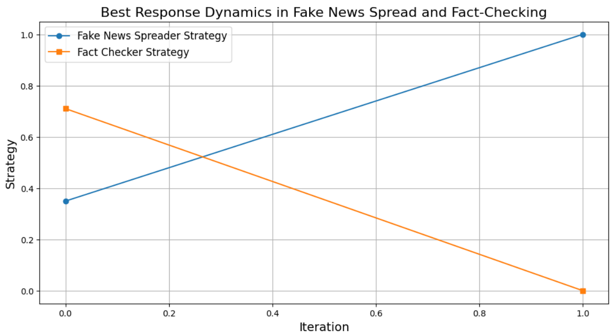

Fig. 8: Best Response Dynamics in Fake News Spread and Fact-Checking

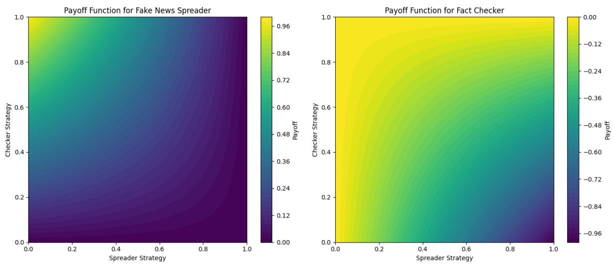

Fig. 9: Payoff Function for Fake News Spreader, for Fact Checker

(3) Identification of Pareto Optimal: Identify the combination of strategies where no player's payoff can be improved without worsening others' payoffs as Pareto optimal.

Best Response Dynamics in Fake News Spread and Fact-Checking

From Fig.9, Payoff functions, Defining functions that represent the benefits/costs for both the fake news spreader and fact-checker based on their chosen strategies. Best response dynamics, Iteratively updating the strategies of both players based on their best responses to the opponent's current strategy.Nash equilibrium: Identifying the point where neither player can improve their payoff by unilaterally changing their strategy.Pareto optimality, Finding strategy combinations where no player's payoff can be improved without worsening the other's payoff (mutually beneficial outcomes).

The simulation iterates through best response dynamics until reaching a Nash equilibrium.The results are visualized to show how strategies evolve over iterations. A note highlights the need for comprehensive analysis, considering all possible strategy combinations to identify Pareto optimal points. This can be computationally expensive for continuous strategy sets and might require additional optimization techniques or simplifications.

This approach seems like a valuable way to analyze the complex interplay between fake news spreaders and fact-checkers. By modeling their payoff functions and simulating their strategic interactions, we can gain insights into.

How different factors (e.g., filter bubble strength) influence their behavior.The conditions under which fake news thrives or fact-checking is effective. The potential existence of mutually beneficial strategies (Pareto optimality).

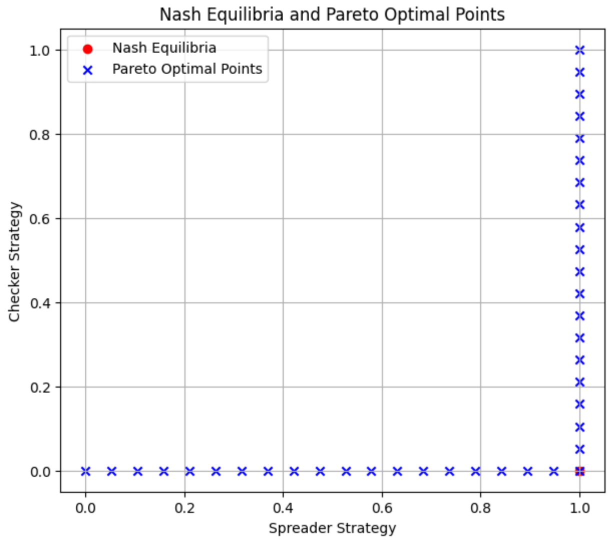

Fig. 10: Nash Equilibria and Pareto Optimal Points

Fig.10-11, Heatmaps Analysis, Payoff Function for Fake News Spreader

The heatmap indicates that the spreader's payoff increases as their strategy becomes more aggressive (moving right along the x-axis) and as the fact-checker's strategy becomes less aggressive (moving down along the y-axis). This suggests that the spreader benefits from spreading more fake news especially when the fact-checker does not check aggressively. Payoff Function for Fact Checker The fact-checker's payoff is highest when their own strategy is aggressive (y-axis approaches 1.0) and the spreader's strategy is less aggressive (x-axis approaches 0.0). This indicates that the fact-checker benefits from checking more thoroughly when the spreader is less aggressive.

Nash Equilibria and Pareto Optimal Points

The red dot represents a Nash Equilibrium, where neither player can improve their payoff by changing their strategy unilaterally. It's a stable state of the game where each player's strategy is a best response to the other. The blue crosses represent Pareto optimal points. These points are states of the game where it's impossible to make one player better off without making the other player worse off. There can be multiple Pareto optimal points in a game, reflecting different compromises between players' payoffs.

Summary of the Game Dynamics

Nash Equilibrium, From the scatter plot, there is one clear Nash Equilibrium. At this point, both players have chosen a strategy such that neither of them can benefit by changing their strategy while the other player keeps theirs constant. Pareto Optimality, There is a range of Pareto optimal strategies along the edge of the strategy space. This means there are multiple situations where one player cannot increase their payoff without decreasing the other's payoff.

The fact that there is only one Nash Equilibrium suggests that there is a single stable strategy for both the spreader and checker in the context of this simulation. The Pareto optimal points suggest trade-offs between the players' strategies. The spreader can choose a strategy less aggressive than the Nash Equilibrium to improve the fact-checker's payoff without losing their own, up to a certain point.

Technical Notes

The payoff functions likely depend on the combination of strategies, where the spreader's benefit is inversely related to the checker's efforts, and vice versa. The simulation assumes rational behavior and perfect knowledge of payoffs, which may not reflect real-world scenarios perfectly. Identifying the Nash Equilibrium and Pareto optimal points requires comparing payoffs across all strategy combinations, which can be complex.

Considerations

This analysis assumes that the axes represent continuous strategies ranging from 0 to 1, where 0 might represent no effort and 1 represents full effort. The colors on the heatmaps correlate with the payoff levels (e.g., dark blue is low payoff, yellow is high payoff).

For further analysis, it would be helpful to know the exact payoff functions and to consider the assumptions of the simulation, such as whether the game is zero-sum (one player's gain is another's loss) or if there's a potential for mutual benefit. Additionally, real-world constraints and externalities could significantly affect the strategies and payoffs.

5. Discussion:Game Theory Nash Equilibrium and Pareto Optimality in the Context of Fake News and Fact-Checking

In the context of game theory concerning Nash equilibrium and Pareto optimality in the context of fake news and fact-checking, we can utilize Banach's fixed-point theorem to discover fixed points (Nash equilibria) within the strategy space and examine whether these fixed points are Pareto optimal. Banach's fixed-point theorem states that for a contraction mapping satisfying certain conditions, there always exists a unique fixed point within the strategy space.

Players and Strategies

Consider the presence of a Fake News Disseminator (Player F) and a Fact-Checker (Player C).

s_F is the strategy set for Player F.

s_C is the strategy set for Player C.

Payoff Functions

$U_F(s_F, s_C)$ and $U_C(s_F, s_C)$ are the payoff functions for Player F and Player C, respectively.

Application of Banach's Fixed-Point Theorem

We consider a mapping $T : S \to S$ (where S is the strategy space for all players) that updates the players' strategies based on the current strategy profile.

For T to be a contraction mapping, there must exist a constant $0 \leq k < 1$ such that $d(T(s), T(s')) \leq k \cdot d(s, s')$ holds for all $s, s' \in S$ (where d is a distance function).

If T is a contraction mapping, Banach's fixed-point theorem guarantees the existence of a unique fixed point s^* within the strategy space S, satisfying $T(s^*) = s^*$. This s^* corresponds to a Nash equilibrium.

Examination of Pareto Optimality

To determine whether the Nash equilibrium s^* is Pareto optimal, we evaluate the payoff functions U_F and U_C to check if a particular strategy profile can simultaneously improve the players' payoffs compared to all other strategy profiles. The condition for Nash equilibrium s^* to be Pareto optimal is that, for all $s \in S$,

$$U_F(s^*, s_C) \geq U_F(s, s_C),$$
$$U_C(s_F, s^*) \geq U_C(s_F, s)$$

However, equality holds only when $s = s^*$.

