Discussion Paper:(1)Case Study of Decision Making and Urban Planning:Loop Flip Errors and Phase Transitions in Local Potential Approximation in Non-trivial Loops(2024), (2)Case Study of Spatio-Temporal Approaches To Syndrome Errors in Decision Making with Applications Such as Bit Phase Flip Errors and Methods Discussion(2024)

Scheme:Inter-Connectivity of Prisoner's Dilemma to Expected Payoff Functions in Cournot Model Application in Information Quality Competition of Fake News

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Abstract: Again, in this paper, when considering noncooperative games broadly in the framework of incomplete information games in the context of fake news and fact checking, players do not have complete information about the choices and types of other players (e.g., whether they are reliable sources of information). An analysis based on expected payoffs is needed. In an era dominated by the rapid spread of both information and misinformation, the strategic decisions made by news providers regarding the quality of their content have far-reaching implications for society. This paper introduces a novel approach by integrating the Cournot model, traditionally used for analyzing quantity competition in oligopolistic markets, with the complex dynamics of prisoner's dilemma and non-complete information games in the realm of media. We focus on the pivotal role of expected payoff functions in shaping the strategies of news providers, amidst the dual challenges of combating fake news and ensuring rigorous fact-checking. The adaptation of the Cournot model to accommodate the qualitative aspect of information, coupled with the strategic uncertainty inherent in non-complete information scenarios, lays the groundwork for a comprehensive analysis. By delving into the intricacies of expected payoff functions and their influence on the strategic choices of news providers, this study aims to shed light on the emergent competitive equilibria and the potential for cooperative outcomes that align with societal welfare. Through this exploration, we seek to unravel the conditions under which news providers might find themselves in a prisoner's dilemma, forced to choose between short-term gains and the long-term trust and reliability of their content. Through a computational experiment informed by numerical simulations, this study aims to uncover the Nash equilibria that define the competitive equilibrium among news providers and to scrutinize the conditions under which the societal optimum—Pareto optimality—is achievable. In doing so, we illuminate the potential prisoner's dilemma scenarios that may arise, offering profound insights into the optimal strategies news providers can adopt to enhance public welfare in the face of fake news and fact-checking challenges.

Keywords: Game Theory, Ultimatum Game, Cournot Model, Prisoner's Dilemma, Bertrand Competition, Fake News, Fact-Checking, Non-Complete Information Game, Nash Equilibrium, Pareto Optimality, Information Quality, Media Strategy, Public Interest

1. Introduction

In this paper, we introduce the Cournot model and the prisoner's dilemma for further discussion. When considering noncooperative games in the context of fake news and fact-checking broadly within the framework of incomplete information games, players do not have complete information about the choices and types of other players (e.g., whether they are reliable sources of information). An analysis based on expected payoffs is needed. We propose a computational experiment using the Cournot model to analyze the competitive dynamics of information quality among news providers under the framework of noncomplete information games, with a particular focus on the interaction between fake news and fact checking. By adapting the traditional Cournot model, which is often used to analyze quantity competition in oligopolistic markets, to the media context, we explore the strategic behavior of news providers and its impact on public welfare. The model considers news providers as players who strategize about the level of quality of information they provide, which in turn affects public welfare. The expected payoff function is defined in terms of information quality, taking into account both the market's valuation of information quality and the cost of producing such quality. Through numerical simulations, we aim to derive Nash equilibria and examine conditions for

Pareto optimality to highlight potential prisoner's dilemma scenarios. While the Cournot model has traditionally been used to analyze quantitative competition in oligopolistic markets, it provides a unique framework for examining "qualitative competition" among news providers in the context of fake news and fact checking. In this study, we extend the Cournot model to the setting of a noncomplete information game, where news providers, acting as players, develop strategies over the quality of the information they disseminate. The main objective is to explore how such strategies affect public welfare and the emergence of a prisoner's dilemma scenario. By conceptualizing information quality as a quantifiable variable, we establish a payoff function for each news provider based on the quality of information in the overall market, the public's assessment of this quality, and the associated production costs. Through this computational experiment, we hope to provide insight into the strategic decisions that news providers must make regarding information quality and their impact on the public good, given the pervasive challenges posed by fake news and the critical role of fact checking.

2. Experimental Plan: Analysis of Fake News and Fact-Checking in the Context of Incomplete Information Games

In the context of incomplete information games, analyzing fake news and fact-checking presents various challenges for both the media (news providers) and recipients (information receivers). These challenges primarily stem from information gaps and communication deficiencies. This research proposal suggests conducting computational experiments on fake news and fact-checking in incomplete information games based on models such as the Cournot model, the Prisoner's Dilemma, the Ultimatum Game, and Bertrand Competition. The objective is to analyze competition regarding the quality of information among news providers and its impact on the public interest.

2.1 Challenges for the Media (News Providers)

2.1.1 Assessing the Authenticity of Information

Media outlets may struggle to accurately assess the authenticity of information, especially when there is a lack of trustworthy information sources or when false information is cleverly crafted.

2.1.2 Bias in Reporting

Individual media outlets with specific perspectives or ideologies may unintentionally or intentionally introduce bias into their reporting, making it challenging for recipients to fully trust the information.

2.1.3 Competitive Pressure

Competition among media outlets can incentivize the provision of sensational and attention-grabbing content, which may lead to bending the truth or disseminating unverified information.

