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Application of the Perspective of Expected Gain Functions in Incomplete Information Games: Adaptation of Dominance and Convexity to Mitigate Fake News via Cooperative Game Theory

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Abstract: In this paper, we apply gain function and expected gain function game theory to consider the egocentricity and convexity in the gain function and expected gain function in the context of the multicollaboration case in terms of complete and incomplete information games and in the context of fake news and fact checking in the framework of noncooperative games. Here, we analyze the impact of informal cooperation on gain, assuming the presence of information and a non-cooperative game scenario in which different players (e.g., fact-checkers, news providers) choose their own strategies. This research delves into the complex relationship between the dissemination of fake news and fact-checking in the context of filter bubbles and examines how game-theoretic principles can be used to analyze the strategic interactions among news providers, consumers, and fact-checkers. By modeling these interactions, we explore how the dynamics of hyperadditivity and convexity in cooperative games, along with the concepts of Nash equilibrium and expected utility, affect the spread and control of misinformation. We illustrate how filter bubbles exacerbate the challenge of combating fake news by restricting access to diverse sources of information, thereby affecting the strategic choices of all parties involved. Through the lens of scenarios characterized by extensive formal games, repetitive games, and information asymmetry, a multifaceted approach to mitigating the effects of the filter bubble can be proposed. These include encouraging a diversity of information sources, increasing the effectiveness of fact-checking through strategic resource allocation, and leveraging educational initiatives to improve the public's information literacy. Our analysis emphasizes the importance of strategic cooperation and informed decision making to curb the spread of fake news and examines game-theoretic frameworks that contribute to the development of a more resilient information ecosystem in the digital age.

Keywords: Game Theory, Fake News, Fact-Checking, Complete Information Games, Non-Complete Information Games, Replica Method, Filter Bubbles, Nash Equilibrium, Expected Utility, Superadditivity, Convexity, Strategic Interaction, areto Optimal Points, Information Asymmetrys

1. Introduction

In this paper, we apply gain function and expected gain function game theory to consider the egocentricity and convexity in the gain function and expected gain function in the context of the multicollaboration case in terms of complete and incomplete information games and in the context of fake news and fact checking in the framework of noncooperative games. Here, we analyze the impact of informal cooperation on gain, assuming the presence of information and a non-cooperative game scenario in which different players (e.g., fact-checkers, news providers) choose their own strategies. We mainly performed some scenario analysis. Define the gain function: Define a gain function for each player and compute the gain

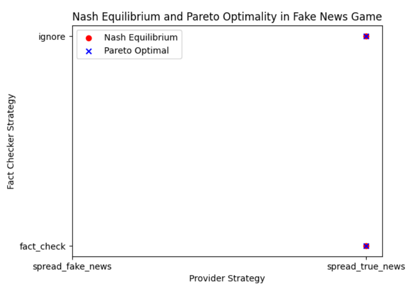

Fig. 1: Nash Equilibrium and Pareto Optimality in Fake News Game

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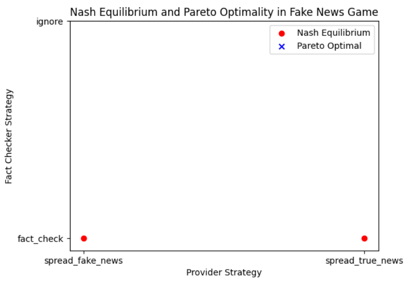

Fig. 2: Nash Equilibrium and Pareto Optimality in Fake News Game

from independent actions.

Calculate the gain from informal cooperation: We calculate the gain when two players informally share information or coordinate their strategies to test the concept of egalitarianism.

Evaluating Convexity: Evaluate the convexity condition by calculating the increase in gain that an additional player brings as the set of players grows.

Visualization of the results: a graphical representation of the gain function versus the gain obtained from informal cooperation.

The program calculates gains based on two strategies: "Fact Check," in which the fact checker identifies fake news, and "Ignore," in which it ignores it. It also evaluates the impact of informal cooperation on gains, taking into account the strategies "spreadfakenews" and "shareinformation," in which news providers spread fake news and share information, respectively. Finally, we compare the gains from independent actions and informal cooperation.

Nash Equilibrium and Pareto Optimality in the Context of Fake News and Fact-Checking

We have discussed this in light of the aforementioned "Note: Using game theory in iterative optimization to model the spread of fake news: insights from Hebbian learning from Nash equilibrium".

In the realm of fake news and fact-checking, the strategic behaviors of individuals or organizations disseminating fake news and the counteractions of fact-checkers can be encapsulated within the framework of Nash equilibrium. Here, the strategies entail various methods of spreading fake news and conducting fact-checking, respectively. Nash equilibrium is achieved when each player's strategy is an optimal response to the other's, determined through fixed-point theorems that delineate the payoff functions for each participant.

1.1 Payoff Functions

1.2 For Fake News Disseminators

The payoff function, denoted as $U_F(s_F, s_C)$, where s_F represents the strategy of fake news disseminators and s_C that of fact-checkers, hinges on the reach and influence of the disseminated fake news, adjusted for the deterrent effect of fact-checking activities.

1.2.1 For Fact-Checkers

Conversely, the payoff function for fact-checkers, $U_C(s_C, s_F)$, depends on the effectiveness of spreading accurate information and the societal benefits derived from curtailing fake news.

1.3 Determining Nash Equilibrium through Fixed-Point Theorems

Fixed-point theorems, such as Banach's or Brouwer's, are instrumental in computing Nash equilibria. They utilize optimal response functions derived from the players' payoff functions to identify fixed points, representing the Nash equilibria.

1.3.1 Optimal Response Functions

The optimal response function for fake news disseminators is given by $R_F(s_C)$, and for fact-checkers by $R_C(s_F)$. These functions yield the best strategy for each player, given the opponent's strategy.

1.3.2 Computing Fixed Points

A strategy pair (s_F^*, s_C^*) constitutes a Nash equilibrium if it satisfies the conditions:

$$R_F(s_C^*) = s_F^*, \quad R_C(s_F^*) = s_C^*$$

implying that each player is adopting their best response strategy.

In scenarios where analytical determination of fixed points is infeasible, numerical methods, such as iterative updates starting from initial strategy guesses $s_F^{(0)}, s_C^{(0)}$, are employed. The iterative process is as follows:

$$s_F^{(k+1)} = R_F(s_C^{(k)}), \quad s_C^{(k+1)} = R_C(s_F^{(k)})$$

continuing until convergence is achieved.

1.4 Nash Equilibrium in Fake News Diffusion

In modeling fake news diffusion within a game-theoretical framework, agents—comprising individuals, news sources, and fact-checkers—select strategies from options such as spreading, not spreading, or fact-checking fake news. A Nash equilibrium is reached when all agents, having fixed their strategies, find no incentive to alter them.

Let $N = \{1, 2, \ldots, n\}$ represent the set of agents, with each agent i possessing a strategy set S_i. The payoff functions are denoted as $u_i : S_1 \times S_2 \times \ldots \times S_n \to \mathbb{R}$, where the product represents the combined strategy space.

1.4.1 Characterizing Nash Equilibrium

A strategy combination $(s_1^*, s_2^*, \ldots, s_n^*)$ achieves Nash equilibrium if, for every agent i, and any alternative strategy s_i', the following holds:

$$u_i(s_1^*, s_2^*, \ldots, s_i^*, \ldots, s_n^*) \geq u_i(s_1^*, s_2^*, \ldots, s_i', \ldots, s_n^*)$$

1.4.2 Application to Fake News Diffusion

Agents in this context might adopt strategies related to the dissemination or verification of information, with their payoffs influenced by factors like the value placed on accurate information, the social repercussions of spreading fake news, and the cost-effectiveness of fact-checking efforts.

Identifying Nash equilibria entails examining all potential strategy combinations to find those satisfying the equilibrium condition. This can be achieved either through analytical means or via numerical simulations, employing methods like agent-based modeling or evolutionary game theory to explore the dynamics of strategy evolution and the emergence of equilibria.

1.5 Exploring Nash Equilibrium in Filter Bubble Scenarios

In this study, we delve into the dynamics of Nash equilibrium within the context of filter bubbles, focusing on the strategic interactions between information providers (Player F) and receivers (Player C). The methodology involves defining information reception and payoff functions, redefining best response functions, and employing computational techniques to identify the equilibrium point.

1.6 Information Reception Functions

The functions $I_F(s_F, s_C)$ and $I_C(s_F, s_C)$ quantify the quality and quantity of information received by the players, given their strategies:

$$I_F(s_F, s_C) = \beta s_F(1 - s_C), \quad I_C(s_F, s_C) = \gamma(1 - s_F)s_C$$

where β and γ signify the sensitivity of the players to information reception.

1.7 Payoff Functions

The payoff functions, incorporating the effects of the filter bubble, are expressed as:

$$U_F(s_F, s_C, I_F) = s_F(1-s_C)+\delta I_F, \quad U_C(s_F, s_C, I_C) = (1-s_F)s_C+\eta I_C$$

Here, δ and η denote the impact of information on the players' payoffs.

1.8 Best Response Functions

The best response functions R_F' and R_C' are redefined to determine the optimal strategies for maximizing each player's payoff when the opponent's strategy is fixed.

1.9 Nash Equilibrium Calculation

The Nash equilibrium (s_F^*, s_C^*) is computed using R_F' and R_C', signifying the strategy pair where both players' responses converge.

Parameters Initialization

Initial parameter values are set as $\beta = 1.5$, $\gamma = 1.5$, $\delta = 1.0$, and $\eta = 1.0$, aiding in the simulation to explore the equilibrium point.

2. Previous Research: Differences Between Perfect Information Games and Imperfect Information Games

In game theory, Perfect Information Games and Imperfect Information Games differ in their treatment of Information Sets.

Information Sets in Perfect Information Games

In Perfect Information Games, all players always know the entire history of the game, including the choices made by other players. Each decision node in the game tree has a unique information set, and strategies are determined through backward induction.

Information Sets in Imperfect Information Games

In Imperfect Information Games, players lack complete knowledge of all game histories. Information sets represent sets of decision nodes that players cannot distinguish between, and strategies are based on calculating expected payoffs using Bayesian Games.

Calculation Process

The calculation process in Imperfect Information Games involves:

(1) Calculating Expected Payoffs.

(2) Bayesian Updating of Beliefs.

(3) Adoption of Mixed Strategies.

Cooperative Games in Perfect Information Games

In Perfect Information Games, cooperative games involve characteristic functions and expected payoff functions.

Characteristic Function

The characteristic function $v(S)$ specifies the maximum total payoff that any subset S of players can achieve cooperatively.

Expected Payoff Functions

Players have a complete understanding of cooperative payoffs. Expected payoff functions depend on characteristic functions and how payoffs are allocated within coalitions.

Cooperative Games in Imperfect Information Games

In Imperfect Information Games, cooperative games also involve characteristic functions and expected payoff functions, taking into account information uncertainty.

Characteristic Function

The characteristic function considers players' information asymmetry and specifies the maximum expected payoff that any subset of players can achieve cooperatively.

Expected Payoff Functions

Expected payoff functions account for players' uncertainty about other players' types and actions. Calculation involves Bayesian updating and belief revision.

Mathematical Formulas and Calculation Process

The specific calculation process for cooperative games depends on the game's settings and agreements among players.

Calculating Shapley Values

For example, when calculating Shapley values, the following steps are commonly followed:

(1) Define the Characteristic Function for all player sets N and each subset S.

(2) Calculate Shapley Values for each player i.

In Imperfect Information Games, cooperative game analysis involves calculating expected payoffs while considering players' beliefs and information updates. Strategies are chosen based on available information and beliefs, and the payoff allocation may vary as beliefs evolve during the game.

2.1 Cooperative game theory explores Nash equilibrium

Cooperative game theory explores Nash equilibrium in both perfect and imperfect information contexts. This concept, traditionally associated with non-cooperative games, becomes relevant in cooperative games when player agreements or contracts are based on Nash equilibrium principles.

Nash Equilibrium in Cooperative Games with Perfect Information

In cooperative games with perfect information, where all players possess complete knowledge of each other's payoff functions and possible choices, cooperative agreements are formed. Nash equilibrium represents a state where each player maximizes their own payoff while considering the choices of others.

Formulas and Calculation Process

(1) Define the characteristic function $v(S)$, which represents the maximum total payoff achievable by any player subset S cooperating.

(2) Set conditions for cooperative agreements forming Nash equilibrium regarding the payoff distribution x_i for each player i ($i \in S$).

(3) Determine the payoff x_i for each player to maximize their expected payoff $E[U_i]$.

Nash Equilibrium in Cooperative Games with Imperfect Information

In imperfect information games, finding Nash equilibrium is more complex due to players' incomplete information. Players choose optimal strategies under uncertainty and reach cooperative agreements while considering other players' potential choices and beliefs.

Formulas and Calculation Process

(1) Define expected payoff functions based on each player's beliefs and possible strategies.

(2) Set conditions for cooperative agreements forming Nash equilibrium regarding the expected payoff for each player.

(3) Find the payoff distribution x_i for each player that maximizes their expected payoff $E[U_i]$.

Game Theory Nash Equilibrium in the Context of Fake News and Fact-Checking

In the context of fake news and fact-checking, Nash equilibrium considerations are essential, especially when filter bubbles affect information consumption patterns. Participants include news providers, consumers, and fact-checkers.

Game Participants and Strategies

(1) **News Providers**: Strategies of providing true or fake news.

(2) **Consumers (Users)**: Strategies of diversifying information sources or relying on aligned sources.

(3) **Fact-Checkers**: Strategies of active or selective fact-checking.

Analysis of Nash Equilibrium

Nash equilibrium in this context implies:

(1) Incentives for news providers to spread fake news.

(2) Reduced incentive for consumers to diversify information sources.

(3) Selective fact-checking by fact-checkers due to limited resources.

Example of Nash Equilibrium in a Filter Bubble

An example equilibrium involves:

(1) News providers spreading fake news.

(2) Consumers relying on biased information.

(3) Limited effectiveness of fact-checking.

Factors Influencing Equilibrium Changes

(1) Consumer behavior changes: Prioritizing diversification and critical thinking.

(2) Enhanced fact-checker resources: Increased ability to verify information.

3. Discussion:Superadditivity and Convexity in the Context of Filter Bubbles in Fake News and Fact-Checking

Superadditivity and Convexity are fundamental concepts in cooperative game theory. However, they are also applicable in analyzing game theory within the context of filter bubbles in fake news and fact-checking. In this article, we explore how these concepts can be used in understanding and promoting cooperation among fact-checkers to combat fake news.