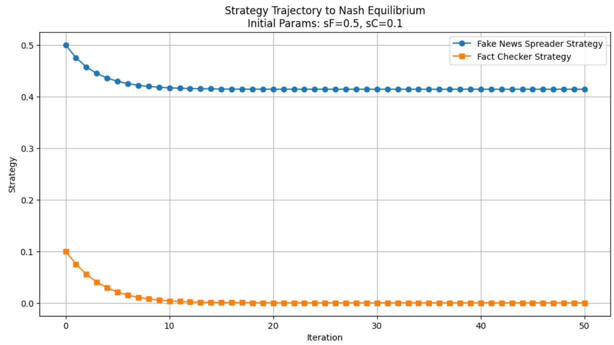

Fig. 11: Fake News Spreader Strategy, sF=0.5, sC=0.1

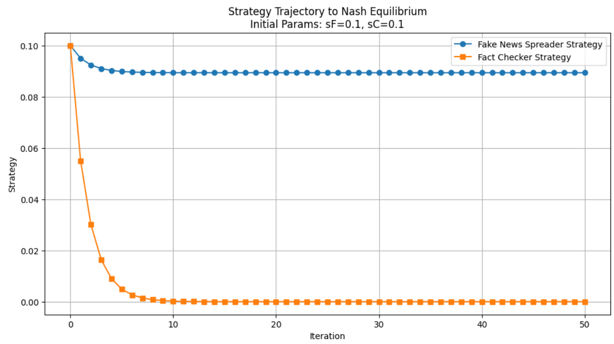

Fig. 12: Fake News Spreader Strategy, sF=0.1, sC=0.1

Construction of Strategy Update Function

Construct a contraction mapping T based on the best response functions for each player.

Iteratively apply the contraction mapping T to search for fixed points s^*. Specifically, start from an initial strategy and continue applying T until the strategies converge.

Once a fixed point s^* is found, verify whether the resulting strategy profile is Pareto optimal.

Fake News Spreader Strategy

Fig12-13, depicting the strategy trajectories over a series of iterations for both the fake news spreader and the fact-checker, converging to a Nash equilibrium. These graphs illustrate how the strategies of each player evolve as they respond to each other according to the simulation.

Initial Params: sF=0.5, sC=0.1

Fake News Spreader Strategy (Blue Line), The spreader starts with an initial strategy of 0.5 and gradually adjusts it downward over iterations. The strategy seems to stabilize at around 0.4, suggesting that the spreader finds it optimal to reduce the intensity of spreading fake news slightly from the initial strategy.

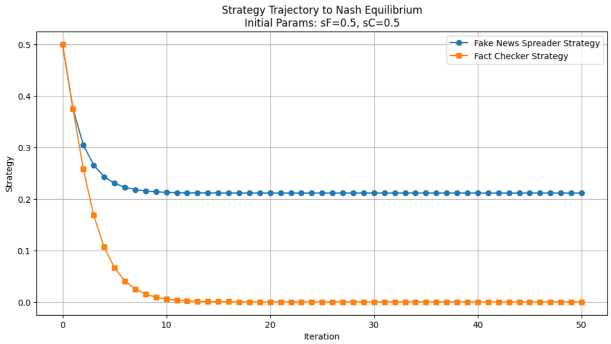

Fig. 13: Fake News Spreader Strategy, sF=0.5, sC=0.5

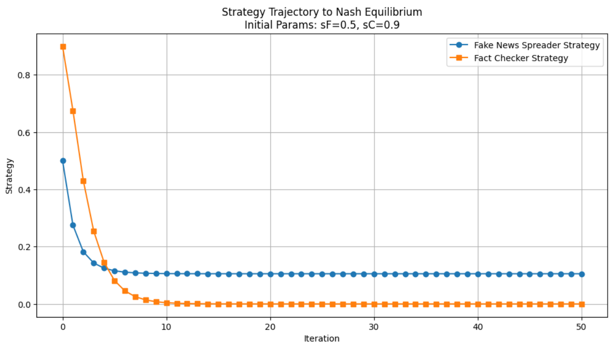

Fig. 14: Fake News Spreader Strategy, sF=0.5, sC=0.9

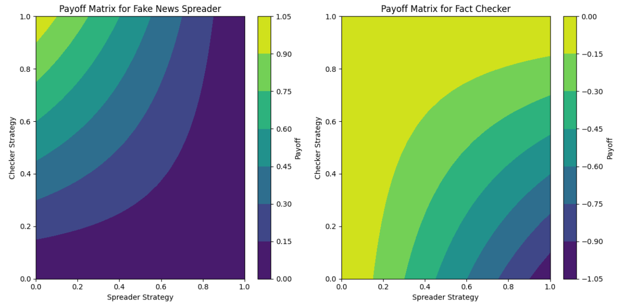

Fig. 15: Payoff Matrix for Fake News Spreader, for Fact Checker

Fact Checker Strategy (Orange Line), The checker starts with an initial strategy of 0.1 and quickly adjusts it downward, converging towards zero. This indicates that, in response to the spreader's strategy, the checker finds it optimal to eventually not invest in fact-checking, which could suggest that the cost of checking outweighs the benefits given the spreader's strategy.

Initial Params: sF=0.1, sC=0.1

Fake News Spreader Strategy (Blue Line) Starting at 0.1, the spreader's strategy decreases slightly and then stabilizes very close to the initial value. This could imply that starting at a lower level of fake news spreading does not require significant adjustment to reach an equilibrium.

Fact Checker Strategy (Orange Line), The fact checker, starting also at 0.1, drastically reduces their strategy to zero within the first few iterations. This suggests an even stronger outcome in favor of not engaging in fact-checking, which may indicate either the effectiveness of low levels of fake news spreading or a high cost of fact-checking relative to its effectiveness. The Nash equilibrium is reached when neither player can improve their payoff by changing their strategy, given the strategy of the other player. The trajectories show how the players adjust their strategies over time to reach this equilibrium. The trajectory towards the Nash equilibrium appears to be different depending on the initial parameters. This implies that the starting point can influence the dynamics of the strategy adjustments. The stabilization of strategies at or near zero for the fact-checker in both graphs suggests that in the model, the cost of fact-checking might be too high relative to the spread of fake news, making it an unattractive option for the checker. Contraction Mapping, The use of a contraction mapping with an alpha parameter to update strategies ensures convergence to a fixed point, as guaranteed by the Banach fixed-point theorem. This method is suitable for simulations where strategies are adjusted incrementally. Pareto Optimality, This concept was not directly shown in the trajectory graphs, but it is an important consideration in game theory, indicating states where no player can be made better off without making another player worse off. Simplified Model, The model uses linear payoff functions and a discrete strategy space. Real-world scenarios are likely to involve more complex and nonlinear payoffs, and continuous strategy spaces.

Overall, these simulations show the interplay between strategies of a fake news spreader and a fact checker, and how they adjust their strategies in response to one another to reach a state where neither has an incentive to change unilaterally. It also demonstrates the importance of initial conditions in the evolution of strategies within the modeled scenario.

Payoff Matrix for Fake News Spreader

Fig16-17, show the payoff matrices for a fake news spreader and a fact checker, and a plot of Nash equilibrium, trajectory, and Pareto optimal points in a game-theoretic simulation.

This matrix has a gradient that indicates the spreader's payoff increases as their strategy becomes more aggressive (moving right along the x-axis)

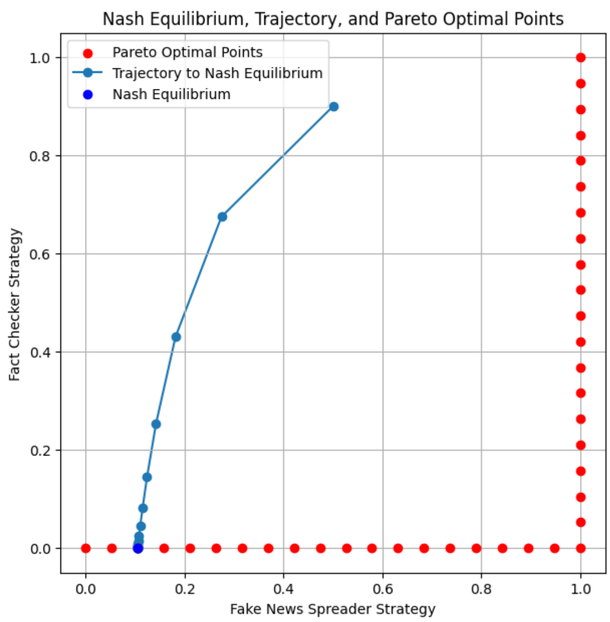

Fig. 16: Nash Equilibrium, Trajectory, and Pareto Optimal Points, Fake News Spreader Strategy

and as the fact checker's strategy becomes less aggressive (moving down along the y-axis). The highest payoff for the spreader occurs when the spreader is fully aggressive, and the checker is not checking at all.