2.1.4 Challenges for Recipients (Information Receivers)

2.1.5 Evaluating the Credibility of Information Sources

Recipients may struggle to distinguish information from various sources and determine which sources are credible. The oversaturation of information and a lack of authenticity further complicate this challenge.

2.2 Cognitive Bias

Individuals tend to be drawn towards information that reinforces their existing beliefs or opinions. This confirmation bias facilitates the spread of misinformation, including fake news.

2.2.1 Information Overload

Information overload increases the risk of recipients missing important information or accepting inaccurate information as truth.

2.2.2 Dilemmas and Nash Equilibrium

In such situations, media outlets and recipients make interdependent strategic decisions. Media outlets anticipate the expectations and reactions of recipients, while recipients speculate about the intentions and credibility of media outlets. This interdependence leads to the formation of dilemmas and Nash equilibria.

2.2.3 Dilemma

Media outlets aim to build trust by providing high-quality information, but the temptation of market competition and sensational content can hinder this goal. On the other hand, recipients seek reliable information but face the risk of accepting misinformation due to information overload and cognitive bias.

2.2.4 Nash Equilibrium

The Nash equilibrium in this game is a state where media outlets and recipients choose their optimal strategies, and there are no gains to be made by changing their strategies. In reality, this equilibrium may involve media outlets pursuing a certain level of sensationalism, and recipients maintaining a certain level of skepticism. This suggests that the problem of fake news may not be completely resolved, and some form of misinformation may persist in society.

2.2.5 Analysis of Incomplete Information Games

The analysis of such incomplete information games can help us understand the strategic interactions between media outlets and recipients. It may also assist in developing more effective approaches to address the issues of fake news and fact-checking.

2.2.6 Players

Player A: News provider A

Player B: News provider B

2.3 Strategies

Level of information quality q_A for Player A

Level of information quality q_B for Player B

2.4 Payoff Functions

News provider A: $\Pi_A(q_A, q_B) = P(Q)q_A C(q_A)$

News provider B: $\Pi_B(q_A, q_B) = P(Q)q_B C(q_B)$

Here, $Q = q_A + q_B$ represents the total quality of information in the market, $P(Q)$ is the public evaluation of information quality, and $C(q)$ is the cost of providing information quality.

2.4.1 Expected Payoff Functions

News provider A: $E[\Pi_A] = P(E[Q])q_A C(q_A)$

News provider B: $E[\Pi_B] = P(E[Q])q_B C(q_B)$

Here, $E[Q]$ is the expected value of the total information quality in the market.

2.4.2 Research Methodology

2.4.3 Maximization of Payoff Functions

Each news provider derives the information quality levels q_A and q_B that maximize their respective expected payoff functions.

2.4.4 Calculation of Nash Equilibrium

The Nash equilibrium is calculated from the optimal response functions of both news providers, determining the information quality levels.

2.4.5 Analysis of Pareto Optimality and the Prisoner's Dilemma

A comparison is made between the situation where news providers cooperate to provide high-quality information and the situation under non-cooperation strategies to examine the Prisoner's Dilemma scenario.

In this computational experiment, numerical analysis software is used to solve the maximization problem of the stated payoff functions and expected payoff functions. Numerical simulations are performed to determine Nash equilibria and Pareto-optimal states.

2.4.6 Expected Outcomes

Through this computational experiment, we expect to gain insights into what strategies news providers should adopt regarding information quality and how these strategies may impact the public interest. Additionally, we hope to derive results that can serve as the basis for policy recommendations to prevent the spread of fake news.

3. Non-Complete Information Games and Game Theory Developments

In this paper, we will first organize some game theory solution examples and the computational experimental design of the development of the application to this problem. First, we will discuss the prisoner's dilemma.

3.1 Prisoner's Dilemma

The Prisoner's Dilemma is a fundamental model in game theory that situations where individual rational choices lead to collectively irrational outcomes.

3.2 Game Setup

Players: Two prisoners, A and B
Strategies: Confess (Defect, D) or Remain Silent (Cooperate, C)

3.3 Payoff Table

The payoffs in the Prisoner's Dilemma are typically represented in a table like the one below:

$A\backslash B$	Cooperate (C)	Defect (D)
Cooperate (C)	$(-1,-1)$	$(-3,0)$
Defect (D)	$(0,-3)$	$(-2,-2)$

Here, the numbers in each cell represent the first number as Player A's payoff and the second number as Player B's payoff.

3.4 Mathematical Model

Let U_A be the payoff function for Player A, and U_B be the payoff function for Player B. This game can be expressed mathematically as follows:

$$U_A(C,C) = U_B(C,C) = -1 \tag{1}$$
$$U_A(C,D) = -3, \quad U_B(C,D) = 0 \tag{2}$$
$$U_A(D,C) = 0, \quad U_B(D,C) = -3 \tag{3}$$
$$U_A(D,D) = U_B(D,D) = -2 \tag{4}$$

3.5 Calculation Process

In the Prisoner's Dilemma, it is common to find the Nash equilibrium, a state where no player can increase their payoff by changing their strategy.

3.5.1 Player A's Optimal Strategy

Assuming that B cooperates, the optimal strategy for A is to defect ($U_A(D,C) > U_A(C,C)$). Similarly, if B defects, the optimal strategy for A is also to defect ($U_A(D,D) > U_A(C,D)$).