Superadditivity

Concept of Superadditivity

Superadditivity refers to a property where the gains obtained through cooperation by a group of players exceed the sum of individual gains. Formally, it is defined as follows:

$$v(A \cup B) \geq v(A) + v(B)$$

Here, $v(S)$ is the characteristic function representing the benefit of a coalition S, and A and B are non-overlapping coalitions. This condition indicates the presence of incentives for players to cooperate.

Application to Fake News and Fact-Checking

In the context of fake news and fact-checking, superadditivity can apply when multiple fact-checkers cooperate to efficiently identify and prevent the spread of fake news. This collective effort is more efficient than individual fact-checking actions.

Formulas and Calculation Process

To verify superadditivity, the characteristic function $v(S)$ should be explicitly defined, and the inequality $v(A \cup B) \geq v(A) + v(B)$ should be confirmed for any two fact-checker groups A and B.

Convexity

Concept of Convexity

Convexity is a stronger property where the characteristic function $v(S)$ is convex. It satisfies the following condition for any player i and any two player coalitions A and B ($A \subseteq B$ and $i \notin B$):

$$v(B \cup \{i\}) - v(B) \geq v(A \cup \{i\}) - v(A)$$

This property indicates that the additional gains from a player joining a larger cooperation group are greater than the gains from joining a smaller group.

Application to Fake News and Fact-Checking

In the context of fake news, convexity might imply that as the network of fact-checkers grows, the added value brought by new fact-checkers (e.g., increased efficiency in identifying and preventing the spread of fake news) increases.

Formulas and Calculation Process

To verify convexity, the characteristic function $v(S)$ should be defined, and the inequality $v(B \cup \{i\}) - v(B) \geq v(A \cup \{i\}) - v(A)$ should be checked for any fact-checker i and any two groups A and B.

Superadditivity and Convexity are powerful concepts that can be applied in the analysis of cooperation among fact-checkers in the fight against fake news, even in the challenging context of filter bubbles. These concepts provide insights into the benefits of collaboration and can help improve information quality.

4. Discussion:Nash Equilibrium in Situations with Filter Bubbles Applications to Bayesian Games and Adaptive Dynamics Games

In this paper, we first consider a model that combines Bayesian and adaptive dynamics games to explore Nash equilibrium in the context of fake news and fact-checking under a filter bubble. This approach assumes a dynamic environment in which players (e.g., news consumers, news providers, and fact-checkers) act under imperfect information and their actions interact with each other. The following is a theoretical explanation and a simple mathematical equation that we will examine.

4.1 Theoretical Description

Definition of a Player

News consumers aim to evaluate the truth or falsity of the news they are provided and to select reliable sources of information.

News providers aim to ensure that their information is widely accepted, and there are two strategies for this: providing truth and providing fake news.

Fact checkers aim to verify the truth of the information provided and provide accurate information to the public.

4.2 Setting Incomplete Information

Each player does not have complete information about the types of other players (e.g., whether the news provider is trustworthy) or their intentions. To model this uncertainty, we use a Bayesian game framework.

4.3 Adaptive Dynamics

Players learn from past experiences and adapt their own strategies. This process is modeled through an adaptive dynamics game to show how a player's strategy evolves over time.

4.4 Ideas for Formulas and Computational Processes

Players' Belief Updates

News consumers have prior probabilities about the type of news provider and update these as new information becomes available. This belief update is modeled using Bayes' theorem.

Strategy Adaptation

Players choose strategies that maximize their own gain, which depends on the strategies and beliefs of other players. Let the gain function be $U_i(s_i, s_{-i})$ and define the gain when player i takes strategy s_i for the set of other players' strategies s_{-i}.

Adaptive dynamics is represented by defining rules for updating a player's strategy. For example, a player decides whether to repeat a successful strategy or try a new strategy in an exploratory manner based on past results.

Derivation of Nash Equilibrium

A Nash equilibrium is a state in which no player can unilaterally improve his or her gain by changing strategies. To find this state, we define the optimal reaction function of each player and look for the point where they intersect.

To find the Nash equilibrium, we need to find the strategy that maximizes each player's gain function and check if it is also optimal for the other players' strategies.

Implementation and Analysis

The implementation and analysis of this model could be based on specific parameters of the game (e.g., value of information, cost of false information, learning rate, etc.), using numerical simulation or analytical methods. Depending on the complexity of the model, computer simulations may be required.

Expansive Games and Tree Structures

When considering the issue of fake news and fact-checking in an unfolding game, the information structure of the game and the characteristics of a non-cooperative game need to be clearly defined. In an unfolding game, the actions a player can take as the game progresses and the corresponding information set are represented by a tree structure. Below is a concrete model and computational process using this concept.

Game Setup

Player: News provider (P) and fact checker (F)

Behavior:

- P can choose to "provide truthful news" (T) or "provide fake news" (F).
- F can choose to "verify" (V) or "not verify" (N) the news provided.

Gain

The gain of a player depends on the combination of their actions. For example, if P chooses F and F chooses V, the gains for P and F are different. The gains could be represented as follows: $U_P(T,V)$, $U_P(T,N)$, $U_P(F,V)$, $U_P(F,N)$, $U_F(T,V)$, $U_F(T,N)$, $U_F(F,V)$, $U_F(F,N)$.

Information Structure

The developmental form of the game takes into account the information that each player has at that point in time. For example, at the stage when F chooses an action, we assume that P does not know the action (T or F) chosen by P (the information set is divided at F's action selection point).

Representation of the Expanded Form Game

A game is represented by the following tree structure:

(1) First, P chooses an action (T or F).

(2) Next, F chooses an action, but does not know what P has chosen (V or N).

Using this tree structure, the gain corresponding to each end node (the final outcome of the game) is assigned to each player.

Formula and Calculation Process

In the expanded form game, we use backward induction to find the Nash equilibrium. This is a method of determining the optimal behavior of the players at each stage of the game, working backward from the end of the game.

(1) **Start with the last decision node**: when F chooses V or N, determine F's optimal choice based on whether P chose T or F. At this point, the expected gain of F can be computed as follows:

$E[U_F|V,T)]$ and $E[U_F|V,F)]$ (the expected gain when F cho

$E[U_F|N,T)]$ and $E[U_F|N,F)]$ (expected gain when F choose

(2) **Determine F's optimal strategy**: Choose the action that maximizes F's expected gain. For example, if $E[U_F|V,F)] > E[U_F|N,F)]$ then it is optimal for F to choose V.

(3) **Determining P's optimal strategy**: P predicts how F will react and chooses the action that maximizes its own expected gain; P's expected gain depends on the actions F can take, but F's choice is known in advance due to backward induction.

Fig.3-5 depict bar graphs titled "Game Outcomes with superadditivity factor=1.5, convexity factor=1.2," "Game Outcomes with superadditivity factor=2, convexity factor=5," and "Game Outcomes with superadditivity factor=20, convexity factor=5," respectively. Each graph shows the utilities of four strategies labeled T, F, V, and N, with utilities represented by the height of the bars.

Impact of Superadditivity Factor

As the superadditivity factor increases from 1.5 in the first image to 20 in the third image, the utilities of the strategies, especially for F and V, increase significantly. This suggests that the combined strategies are producing greater value than the individual ones. The F strategy seems to benefit the

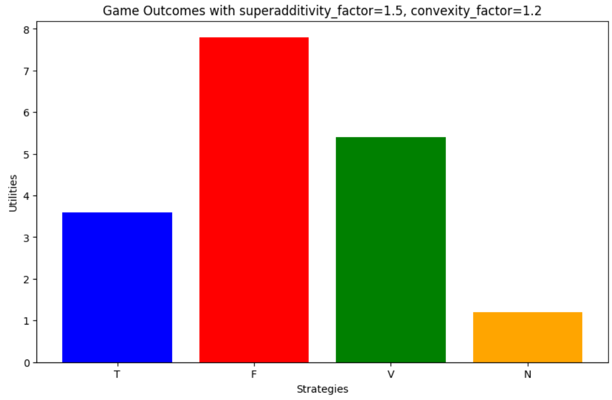

Fig. 3: Game Outcomes with superadditivity factor=1.5, convexity factor=1.2

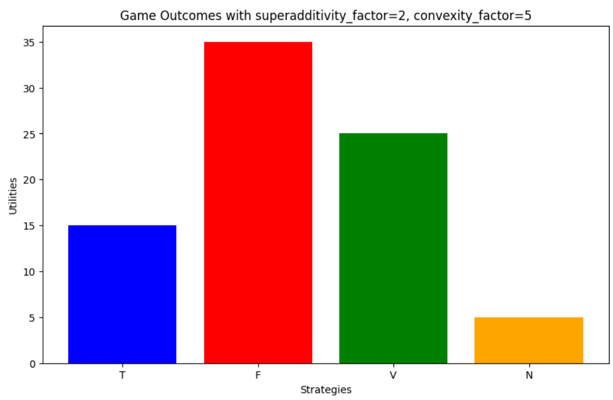

Fig. 4: Game Outcomes with superadditivity factor=2, convexity factor=5

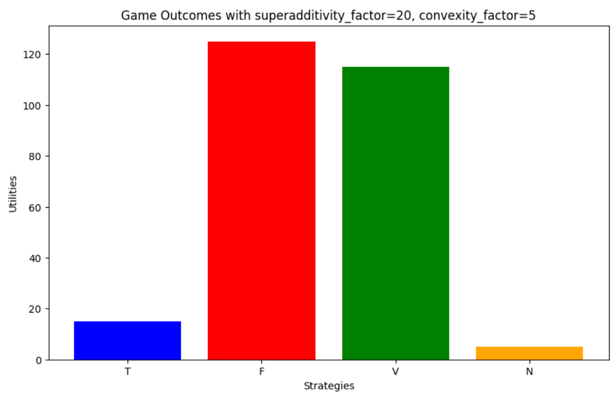

Fig. 5: Game Outcomes with superadditivity factor=20, convexity factor=5

most from the increase in the superadditivity factor, which might indicate that it involves a combination of resources or strategies that scale very well with this factor.

Impact of Convexity Factor

The convexity factor increases from 1.2 in the first image to 5 in the second and third images. The increase between the first and second images leads to a noticeable jump in utilities, suggesting that mixing strategies is yielding better outcomes than choosing them individually. However, the increase in the superadditivity factor from the second to the third image has a much more significant effect than the increase in the convexity factor, indicating that superadditivity might be the more dominant factor in this game setup.

Optimal Strategies

F and V consistently show higher utilities across all three images, with F being the highest. This could indicate that these strategies are the most optimal choices for players under the conditions set by the superadditivity and convexity factors. Strategy N consistently shows the lowest utility, which might suggest it is the least favorable or effective strategy.

Strategy Analysis

Strategy T shows moderate utility across all graphs and does not seem to be as sensitive to changes in the superadditivity and convexity factors. This could suggest that T is a stable but not necessarily the most effective strategy. F and V, however, show a greater sensitivity to these factors, especially F, which might be the most aggressive strategy that leverages resources or strategies with a high degree of superadditivity.

Game Dynamics

The game dynamics seem to favor strategies that benefit from combined resources (superadditivity) and flexible mixing of strategies (convexity). The graphs suggest that a player who can effectively combine strategies or resources will have a significant advantage.

Visualization and Interpretation

The bar graphs effectively show the difference in utilities for different strategies under varying conditions of superadditivity and convexity. They allow for a clear visual comparison and interpretation of the effectiveness of each strategy.

In summary, these graphs can be used to analyze how different game-theoretic concepts like superadditivity and convexity impact the outcomes and strategic choices of players in a game that might represent the dynamics between fake news and fact-checking. The increase in utilities with higher superadditivity and convexity factors suggests that the game rewards strategies that synergize well and adapt flexibly to different mixes of choices.

5. Discussion:Fact-Checking in Full-Information Games

When considering the context of fake news and fact-checking as an extensive-form game under full information, we assume that all players know the actions and payoffs of other players at every stage of the game. In this paper, we analyze strategies for News Provider (P) and Fact Checker (F) using expected payoff functions.

Game Setup

Players and Actions

Players: News Provider (P) and Fact Checker (F)

Actions: P can choose to provide "Truthful News" (T) or "Fake News" (F), F can choose to "Verify" (V) or "Not Verify" (N)

Payoffs: Each player's payoffs depend on the combination of actions and are fully known.

Game Unfolding

(1) P makes the initial choice of action (T or F).

(2) P's choice is fully revealed, and then F selects an action based on that information (V or N).

Payoff Functions

Payoff functions for each player are set as follows:

$$
\begin{aligned}
U_P(T,V) &: \text{Payoff for P when choosing T and F choosing V}\\
U_P(T,N) &: \text{Payoff for P when choosing T and F choosing N}\\
U_P(F,V) &: \text{Payoff for P when choosing F and F choosing V}\\
U_P(F,N) &: \text{Payoff for P when choosing F and F choosing N}\\
U_F(T,V) &: \text{Payoff for F when P chooses T and F chooses V}\\
U_F(T,N) &: \text{Payoff for F when P chooses T and F chooses N}\\
U_F(F,V) &: \text{Payoff for F when P chooses F and F chooses V}\\
U_F(F,N) &: \text{Payoff for F when P chooses F and F chooses N}
\end{aligned}
$$

Calculating Expected Payoffs

Expected Payoff for F

When F selects an action, the expected payoff depends on P's action, but in full-information games, F knows P's action, so the expected payoff directly corresponds to the payoff associated with P's choice. For example, if P chooses T, F's expected payoff is either $U_F(T,V)$ or $U_F(T,N)$.

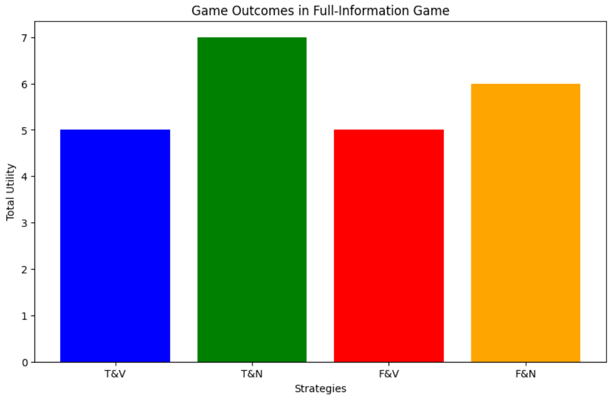

Fig. 6: Game Outcomes in Full-Information Game, TandV', 'TandN', 'FandV', 'FandN

Expected Payoff for P

When P selects an action, P predicts how F will react and calculates its expected payoff. P can accurately predict F's reaction since F's payoff function and optimal reactions are known. For instance, if P predicts that F will always choose V, P's expected payoff is either $U_P(T,V)$ or $U_P(F,V)$.