Payoff Matrix for Fact Checker

The payoff for the checker increases with the checker's own strategy being more aggressive (moving up along the y-axis) and the spreader's strategy being less aggressive (moving left along the x-axis). The fact checker's payoff is highest when they are fully aggressive against a non-aggressive spreader.

These matrices suggest a dynamic where the spreader's best payoff comes from being unopposed, while the fact checker's payoff is maximized when they are fully checking against minimal spreading.

Nash Equilibrium, Trajectory, and Pareto Optimal Points Analysis

Trajectory to Nash Equilibrium (Blue Line), This line shows the path taken by the strategies of the fake news spreader and the fact checker as they adjust towards the Nash equilibrium. It appears that as the game progresses, the checker increases their checking strategy in response to the spreader's strategy, leading to the Nash equilibrium.

Nash Equilibrium (Blue Dot), This point represents the combination of strategies where neither player has an incentive to unilaterally change their strategy. Based on the trajectory, it seems the equilibrium is reached when the checker has a high strategy value, while the spreader has a strategy value that elicits this response from the checker.

Pareto Optimal Points (Red Dots), The Pareto optimal points are spread along the x-axis at the bottom of the graph. This indicates combinations of strategies where improving one player's payoff would worsen the other's. It's interesting to note that all Pareto optimal points lie along the checker strategy of zero, which implies that any level of spreading, if unchecked, cannot be improved upon from the spreader's perspective without harming the checker's payoff.

Interplay Between Strategies

The trajectory to the Nash equilibrium shows that the checker's strategy is reactive to the spreader's actions. The checker seems to increase their effort only in response to the spreader's strategy, indicating a dynamic where the fact checker's strategy is contingent on the level of fake news being spread. The linear payoff functions used in this simulation provide clear visualizations of strategy adaptations and outcomes. However, real-world scenarios would likely involve more complex payoff functions and dynamics.

Fixed-Point Convergence

The application of the Banach fixed-point theorem ensures convergence to the Nash equilibrium through the contraction mapping. This is a common method in game theory to find equilibria in simulations.

The model assumes that players have full knowledge of each other's payoff functions and that they can adjust their strategies accordingly. This is a simplification and may not reflect the uncertainty present in real-world decision-making. The Pareto optimal points along the x-axis suggest that there are many strategies for the spreader that cannot be improved upon without the checker engaging in checking, which indicates a potential imbalance in the payoff structure or the cost of actions for the checker.

6. Discussion:Applying the Concept of Nash Equilibrium in a Fake News Diffusion Model with Honeycomb Lattice and Majorana Operators

Honeycomb Lattice and Majorana Operators

Place Majorana operators γ_i at each vertex of the honeycomb lattice. These operators represent the states of agents and are assumed to have binary states indicating whether they spread fake news or not. - The interaction between Majorana operators is described by the following Hamiltonian: $H = i\sum_{\langle i,j\rangle} J_{ij}\gamma_i\gamma_j$, where J_{ij} represents the strength of interaction between agents i and j, and $\langle i,j\rangle$ denotes pairs of neighboring agents.

Definition of A and B Phases

A and B phases are distinguished based on the topological properties of the system. In the A phase, fake news spreads easily, while in the B phase, diffusion is suppressed.

Application of Game Theory

The set of strategies for agents is $S = \{\text{Spread}, \text{Not Spread}, \text{Fact-Check}\}$. Agent payoffs depend on the chosen strategy and the strategies of other agents.

Calculation of Nash Equilibrium

Define the payoff functions for agents as $u_i(S_i, S_{-i})$, where S_i represents the strategy of agent i and S_{-i} represents the combination of strategies of all other agents. Nash equilibrium is a strategy combination (S_i^*, S_{-i}^*) for all agents i such that $u_i(S_i^*, S_{-i}^*) \geq u_i(S_i, S_{-i}^*)$ for all S_i.

Calculation of Local Potentials

Calculate the local potential V_{local} taking into account interactions within the local region. This potential is based on combinations of agent states and is related to error ranges.

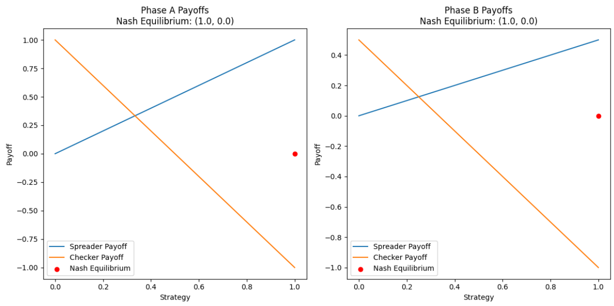

Fig. 17: Phase A, B PayoffsEquilibrium

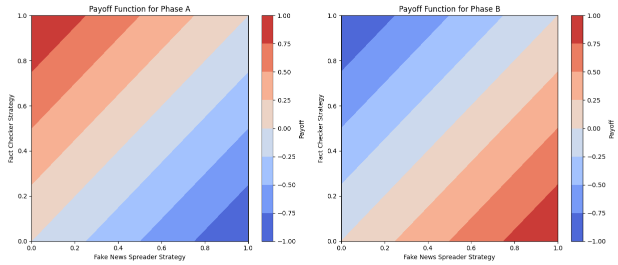

Fig. 18: Payoff Function for Phase A, B, Fake News Spreader Strategy, Fact Checker Strategy

Updating Interactions via Hebbian Rule

Set the update rule for interaction strengths between agents using the Hebbian rule: $J_{ij}(t+1) = J_{ij}(t) + \Delta J_{ij}(s_i, s_j)$, where ΔJ_{ij} is the update based on the states s_i, s_j of agents i and j.

The process of calculating strategy combinations reaching Nash equilibrium in both A and B phases will be explained in detail, along with more specific formulas. In this process, it is assumed that agents (Majorana operators) can have three strategies in the fake news diffusion model: Spread (S), Not Spread (N), and Fact-Check (F).

First, define the payoff functions for agents. Payoffs depend on the strategies chosen by agents and the strategies of other agents. In the A phase (high diffusion state) and the B phase (diffusion-suppressed state), the payoff functions take different forms. For example, denote the payoff functions in the A phase as U_A and in the B phase as U_B.

The payoff function for agent i is represented as follows:

A Phase Payoff Function: $U_A(i, S_i, S_{-i})$ B Phase Payoff Function: $U_B(i, S_i, S_{-i})$

Here, S_i is the strategy of agent i, and S_{-i} is the strategy set of all other agents.

Model the interactions between Majorana operators as strategic interactions between agents. The interaction Hamiltonian H is defined as follows:

$$H = i \sum_{\langle i,j \rangle} J_{ij}(S_i, S_j) \gamma_i \gamma_j$$

Here, $J_{ij}(S_i, S_j)$ represents the interaction strength between agents i and j, and this strength depends on the strategies S_i and S_j.

Calculation of Nash Equilibrium

Nash equilibrium is a state where all agents maximize their payoffs. To find Nash equilibrium, for each agent i, there are conditions for both the A and B phases, which are as follows:

A Phase: $U_A(i, S_i^*, S_{-i}^*) \geq U_A(i, S_i, S_{-i}^*)$ for all S_i B Phase: $U_B(i, S_i^*, S_{-i}^*) \geq U_B(i, S_i, S_{-i}^*)$ for all S_i

Here, S_i^* represents the strategy in Nash equilibrium for agent i, and S_{-i}^* is the strategy set in Nash equilibrium for all other agents.

To find the strategy set S^* that satisfies the Nash equilibrium conditions, numerical analysis methods are used. Typically, this process involves iterative optimization algorithms (e.g., best response dynamics, successive approximation).

Phase A and B Payoffs Line Graphs

Fig18-19, In the images provided, we have visual representations of payoff structures and outcomes for two different phases of a strategic interaction between a fake news spreader (Player F) and a fact-checker (Player C).