3.5.2 Player B's Optimal Strategy

Assuming that A cooperates, the optimal strategy for B is to defect ($U_B(C,D) > U_B(C,C)$). Similarly, if A defects, the optimal strategy for B is also to defect ($U_B(D,D) > U_B(D,C)$).

Therefore, the Nash equilibrium in this game is both A and B defecting (D, D).

Through this calculation process, it is demonstrated that in the Prisoner's Dilemma, individual rational choices (defecting) lead to collectively irrational outcomes (both players defecting and receiving lower payoffs).

3.6 Prisoner's Dilemma in the Context of Fake News

In the context of fake news and fact-checking, we can consider the Prisoner's Dilemma as an incomplete information game, where news providers have uncertain information about whether their counterparts will provide fake news or report the truth. This game aims to explore what strategies each news provider should adopt under this uncertainty through a computational experiment.

3.7 Game Setup

- Players: News provider A and B - Strategies: Provide truthful information (C: Cooperation) or provide fake news (D: Defection)

3.8 Payoff Table

In an incomplete information game, each player's payoffs depend not only on their opponent's strategy but also on their own beliefs about that strategy. Let's consider the following payoff table, but actual payoffs may vary based on player beliefs:

$A\backslash B$	Cooperate (C)	Defect (D)
Cooperate (C)	$(3,3)$	$(0,5)$
Defect (D)	$(5,0)$	$(1,1)$

3.9 Expected Payoff Functions

The expected payoff for Player A depends on the probability p that they believe B will cooperate (C) and the probability $1-p$ that they believe B will defect (D):

When A cooperates: $E[U_A(C)] = 3p + 0(1-p) = 3p$
When A defects: $E[U_A(D)] = 5p + 1(1-p) = 5p + 1 - p$

A similar expected payoff function can be established for Player B, assuming that B believes A will cooperate with probability q.

3.10 Calculation Process

1. Deriving Optimal Strategies for Each Player: Find the strategies that maximize the expected payoffs for each player. For example, if Player A believes that B is likely to cooperate (p is high), there is a higher probability that $E[U_A(D)] > E[U_A(C)]$, favoring the defection strategy. 2.Identifying Nash Equilibrium: Look for strategy combinations where both players have chosen their optimal strategies based on their beliefs about the opponent's strategy. This is the Nash equilibrium. 3. Updating Beliefs: In practice, players can obtain additional information about their opponent's strategy during the game, so it's essential to consider the process of updating beliefs p and q.

Applying this analysis to the context of fake news and fact-checking can help us understand how news providers make decisions about their strategies and under what conditions they may engage in cooperative or non-cooperative actions. Additionally, considering the process of belief updating, insights into improving information quality and the effectiveness of policies to prevent the spread of fake news can be gained.

4. Non-Full information game and Prisoner's Dilemma in the context of Fake News and Fact-Checking

In the context of Fake News and Fact-Checking in a non-full information game, let's consider the application of the concepts of superadditivity and convexity to the Prisoner's Dilemma using formulas and the calculation process. The Prisoner's Dilemma illustrates a classic game theory scenario where cooperation would benefit the overall outcome, but individual incentives promote non-cooperation.

4.1 Game Setup

Players: News Provider (Player 1) and Fact-Checker (Player 2).

Strategies: Cooperation (C) or Non-Cooperation (D). For News Provider, cooperation means providing true news, while non-cooperation means providing fake news. For Fact-Checker, cooperation means fact-checking, and non-cooperation means not fact-checking.

4.2 Utility Functions

We define utility functions as follows:

$U_1(C, C)$: News Provider cooperates.

$U_1(C, D)$: News Provider cooperates, Fact-Checker doesn't

$U_1(D, C)$: News Provider doesn't cooperate, Fact-Checkers

$U_1(D, D)$: News Provider doesn't cooperate.

$U_2(C, C)$: Fact-Checker cooperates.

$U_2(C, D)$: Fact-Checker cooperates, News Provider doesn't

$U_2(D, C)$: Fact-Checker doesn't cooperate, News Provider cooperates

$U_2(D, D)$: Fact-Checker doesn't cooperate.

4.3 Application of Superadditivity and Convexity

4.3.1 Superadditivity

Superadditivity refers to the property where the gains from cooperation are greater than the sum of individual gains achieved by acting independently. In this game, superadditivity can be expressed through the following inequality:

$$U_1(C, C) + U_2(C, C) >$$
$$\max(U_1(C, D) + U_2(C, D), U_1(D, C) + U_2(D, C))$$

4.3.2 Convexity

Convexity refers to the property where the additional gains from the cooperation of more players increase significantly. In this game, assuming that the highest social welfare is achieved when all players cooperate, convexity conditions are automatically satisfied.

(1) **Calculating payoffs for each strategy**: Based on the given utility functions, compute the payoffs for all possible combinations of strategies.

(2) **Verifying superadditivity**: Use the above superadditivity inequality to determine if cooperation yields a better outcome than individual actions.

(3) **Identifying Nash Equilibrium**: Each player selects a strategy that maximizes their utility, considering the strategies chosen by other players. If this selection is a best response to others' choices, the combination of strategies is a Nash equilibrium.

Through this calculation process, we can understand players' strategic choices and their consequences in the context of a non-full information game concerning Fake News and Fact-Checking. The concepts of superadditivity and convexity are employed to emphasize the importance of cooperation and its societal benefits.