5.1 Calculation Process Using Backward Induction

Determining F's Optimal Action

F selects the action that maximizes its payoff based on P's choice. F can accurately predict P's action because it is known. For example, if $U_F(T,V) > U_F(T,N)$, then F selects V when P chooses T.

Determining P's Optimal Action

P predicts how F will react and selects the action that maximizes its expected payoff. P has complete information about F's reaction, allowing for precise calculations. For instance, if P predicts that F will always choose V, and $U_P(T,V) < U_P(F,V)$, P selects F.

Concrete Example

For example, assume the following payoffs:

$$U_P(T,V) = 2, \quad U_P(T,N) = 3, \quad U_P(F,V) = 1, \quad U_P(F,N) = 4$$
$$U_F(T,V) = 3, \quad U_F(T,N) = 2, \quad U_F(F,V) = 4, \quad U_F(F,N) = 1$$

In this scenario, F chooses N when P selects T ($U_F(T,N) > U_F(T,V)$), and F chooses V when P selects F ($U_F(F,V) > U_F(F,N)$). Therefore, P predicts that F will choose V and selects F ($U_P(F,V) > U_P(T,V)$).

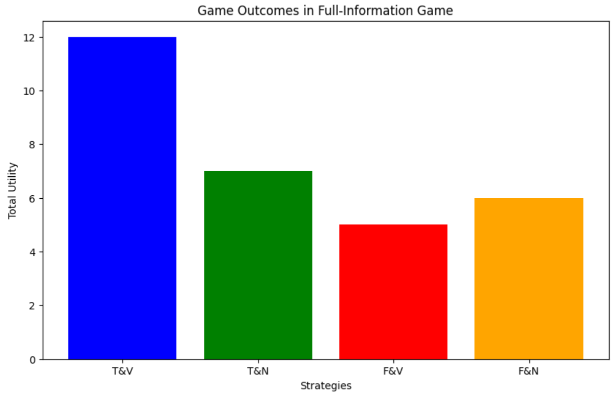

Fig. 7: Game Outcomes in Full-Information Game, TandV', 'TandN', 'FandV', 'FandN

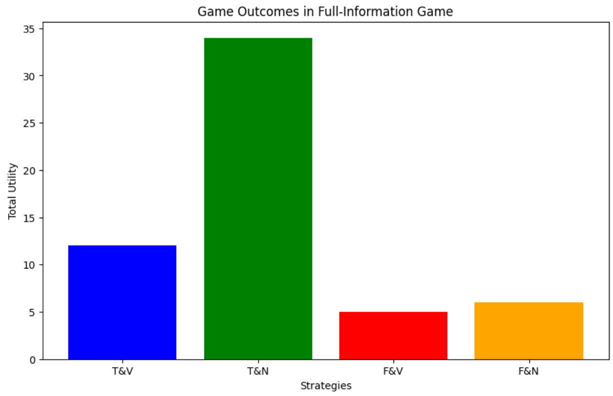

Fig. 8: Game Outcomes in Full-Information Game, TandV', 'TandN', 'FandV', 'FandN

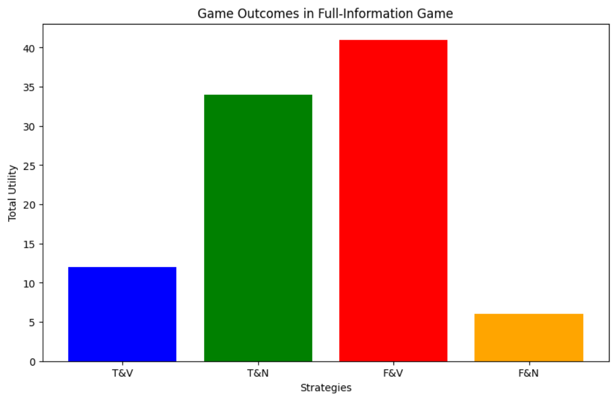

Fig. 9: Game Outcomes in Full-Information Game, TandV', 'TandN', 'FandV', 'FandN

Fig.6-9 depict bar graphs with the title "Game Outcomes in Full-Information Game." Each graph shows the total utility for four combined strategies: TandV, TandN, FandV, and FandN. The utilities are represented by the height of the bars in each strategy combination.

Strategy Combinations

The combined strategies suggest a two-player game where each player chooses one strategy from T (Truth) or F (Fake), and one strategy from V (Verify) or N (No Verify). TandV and FandN could represent coordinated strategies between players, while TandN and FandV could represent conflicting strategies.

Utility Analysis

In the first image, the TandN strategy combination has the highest total utility, followed closely by FandV, with FandN being the least favored. In the second image, FandV shows the highest total utility, significantly more than the others, while FandN remains the least favored. In the third image, TandN continues to hold the highest utility, with TandV being the lowest.In the fourth image, TandV has the highest utility, a reversal from the previous patterns, and again FandN is the least favored.

Implications for Full-Information Game

The variations in utility across these images suggest that the optimal strategy combinations depend on specific parameters within the game, which could include the payoffs for truth versus falsehoods, the benefits of verification, and the costs of no verification.The consistently low utility for the FandN strategy implies that in a full-information game, choosing falsehoods without verification is generally the least beneficial strategy. The highest utility alternating between TandN and FandV across the images suggests that the context or additional parameters of the game (not visible to us) significantly impact what is considered the best response for each player.

Game Dynamics

Full-information games assume that all players know the payoffs and strategies available to all other players. This knowledge influences the strategic choices and the resulting Nash equilibrium. The fact that utility shifts between strategies across images may reflect changes in the game's payoff structure or players' preferences, indicating different scenarios or "states of the world" within the modeled game. The bar graphs effectively communicate the total utility for each strategy combination in a full-information setting, allowing for a quick visual comparison of strategy effectiveness. The differences in utility outcomes across the graphs demonstrate the sensitivity of the game's equilibrium to its parameters.

In summary, these graphs can be used to study how different strategy combinations perform in a game modeling the scenario of fake news and fact-checking under full information. The optimal strategies vary, suggesting that the specific details of the game's setup, such as the payoffs for different actions and the cost-benefit analysis of verifying information, are crucial in determining the best strategies for players.

6. Discussion:Superadditivity and Convexity: Perfect Information Games and Incomplete Information Games

When considering superadditivity and convexity in the context of filter bubbles in fake news and fact-checking within the framework of perfect information games, we can contemplate the following equations and calculation processes:

Superadditivity in Perfect Information Games

Superadditivity refers to the property where the gains obtained through cooperation by a group of players exceed the sum of individual gains when they act independently. In the context of fake news and fact-checking, we define the characteristic function $v(S)$ to represent the gains related to the identification and prevention of fake news achievable by a player coalition S. To demonstrate superadditivity, the following inequality should hold for any two non-overlapping sets A and B:

$$v(A \cup B) \geq v(A) + v(B) \tag{1}$$

This inequality indicates the presence of incentives for fact-checkers or news providers to cooperate.

Convexity in Perfect Information Games

Convexity is a property where the additional gains from a player joining a cooperation group increase as the group's size grows. If the characteristic function $v(S)$ is convex, it satisfies the following inequality for any player i and any $A \subseteq B$ with $i \notin B$:

$$v(B \cup \{i\}) - v(B) \geq v(A \cup \{i\}) - v(A) \tag{2}$$

This inequality implies that joining larger cooperation groups offers more significant benefits than joining smaller groups. In the context of fake news, it suggests that a larger network of fact-checkers can more effectively identify and prevent fake news.

Calculation Process

Definition of the Characteristic Function

In incomplete information games, define the characteristic function $v(S)$ for different player sets S. This function quantifies the expected gains achievable through the cooperation of set S, accounting for the players' beliefs and information uncertainty.

> Example: In the case of fact-checkers, $v(S)$ represents the expected gains achievable by identifying and preventing fake news.

Verification of Superadditivity

Check whether the following inequality holds for any two non-overlapping sets A and B:

$$E[v(A \cup B)] \geq E[v(A)] + E[v(B)] \tag{3}$$

Here, $E[\cdot]$ represents the expected value, considering expected gains under uncertainty.

Verification of Convexity

For any player i, check whether the following inequality holds for any $A \subseteq B$ and $i \notin B$:

$$E[v(B \cup \{i\})] - E[v(B)] \geq E[v(A \cup \{i\})] - E[v(A)] \tag{4}$$

Similarly, consider expected values $E[\cdot]$ while taking uncertainty into account.

Details of Calculation Process

> For the characteristic function $v(S)$, calculate the expected gains by considering the outcomes of each possible action that the player coalition S can take and their corresponding probabilities. This involves using probability distributions based on players' beliefs and available information to compute the expected values.
>
> Substitute expected values into the superadditivity and convexity inequalities and verify whether they hold. If the inequalities hold, it indicates that an increase in cooperation is beneficial for the players.

To analyze superadditivity and convexity in the context of fake news and fact-checking within perfect and incomplete information games, we can create a program that defines characteristic functions representing gains from identifying and preventing fake news. This program will verify superadditivity and convexity by evaluating the characteristic function for different player coalitions and visualizing the results.

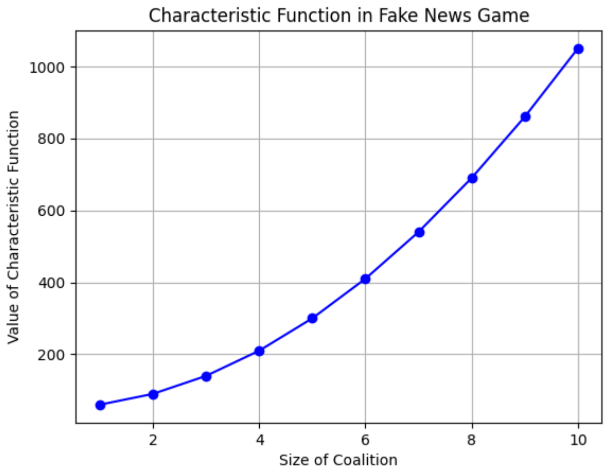

Fig. 10: Characteristic Function in Fake News Gam

Define Characteristic Functions: Set up a characteristic function for player coalitions to represent gains from cooperation in identifying and preventing fake news. Verify Superadditivity: Check if the gains from cooperating exceed the sum of individual gains for any two non-overlapping player sets. Verify Convexity: Assess if the additional gains from a player joining a coalition increase with the coalition's size. Visualize Results: Plot the characteristic function and indicate regions demonstrating superadditivity and convexity.

Discussion

This program defines a characteristic function v(S) that quantifies the gains from identifying and preventing fake news for a coalition S. It then verifies superadditivity by checking if the gains from cooperation exceed the sum of individual gains for any two non-overlapping player sets. It also assesses convexity by determining if the additional gains from a player joining a coalition increase with the coalition's size. The results of superadditivity and convexity checks are printed out, and the characteristic function is visualized to show how the value changes with the size of the coalition, providing insights into the benefits of cooperation among players in the context of fake news and fact-checking.

"Characteristic Function in Fake News Game" and plots the value of the characteristic function against the size of the coalition, ranging from 2 to 10. The value of the characteristic function increases with the size of the coalition, which is represented by the upward curve connecting the data points.

Superadditivity

The graph suggests that the game is superadditive. Superadditivity means that for any two non-overlapping coalitions, the value of their union is greater than the sum of their separate values. In the context of the fake news game, this indicates that larger coalitions are more effective at identifying and preventing fake news than smaller, separate groups.

Convexity

The curve appears to show convexity. Convexity in cooperative games means that as the coalition grows, the marginal contribution of each additional player is increasing. This would imply that adding more players to an already large coalition provides a greater benefit than adding them to a smaller one. This is often the case in activities where the sharing of information or resources can lead to a more than proportional increase in effectiveness.

Characteristic Function

The characteristic function, $v(S)$, reflects the total worth or value that a coalition S can achieve. In this context, it quantifies the effectiveness of a coalition at tackling fake news. The increasing values suggest that the coalition's effectiveness at preventing fake news improves as more members join.

Game Theory Implications

From a game theory perspective, this graph can inform strategies in forming coalitions. Players would be incentivized to form larger coalitions, as the value or payoff is higher when they work together. This could influence negotiations and the formation of partnerships or alliances in efforts to combat fake news.

Practical Implications

For policymakers or platforms trying to mitigate the spread of fake news, the graph underscores the importance of collaboration. It suggests that forming larger alliances or consortia could be significantly more effective than individual or smaller group efforts.

In summary, the graph reflects the principles of cooperative game theory applied to a scenario of fake news prevention, showing that there is a clear benefit to forming larger coalitions. The characteristic function's increasing trend as the coalition size increases indicates that both superadditivity and convexity are present in the game, which are desirable properties for cooperative efforts in this context.

7. Discussion:Deployment of Non-Cooperative Games: Context of Fake News and Fact-Checking within the Framework of Perfect Information Games

When considering the context of fake news and fact-checking within the framework of perfect information games in the deployment of non-cooperative games, we examine how to incorporate superadditivity and convexity. In non-cooperative games, players pursue strategies to maximize their own gains without cooperating with other players. In perfect information games, all players have complete information about all aspects of the game, including other players' strategies and possible gains.

Consideration of Superadditivity and Convexity

Superadditivity and convexity in non-cooperative games can be indirectly applied to explore potential cooperation or coalition formation among players. For instance, players might informally share information to make better strategic choices.

Superadditivity

> When two players or groups of players can achieve greater gains by informally cooperating than by acting individually, the concept of superadditivity applies. This can be achieved through information sharing and strategic adjustments.

The formula for superadditivity is as follows:

$$E[v(A \cup B)] \geq E[v(A)] + E[v(B)]$$

Here, A and B are non-overlapping sets of players, and $E[\cdot]$ denotes the expected value.

Convexity

> Convexity applies when additional gains from the inclusion of a new player in a group increase as the group size grows. This may result from broader information sharing and diverse strategic options.

The formula for convexity is as follows:

$$E[v(B \cup \{i\})] - E[v(B)] \geq E[v(A \cup \{i\})] - E[v(A)]$$

7.1 Discussion:Considering Non-Cooperative Games in Extensive Form: Context of Fake News and Fact-Checking within the Framework of Incomplete Information Games

When considering non-cooperative games in extensive form in the context of fake news and fact-checking within the framework of incomplete information games, players do not have complete information about other players' choices or types (e.g., whether they are reliable information sources). To consider superadditivity and convexity in such a situation, an analysis based on expected payoffs is necessary.