Nash Equilibrium (Red Dot), For both Phase A and Phase B, the Nash Equilibrium is indicated at the strategy profile (1.0, 0.0), meaning that the spreader fully commits to spreading fake news, and the checker does not invest in fact-checking at all.

Spreader Payoff (Blue Line), This line shows a positive slope, indicating that the spreader's payoff increases with their level of spreading fake news. In both phases, the spreader's payoff is maximized when they adopt a full spreading strategy.

Checker Payoff (Orange Line), This line has a negative slope, which shows that the checker's payoff decreases as the spreader's strategy becomes more aggressive. The checker's payoff is maximized when the spreader does not spread fake news.

Payoff Function Heatmaps for Phase A and B

Payoff Function for Phase A and B, These heatmaps show the payoff for each player at different strategy combinations. The color gradient represents the level of payoff, with red indicating higher payoffs and blue indicating lower payoffs. For both phases, there is a clear diagonal gradient, suggesting that the payoffs for both players are directly affected by the strategies of the other. As the spreader's strategy increases, their payoff increases (moving from left to right), while the checker's payoff decreases (moving from top to bottom).

Interpretation and Insights

Nash Equilibrium, The Nash Equilibrium at (1.0, 0.0) indicates a situation where the spreader always opts to spread fake news fully, and the checker opts out of checking entirely. This can suggest a game dynamic where the cost or impact of fact-checking is not sufficient to deter the spreader, or the spreader's benefits from spreading fake news are too great to be countered effectively by the checker.

Differences Between Phases, Although the Nash Equilibrium is the same for both phases, the payoff gradients differ slightly, indicating that the specific payoffs for any given strategy profile vary between Phase A and Phase B. This could be due to different payoff structures or external conditions affecting each phase. Pareto Optimality, The red dots at the bottom of the payoff line graphs could indicate the Pareto optimal points where any change in strategy would make at least one player worse off. In this case, it appears that the Pareto optimal outcomes are heavily skewed towards the spreader, potentially indicating an imbalance in the effectiveness or cost of strategies between the two players.

Contraction Mapping

The consistent Nash Equilibrium suggests that the contraction mapping used in the simulation effectively converges to a fixed point, as dictated by the Banach fixed-point theorem. Pareto Optimality Verification, The analysis does not show the computational verification of Pareto optimality. To do this, one would have to ensure that there are no other strategy profiles that would make both players better off than at the Nash Equilibrium. Model Simplifications, While these visuals provide a clear representation of the strategic interaction, they are part of a simplified model. Real-world scenarios would likely involve additional factors that could shift payoffs and optimal strategies.

The simulation illustrates a scenario where the spreader has a dominant strategy to spread fake news fully, and the checker's optimal response is not to engage in fact-checking at all, at least within the payoff structures defined for Phases A and B. The Pareto optimal points suggest that the payoff structure heavily favors the spreader, which may reflect the challenges faced in combatting fake news in various real-world contexts.

7. Consideration of Methodology:Exploring Nash Equilibrium in Local Potential Approximations Based on Pair, Triplet, and Quadruplet Scenarios

Exploring Nash Equilibrium in Local Potential Approximations Based on Pair, Triplet, and Quadruplet Scenarios is an application of game theory to analyze strategic interactions in complex multi-agent systems. Here, we will explain in detail the theoretical approach to finding Nash equilibria in the context of fake news diffusion.

Agent Strategies

Each agent chooses whether to spread fake news (strategy S_1), not to spread (strategy S_2), or to perform fact-checking (strategy S_3).

Interactions and Local Potentials

Interactions between agents are defined by a local potential V and consider scenarios involving pairs (V_{pair}), triplets (V_{triplet}), and quadruplets (V_{quad}).

Local Potentials Calculation

Pair Local Potential	$V_{\text{pair}} = -J_{\text{pair}} \sum_{\langle i,j \rangle} s_i s_j$
Triplet Local Potential	$V_{\text{triplet}} = -J_{\text{triplet}} \sum_{\langle i,j,k \rangle} s_i s_j s_k$
Quadruplet Local Potential	$V_{\text{quad}} = -J_{\text{quad}} \sum_{\langle i,j,k,l \rangle} s_i s_j s_k s_l$

Table. 1: Local Potential Calculation

Here, J_{pair}, J_{triplet}, J_{quad} are the strengths of pair, triplet, and quadruplet interactions, and s_i represents the strategy state of agent i.

Payoff Functions

The payoff function U_i for each agent is defined based on the local potential and the agent's strategy.

Nash Equilibrium Search

Nash equilibrium is a combination of strategies where no agent can unilaterally change their strategy to increase their payoff. This condition is expressed for each agent i.

Nash Equilibrium Conditions

$$
\begin{aligned}
U_i(S_1^*, S_{-i}^*) &\geq U_i(S_1, S_{-i}^*) \\
U_i(S_2^*, S_{-i}^*) &\geq U_i(S_2, S_{-i}^*) \\
U_i(S_3^*, S_{-i}^*) &\geq U_i(S_3, S_{-i}^*)
\end{aligned}
$$

Here, S_i^* represents the Nash equilibrium strategy for agent i, and S_{-i}^* is the Nash equilibrium strategy set for all other agents.

Numerical methods and game theory algorithms are used to search for Nash equilibrium strategy combinations that satisfy the Nash equilibrium conditions. During the simulation and optimization stage, the dynamics of the system are modeled, and numerical methods or game theory algorithms are employed to evolve the system such that each agent chooses the optimal strategy.

Analysis of Dynamics via Simulation

In simulations, the system is initiated from an initial state, and the optimization process described above is carried out over time.

At each time step, the following information is calculated and recorded:

Fake news diffusion rate: $\rho(t) = \frac{1}{N} \sum_i s_i(t)$

Error range: Average size of clusters of consecutive fake news diffusion agents.

Simulations are executed for different scenarios (pair, triplet, quadruplet), and the diffusion patterns of fake news and changes in error ranges are compared and analyzed.

8. Discussion:Exploring Nash Equilibrium in Scenarios

Scenario 1

Expressing the changes in the payoff matrix when one of the players continues to spread fake news excessively and fact-checking is conducted against it using a general formula.

Let Player 1 (Information Provider) have a probability of spreading fake news excessively as $p_1(t)$, and Player 2 (Information Receiver) have a probability of conducting fact-checks as $p_2(t)$. The general elements of the payoff matrix are denoted as $U1(p_1(t), p_2(t))$ and $U2(p_1(t), p_2(t))$.

In this case, when the probability of spreading fake news excessively increases over time t, it can be expressed in a generalized formula as follows:

$$p_1(t) = f(t) \cdot p_{1_0}$$

Here, p_{1_0} is the initial probability of spreading fake news, and $f(t)$ is a function that varies over time. $f(t)$ represents the occurrence of excessive fake news and has a specific functional form with respect to t.

On the other hand, the probability of conducting fact-checks, $p_2(t)$, may also vary over time. The generalized formula for this is as follows:

$$p_2(t) = g(t) \cdot p_{2_0}$$

Here, p_{2_0} is the initial probability of fact-checking, and $g(t)$ is a function that changes over time. $g(t)$ represents the increase or decrease in fact-checking and has a specific functional form with respect to t.

And, when expressing the elements of the payoff matrix in a general manner, it becomes as follows:

$$
\begin{aligned}
U1(p_1(t), p_2(t)) &= U_{1_0} + h(t) \\
U2(p_1(t), p_2(t)) &= U_{2_0} + i(t)
\end{aligned}
$$

Here, U_{1_0} and U_{2_0} represent the initial payoffs, and $h(t)$ and $i(t)$ are functions that change over time. These functions reflect how they change over time due to the players' strategies.

Fig.20-21, scenario described, we can analyze the changes in payoffs over time for two players involved in a situation where one spreads fake news and the other conducts fact-checking.