5. Full information game and Prisoner's Dilemma in the context of Fake News and Fact-Checking

In the context of fake news and fact-checking, we will explore the application of the concepts of superadditivity and convexity to the prisoner's dilemma problem when considering both complete information games and non-cooperative games in an extensive form. In this scenario, the primary players are news providers (referred to as Player A) and fact-checkers (referred to as Player B).

6. Game Setup

6.1 Players and Strategies

Player A can disseminate true news (T) or fake news (F).

Player B can choose to verify (V) or not verify (N) the news.

Both players' payoffs depend on each other's choices, but all information is known at all stages of the game (complete information game).

6.2 Payoff Table

The following table shows the payoffs based on the combinations of actions by each player. The values are based on assumptions.

AB	Verify (V)	Not Verify (N)
True (T)	(3, 2)	(2, 1)
Fake (F)	(-1, 3)	(4, -1)

Here, each payoff pair is interpreted as (Player A's payoff, Player B's payoff).

6.3 Verification of Superadditivity

Superadditivity refers to the property where the gains from cooperation are greater than the sum of individual gains achieved by acting independently. However, in this case, since we are dealing with a non-cooperative game, the direct application of superadditivity is challenging. Instead, the goal is to find the combination of strategies where both players maximize their benefits (for example, Player A providing true news, and Player B verifying).

6.4 Verification of Convexity

Convexity refers to the property where the additional gains from the cooperation of more players increase significantly as the size of the player set grows. However, in this scenario, there are only two players, and directly applying the concept of convexity is not possible.

6.5 Relation to the Prisoner's Dilemma

When likening this scenario to the prisoner's dilemma, the choice of both Player A and B not to cooperate (Player A providing fake news, and Player B not verifying) could lead to undesirable outcomes (the spread of fake news), similar to the situation in the prisoner's dilemma where both prisoners pursuing self-interest results in the worst possible outcome.

6.6 Identification of Nash Equilibrium

Find combinations where each player's strategy is the optimal response to the other player's strategy. In this example, (F, N) could be a Nash equilibrium.

6.7 Maximization of Social Welfare

Examine combinations of strategies where the sum of players' payoffs is maximized. In this example, (T, V) results in a socially desirable outcome. Through this analysis, insights into the issue of fake news and fact-checking can be gained, and an understanding of strategic interactions in the context of information dissemination and the role of media can be obtained.

7. Both complete information games and Cooperative games and Prisoner's Dilemma in the context of Fake News and Fact-Checking

both complete information games and cooperative games In the context of fake news and fact-checking, let's consider the framework of both complete information games and cooperative games and explore the application of the concepts of superadditivity and convexity to the prisoner's dilemma problem, along with some example scenarios.

7.1 Game Setup

7.2 Players and Strategies

Players: News providers A and B. Strategies: Provide true news (C: cooperate) or provide fake news (D: deceive).

7.3 Payoff Table

The typical payoff table for the prisoner's dilemma is as follows, but in this context, cooperation (C) means providing true news, and deception (D) means providing fake news.

	C	D
C	$(3,3)$	$(0,5)$
D	$(5,0)$	$(1,1)$

Here, payoffs are shown in the form (A's payoff, B's payoff). For example, if both cooperate (provide true news), each receives a payoff of 3.

7.4 Verification of Superadditivity

Superadditivity refers to the property where the total gains from cooperation are greater than the sum of individual gains achieved by acting independently. In this scenario, when both players cooperate (provide true news), the total payoff is 6, which is greater than the sum of payoffs from any other combination of strategies. Therefore, superadditivity holds.

7.5 Verification of Convexity

Convexity is the property where the additional gains from the cooperation of more players increase as the size of the cooperating group grows. In this context, since only cooperation (C) is considered, there is no direct application of convexity. However, from the perspective that cooperation results in the most desirable outcome for the entire society, the value of cooperation is emphasized.

7.6 Relation to the Prisoner's Dilemma

In the prisoner's dilemma, there is a paradox where individual optimal strategies (D: providing fake news) lead to an undesirable outcome (1, 1) for the entire group. In the context of a cooperative game, the presence of superadditivity suggests that players have an incentive to cooperate (provide true news), potentially maximizing the overall welfare of society.

7.7 Calculation Process

1. Calculate the payoffs for each strategy combination. 2. To verify superadditivity, compare the total payoffs when cooperating. 3. Identify Nash equilibria by considering each player's optimal strategy.

8. Introduction of Cournot Models and Cournot-Nash in Game Theory

8.1 Cournot Model: Analysis of Quantity Competition

The Cournot model is an economic model used to analyze quantity competition among firms in an oligopoly market.

In Cournot competition, each firm determines the quantity it will produce, taking into account the quantities provided by other firms, in order to maximize its own profits. Here, we will also touch upon the Cournot Nash equilibrium.

8.2 Basic Setup of the Cournot Model

Firms produce the same product and supply it to the market. The objective of each firm is to maximize its own profit. Firms independently determine their production quantities without affecting each other's production quantities.

8.3 Mathematics and Calculation Process

8.3.1 Market Demand Function

$P(Q) = a - bQ$ - $P(Q)$ is the market price, a and b are constants, and Q is the total market supply ($Q = q_1 + q_2 + \ldots + q_n$).