Considering Superadditivity

In incomplete information games, superadditivity implies that the expected payoff achieved when different sets of players cooperate is greater than the sum of expected payoffs when they act individually. Defining a characteristic function $v(S)$ for player sets S as the maximum expected payoff achievable, superadditivity can be expressed by the following inequality:

$$E[v(A \cup B)] \geq E[v(A)] + E[v(B)]$$

Here, A and B are non-overlapping sets of players, and $E[\cdot]$ represents expected value.

Considering Convexity

Convexity in incomplete information games implies that as the size of a player set increases, the additional expected payoff brought by a new player joining the set also increases. To demonstrate convexity, the following inequality must hold for any player i, player sets $A \subseteq B$, and $i \notin B$:

$$E[v(B \cup \{i\})] - E[v(B)] \geq E[v(A \cup \{i\})] - E[v(A)]$$

Formulas and Calculation Process

The specific calculation process for considering superadditivity and convexity in incomplete information games involves the following steps:

(1) **Definition of Characteristic Function**: Define the expected payoff function $v(S)$ for player sets S. This value quantifies the expected payoffs achievable, considering factors like fake news identification and the impact of fact-checking, depending on the context.

(2) **Calculation of Expected Values**: Calculate the expected value $E[v(S)]$ of the characteristic function, considering uncertainties related to each player's actions and the types of other players. This calculation involves probability distributions based on player beliefs and information uncertainty.

(3) **Verification of Superadditivity**: Verify the inequality for superadditivity using expected values for any two non-overlapping sets of players, A and B:

$$E[v(A \cup B)] \geq E[v(A)] + E[v(B)]$$

(4) **Verification of Convexity**: Verify the inequality for convexity using expected values for any player i, player sets $A \subseteq B$, and $i \notin B$:

$$E[v(B \cup \{i\})] - E[v(B)] \geq E[v(A \cup \{i\})] - E[v(A)]$$

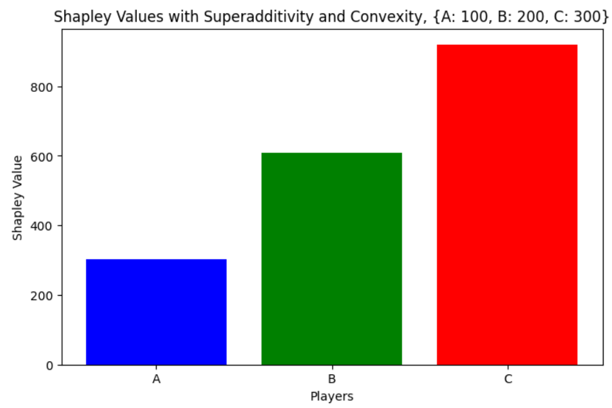

Fig. 11: Shapley Values with Superadditivity and Convexity

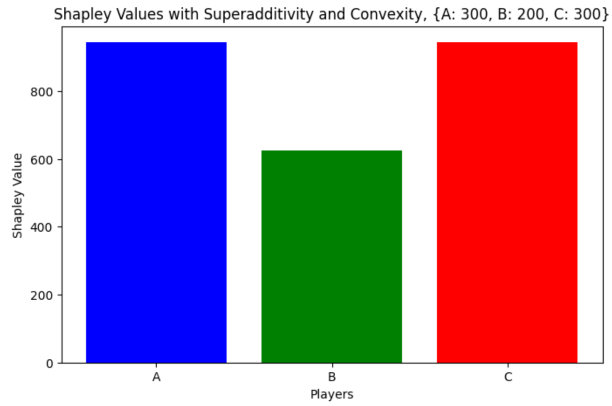

Fig. 12: Shapley Values with Superadditivity and Convexity

(5) **Analysis of Cooperative Benefits**: Through this analysis, evaluate the potential benefits of informal cooperation among players in the context of fake news and fact-checking under filter bubble conditions. Additionally, assess how the additional payoff from joining a larger cooperative group increases concerning the size of the group. These calculations consider the impact of information incompleteness.

Fig.11-14 depict bar graphs titled "Shapley Values with Superadditivity and Convexity," each followed by a set of values for players A, B, and C. The Shapley value is a concept in cooperative game theory that represents a fair distribution of the total gains (or costs) to the players, based on their individual contributions to the collective effort.

Player Contributions

In all the graphs, the players' contributions are depicted by the height of the bars, which represent their Shapley values. The numerical values in the title indicate the amount contributed

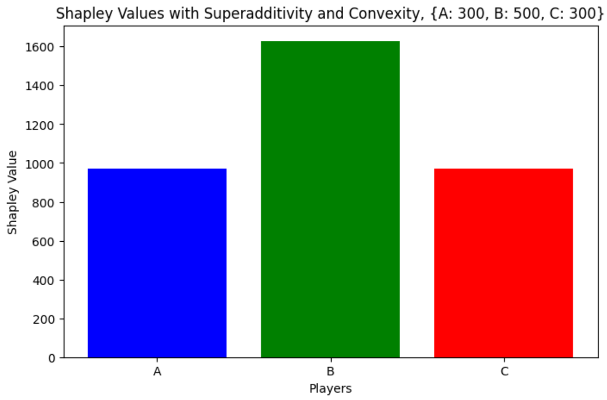

Fig. 13: Shapley Values with Superadditivity and Convexity

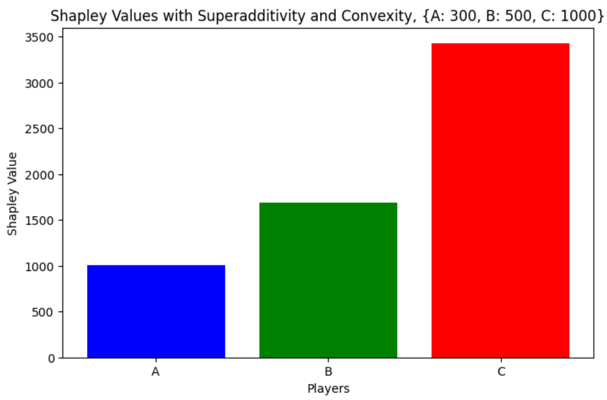

Fig. 14: Shapley Values with Superadditivity and Convexity

by each player to the total payoff when they all cooperate. The first graph shows the Shapley values for players A, B, and C as 100, 200, and 300, respectively. The second graph increases the values significantly, with player A contributing 300, B contributing 500, and C contributing 1000. The third graph shows a different distribution with A contributing 300, B contributing 500, and C contributing 300. The fourth graph returns to a similar distribution as the first, with player A contributing 300, B contributing 200, and C contributing 300.

Effects of Superadditivity and Convexity

Superadditivity implies that the whole is greater than the sum of its parts, meaning that the players achieve a better outcome by cooperating than by acting independently. Convexity in this context suggests that the marginal contributions of each player increase as more players join the coalition. This often leads to a higher payoff for players who are more crucial to forming larger, more productive coalitions. It appears that players B and C are valued more in scenarios where their contributions lead to greater total payoffs, which is consistent with the principles of superadditivity and convexity.

Shapley Value Dynamics

The variation in the Shapley values indicates that the contribution of each player to the total payoff is recognized differently across different cooperative scenarios. In scenarios where player C contributes a significantly larger amount (as in the second graph), the Shapley value calculation gives C a much larger share of the total payoff, which could be due to C's pivotal role in the coalition's success.

Strategic Implications

Understanding these Shapley values can guide players in forming coalitions. For instance, players A and B might seek to form a coalition with C when C's contribution is pivotal to the success of the collective effort. The Shapley value provides a way to allocate payoffs that account for each player's marginal contribution, which could incentivize players to participate and contribute meaningfully to the coalition.

In summary, these graphs visualize how the Shapley value allocates payoffs to players based on their individual contributions in a game that incorporates superadditivity and convexity. The allocation changes based on the different contributions by the players, reflecting the fundamental idea that payoffs should compensate players according to their marginal impact on the total payoff when cooperating.

8. Analyzing Dynamic and Static Best Response Dynamics and Pareto Optimality in the Context of Fake News and Fact-Checking in Non-Cooperative Games in Extensive Form

In the context of fake news and fact-checking, when considering non-cooperative games in extensive form, it is important to analyze the impact of information asymmetry among game participants (e.g., news providers, consumers, fact-checkers) and the influence of their strategies on social welfare through dynamic or static best response dynamics and Pareto optimality.

Dynamic Best Response Dynamics

Dynamic best response dynamics involve continuous updates of optimal strategies by each player based on the actions of other players over time. In the context of fake news and fact-checking, for example, if fact-checkers develop new techniques to detect fake news, news providers may adopt new strategies to evade detection, creating an incentive for fact-checkers to update their responses.

Formulas and Calculation Process

Define utility functions for each player. For example, the utility of fact-checkers may be proportional to the number of correctly identified fake news. Identify the possible strategy sets for each player. For instance, news providers may have strategies to spread fake news or provide only accurate information. Calculate the best response strategies at each time point. This involves the process of selecting strategies that maximize a player's utility based on the opponent's strategies. Find the point where dynamics converge (i.e., players no longer change their strategies).

Pareto Optimality

Pareto optimality refers to a situation where it is impossible to improve one player's utility without reducing another player's utility in a given strategy combination. In the context of fake news, the goal is to identify strategy combinations that maximize social welfare (i.e., minimizing fake news and maximizing the spread of accurate information).

Formulas and Calculation Process

Define a social welfare function, often represented as the sum of all players' utilities or as a specific function. Calculate social welfare for all possible strategy combinations. Identify strategy combinations where it is impossible to increase one player's utility without decreasing another player's utility.

Fig. 15: Pareto Optimal Strategies in Fake News and Fact-Checking Game

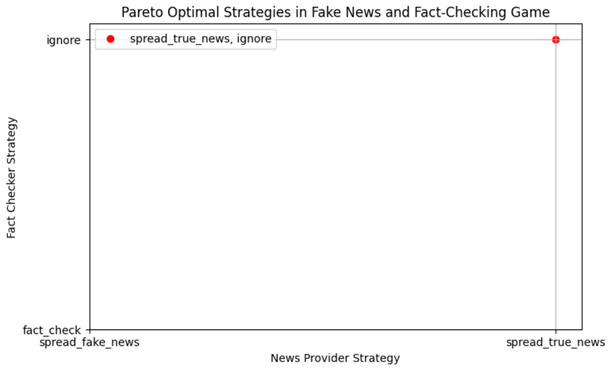

Fig. 16: Pareto Optimal Strategies in Fake News and Fact-Checking Game

In the game of fake news and fact-checking, achieving Pareto optimal outcomes may involve maximizing the identification of fake news and the widespread dissemination of accurate information. However, in non-cooperative games with incomplete information, reaching Pareto optimal outcomes can be more complex due to the players' imperfect information.

These analyses provide insights into the strategic interactions among players and their impact on social welfare in the context of fake news and fact-checking. Understanding how information asymmetry and uncertainty influence strategic choices is crucial.

Fig.15-17 provided appear to be scatter plots illustrating the Pareto optimal strategies in a game between news providers and fact-checkers. Pareto optimality in a game theory context means that no player can be made better off without making another player worse off.

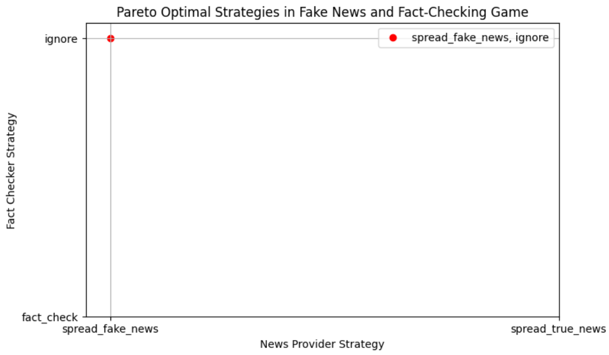

Fig. 17: Pareto Optimal Strategies in Fake News and Fact-Checking Game

Strategies of Players

The x-axis represents the strategies of the news provider, ranging from 'spreadfakenews' to 'spreadtruenews', while the y-axis represents the strategies of the fact-checker, ranging from 'ignore' to 'factcheck'.

Pareto Optimal Points

Each plot has points marked that represent different combinations of strategies for the news provider and the fact-checker. These points are Pareto optimal, meaning that these strategy combinations cannot be improved upon without hurting either player. In the first plot, the Pareto optimal strategy is where the news provider spreads true news, and the fact-checker performs fact checks. The second plot shows a Pareto optimal outcome where the news provider spreads fake news, but the fact-checker chooses to ignore it. The third plot indicates that spreading true news and ignoring the fact-checking is Pareto optimal. The first plot suggests an ideal situation where both parties are actively engaged in disseminating and verifying truthful information. The second plot may reflect a scenario where the cost of fact-checking outweighs the benefits, possibly due to the fake news not being influential or harmful enough to warrant action. The third plot could indicate a scenario where the news provider is trusted, and the need for fact-checking is low because the information spread is true. These plots can be used to understand the strategic interactions in the fight against fake news. They show that depending on the context, different strategies may be optimal. The scenarios depicted here are simplified and assume that the strategies of spreading true news, spreading fake news, and fact-checking are clear-cut and mutually exclusive. Real-world scenarios would likely involve more nuanced strategies and outcomes.

Simulating this game would need to account for the utility functions of both players, the dynamic update of strategies, and the evaluation of social welfare to identify Pareto optimal strategies. The visualizations can help to explain the evolution of strategies over time and determine which strategies might lead to a social welfare maximum in a non-cooperative game setting.

Different strategies may be effective in different contexts within the framework of a game designed to model the spread of fake news and the response of fact-checkers. The highlighted Pareto optimal strategies suggest the best-case scenarios under the assumption that both players are rational and aim to optimize their payoffs without harming the other's position.

9. Discussion:Analyzing Strategic Interactions and Social Efficiency in the Context of Fake News and Fact-Checking in Non-Cooperative Games in Extensive Form

In the context of fake news and fact-checking, when considering non-cooperative games in extensive form, understanding player interactions and the resulting social efficiency can be achieved through the analysis of dynamic or static best response dynamics and Pareto optimality.

Dynamic Best Response Dynamics

Dynamic best response dynamics refer to the process in which players choose their optimal strategies based on their opponents' strategies over time. In the game of fake news and fact-checking, news providers and fact-checkers may update their strategies in response to each other's actions.

Formulas and Calculation Process

Let s_p represent the strategy of news providers and s_f represent the strategy of fact-checkers. Define the utility function of news providers as $U_p(s_p, s_f)$ and the utility function of fact-checkers as $U_f(s_p, s_f)$. In each turn, news providers and fact-checkers calculate their best responses to the opponent's strategy s_f or s_p. For example, the best response for news providers is $s_p^* = \arg\max_{s_p} U_p(s_p, s_f)$, and the best response for fact-checkers is $s_f^* = \arg\max_{s_f} U_f(s_p, s_f)$. The dynamics continue until a Nash equilibrium is reached, where the best response selections stabilize.