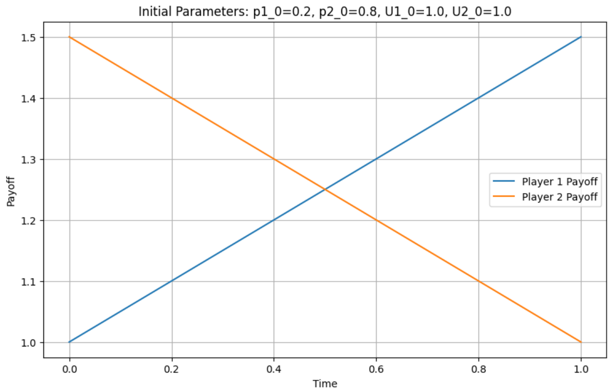

Fig. 19: Filter Bubble Scenario 1, Game Theory Applications

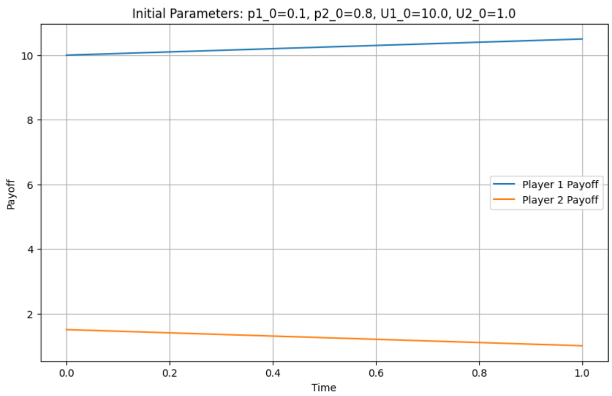

Fig. 20: Filter Bubble Scenario 1, Game Theory Applications

Fig.20 (Initial Parameters: $\mathbf{p1}_0 = 0.2, p2_0 = 0.8, U1_0 = 1.0, U2_0 = 1.0$)

The graph shows a linear relationship between the payoffs of the two players over time.Player 1's payoff (Information Provider) increases linearly with time, while Player 2's payoff (Information Receiver) decreases linearly. The crossing point suggests a moment in time where both players have the same payoff before their payoffs diverge.

Fig.21 (Initial Parameters: $\mathbf{p1}_0 = 0.1, p2_0 = 0.8, U1_0 = 10.0, U2_0 = 1.0$)

This graph presents a different scenario where Player 1 starts with a significantly higher payoff than in the first graph, while Player 2's payoff remains the same as before. The payoffs for both players remain constant over time, indicating that the functions $f(t)$ and $g(t)$ may be constant (i.e., $f(t) = 1$ and $g(t) = 1$), and the changes in strategies do not impact their payoffs.

Interpretation of the Scenario

The first graph represents a dynamic system where the strategies and payoffs change over time. The linear increase and decrease in payoffs suggest that the functions $h(t)$ and $i(t)$ may be linear with respect to time, implying that the consequences of the strategies linearly affect the players' payoffs. The second graph indicates a scenario where, despite the presence of fake news and fact-checking, the payoffs of the players do not change over time, which might suggest that the strategies of spreading fake news or fact-checking are ineffective, or that the initial advantage of Player 1 is so significant that it remains unchallenged over the observed period.

Implications for the Payoff Matrix

The general formulas provided define how the players' strategies evolve over time and how these strategies affect their payoffs. In both cases, the payoff for the information provider (Player 1) is tied to the function $h(t)$, which might represent the benefits of spreading fake news, while the payoff for the information receiver (Player 2) is tied to the function $i(t)$, which might represent the costs or effects of fact-checking. If $h(t)$ increases and $i(t)$ decreases over time, it indicates that the act of spreading fake news becomes more profitable while fact-checking becomes less effective or more costly. Fig.20 suggests a competitive scenario where the strategies of the two players have inverse effects on their payoffs over time. Fig.21 implies a static situation where the initial conditions dictate the outcomes, and there is no change in payoffs over time.The differences in the initial parameters significantly impact the dynamics between the players. Player 1's high initial payoff in the second graph suggests that they are in a dominant position that is not affected by the passage of time or the actions of Player 2.

Scenario 2

In Scenario 2, if both players continue to excessively spread fake news and no fact-checking is conducted, there is a possibility that their payoff matrices will be evenly balanced. To express this general trend, the following general formula is provided.

Let Player 1 (Information Provider) have a probability of excessively spreading fake news as $p_1(t)$, and Player 2 (Information Receiver) also have a probability of believing in fake news excessively as $p_2(t)$. Additionally, in the case where no fact-checking is conducted, the probability of fact-checking is assumed to be zero.

If the strategy probabilities for Player 1 and Player 2 do not change over time, that is, if $p_1(t) = p_{1_0}$ and $p_2(t) = p_{2_0}$ hold, the general formula for the payoff matrices is as follows:

$$U1(p_1(t), p_2(t)) = U_{1_0}$$
$$U2(p_1(t), p_2(t)) = U_{2_0}$$

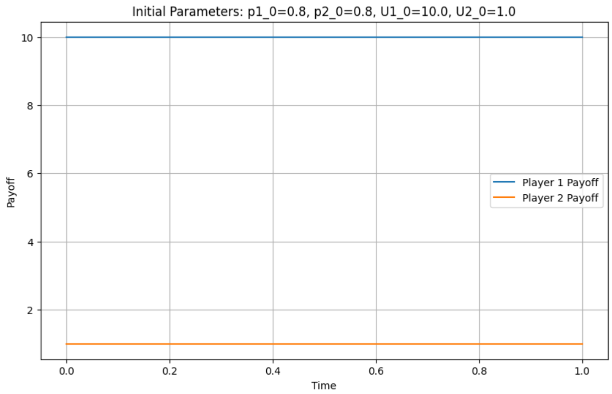

Fig. 21: Filter Bubble Scenario 2, Game Theory Applications

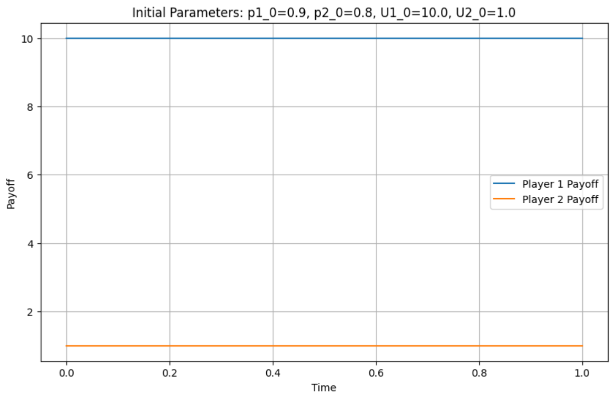

Fig. 22: Filter Bubble Scenario 2, Game Theory Applications

Here, U_{1_0} and U_{2_0} represent the initial payoffs, and it indicates that the payoffs do not change over time. In other words, as long as both continue to excessively spread fake news, and the probability of accepting it remains unchanged, the payoffs do not change.

However, this situation is unrealistic, and typically, the payoff matrix would change due to information receivers, external factors (e.g., strengthening fact-checking, introducing regulations, changes in information providers, etc.).

Fig.22-24, depicting the payoff evolution over time for two players under different initial conditions. In these scenarios, the players' strategies involve spreading and receiving fake news, with no fact-checking conducted.

Graphs with Static Payoffs (Fig.22-23)

These graphs illustrate scenarios where the payoffs for both players do not change over time, suggesting that the probabilities $p_1(t)$ and $p_2(t)$ remain constant. Player 1 (Information Provider) has a significantly higher constant payoff compared to Player 2 (Information Receiver). This may reflect the benefit that Player 1 gains from spreading fake news without the presence of fact-checking. The constant payoff for Player 2 suggests that the impact of believing in fake news does not change their payoff over time in these scenarios.

Graph with Dynamic Payoffs (Fig24)

This graph shows a scenario where the payoffs change over time, indicating that $p_1(t)$ and $p_2(t)$ may be dynamic, or other factors are at play which

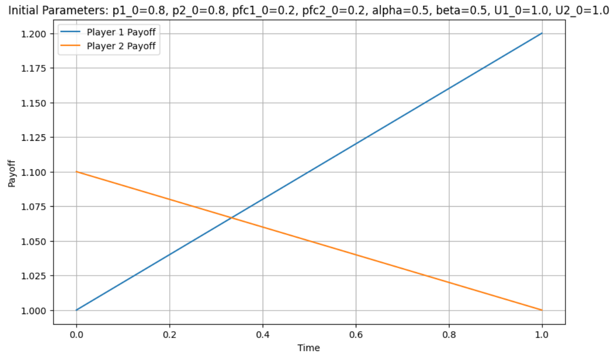

Fig. 23: Filter Bubble Scenario 2, Game Theory Applications

influence the payoffs. The payoff for Player 1 increases over time while the payoff for Player 2 decreases, implying a shift in the benefits and costs associated with the strategies of spreading and receiving fake news, respectively. The trends suggest a competitive interaction where the gain for Player 1 comes at the expense of Player 2.