8.3.2 Profit Function

$\Pi_i = P(Q)q_i - C_i(q_i)$ - Π_i is the profit of firm i, q_i is the production quantity of firm i, and $C_i(q_i)$ is the cost function of firm i.

Deriving the Best Response Function: Differentiate the profit function of firm i with respect to q_i and find the points where the first derivative equals zero. - $\frac{\partial \Pi_i}{\partial q_i} = \frac{\partial (P(Q)q_i)}{\partial q_i} - \frac{\partial C_i(q_i)}{\partial q_i} = 0$

Calculating the Cournot Nash Equilibrium: Solve for the production quantities of each firm in the Cournot Nash equilibrium by setting up and solving the simultaneous best response functions of all firms.

The Cournot model is one of the game theory models used to analyze quantity competition among firms. Cournot competition involves each firm determining its quantity of production, taking the production quantities of other firms as given. This model is primarily applied in the analysis of competition in oligopoly markets.

Market Demand Function: The market price P is expressed as a function of the total production quantity Q, often represented using the linear demand function $P(Q) = a - bQ$, where a and b are positive constants.

Firm's Profit Function: The profit Π_i of firm i depends on its own production quantity q_i and the total production quantity Q of all firms. Profit is calculated as the difference between revenue $P(Q) \times q_i$ and cost $C_i(q_i)$. In other words, $\Pi_i(q_i, Q) = P(Q) \times q_i - C_i(q_i)$. ion Quantity: $Q = \sum_{i=1}^{n} q_i$, where n is the total number of firms in the market.

Derivation of Each Firm's Best Response Function: Each firm i determines the quantity q_i that maximizes its profit, taking the production quantities of other firms as given. To find this, the profit function Π_i is differentiated with respect to q_i, and the result is set to zero, i.e.,

$$\frac{\partial \Pi_i}{\partial q_i} = \frac{\partial (P(Q)q_i)}{\partial q_i} - \frac{\partial C_i(q_i)}{\partial q_i} = 0$$

Calculation of Cournot Nash Equilibrium: After obtaining the best response functions for all firms, the Cournot Nash equilibrium in terms of production quantities for each firm is determined by solving this system of equations. This represents a state where all firms are taking optimal strategies with respect to each other.

Consider a situation with two firms competing in the market. The market demand function is $P(Q) = 100 - Q$, and both firms have cost functions $C_1(q_1) = C_2(q_2) = 20q$.

The profit function for firm 1 is $\Pi_1 = (100-(q_1+q_2))q_1 - 20q_1$. This is differentiated with respect to q_1, and the result is set to zero to solve for q_1.

Similarly, the profit function for firm 2 is $\Pi_2 = (100 - (q_1+q_2))q_2 - 20q_2$. This is differentiated with respect to q_2, and the result is set to zero to solve for q_2.

By solving the resulting best response functions as a system of equations, the production quantities for each firm in the Cournot Nash equilibrium can be determined.

Using the Cournot model in this way allows for the analysis of quantity competition among firms, particularly in oligopoly markets.

8.4 Case Study

Consider a market with two firms. The market demand function is $P(Q) = 120 - Q$, and the cost functions for firms 1 and 2 are $C_1(q_1) = 40q_1$ and $C_2(q_2) = 40q_2$, respectively.

1. The profit function for firm 1 is $\Pi_1 = (120 - (q_1 + q_2))q_1 - 40q_1$, and for firm 2, it is $\Pi_2 = (120-(q_1+q_2))q_2 - 40q_2$. 2. To derive the best response function for firm 1, differentiate Π_1 with respect to q_1 and set it equal to zero. Similarly, do the same for firm 2. 3. Solve the resulting system of equations to find the production quantities for each firm in the Cournot Nash equilibrium.

When applying this model to the context of fake news and fact-checking, one can consider news providers as firms and model the "quality" of information as production quantities. The market demand function can represent how the total quality of information affects the public interest or viewer trust. Each news provider aims to maximize its profit by determining the quality of information it provides.

9. Deployment of Non-Cooperative Games: Context of Fake News and Fact-Checking within the Framework of Perfect Information Games

The proliferation of fake news and the counteracting role of fact-checking have become central concerns in the information age. This paper examines these phenomena through the lens of non-cooperative game theory, specifically focusing on the strategic implications of information completeness and the potential for informal cooperation among rational actors. In the realm of non-cooperative games, particularly those with perfect information, all participants are fully aware of the game's structure, including the strategies and payoffs available to their opponents. This section delves into how these concepts apply to the dissemination of fake news and the efforts to combat it through fact-checking, employing the principles of superadditivity and convexity to discuss potential informal cooperation strategies.

9.1 Consideration of Superadditivity and Convexity

The concepts of superadditivity and convexity are crucial in understanding the potential for cooperative behavior in a non-cooperative setting. Superadditivity suggests that the combined effort of two or more players can lead to greater gains than if they acted independently, while convexity indicates that the marginal benefit of adding more participants to a coalition increases with its size.

9.1.1 Superadditivity

Superadditivity is illustrated when the collective action of players, through strategies like information sharing, leads to better outcomes than individual efforts. The formal representation is given by:

$$E[v(A \cup B)] \geq E[v(A)] + E[v(B)] \tag{5}$$

where A and B are distinct sets of players, and $E[\cdot]$ denotes the expected value.