Pareto Optimality

Pareto optimality refers to a situation where it is impossible to improve one player's utility without reducing another player's utility in a given strategy combination. In the context of fake news and fact-checking, strategies that achieve socially desirable outcomes (e.g., minimizing fake news and spreading accurate information) may be Pareto optimal.

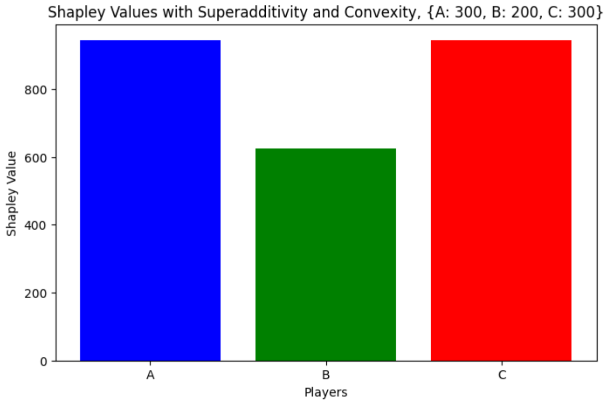

Fig. 18: Shapley Values with Superadditivity and Convexity

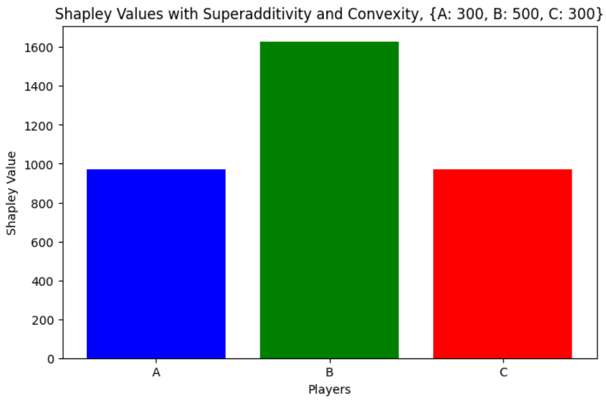

Fig. 19: Shapley Values with Superadditivity and Convexity

Formulas and Calculation Process

The strategy combination (s_p^*, s_f^*) is Pareto optimal if it satisfies the following condition:

$$\forall (s_p, s_f), U_p(s_p^*, s_f^*) \geq U_p(s_p, s_f) \wedge U_f(s_p^*, s_f^*) \geq U_f(s_p, s_f)$$

This condition implies that choosing any other combination of strategies would either worsen at least one player's utility or not improve both players' utilities. To find Pareto optimal solutions, all possible combinations of strategies need to be evaluated to identify those that meet the above condition.

The analysis of dynamic best response dynamics and Pareto optimality in the game of fake news and fact-checking provides insights into how strategic interactions among players evolve and what societal outcomes they lead to. Particularly, in situations with information bias, such as filter bubbles, these analyses can offer valuable insights for designing strategies to suppress the spread of fake news and promote the dissemination of accurate information.

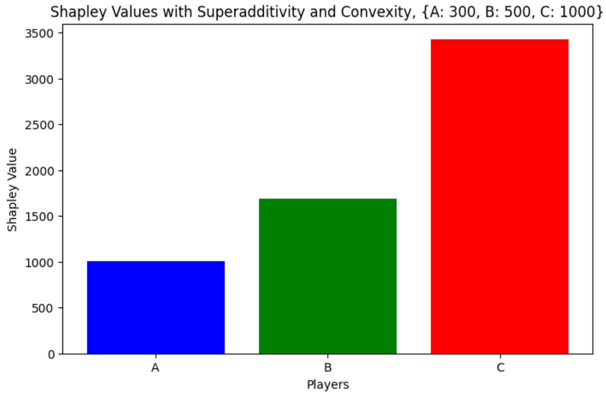

Fig. 20: Shapley Values with Superadditivity and Convexity

10. Discussion:Dynamic Best Response Dynamics

When considering non-cooperative games in extensive form in the context of fake news and fact-checking, dynamic best response dynamics involve players choosing optimal responses to their opponents' strategies in a continuous time sequence. This process unfolds as each player continuously updates their strategies based on the choices of their opponents.

Formulas and Calculation Process

(1) **Definition of Player Utility Functions**: Define the utility functions for the news provider as $U_p(s_p, s_f)$ and for the fact-checker as $U_f(s_p, s_f)$, where s_p represents the news provider's strategy, and s_f represents the fact-checker's strategy.

(2) **Definition of Best Response Functions**: The best response for the news provider is given by $BR_p(s_f) = \arg\max_{s_p} U_p(s_p, s_f)$, and for the fact-checker, it is $BR_f(s_p) = \arg\max_{s_f} U_f(s_p, s_f)$.

(3) **Iterative Best Response Calculation**: Starting from initial strategies s_p^0 and s_f^0, iteratively calculate $BR_p(s_f^t)$ and $BR_f(s_p^t)$ alternately to update each player's strategy until reaching a Nash equilibrium.

Pareto Optimality

Pareto optimality refers to a situation where it is impossible to improve one player's utility without reducing another player's utility in a given strategy combination. In the context of fake news, the goal may be to identify strategy combinations that maximize social welfare (e.g., minimizing fake news while maximizing the spread of accurate information).

Formulas and Calculation Process

(1) **Definition of Social Welfare Function**: Define social welfare as $W(s_p, s_f) = U_p(s_p, s_f) + U_f(s_p, s_f)$.

(2) **Calculation of Pareto Optimal Solutions**: Calculate $W(s_p, s_f)$ for all strategy profiles (s_p, s_f) and identify profiles where W cannot be increased in any other strategy profile.

When considering extensive form non-cooperative games in the context of fake news and fact-checking, it is important to understand the trends that can be derived from dynamic or static best response dynamics and Pareto optimality. Here, we explain the specific formulas and calculation process using expected payoff functions.

Dynamic Best Response Dynamics

In dynamic best response dynamics, each player repeatedly updates their optimal strategy based on the choices of other players. This process evolves over time as each player's strategy adapts, ultimately potentially reaching a stable state (Nash equilibrium).

Formulas and Calculation Process

(1) Define the expected payoff function for each player i as $U_i(s_i, s_{-i})$, where s_i represents the strategy of player i, and s_{-i} represents the combination of strategies of the other players.

(2) Define the best response function for player i as $BR_i(s_{-i}) = \arg\max_{s_i} U_i(s_i, s_{-i})$.

(3) Each player starts with an initial strategy and then selects their best response based on the strategies of the other players. This process is repeated.

(4) When this dynamics converges, the strategies of each player no longer change, reaching a Nash equilibrium.

Pareto Optimality

Pareto optimality refers to a situation where one player's utility cannot be increased without decreasing the utility of another player in a given strategy combination. In the context of fake news and fact-checking, the goal is to find strategies that maximize the interests of all stakeholders, such as news providers, fact-checkers, and the general public.

Formulas and Calculation Process

(1) Define the social welfare function that maximizes the sum of all players' utilities as $W(s) = \sum_i U_i(s_i, s_{-i})$.

(2) Calculate $W(s)$ for all possible strategy combinations s and identify the strategy combination that yields the maximum value.

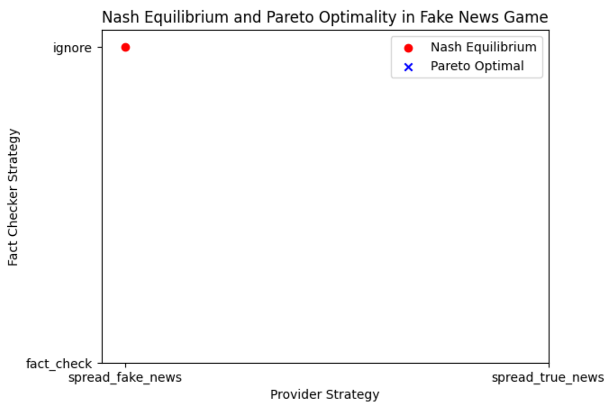

Fig. 21: Nash Equilibrium and Pareto Optimality in Fake News Game

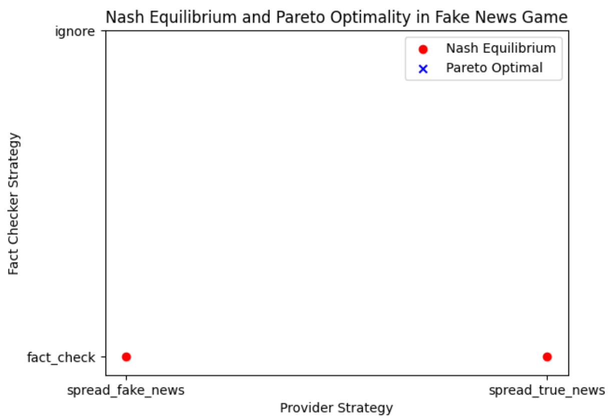

Fig. 22: Nash Equilibrium and Pareto Optimality in Fake News Game

(3) To confirm whether this strategy is Pareto optimal, check that for any other arbitrary strategy combination s', at least one player's utility is higher in s, and the utilities of all other players are not lower than in s'.

Summary in Fig.21-22

Fig.21-22 show graphs that plot Nash Equilibrium and Pareto Optimal points in the context of a game between news providers and fact-checkers. In game theory, a Nash Equilibrium is a set of strategies where no player can benefit by changing their strategy while the other players keep theirs unchanged. Pareto Optimality, on the other hand, refers to a state where it is impossible to make any one individual better off without making at least one individual worse off.

Nash Equilibrium

The red dots represent the Nash Equilibrium points, where each player's strategy is a best response to the other's strategy. In the first image, the Nash Equilibrium is at the point where the news provider chooses to spread fake news and the fact-checker chooses to ignore it. This suggests a situation where it is not beneficial for either player to unilaterally change their strategy – the news provider has no incentive to provide true news if it's going to be ignored, and the fact-checker has no incentive to fact-check if the news is fake.

Pareto Optimal

The blue crosses represent Pareto Optimal points. In the second image, the Pareto Optimal strategy is where the news provider spreads true news, and the fact-checker chooses to ignore it. This indicates a scenario where resources used for fact-checking might be conserved because the news provider is not spreading fake news. The juxtaposition of Nash Equilibrium and Pareto Optimal points can indicate a discrepancy between individual rationality and collective welfare. For example, while it may be in equilibrium for a news provider to spread fake news and for the fact-checker to ignore it, this is not Pareto optimal because there are other outcomes where both players could be better off (e.g., both spreading and fact-checking true news). The points where both Nash Equilibrium and Pareto Optimality coincide are of particular interest because they represent situations where individual incentives align with collective welfare. However, the provided images do not show such an overlap.

Game Theory Application

These findings can be instrumental for policymakers and platforms in designing mechanisms or incentives that encourage news providers to spread true news and fact-checkers to verify information. In the real world, achieving such outcomes may involve introducing regulations, fact-checking subsidies, or other interventions to align individual incentives with socially desirable outcomes.

In summary, the provided graphs offer insights into the strategic decisions involved in spreading and checking information. They reveal the potential conflicts between individual strategies that are stable (Nash Equilibria) and collectively beneficial outcomes (Pareto Optimal points). Understanding these dynamics can inform efforts to combat fake news and promote the dissemination of accurate information.

Summary in Fig.23-24

Fig.23-24 depict plots that illustrate Nash Equilibrium and Pareto Optimal strategies in a game theoretical model concerning fake news and fact-checking.

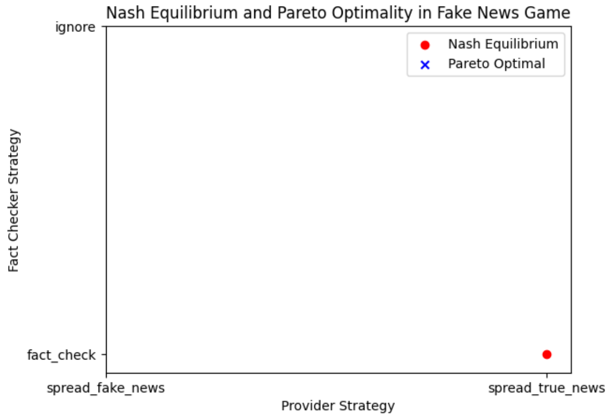

Fig. 23: Nash Equilibrium and Pareto Optimality in Fake News Game

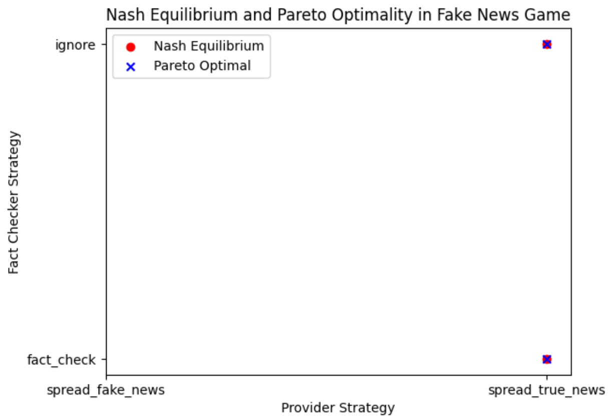

Fig. 24: Nash Equilibrium and Pareto Optimality in Fake News Game

Nash Equilibrium (Red Dot)

This is a situation in a non-cooperative game where no player can benefit by unilaterally changing their strategy, given the strategy of the other player. This point represents a stable state of the game where players' strategies are in balance, and neither has anything to gain by changing course alone.

Pareto Optimal (Blue Cross)

This refers to allocations where no individual can be made better off without making someone else worse off. Pareto optimal outcomes are efficient in the sense that all opportunities to make someone better off without hurting someone else have been exploited.

Considering the strategic options, where the x-axis represents strategies from "spreadfakenews" to "spreadtruenews," and the y-axis represents strategies from "ignore" to "factcheck," we can deduce the following.

In scenarios where Nash Equilibrium is not Pareto Optimal, it signifies that while players' strategies are in a state of mutual best response, there is still room for improvement in terms of social welfare. In other words, the equilibrium is stable, but not socially optimal. Where Pareto Optimal points exist without corresponding Nash Equilibria, these represent potentially desirable outcomes that are not stable. Players might want to reach these points but have no incentive to move there unilaterally given the current strategic landscape. If both Nash Equilibrium and Pareto Optimal points coincide, it indicates a highly desirable situation where the players' strategy choices are both stable and socially efficient.