Interpretation of the Scenarios

In the static payoff scenarios, the unchanging payoffs may reflect a situation where both players are locked into their strategies, and the spreading and receiving of fake news have become normalized without any intervention or changes in behavior. The dynamic payoff scenario implies a system where consequences evolve over time. For Player 1, this could mean increasing benefits from spreading fake news, possibly due to a growing audience or more sophisticated tactics. For Player 2, the decreasing payoff could represent the cumulative cost of believing in fake news, such as loss of credibility or other long-term detriments.

Implications for the Payoff Matrix

The constant payoffs in the first two graphs suggest that the system is in a state of equilibrium where the strategies of both players are stable and unresponsive to each other's actions. The changing payoffs in the third graph suggest that the equilibrium is either not reached or is being disrupted by external factors or evolving strategies. When no fact-checking is conducted, and both players continue to engage with fake news, the payoff matrix initially remains static, suggesting a stalemate or an accepted status quo. Over time, external influences or intrinsic changes in the system may alter this balance, leading to a dynamic situation where the payoffs for spreading and receiving fake news change, potentially requiring new strategies or interventions to address the evolving landscape.

To provide a more detailed analysis, additional information about the specific functional forms of $p_1(t)$, $p_2(t)$, and any external factors influencing the payoffs would be necessary.

Scenario 3

In Scenario 3, one of the players continues to alternate between lies (L) and truths (T), and fact-checking is conducted against them. There is a possibility that both players' payoff matrices will be evenly balanced. Such a general trend can be expressed with mathematical equations. Below, I provide the idea for a general formula.

Let Player 1 (Information Provider) have a probability of alternating between lies and truths as $p_1(t)$, and Player 2 (Information Receiver) also have a probability of believing in lies and truths as $p_2(t)$. Since the probability of fact-checking may vary for each player, we denote it as $p_{fc1}(t)$ and $p_{fc2}(t)$.

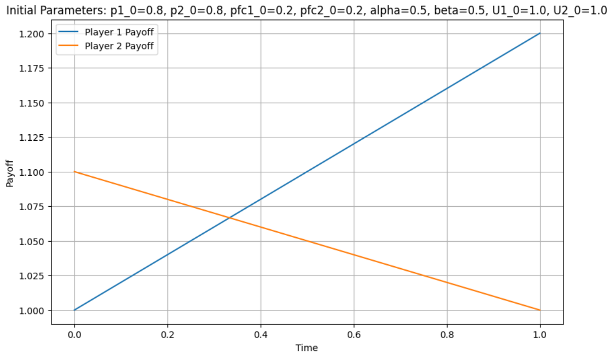

Fig. 24: Filter Bubble Scenario 3, Game Theory Applications

Here, the strategy probabilities for Player 1 and Player 2 at time t can be expressed as follows:

$$p_1(t) = \alpha \cdot p_{1_0} + (1-\alpha) \cdot p_{fc1}(t)$$
$$p_2(t) = \beta \cdot p_{2_0} + (1-\beta) \cdot p_{fc2}(t)$$

Here, α and β are parameters that control to what extent Player 1 and Player 2 employ the strategies of lies and truths. p_{1_0} and p_{2_0} are the initial strategy probabilities.

The general formula for the payoff matrices is as follows:

$$U1(p_1(t), p_2(t)) = U_{1_0} + f(t)$$
$$U2(p_1(t), p_2(t)) = U_{2_0} + g(t)$$

Here, U_{1_0} and U_{2_0} represent the initial payoffs, and $f(t)$ and $g(t)$ are functions that change over time. These functions depend on the players' strategies and the results of fact-checking. For example, if $f(t)$ and $g(t)$ have appropriate functional forms that compete with each other, there is a possibility that the payoff matrices will be evenly balanced.

Fig.25 representing the evolution of payoffs over time for two players engaged in a scenario where one alternates between spreading lies and truths, and the other conducts fact-checking. The described mathematical model considers the interplay between spreading misinformation and fact-checking, and how these dynamics affect each player's payoff.

Player 1 Payoff (Blue Line)

Player 1's payoff increases over time, suggesting that the strategy of alternating between lies and truths (possibly in response to fact-checking) is becoming increasingly effective, or that the cost of being caught in a lie decreases over time due to $f(t)$.

Player 2 Payoff (Orange Line)

Player 2's payoff decreases over time, indicating that the strategy of believing in lies and truths and conducting fact-checking is becoming less effective or more costly due to $g(t)$.

Interpretation of the Scenario

The parameters α and β likely represent the extent to which players stick to their initial strategies versus adjusting in response to the other player's actions. The functions $f(t)$ and $g(t)$ reflect the temporal evolution of the payoffs, potentially incorporating the effects of external factors like increased public awareness, changing credibility, or the introduction of regulatory measures. The increasing payoff for Player 1 suggests that they are successfully navigating between spreading lies and truths to maximize their benefit, possibly by adapting to the fact-checking efforts of Player 2. The decreasing payoff for Player 2 suggests that fact-checking is not sufficiently mitigating the negative impact of misinformation, or the cost of fact-checking (in terms of resources or credibility) is rising.

Implications for the Payoff Matrix

The graph indicates that the strategies and effectiveness of fact-checking evolve over time, potentially due to learning, adaptation, or changes in the information environment. The model suggests that a balance may be reached if the functional forms of $f(t)$ and $g(t)$ are appropriately chosen, which could lead to a stable outcome where neither player has an incentive to unilaterally change their strategy. The described scenario implies a dynamic competition where the effectiveness of fact-checking and the strategy of alternating between lies and truths influence the players' payoffs. The balance of the payoff matrices will depend on how each player adapts their strategy in response to the other, as well as on external factors that influence the efficacy of fact-checking and the cost of spreading misinformation. To fully understand the dynamics at play, additional details on the specific forms of $f(t)$ and $g(t)$, as well as the values of α and β, would be required. These would offer insight into the specific mechanisms through which the strategies affect the payoffs and how these are expected to evolve over time.

Scenario 4

In Scenario 4, if one of the players continues to present inconvenient truths (F), and fact-checking is conducted against them, there is a possibility that both players' payoff matrices will be evenly balanced. Such a general trend can be expressed with mathematical equations. Below, I provide the idea for a general formula.

Let Player 1 (Information Provider) have a probability of presenting inconvenient truths (F) as $p_1(t)$, and Player 2 (Information Receiver) also have a probability of believing in inconvenient truths as $p_2(t)$. Since the probability of fact-checking may vary for each player, we denote it as $p_{fc1}(t)$ and $p_{fc2}(t)$.

Here, the strategy probabilities for Player 1 and Player 2 at time t can be expressed as follows:

$$p_1(t) = \alpha \cdot p_{1_0} + (1-\alpha) \cdot p_{fc1}(t)$$
$$p_2(t) = \beta \cdot p_{2_0} + (1-\beta) \cdot p_{fc2}(t)$$

Here, α and β are parameters that control to what extent Player 1 and Player 2 present inconvenient truths. p_{1_0} and p_{2_0} are the initial strategy probabilities.

The general formula for the payoff matrices is as follows:

$$U1(p_1(t), p_2(t)) = U_{1_0} + f(t)$$
$$U2(p_1(t), p_2(t)) = U_{2_0} + g(t)$$

Here, U_{1_0} and U_{2_0} represent the initial payoffs, and $f(t)$ and $g(t)$ are functions that change over time. These functions depend on the players' strategies and the results of fact-checking. For example, if $f(t)$ and $g(t)$ have appropriate functional forms that compete with each other, there is a possibility that the payoff matrices will be evenly balanced.

Fig.26 showing the payoffs for two players over time under specific initial conditions and strategy probabilities related to presenting and believing in inconvenient truths (F), with a component of fact-checking involved.

Player 1 Payoff (Blue Line)

Player 1's payoff remains constant over time, suggesting that their strategy or the effectiveness of presenting inconvenient truths is stable and unaffected by the dynamics of the game within the observed time frame.