9.1.2 Convexity

Convexity becomes relevant when the incremental benefits of adding a new player to a coalition surpass the benefits of the existing coalition size. This is mathematically expressed as:

$$E[v(B \cup \{i\})] - E[v(B)] \geq E[v(A \cup \{i\})] - E[v(A)] \tag{6}$$

signifying the growing advantage of coalition expansion.

9.1.3 Non-Cooperative Games in Extensive Form: Incomplete Information in Fake News and Fact-Checking

Transitioning to extensive-form games under incomplete information, we encounter a scenario where players lack full knowledge about others' actions or types, such as the reliability of information sources. This uncertainty necessitates a focus on expected payoffs to navigate the strategic landscape effectively.

9.1.4 Superadditivity in Incomplete Information Games

In the context of incomplete information, superadditivity implies that the expected payoff from collaboration exceeds the sum of individual efforts, even when full knowledge of other players' strategies is not available. The characteristic function $v(S)$, representing the maximum expected payoff for a coalition S, underpins this analysis.

9.1.5 Convexity in Incomplete Information Games

Convexity, within the framework of incomplete information, suggests an increasing return on the inclusion of additional players into a coalition, emphasizing the value of expanding participant numbers in the face of strategic uncertainty.

9.2 Dynamic and Static Best Response Dynamics in Fake News and Fact-Checking

The interplay between dynamic and static best response strategies offers a rich vein of analysis in the study of fake news and fact-checking. This section explores how players adapt their strategies over time in response to the evolving tactics of their opponents, seeking equilibrium states that balance individual and collective interests.

9.3 Dynamic Best Response Dynamics

Dynamic best response dynamics highlight the iterative process of strategy adaptation, where each player continuously refines their approach based on the observed actions of others, aiming for a Nash equilibrium where no player has an incentive to deviate unilaterally.

9.4 Pareto Optimality and Social Welfare

The concept of Pareto optimality provides a benchmark for evaluating the efficiency of strategy combinations, identifying scenarios where any improvement for one player would necessitate a loss for another. This principle guides the search for socially optimal outcomes that balance the spread of accurate information against the need to counteract fake news.

This paper has explored the application of non-cooperative game theory to the challenges posed by fake news and fact-checking, highlighting the nuanced interplay between strategic behavior, information asymmetry, and the potential for informal cooperation. Through the lens of perfect and incomplete information games, we gain insights into the mechanisms that drive the dissemination of information in the digital age and the collective efforts to safeguard the integrity of public discourse.

10. Application of Bertrand Competition in Game Theory

Bertrand Competition is a game-theoretic model used to analyze price competition among firms. In this model, each firm determines the price of its product with knowledge of the pricing of other firms and usually assumes that the products are perfect substitutes. The fundamental feature of Bertrand competition is that the price can be only slightly lower than marginal cost to capture the full market demand. This increases price competition, resulting in convergence to the price in a perfectly competitive market, i.e., marginal cost.

10.1 Setup of Model

Players: Firms A and B Strategy: Price of the product set by each firm p_A and p_B Gain function: The profit of each firm depends on the difference between the price and marginal cost and the volume of sales.

10.2 Setup of the formula

Profit function for firm A: $\Pi_A = (p_A - c) \times q_A$ Profit function for firm B: $\Pi_B = (p_B - c) \times q_B$

where c is the marginal cost and q_A and q_B are the quantities of products sold by firms A and B, respectively. The model assumes that the firm that sets the lower price will capture all market demand because its product is a perfect substitute. That is, if $p_A < p_B$ then $q_A > 0$ and $q_B = 0$; similarly, if $p_B < p_A$ then $q_B > 0$ and $q_A = 0$. If prices are equa$p_A = p_B$, assume that the market is equally divided between the two firms.

10.3 Computational Process

Derivation of Nash Equilibrium

Firms A and B each set their prices so as to maximize their profits. In the Nash equilibrium, both firms will set prices equal to their marginal costs, and profits will be zero ($p_A^= p_B^= c$).

Equilibrium Stability Analysis

In the Bertrand equilibrium, there is always an incentive for one of the firms to try to gain the entire market by lowering its price slightly. However, this is because the other firm has an incentive to lower its price as well, so both firms settle at a price that converges to the marginal cost.

10.4 example

If the marginal costs of firms A and B are $c_A = 10$ and $c_B = 10$, respectively, then in Nash equilibrium both firms set a price of $p_A^= p_B^= 10$. With this pricing, neither firm can earn a profit.

The key insight of this model is that price competition is likely not profitable for the firms. Therefore, in real markets, competition is often driven by factors other than price (e.g., brand value, product differentiation, etc.).

11. Application of Game Theory: Ultimatum Game

The Ultimatum Game is one of the commonly used games in experimental economics within the field of game theory. This game involves two players and is a simple bargaining game that analyzes how rational individuals allocate resources.

11.1 Game Rules

(1) There are two players: the Proposer and the Responder.

(2) The Proposer suggests how to divide a sum of money (e.g., 10 dollar) between themselves and the Responder. For example, the Proposer may propose to keep 6dollar) for themselves and offer 4dollar) to the Responder.

(3) The Responder chooses to either "accept" or "reject" the proposal.

If they choose "accept," the money is divided as proposed.

If they choose "reject," both players receive nothing.

11.2 Mathematical Setup

$$\text{Proposer's payoff: } U_P = X - P$$
$$\text{Responder's payoff: } U_R = P$$

Here, X represents the total amount of money, and P is the amount proposed to the Responder.