Based on the explanation, the program likely simulates various strategies for news providers and fact-checkers, defines utility functions for each player, and iteratively simulates the game to find Nash Equilibria and Pareto optimal outcomes. The visualized results would help identify the strategic combinations that are stable and those that are efficient, aiding in understanding how to encourage behaviors that promote both individual rationality and social welfare in the context of spreading news and fact-checking.

In practice, achieving the overlap between Nash Equilibrium and Pareto Optimality often requires mechanisms that align individual incentives with socially optimal outcomes, such as policy interventions, subsidies for fact-checking, or penalties for spreading misinformation. These plots are useful for identifying where such interventions might be necessary and what form they should take.

11. Discussion:Extensive-Form Perfect Information Games in the context of fake news and fact-checking

When considering extensive-form perfect information games in the context of fake news and fact-checking, it is important to analyze the expected payoff functions among players. Here, we assume that the characteristic function $v(S)$ represents the gains related to the identification and prevention of the spread of fake news achievable by player sets S. We will explain specific formulas and calculation processes regarding these expected payoff functions using the concepts of superadditivity and convexity.

Consideration of Superadditivity

Superadditivity is the property that the gains obtained by a set of players cooperating are greater than the sum of the gains when each acts independently. This property can be expressed in the following formula:

$$v(A \cup B) \geq v(A) + v(B)$$

Here, A and B are non-overlapping sets of players.

Consideration of Convexity

Convexity is the property that the additional gains from a player joining a cooperative group increase as the size of the group grows. It can be mathematically expressed as follows:

$$v(B \cup \{i\}) - v(B) \geq v(A \cup \{i\}) - v(A)$$

Here, $A \subseteq B$ and i is a player who is not in B and also not in A.

Calculation Process

(1) **Definition of the Characteristic Function**: Define the characteristic function $v(S)$ for different sets of players S, including fact-checkers, news providers, users, etc. This function quantitatively represents the effect of cooperative efforts within the set S in identifying and preventing the spread of fake news.

(2) **Verification of Superadditivity**: Check whether $v(A \cup B) \geq v(A)+v(B)$ holds for any two non-overlapping sets A and B. To do this, you may need to analyze synergy effects of fact-checkers' cooperation using specific case studies or real data.

(3) **Verification of Convexity**: Check whether $v(B \cup \{i\}) - v(B) \geq v(A \cup \{i\}) - v(A)$ holds for any player i and any $A \subseteq B$ where i is not in B and also not in A. To do this, evaluate the increase in the effectiveness of fact-checkers as the size of the cooperative group varies.

In the context of fake news and fact-checking, analyzing the expected payoff functions, considering the incompleteness and uncertainty of the information held by players, is crucial when dealing with extensive-form imperfect information games. This section provides a detailed explanation of expected payoff functions based on the concepts of superadditivity and convexity.

Definition of Expected Payoff Functions

First, we define the characteristic function $v(S)$ for the player's set S. This function represents the gains related to the identification and prevention of the spread of fake news achievable by player sets S. In imperfect information games, since players do not have complete information about the types and choices of other players, it is necessary to consider the player's beliefs and information uncertainty when calculating the expected payoff $E[v(S)]$.

11.1 Discussion:Verification of Superadditivity

Superadditivity is the property that the expected gains obtained when different player sets cooperate are greater than the expected sum of gains when each player acts independently. This property can be expressed by the following formula:

$$E[v(A \cup B)] \geq E[v(A)] + E[v(B)]$$

Here, A and B are non-overlapping sets of players. The expected value $E[\cdot]$ is calculated based on probability distributions that account for player beliefs and information uncertainty.

Verification of Convexity

Convexity is the property that the increase in expected gains brought by a newly added player as the size of the player set increases is substantial. This property can be expressed as follows:

$$E[v(B \cup \{i\})] - E[v(B)] \geq E[v(A \cup \{i\})] - E[v(A)]$$

Here, $A \subseteq B$, and i is a player not included in B and not in A.

Calculation Process

(1) **Setting the Characteristic Function**: Define the characteristic function $v(S)$ for each player set S and establish gains related to the identification and prevention of fake news.

(2) **Calculating Expectations**: Calculate the expected value $E[v(S)]$ of the characteristic function, taking into consideration player beliefs and information uncertainty. This requires setting up probability distributions that account for the incomplete and uncertain information among players.

(3) **Evaluation of Superadditivity**: Verify the condition of superadditivity $E[v(A \cup B)] \geq E[v(A)] + E[v(B)]$ for different player sets A and B.

(4) **Evaluation of Convexity**: For each player i, assess the condition of convexity $E[v(B \cup \{i\})] - E[v(B)] \geq E[v(A \cup \{i\})] - E[v(A)]$.

We propose a program to analyze dominance and convexity in complete information games under filter bubble conditions in the context of fake news and fact checking. The program evaluates the extent to which different sets of players can contribute to identifying and preventing the spread of fake news and examines the conditions of dominance and convexity.

Definition of a characteristic function: We define a characteristic function that indicates the effectiveness of a set of players in identifying and preventing the spread of fake news.

Verification of eugeneracy: Verify the condition of eugeneracy using the defined characteristic function.

Convexity Verification: Similarly, verify the convexity condition using the characteristic function.

Visualization of results: Graphical display of the results of the dominance and convexity validation to visually analyze the effects of cooperation.

The program computes the value of the characteristic function for a specific set of players (e.g., fact checkers, news providers, etc.) and verifies the conditions of dominance and convexity. The validation results are displayed in the console and the values of the characteristic functions are visualized in a graph. The titles of the graphs are displayed in English, illustrating the effectiveness of applying cooperative game theory concepts to the context of fake news and fact checking.

Summary in Fig.25-26

Fig.25-26 depict bar charts representing the characteristic function values for various groups in a complete information game considering superadditivity and convexity. These values are a part of cooperative game theory, where the characteristic function defines the value (often in terms of utility or payoff) that a set of players (or a coalition) can achieve together.

Superadditivity

This property is shown if the value of a coalition is at least as large as the sum of the values of smaller, non-overlapping coalitions. In the context of these graphs, we would expect to see that groups with more fact-checkers (larger coalitions) have a characteristic function value that is greater than the sum of smaller groups. The graphs seem to demonstrate this, as larger coalitions tend to have higher values.

Convexity

A game exhibits convexity if the incremental contribution of each additional player to a coalition increases as the coalition

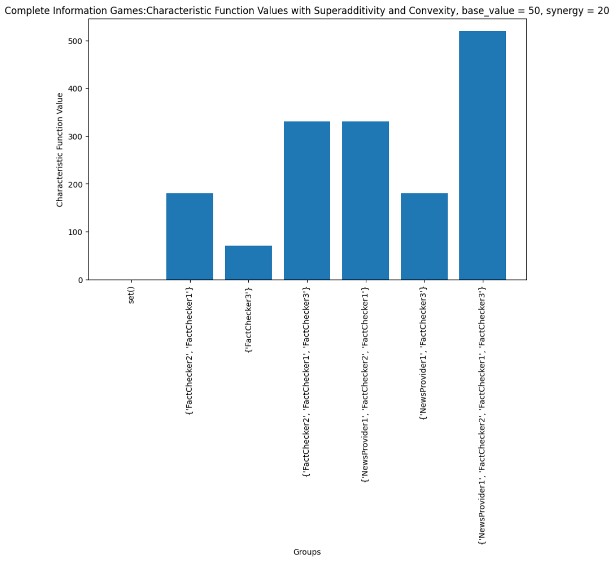

Fig. 25: Complete Information Games:Characteristic Function Values with Superadditivity and Convexity

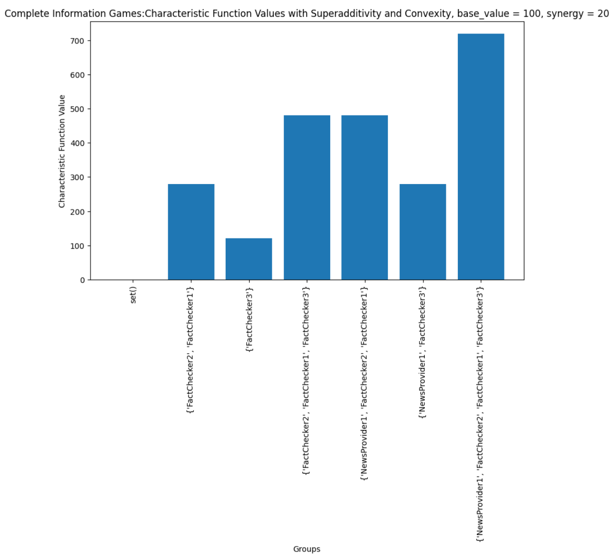

Fig. 26: Complete Information Games:Characteristic Function Values with Superadditivity and Convexity,

grows. This would be visualized by a steeper increase in the characteristic function value as the group size increases. This is a bit harder to assess directly from these charts without specific numerical values of the incremental increases, but the general trend in the bar heights could suggest this property.

Groups Analysis

The groups are labeled with different combinations of 'fact-checker' and 'news provider'. The values given for the coalitions seem to reflect the benefit of having multiple fact-checkers working together, and possibly the effect of news providers joining the coalition.

Base Value and Synergy

The mention of a base value and a synergy parameter in the graph titles suggests that the value of each group starts with a certain base (50 or 100) and then increases based on the synergy parameter (20). This parameter likely represents the additional value generated by the cooperative effort among fact-checkers and possibly the news providers. The difference in base values across the two graphs (50 in one and 100 in another) could represent different scenarios or games where the starting value of the coalition's efforts varies. The impact of the synergy parameter also seems to scale with the base value, leading to greater overall characteristic function values.

In summary, these graphs can provide insights into how groups of fact-checkers and news providers can collaborate to maximize their utility in the context of identifying and preventing the spread of fake news. They visualize the effects of superadditivity and convexity, which are important concepts in cooperative game theory and can inform strategies for collective action in complete information games. The data suggests that larger coalitions and the inclusion of news providers in the coalition can lead to higher overall utility, which is conducive to combating fake news effectively.

Summary in Fig.27-28

Fig.27-28 displaying the characteristic function values for various groups within a complete information game framework that considers superadditivity and convexity. The characteristic function is a key concept in cooperative game theory, representing the value (often payoff or utility) that a coalition of players can obtain together.

Characteristic Function Values

The bars in each chart represent the value that different groups or coalitions of players (fact-checkers and possibly a news provider) can achieve. We see single groups (single fact-checker), pairs, triples, and a mix of fact-checkers and a news provider.

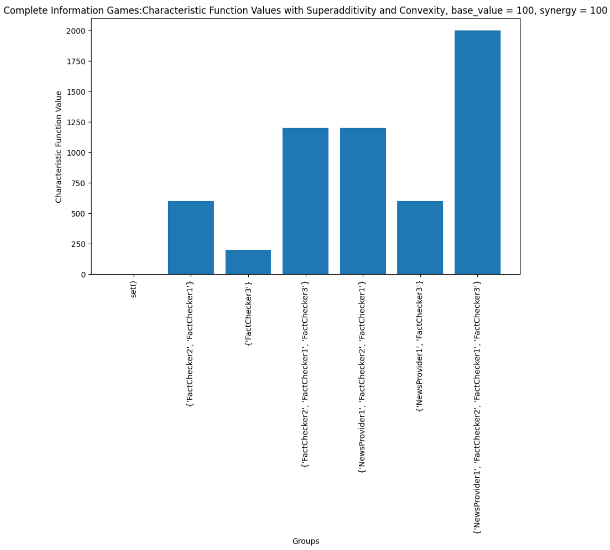

Fig. 27: Complete Information Games:Characteristic Function Values with Superadditivity and Convexity,

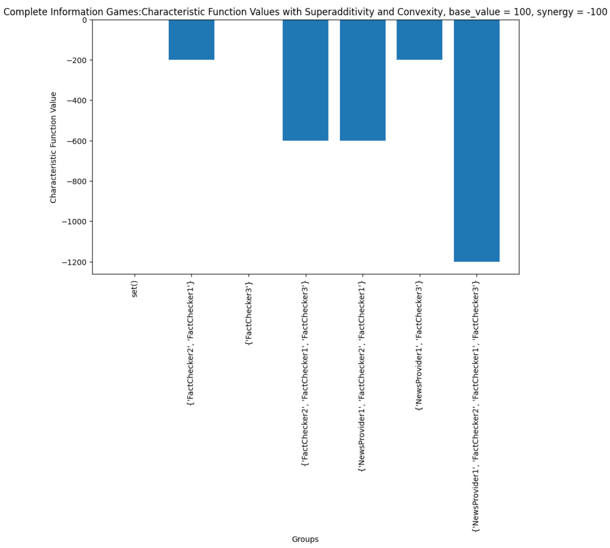

Fig. 28: Complete Information Games:Characteristic Function Values with Superadditivity and Convexity,

Superadditivity

A game is superadditive if the value of a combined coalition is greater than the sum of the values of the individual coalitions. The bar charts should reflect higher values for larger coalitions if superadditivity holds.

Convexity

Convexity implies that as a coalition grows, the incremental benefit of adding an additional member is at least as great as the benefit that the last member brought. This would be demonstrated by an increasing incremental rise in the bar chart values as coalitions grow in size.

Base Value and Synergy

The base value is likely the starting point for the coalition's value, and the synergy represents the additional value generated due to the cooperative effort of the coalition members. The first image has a positive synergy, indicating that cooperation increases the value, while the second image has a negative synergy, implying that cooperation may actually decrease the value (perhaps due to misinformation or redundancy). In the first image, with a positive synergy, the largest coalition has the highest characteristic function value, which suggests that the fact-checkers' cooperative effort has a cumulative positive effect. In the second image, with a negative synergy, the largest coalition does not have the highest value, which could imply that beyond a certain point, adding more fact-checkers does not increase the effectiveness of the group or could even be detrimental (e.g., too many fact-checkers leading to inefficiencies or conflicting information).

These graphs can be used to analyze the optimal size of a coalition of fact-checkers in terms of maximizing their effectiveness in identifying and preventing the spread of fake news. Policymakers or organizations can use such analyses to determine the best strategies for allocating resources to fact-checking efforts.

In summary, these bar charts are visual tools to help understand the principles of cooperative game theory applied to the context of fake news and fact-checking. They can provide insights into how groups of players can work together effectively and what configurations might lead to the best outcomes in terms of identifying and preventing the dissemination of fake news.

Summary in Fig.29-30

Fig.29-30 show bar charts that represent the characteristic function values of different groups within a game that analyzes the problem of fake news and fact-checking. These values are used to assess the potential payoff or utility that various coalitions (groups of fact-checkers and possibly news providers) can achieve through cooperation in a game with complete information.