Player 2 Payoff (Orange Line)

Similarly, Player 2's payoff is also constant over time, indicating that their strategy or the impact of believing in inconvenient truths does not vary over

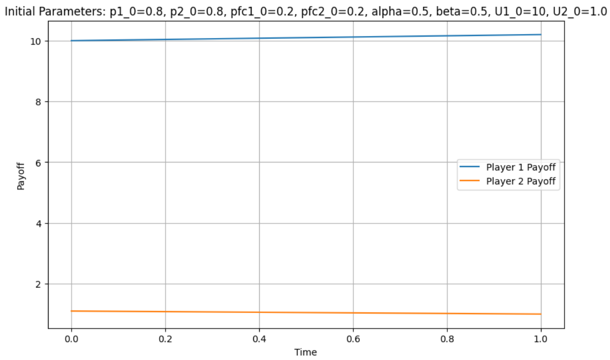

Fig. 25: Filter Bubble Scenario 4, Game Theory Applications

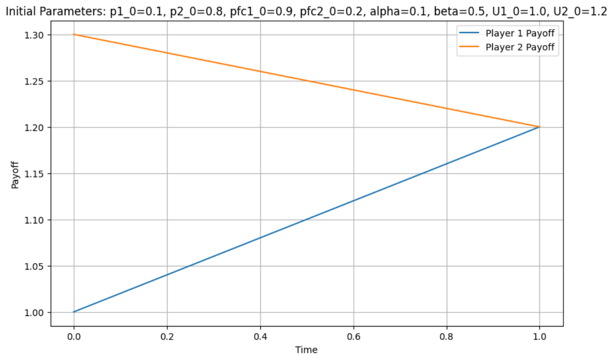

Fig. 26: Filter Bubble Scenario 5, Game Theory Applications

time, or that fact-checking efforts are neither beneficial nor harmful to their payoff.

Interpretation of the Scenario

The constants α and β suggest the degree to which each player's strategy is influenced by their own actions versus the fact-checking activities. The fact that payoffs are constant may indicate that α and β are such that the strategies are stable, or that $p_{fc1}(t)$ and $p_{fc2}(t)$ are constant and do not impact the outcome. The functions $f(t)$ and $g(t)$ are meant to reflect changes in payoffs over time. However, the constant nature of the payoffs in this graph suggests that these functions are either absent or zero, implying no change or impact from external factors or strategic shifts over the observed time.

Implications for the Payoff Matrix

Given that the payoffs do not change, this scenario might represent a system in equilibrium where the current strategies are optimal for both players, or where the potential benefits and costs of changing strategies are perfectly balanced. The scenario assumes no evolution in the strategies of presenting and believing in inconvenient truths, which could be interpreted as the players having reached a stable understanding or agreement on how to handle these truths. The graph suggests a static situation where the initial strategies and payoffs are maintained over time. This could either be due to a lack of incentive for either player to deviate from their strategy or due to external constraints that prevent the players from altering their strategies. To determine why the payoffs are static and to predict future changes, one would need to consider additional information about the nature of the inconvenient truths, the context in which they are presented and received, and the effectiveness and intensity of the fact-checking efforts.

Scenario 5

In Scenario 5, if one of the players continues to provide accurate information (T), and no fact-checking is performed, there is a possibility that both players' payoff matrices will be evenly balanced. This general trend can be expressed using mathematical equations. Below, I provide the idea for a general formula.

Let Player 1 (Information Provider) have a probability of continuously providing accurate information (T) as $p_1(t)$, and Player 2 (Information Receiver) also have a probability of believing in accurate information as $p_2(t)$. Since the probability of fact-checking may vary for each player, we denote it as $p_{fc1}(t)$ and $p_{fc2}(t)$.

Here, the strategy probabilities for Player 1 and Player 2 at time t can be expressed as follows:

$$p_1(t) = \alpha \cdot p_{1_0} + (1 - \alpha) \cdot p_{fc1}(t)$$

$$p_2(t) = \beta \cdot p_{2_0} + (1 - \beta) \cdot p_{fc2}(t)$$

Here, α and β are parameters that control to what extent Player 1 and Player 2 provide accurate information. p_{1_0} and p_{2_0} are the initial strategy probabilities.

The general formula for the payoff matrices is as follows:

$$U1(p_1(t), p_2(t)) = U_{1_0} + f(t)$$
$$U2(p_1(t), p_2(t)) = U_{2_0} + g(t)$$

Here, U_{1_0} and U_{2_0} represent the initial payoffs, and $f(t)$ and $g(t)$ are functions that change over time. These functions depend on the players' strategies and the results of fact-checking. For example, if $f(t)$ and $g(t)$ have appropriate functional forms that lead to disagreements at some point, there is a possibility that the payoff matrices will be evenly balanced.

Fig.27 depicts the payoffs for two players over time within a specific scenario where Player 1 (Information Provider) is continuously providing accurate information, and Player 2 (Information Receiver) believes in this information. In this scenario, no fact-checking is being performed. The mathematical equations provided are used to model the strategy probabilities and payoffs over time.

Player 1 Payoff (Blue Line)

Player 1's payoff increases over time, which indicates that continuously providing accurate information is becoming increasingly beneficial for Player 1. The payoff starts at 1.0 and grows steadily, suggesting that the value of truthfulness in the information provided by Player 1 increases over time, perhaps due to building trust or credibility.

Player 2 Payoff (Orange Line)

Player 2's payoff decreases over time, starting higher than Player 1's but dropping below it as time progresses. This may suggest that while initially, there is a benefit to believing accurate information, over time, the lack of fact-checking might lead to a decrease in Player 2's ability to discern the truth or perhaps an increase in the cost associated with accepting information without verification.

Interpretation of the Scenario

Strategy Probabilities, The probabilities $p_1(t)$ and $p_2(t)$ represent the likelihood of Players 1 and 2 sticking to their strategies of providing and believing in accurate information, respectively. Since α and β are low (0.1 and 0.5, respectively), this suggests that fact-checking plays a small role in their strategies. The payoff functions $U1(p_1(t), p_2(t))$ and $U2(p_1(t), p_2(t))$ are affected by the functions $f(t)$ and $g(t)$, which represent the time-dependent change in payoffs due to the players' strategies and possibly other external factors. In a scenario without fact-checking, the players' payoffs change as a direct consequence of their strategies and the changing environment in which the information is provided and received. The increasing payoff for Player 1 suggests that providing accurate information without the need for fact-checking is rewarded over time, possibly due to establishing a reputation for reliability. The decreasing payoff for Player 2 indicates a potential cost associated with not verifying information, even if it is accurate. Over time, the information landscape may change, or there may be a need for more scrutiny, leading to lower payoffs for simply accepting information at face value.

The graph and scenario highlight the dynamic nature of trust and credibility in information exchange. While initially, there may be high trust and lower verification costs, over time, the need for fact-checking and verification may become more apparent, affecting the payoffs for both the information provider and receiver. The crossing point of the payoffs suggests a critical moment where the benefit of providing accurate information equates to the cost of believing it without verification. This model serves as a simplified representation of the complex dynamics involved in information dissemination and consumption.

Scenario 6

In Scenario 6, if one of the players continues to provide false information (L) without fact-checking, over time, the fact-checker may also continue to provide correct information until an error occurs at some point, leading to a divergence in opinions. We consider the possibility of such a general trend being expressed using mathematical equations. Below, I provide the idea for a general formula.

Let Player 1 (Information Provider) have a probability of continuously providing false information (L) as $p_1(t)$, and Player 2 (Information Receiver) also have a probability of believing in lies as $p_2(t)$. Since the probability of fact-checking may vary for each player, we denote it as $p_{fc1}(t)$ and $p_{fc2}(t)$.

Here, the strategy probabilities for Player 1 and Player 2 at time t can be expressed as follows:

$$p_1(t) = \alpha \cdot p_{1_0} + (1 - \alpha) \cdot p_{fc1}(t)$$
$$p_2(t) = \beta \cdot p_{2_0} + (1 - \beta) \cdot p_{fc2}(t)$$

Here, α and β are parameters that control to what extent Player 1 and Player 2 provide false information. p_{1_0} and p_{2_0} are the initial strategy probabilities.

The general formula for the payoff matrices is as follows:

$$U1(p_1(t), p_2(t)) = U_{1_0} + f(t)$$
$$U2(p_1(t), p_2(t)) = U_{2_0} + g(t)$$

Here, U_{1_0} and U_{2_0} represent the initial payoffs, and $f(t)$ and $g(t)$ are functions that change over time. These functions depend on the players' strategies and the results of fact-checking. For example, if $f(t)$ and $g(t)$ have appropriate functional forms that lead to errors occurring at some point, causing a divergence in opinions, there is a possibility that the payoff matrices will be evenly balanced.