(1) **Proposer's Strategy**: The Proposer aims to maximize their own payoff U_P while considering the minimum amount P_{min} that the Responder would accept. In theory, the Proposer can propose the lowest possible amount as long as the Responder accepts it to maximize their payoff.

(2) **Responder's Strategy**: In practice, the Responder not only considers the amount but also factors in notions of fairness and the possibility of retaliation. Consequently, very unfair proposals are often rejected by the Responder. Hence, the Proposer needs to make a "fair" proposal that the Responder is likely to accept.

11.3 Example

Let X =10 and suppose the Proposer suggests P =1 for the Responder. In this case, the Proposer's payoff is U_P =9 and the Responder's payoff is U_R =1. However, many experiments have shown that such unequal proposals are often rejected by the Responder.

Empirical studies have found that proposals offering the Responder approximately 40% to 50% of the total amount are generally considered "fair" and are typically accepted by the Responder.

The Ultimatum Game illustrates that people not only seek to maximize their self-interest but also place importance on fairness and social norms, highlighting aspects of human behavior that cannot be fully explained by pure economic theory.

12. Application of Bertrand Competition Model to Information Reliability

In the context of fake news and fact-checking, one can apply the Bertrand Competition Model to analyze the strategic interaction among news providers, considering "information reliability" as a form of competition analogous to price competition.

12.1 Model Setup

There are two news providers, A and B, each determining the reliability of the information they provide.

Consumers (viewers) choose news providers based on the reliability of the information provided.

Higher information reliability is assumed to come at higher costs.

News providers do not have complete information about the information reliability of other providers (imperfect information game).

12.2 Equations and Computational Process

12.2.1 Profit Functions

The profit functions for News Providers A and B are expressed as follows:

$$\Pi_A = r_A \cdot D_A(r_A, r_B) - C(r_A)$$

$$\Pi_B = r_B \cdot D_B(r_A, r_B) - C(r_B)$$

Here, D_A and D_B represent the number of viewers (market share) each news provider attracts.

12.2.2 Best Response Functions

Each news provider selects the information reliability that maximizes their own profit, taking into account the other's information reliability. In an imperfect information game, each news provider has probabilistic expectations regarding the other's information reliability and calculates the best response based on this.

12.2.3 Nash Equilibrium

The Nash equilibrium is found where the best response functions of each news provider intersect. It signifies a state where neither news provider can unilaterally change their strategy to increase their profit.

12.2.4 Computational Process

Specific market share functions D_A and D_B, and cost functions $C(r)$ need to be defined. For example, higher information reliability may lead to a larger market share but also increasing costs to maintain that reliability. These functions are then substituted into the profit functions, and the information reliability levels r_A and r_B that maximize the profit functions are determined. In this process, expectations or probability distributions regarding the other's information reliability are considered, and information reliability levels that maximize expected profits are calculated.

By applying the Bertrand Competition Model in this context, it becomes possible to analyze how news providers strategically compete in terms of information reliability. The computational process, especially considering the characteristics of an imperfect information game, allows for a more accurate reflection of the complexity of the real media market.

13. Bertrand Competition Model in the Context of Fake News and Fact-Checking

When considering the Bertrand Competition as a complete information game among news providers in the context of fake news and fact-checking, it is assumed that each news provider has full knowledge of the strategies of other news providers. In this scenario, news providers compete in terms of the "price" of information, which corresponds to the level of reliability or accuracy. As a complete information game, each provider knows how much reliability others offer and is expected to decide their strategies accordingly.

13.1 Game Setup

Players: News Provider A and B

Strategies: Levels of information reliability (analogous to price), p_A and p_B

Payoff Functions: The profit earned by news providers depends on the level of reliability and the extent to which that reliability is accepted by the market.

13.2 Payoff Functions

The profit of news providers is determined by the level of information reliability they provide and the extent to which that reliability is accepted by the market. The payoff functions are set as follows:

$$\text{Profit of News Provider A:}\quad \Pi_A(p_A, p_B) = D_A(p_A, p_B) \cdot p_A - C_A(p_A)$$
$$\text{Profit of News Provider B:}\quad \Pi_B(p_A, p_B) = D_B(p_A, p_B) \cdot p_B - C_B(p_B)$$

Here, $D_A(p_A, p_B)$ and $D_B(p_A, p_B)$ are the demand functions of the market for the information provided by News Providers A and B, respectively. $C_A(p_A)$ and $C_B(p_B)$ represent cost functions for maintaining the reliability levels p_A and p_B.

13.3 Calculation of Nash Equilibrium

To find the Nash equilibrium, we look for strategies that maximize the profit functions of each player. This corresponds to solving the following optimization problems:

$$\max_{p_A} \Pi_A(p_A, p_B) \quad \text{for } p_A^*$$
$$\max_{p_B} \Pi_B(p_A, p_B) \quad \text{for } p_B^*$$

The solutions to the optimization problems must satisfy the first-order conditions (FOC):

$$\frac{\partial \Pi_A(p_A, p_B)}{\partial p_A} = 0$$
$$\frac{\partial \Pi_B(p_A, p_B)}{\partial p_B} = 0$$

13.4 Consideration of Entanglement

In this scenario, entanglement arises due to the fact that the strategies of news providers depend on each other. That is, the choice of reliability level p_A by A affects the payoff of B, and vice versa. This interdependence is expressed through the demand functions $D_A(p_A, p_B)$ and $D_B(p_A, p_B)$.