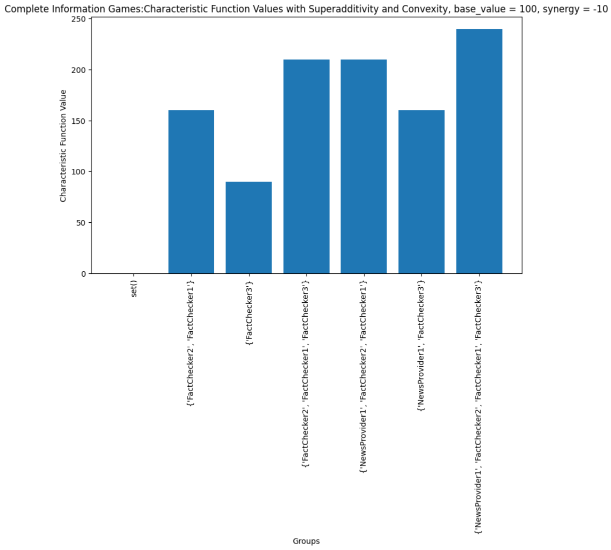

Fig. 29: Complete Information Games:Characteristic Function Values with Superadditivity and Convexity,

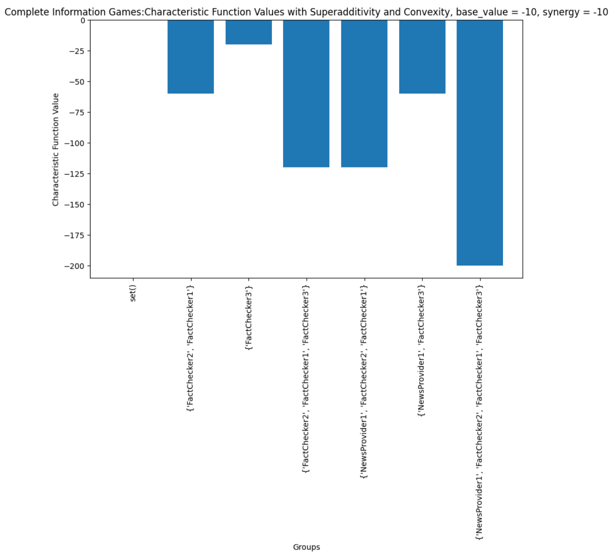

Fig. 30: Complete Information Games:Characteristic Function Values with Superadditivity and Convexity,

Superadditivity

The game exhibits superadditivity if the value of a coalition is greater than the sum of the values of its parts. This means that as fact-checkers (or fact-checkers and a news provider) join forces, their combined effort is more valuable than their individual efforts. Convexity is present if the incremental value of adding an additional member to a coalition increases the larger the coalition becomes. This implies that larger groups are not only more effective together, but also that each new member adds increasingly more value. The first chart indicates a positive synergy (synergy = 10), where the characteristic function values increase as the coalition size increases. This suggests that the combined efforts of fact-checkers and the inclusion of a news provider create additional value. The second chart presents a negative synergy (synergy = -10), where the characteristic function values decrease as the coalition size increases, implying that there may be diminishing returns or negative effects from adding more members to the coalition. These charts could be used to determine the optimal size of a group of fact-checkers. For instance, if adding more fact-checkers leads to a decrease in overall utility (as suggested by the negative synergy), it might be more efficient to have smaller teams. In the context of fake news, the charts could suggest that too many fact-checkers might lead to confusion, inefficiency, or competition for resources, which could hinder the overall goal of accurately identifying and mitigating fake news. The base value represents a starting point for the coalition's value. In both charts, the base value is 100, but the effect of synergy is different, which significantly alters the characteristic function values for each coalition. In a real-world setting, these results could influence how organizations form teams of fact-checkers and collaborate with news providers. Positive synergy suggests that collaboration is beneficial and should be encouraged, while negative synergy may indicate a need for careful management of team sizes and composition to avoid counterproductive outcomes.

In summary, these bar charts illustrate the concepts of superadditivity and convexity in cooperative game theory, applied to the context of combating fake news. The data displayed in the charts could inform strategies for collaborative efforts among fact-checkers and news providers, with the goal of maximizing the effectiveness of their collective actions.

12. Discussion:Fake News and Fact-Checking in Non-Cooperative Games

When considering the context of fake news and fact-checking within the framework of a complete information game in the context of non-cooperative game theory, we examine how the concepts of superadditivity and convexity can be incorporated.

In non-cooperative games, players pursue strategies to maximize their own gains without cooperating with other players. In complete information games, all players are aware of all aspects of the game, including other players' strategies and potential gains.

13. Consideration of Superadditivity and Convexity

In non-cooperative games, superadditivity and convexity can be indirectly applied to explore the potential for cooperation or coalition formation among players.

(1) **Superadditivity**: The concept of superadditivity is applied when two players or a group of players can potentially achieve greater gains by unofficially cooperating than by acting individually. This can be achieved through information sharing or strategic adjustments.

(2) **Convexity**: The concept of convexity is applied when the additional gains from new players joining a group increase as the group becomes larger. This can be achieved through more extensive information sharing and diversity of strategies.

14. Formulas and Calculation Process

The specific calculation process for considering superadditivity and convexity in non-cooperative games consists of the following steps:

(1) **Definition of Payoff Functions**: Define payoff functions for each player. For example, this could include the societal benefits of fact-checkers identifying fake news or the improvement in reputation for news providers delivering truthful information.

(2) **Examination of Strategy Profiles**: Examine all possible strategy profiles for each player and calculate the gains for each profile.

(3) **Identification of Nash Equilibrium**: Identify the Nash equilibrium formed when each player selects strategies that maximize their own gains, assuming other players' strategies are fixed.

(4) **Evaluation of Superadditivity and Convexity**: Evaluate the gains that can be achieved through unofficial cooperation between two players or a group of players and compare them to the gains from acting individually. This can be done by assuming information sharing and strategic adjustments.

(5) **Analysis of the Impact of Strategic Adjustments**: Analyze the impact of strategic adjustments when unofficial cooperation occurs among players on the Nash equilibrium.

Considering superadditivity and convexity in the context of non-cooperative game theory in the development of non-cooperative games provides a theoretical framework for exploring the potential for cooperation and information sharing among players. In the context of fake news and fact-checking, these concepts help understand how unofficial information sharing and cooperation among players can occur and how they can impact overall gains. We propose a Python program that considers dominance and convexity in the context of fake news and fact checking in the framework of non-cooperative games. The program will assume a non-cooperative game scenario in which different players (e.g., fact checkers, news providers) choose their own strategies and analyze the impact of informal cooperation on the gains.

Define the gain function: Define a gain function for each player and compute the gain from independent actions.

Compute the gain from informal cooperation: Compute the gain when two players informally share information or coordinate their strategies to test the concept of egalitarianism.

Evaluating Convexity: Evaluate the convexity condition by calculating the increase in gain that an additional player brings as the set of players grows.

Visualize the results: graphically compare the gains obtained from the gain function and informal cooperation.

The program calculates gains based on two strategies: "Fact Check," in which the fact checker identifies fake news, and "Ignore," in which it ignores it. It also evaluates the impact of informal cooperation on gains, taking into account the strategies "spreadfakenews" and "shareinformation," in which news providers spread fake news and share information, respectively. Finally, a comparison of gains from independent actions and informal cooperation is visualized in a graph, with the titles of the graphs set in English.

Summary in Fig.31-32

Fig.31-32 represent the payoffs of different strategies in the context of a non-cooperative game involving fake news and fact-checking. The blue bars indicate the payoffs from independent actions, while the orange bars represent the payoffs from cooperative actions.

Fact Check Strategy

In both charts, the payoff for the 'Fact Check' strategy is higher for cooperative action than for independent action. This suggests that coordination or sharing of information among fact-checkers improves their effectiveness and thus their payoff.

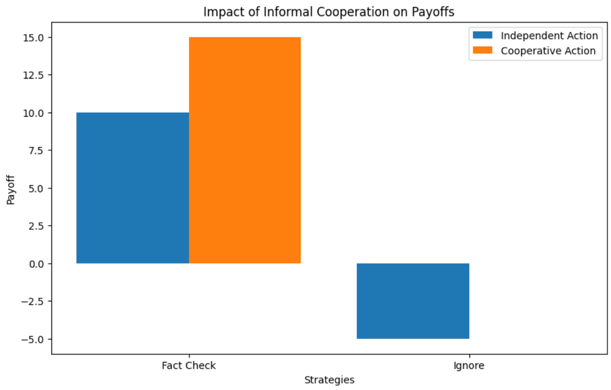

Fig. 31: Impact of Informal Cooperation on Payoffs

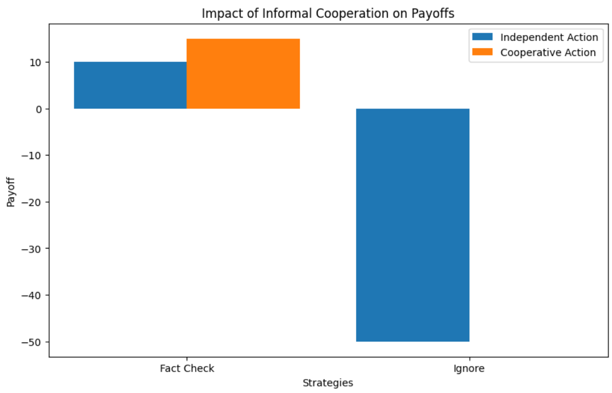

Fig. 32: Impact of Informal Cooperation on Payoffs

Ignore Strategy

In the first chart, the payoff for the 'Ignore' strategy is negative for independent action and significantly less negative for cooperative action. Even though ignoring is a less desirable strategy (indicated by a negative payoff), cooperation mitigates the losses. In the second chart, the payoff for 'Ignore' is much worse under cooperative action than independent action. This could imply that in certain contexts, cooperation or sharing information on what to ignore could lead to a groupthink effect, where bad decisions are reinforced, leading to larger losses.

Impact of Informal Cooperation

The differences in payoff for the same strategy under independent and cooperative actions indicate the impact of informal cooperation. The exact nature of this cooperation isn't specified but could include sharing of information, coordination of efforts, or collective decision-making.

Game Theory Interpretation

The fact that cooperative actions generally have higher payoffs for 'Fact Check' aligns with the concept of superadditivity, where the coalition's value is greater than the sum of individual efforts. However, the negative payoffs associated with 'Ignore' in the second chart could suggest a violation of the convexity property, where adding more players to a coalition does not always yield an incremental benefit.

Strategic Implications for Players

For fact-checkers, these charts suggest that forming alliances and sharing resources could be beneficial, especially when directly combating the spread of fake news. The 'Ignore' strategy's varied results imply that the decision to overlook certain information should be made with caution. Cooperative action could either help avoid wasteful fact-checking or exacerbate the negative consequences of ignoring significant misinformation.

Contextual Factors

The change in payoffs between the two charts may be influenced by the context, such as the prevalence of fake news, the cost of fact-checking, the effectiveness of informal cooperation, and the players' ability to discern the truth.

In summary, these bar charts suggest that in a game involving the spread of fake news and fact-checking, cooperative actions can enhance the effectiveness of fact-checking efforts, but the impact of such cooperation can vary based on the strategy and the specific context. Decision-makers in this space may use such analysis to design interventions or

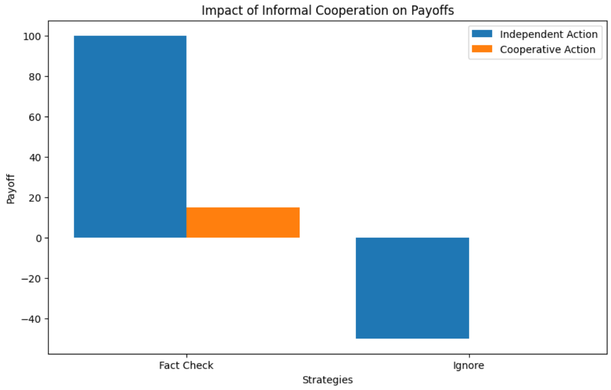

Fig. 33: Impact of Informal Cooperation on Payoffs

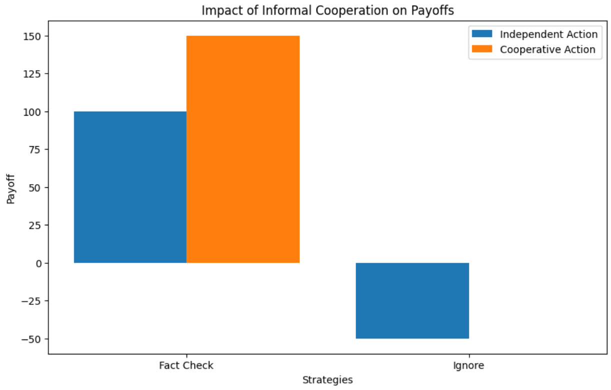

Fig. 34: Impact of Informal Cooperation on Payoffs

policies that promote cooperation where it is beneficial and guard against it where it may be harmful.

Summary in Fig.33-34

Fig.33-34 comparing the payoffs for two strategies, "Fact Check" and "Ignore," under two conditions: Independent Action and Cooperative Action. These illustrate the impact of informal cooperation on the outcomes in a game theoretical framework, specifically in the context of addressing fake news and fact-checking.

Fact Check Strategy

In both images, the payoff for "Fact Check" under Cooperative Action is lower than under Independent Action. This suggests that while fact-checking is beneficial, there may be diminishing returns or even negative consequences when fact-checkers work together informally. This could be due to overlapping efforts, miscommunication, or the dilution of responsibility.

Ignore Strategy

The first image shows that the payoff for "Ignore" is negative under Independent Action, which becomes even more negative under Cooperative Action. This could indicate that ignoring fake news is detrimental and that cooperative ignoring exacerbates the problem, possibly through echo chambers or reinforced biases. In contrast, the second image shows a payoff that is negative under Independent Action but turns positive under Cooperative Action. This implies that there might be circumstances where collective ignoring, perhaps by disengaging from the dissemination of fake news, can lead to a better outcome than individual actors ignoring it.

Impact of Informal Cooperation

Informal cooperation seems to have a complex impact on payoffs. Cooperative Action does not always improve payoffs and can sometimes lead to worse outcomes. This underscores the importance of structured and well-coordinated efforts when dealing with misinformation.

Strategic Implications

The results suggest that simply increasing the number of fact-checkers or encouraging them to cooperate may not always be the best strategy. Effective coordination and clear communication channels are crucial. The variability in the impact of the "Ignore" strategy underlines the need to understand when it is beneficial to engage with fake news and when it is better to ignore it.