Fig.28-29, representing the payoffs for two players over time in a scenario where one player provides false information without fact-checking and the other player, a fact-checker, may provide correct information until a divergence occurs due to an error.

Player 1 Payoff (Blue Line)

Player 1 starts with a higher payoff that increases over time. This suggests that providing false information without the presence of fact-checking initially benefits Player 1, and this benefit grows over time, possibly due to the accumulation of influence or trust built on the unverified information.

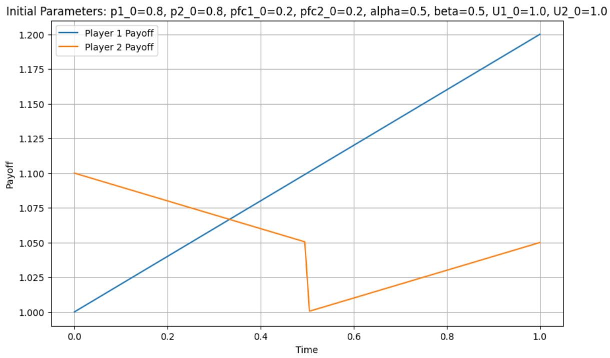

Fig. 27: Filter Bubble Scenario 6, Game Theory Applications

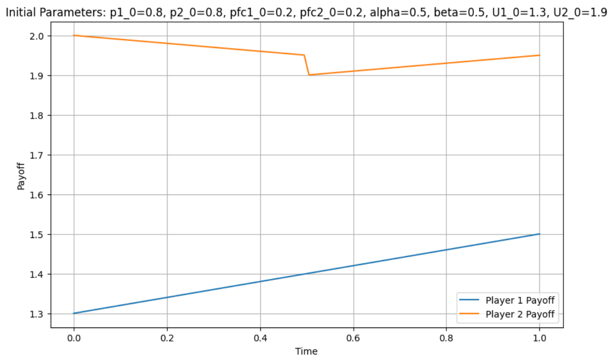

Fig. 28: Filter Bubble Scenario 6, Game Theory Applications

Player 2 Payoff (Orange Line)

Player 2's payoff begins lower than Player 1's and shows a slight decrease before stabilizing. The initial decline might reflect the initial costs or consequences of believing false information. The subsequent stability could indicate that Player 2 either starts to detect falsehoods or that the costs associated with false beliefs have reached a steady state.

Interpretation of the Scenario

The strategy probabilities, represented by $p_1(t)$ and $p_2(t)$, are influenced by the parameters α and β, which determine how much the players stick to their initial strategies in the presence of fact-checking activities. The functions $f(t)$ and $g(t)$ in the payoff formulas are designed to model the time-dependent change in payoffs, which could include the impact of external factors such as societal reactions to misinformation or changes in the information environment.

Implications for the Payoff Matrix

The increasing payoff for Player 1 despite providing false information suggests that the negative consequences of spreading falsehoods have not yet materialized significantly within the model's timeframe or that these consequences are outweighed by the benefits gained. The fact that Player 2's payoff stabilizes suggests that the negative impact of false information may be mitigated over time, possibly due to adaptation by Player 2 or other entities in the system that begin to counteract the misinformation. Fig.28-29 demonstrates a scenario where misinformation initially leads to an advantage for the provider but does not result in a continuous decline for the receiver, suggesting a complex interplay between the spread of misinformation and the response of the information environment. The dynamics between the players' strategies and the resulting payoffs would be influenced by additional factors in a real-world context, such as the ability of the wider community to recognize and challenge false information, or the introduction of penalties or other disincentives for spreading falsehoods. To fully understand the evolution of the payoffs and the potential for errors causing a divergence in opinions, further details about the nature of the false information, the effectiveness of fact-checking, and the potential for correcting misinformation would be needed.

9. Note:Exploring Nash Equilibrium in Scenarios

In this computational experiment, we aim to find Nash equilibrium on the filter bubble by performing the following calculations:

Definition of Information Reception Functions

We define the information reception functions $I_F(s_F, s_C)$ and $I_C(s_F, s_C)$, which represent the quality and quantity of information received by players based on their strategies.

Definition of Payoff Functions

We define the payoff functions U_F and U_C that are influenced by the filter bubble.

Redefinition of Best Response Functions

We redefine the best response functions R'_F and R'_C for Player F and Player C.

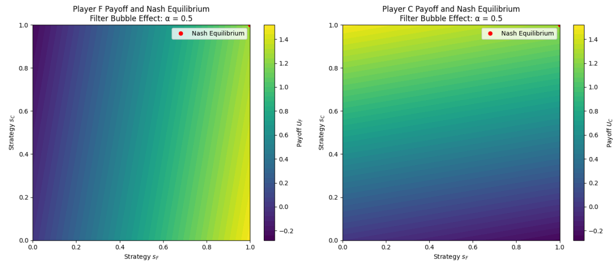

Fig. 29: Filter Bubble Scenario:Player F and C Payoff Function

Calculation of Nash Equilibrium

Using R'_F and R'_C, we calculate the strategy pair (s_F^*, s_C^*) at Nash equilibrium.

In this simulation, specific forms of information reception functions and payoff functions have not been defined yet. We will assume appropriate functional forms to find the Nash equilibrium.

As an assumption, information reception functions are defined as functions of strategies:

$$I_F(s_F, s_C) = \beta s_F (1 - s_C)$$
$$I_C(s_F, s_C) = \gamma (1 - s_F) s_C$$

Here, β and γ are parameters representing players' sensitivity to receiving information.

The payoff functions are defined by adding the influence of information reception functions to basic payoffs:

$$U_F(s_F, s_C, I_F) = s_F(1 - s_C) + \delta I_F$$
$$U_C(s_F, s_C, I_C) = (1 - s_F)s_C + \eta I_C$$

δ and η are parameters representing how much the impact of information is reflected in payoffs. Player's best response functions R'_F and R'_C are functions that return the strategy that maximizes their own payoff when the opponent's strategy is fixed. We will implement a Python simulation to explore the Nash equilibrium. Set the initial values for parameters as follows:

$$\beta = 1.5, \gamma = 1.5, \delta = 1.0, \eta = 1.0$$

The Nash equilibrium is the point where both players' best response strategies intersect. Use numerical analysis to find this point.

In this computational experiment, we have outlined the methodology for finding Nash equilibrium on the filter bubble. The actual implementation and results will depend on the specific functional forms chosen for information reception and payoff functions.

Fig.30, depict a game-theoretic model concerning the dissemination and reception of information within the context of a filter bubble effect. The model includes Player F (the information provider) and Player C (the information receiver), with strategies s_F and s_C, respectively. The filter bubble effect parameters α, β, δ, and γ influence how players receive and value information.

Player F Payoff Function

The heatmap indicates that Player F's payoff increases as their strategy s_F approaches 1, regardless of Player C's strategy. This suggests that Player F benefits more from fully committing to their strategy of information provision within the filter bubble.

Player C Payoff Function

Conversely, Player C's payoff is maximized when s_C is higher, meaning they benefit from fully engaging with the information they receive. This could be interpreted as Player C gaining more when they are open to the information provided by Player F.

Nash Equilibrium

The Nash Equilibrium, indicated by the red dot at the coordinates (1.00, 1.00) on both heatmaps, represents the strategy profile where Player F fully commits to their information strategy and Player C fully engages with the information. At this point, neither player has an incentive to change their strategy unilaterally.

Interpretation of the Scenario

The heatmaps suggest a scenario where both players find it best to engage fully with their respective strategies. The filter bubble effect appears to reinforce the players' tendencies to stick to their strategies, leading to a situation where both are locked into their behavior.

Implications for Payoff Matrix

The players' payoff functions, including the terms δI_F and ηI_C, suggest that the payoff is not only determined by the strategies themselves but also by the quality of information reception within the filter bubble. The best response functions R'_F and R'_C will mathematically determine the players' optimal strategies given the opponent's strategy. These functions likely reflect a dynamic system where players adjust their strategies based on the payoffs and the information received. The model outlined and the associated heatmaps represent a simplified view of the complex interactions within a filter bubble. In this environment, the players are incentivized to fully engage with their strategies, leading to a Nash Equilibrium where both players fully commit to their roles as information provider and receiver. This scenario can be illustrative of online social platforms where users and content creators become increasingly entrenched in their respective echo chambers, potentially leading to polarization.

To validate these insights, an actual implementation of the simulation with the specified parameters would be necessary, considering the functional forms for the information reception and the impact on the payoffs.

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