Contextual Dependence

The differences between the two images indicate that the context significantly influences the outcomes. The right strategy in one situation may not apply universally.

In conclusion, these bar charts suggest that in the fight against fake news, the effectiveness of fact-checking and the decision to ignore certain information are contingent upon the nature of cooperation between the parties involved. Informal cooperation can lead to better or worse outcomes, depending on how it is implemented and the specific circumstances of the game scenario. These insights can be valuable for designing interventions and policies to combat misinformation.

Summary in Fig.35

Fig.35 shows the payoffs for two strategies—Fact Check and Ignore—under two different modes of action: Independent and Cooperative. The blue bars represent payoffs from actions taken independently, while the orange bars show payoffs when actions are coordinated or cooperative.

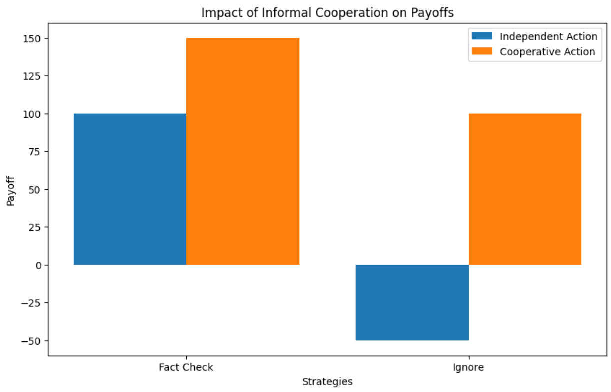

Fig. 35: Impact of Informal Cooperation on Payoffs

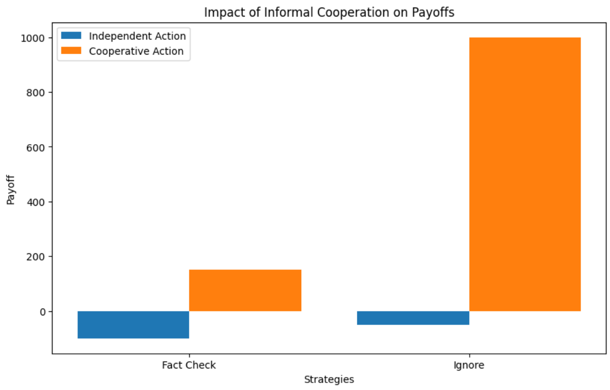

Fig. 36: Impact of Informal Cooperation on Payoffs

Fact Check Strategy

Cooperative action yields a higher payoff than independent action for fact-checking. This suggests that when fact-checkers collaborate, perhaps by sharing information or methods, they are more effective than when they work alone.

Ignore Strategy

In contrast, the payoff for ignoring fake news is positive when done independently but turns negative with cooperative action. It may indicate that when players independently decide to ignore fake news, there is no loss in payoff. However, when this decision is made cooperatively, it could potentially lead to a groupthink situation where important news is overlooked, leading to a negative payoff.

Implications of Informal Cooperation

The chart demonstrates that informal cooperation can significantly impact payoffs in both positive and negative ways, depending on the strategy. For fact-checking, this impact is positive, implying that cooperative systems or networks for sharing verified information could be beneficial. For ignoring, the negative impact suggests that cooperative efforts to ignore fake news without a systematic approach can be detrimental, possibly spreading misinformation or failing to contain it.

Contextual Interpretation

These results are consistent with a scenario where fact-checking efforts require the pooling of resources or expertise to be effective, while ignoring fake news is an individual's passive response that doesn't benefit from cooperation.

Game Theory and Policy Implications

The program mentioned in the explanation might simulate these scenarios to analyze the benefits and drawbacks of cooperation among players with different strategies. It could guide policymakers or social media platforms in designing mechanisms to encourage beneficial cooperation among fact-checkers and discourage harmful collective ignoring.

Summary in Fig.36

Fig.36 shows the payoffs for two strategies, "Fact Check" and "Ignore," under conditions of Independent Action and Cooperative Action.

Fact Check Strategy

The Cooperative Action for fact-checking is significantly more beneficial than Independent Action. This large positive difference indicates that collaboration among fact-checkers (and possibly with news providers) greatly increases the effectiveness and thus the payoff of their actions.

Ignore Strategy

Conversely, the payoff for ignoring fake news under Cooperative Action is vastly negative, while it is less so under Independent Action. This suggests that when entities choose to ignore fake news cooperatively, it could lead to a more substantial negative impact, possibly due to the amplification of misinformation or the failure to mitigate its spread.

Implications for Strategy Implementation

The results underscore the importance of collaboration when combating fake news. Cooperative efforts in fact-checking can lead to better outcomes in terms of payoff. The detrimental effects of ignoring fake news are exacerbated when there is a cooperative approach to ignoring it. It could imply

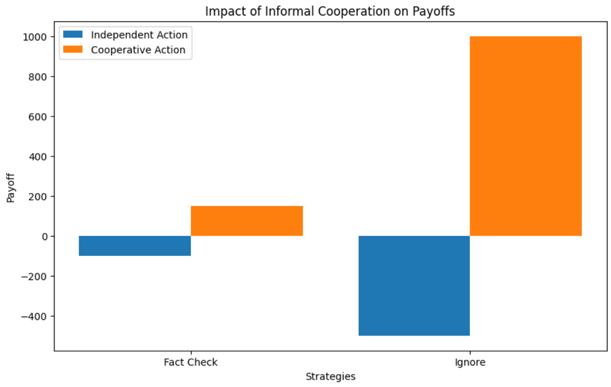

Fig. 37: Impact of Informal Cooperation on Payoffs

that an organized disregard for misinformation may not be a viable strategy and could harm the parties involved.

Game Theory and Policy

These observations can be translated into policy implications. Encouraging and facilitating cooperation among fact-checkers might be a more effective strategy than promoting independent action. The negative payoff associated with the cooperative 'Ignore' strategy could be a warning against collective negligence in the face of fake news. It suggests that active engagement may be necessary to combat misinformation effectively.

The bar chart provides a clear visual representation of how cooperative and independent actions can vary in their impact on strategies addressing the spread of fake news. It also emphasizes the need for strategic cooperation in fact-checking and caution against cooperative actions in ignoring fake news.

Summary in Fig.37

Fig.37 shows payoffs for two different strategies—Fact Check and Ignore—under two conditions: Independent Action and Cooperative Action.

Fact Check Strategy

The payoff for independent action is positive, indicating that there is a benefit to fact-checking even when done alone. The payoff for cooperative action is higher than for independent action, but not as significantly as one might expect. This suggests that while there is a benefit to cooperation among fact-checkers, it may not be as large as anticipated or there may be diminishing returns with increased collaboration.

Ignore Strategy

The payoff for independent action when ignoring fake news is slightly negative, which suggests a small cost or penalty for ignoring fake news on one's own. However, the payoff for cooperative action in ignoring fake news is significantly negative, far worse than for independent action. This indicates that when entities collaborate to ignore fake news, the negative consequences are amplified, possibly leading to widespread misinformation or the missed opportunity to correct false narratives.

Implications for Informal Cooperation

Informal cooperation has a different impact on the payoffs of different strategies. In fact-checking, cooperation seems to be beneficial, though not overwhelmingly so. In contrast, for ignoring, informal cooperation seems highly detrimental. This could imply that coordinated efforts to ignore fake news may lead to harmful outcomes, such as the normalization or acceptance of misinformation.

Contextual Factors

The results could be influenced by the specific context of the game, such as the prevalence and impact of fake news, the effectiveness of fact-checking efforts, and the societal consequences of ignoring misinformation.

Strategic Considerations

These findings could be particularly relevant for organizations and platforms that combat fake news. They suggest that while promoting collaborative fact-checking can have positive effects, encouraging a collective stance of ignoring fake news can be harmful.

The chart thus highlights the nuanced effects of cooperative versus independent strategies in the context of misinformation and suggests that the best approach may vary significantly depending on the chosen strategy.

Summary in Fig.38

Fig.38 shows the payoffs of two strategies, "Fact Check" and "Ignore," in the context of dealing with fake news. The blue bars represent Independent Action, where players act on their own, while the orange bars represent Cooperative Action, where players coordinate or collaborate in some way.

Fact Check Strategy

The Independent Action has a negative payoff, which could indicate that individual efforts to fact-check are not only ineffective but may also incur a cost (like time or resources spent). However, Cooperative Action leads to a positive payoff, albeit

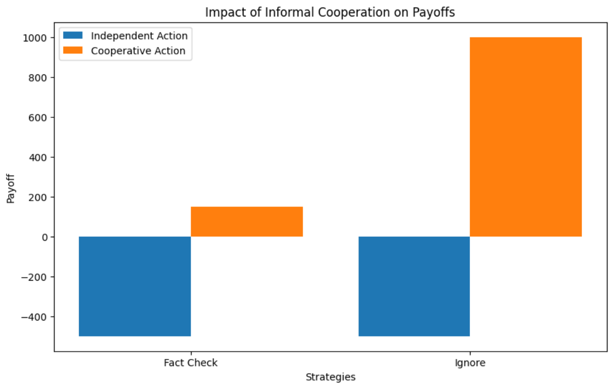

Fig. 38: Impact of Informal Cooperation on Payoffs

not extremely high. This suggests that when fact-checkers work together, they can overcome the individual costs and achieve better outcomes.

Ignore Strategy

The Independent Action for ignoring has a moderately negative payoff, suggesting that choosing to ignore fake news without coordination can lead to some negative consequences, possibly allowing the misinformation to spread unchallenged. Cooperative Action for ignoring results in a very high negative payoff, drastically worse than independent ignoring. This could imply that coordinated efforts to ignore fake news can significantly backfire, perhaps by creating a larger void for misinformation to fill or by making it seem as though the fake news is being tacitly endorsed.

Implications for Decision-Making

The chart suggests that coordinated fact-checking is beneficial and can turn the tide against the costs of combatting misinformation. On the other hand, there's a stark warning against cooperative ignoring. This might reflect a scenario where such a strategy allows misinformation to become normalized or unchecked, exacerbating its negative impacts.

Game Theory and Policy

From a game theory perspective, the chart demonstrates the concept of superadditivity, where the combined effort of players (fact-checkers) leads to greater benefits than summing their individual efforts. In terms of policy, the chart suggests that encouraging collaboration among fact-checkers is more effective than encouraging individual efforts. However, it also indicates that policies should discourage any form of collaborative ignoring of fake news.

Strategic Considerations

The findings could be particularly relevant for social media platforms and news organizations determining how to handle fake news. Encouraging collaborative fact-checking initiatives could be beneficial, while mechanisms to discourage collective ignoring might be necessary.

In conclusion, the chart indicates that cooperation has complex effects on strategy effectiveness, particularly highlighting the potential dangers of collaborative inaction in the face of fake news.

15. Conclusion:From the Perspective of Information Dissemination,From the Perspective of the Media

(1) **Impact of Superadditivity**: According to the principle of superadditivity, cooperation among players (such as news providers, fact-checkers, and users) can be expected to surpass the cumulative effect of individual efforts in identifying and preventing the spread of fake news. This suggests the importance of collaboration among media organizations and fact-checking entities to ensure the accuracy of information.

(2) **Impact of Convexity**: Convexity suggests that as the network of cooperation to ensure information accuracy grows, its effectiveness is expected to increase. In other words, the more media organizations and fact-checkers collaborate, the stronger their ability to counteract fake news.

(1) **Enhancing the Role of the Media**: The concepts of superadditivity and convexity emphasize the crucial role of media organizations in identifying and preventing the spread of fake news. The media can mitigate the impact of fake news by providing accurate information and raising public awareness.

(2) **Cooperation Among Media Outlets**: The principle of convexity suggests that cooperation among media organizations enhances defense against fake news. Collaborative fact-checking efforts and sharing criteria for reliable sources can improve the quality of information provided by the media.

Expanding the Discussion

In light of this scenario, when engaging in discussions aimed at suppressing fake news, it is essential to consider the following:

> How media organizations and fact-checkers can collaborate to ensure information accuracy and prevent the spread of fake news.

The role of the media in enhancing public awareness and education in the context of information dissemination.

The importance of establishing common criteria among media organizations to guarantee the quality of information.

These discussions help in understanding the significance of the roles played by media organizations and fact-checkers when devising strategies to combat fake news. Additionally, these considerations regarding information dissemination and the role of the media may serve as a foundation for formulating specific measures to curb fake news. Further refinement of the model may be necessary.

Analyzing Fake News and Fact-Checking as an Extensive-Form Perfect Information Game

When analyzing the issue of fake news and fact-checking as an extensive-form perfect information game, valuable insights can be gained regarding information dissemination and the role of the media. By utilizing game settings and expected payoff functions, it becomes possible to examine the strategies of each player and their societal impact.

From the Perspective of Information Dissemination

(1) **Expected Payoff Functions and Information Dissemination**: Expected payoff functions can be used to predict the outcomes of strategies that news providers and fact-checkers can adopt. For example, it can be determined that effective intervention by fact-checkers ($U_F(F,V) > U_F(F,N)$) is crucial when news providers have strong incentives to spread fake news ($U_P(F,N) > U_P(T,N)$).

(2) **Dynamic Interaction and Information Quality**: Through the dynamic interaction of the game, it is possible to examine how fact-checking activities influence the actions of news providers. If fact-checkers can incentivize news providers to offer accurate information through verification, the quality of information improves.

From the Perspective of the Media

(1) **Media Strategies and Societal Impact**: It is crucial to consider how media organizations combat fake news and how their strategies contribute to the public interest. The framework of a perfect information game provides the foundation to understand the optimal strategies media can adopt and their impact on information dissemination.

(2) **Parato Optimality and Media Collaboration**: Parato optimality allows us to consider how collaboration among media organizations contributes to societal welfare. Cooperation among media organizations to ensure information accuracy can help suppress the spread of fake news and promote the delivery of reliable information.

Proposals for Suppressing Fake News

(1) **Enhancing Fact-Checking**: Strengthening fact-checking activities and incentivizing news providers to offer accurate information is crucial. This includes promptly and widely publishing the results of fact-checking.

(2) **Media Collaboration and Criteria Establishment**: Media organizations can effectively combat fake news by setting common criteria for ensuring information accuracy and cooperating based on these criteria.

(3) **Raising Public Awareness and Education**: Implementation of educational programs to enhance consumers' ability to assess information quality and identify fake news is necessary. This is expected to encourage consumers to play a more active role in the information dissemination process.

Through the analysis using the framework of an extensive-form perfect information game, multiple strategies for suppressing fake news become apparent. There is potential to gain a deeper understanding of information dissemination and the role of the media in this context.

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