13.5 Specific Example of Equations and Computational Process

Specific demand functions and cost functions need to be defined. For instance, assuming linear demand functions and linear cost functions would simplify the calculations. However, to more accurately reflect real market dynamics and news provider strategies, more complex functional forms are often adopted. As for solving the optimization problem, both analytical and numerical methods are possible approaches, depending on the complexity of the problem.

To apply the Bertrand Competition Model effectively, one should carefully choose the appropriate functions and methods to analyze the competition among news providers in terms of information reliability within the context of fake news and fact-checking.

14. Application of Game Theory: Ultimatum Game in the Context of Fake News and Fact-Checking

In the context of fake news and fact-checking, one can consider the interaction between news providers as a game of incomplete information using the framework of the Ultimatum Game. In this scenario, the participants in the game are news providers (Proposers) and the public (Responders). In this setting, news providers decide how much "truthful information" to provide to the public (or how much to reduce fake news), and the public decides whether to "accept" or "reject" that information.

14.1 Game Setup

Player 1: News Provider (Proposer)

Player 2: Public (Responder)

Proposal: The level of truthfulness of the information provided by the news provider (e.g., ranging from 0% to 100%)

14.2 Mathematical Setup

News Provider's Payoff: $U_{NP} = V - C(T)$

Public's Payoff: $U_P = T$

Here, T represents the level of truthfulness of the provided information, V is the benefit received by the news provider (e.g., viewership, advertising revenue), and $C(T)$ represents the cost of providing truthful information.

14.3 Computational Process

(1) **News Provider's Strategy**: The news provider predicts the minimum level of truthfulness T_{min} that the public would accept and chooses the level of truthfulness T^* that maximizes their own payoff U_{NP}. Here, the news provider may seek to maximize their profit by providing information (including fake news) at a lower cost, but they must consider whether the public will accept the information.

(2) **Public's Strategy**: The public evaluates the truthfulness of the provided information and decides whether to "accept" or "reject" it. If the truthfulness of the information

falls below a threshold $T_{threshold}$, the public may reject the information and consider it as fake news.

14.4 Example

Suppose the news provider offers information with a truthfulness level of $T = 70\%$, $V = 100$ units, and $C(70\%) = 50$ units of gain. In this case, the news provider's payoff is $U_{NP} = 100 - 50 = 50$ units.

If the public has set $T_{threshold} = 60\%$, they will accept this offer, and the public's payoff will be $U_P = 70\%$.

This application example analyzes the strategic interaction between news providers' incentives to reduce fake news and provide truthful information, and the public's demand for truthfulness. It also considers the incentives for news providers to incur costs to provide truthful information and the public's decision to accept or reject information. This analysis can provide insights into strategies and policies to prevent the spread of fake news.

15. Application of Game Theory: Understanding Perfect Information Games

When considering a perfect information game among news providers in the context of fake news and fact-checking, the participants in the game are multiple news providers. It is assumed that they know the level of truthfulness of the information they provide to each other. In this scenario, each news provider strives to enhance public trust and their own reputation by reducing fake news and offering more truthful information, but they may also try to avoid increasing costs simultaneously.

15.1 Game Setup

Players: News Provider A, B (other news providers can also be considered)

Strategies: Level of truthfulness of the information they provide T_A, T_B (ranging from 0% to 100%)

15.2 Mathematical Setup

Let:

$$U_A = V_A(T_A, T_B) - C_A(T_A)$$
$$U_B = V_B(T_A, T_B) - C_B(T_B)$$

where $V_A(T_A, T_B)$ and $V_B(T_A, T_B)$ represent the benefits received by News Providers A and B, respectively, depending on the truthfulness level of the information they provide and the truthfulness level of information provided by competing firms. $C_A(T_A)$ and $C_B(T_B)$ represent the costs of providing truthful information.

15.3 Computational Process

(1) **Definition of Payoff Functions**: Define payoff functions for each news provider. The payoff functions depend on the truthfulness level of the information they provide, the truthfulness level of information provided by competing firms, and the cost of providing truthful information.

(2) **Derivation of Nash Equilibrium**: Search for the combination of strategies that arises when each news provider chooses strategies that maximize their own payoffs. This combination is known as the Nash equilibrium.

(3) **Calculation of Optimal Strategies**: To find the Nash equilibrium, calculate the optimal truthfulness levels T_A^* and T_B^* that maximize the payoff functions for each news provider.

15.4 Example

Suppose News Providers A and B provide information with truthfulness levels T_A and T_B and have linear cost functions, $C_A(T_A) = k_A T_A$ and $C_B(T_B) = k_B T_B$. Additionally, assume profit functions are $V_A(T_A, T_B) = a_A T_A + b_A T_B$ and $V_B(T_A, T_B) = a_B T_A + b_B T_B$. In this case, to find the Nash equilibrium, you need to solve the following optimization problems:

$$\text{A's optimization problem:} \quad \max_{T_A}(a_A T_A + b_A T_B - k_A T_A)$$
$$\text{B's optimization problem:} \quad \max_{T_B}(a_B T_A + b_B T_B - k_B T_B)$$

Solving these problems will yield the optimal truthfulness levels T_A^* and T_B^* for each news provider.

This approach provides insights into the strategic interactions among news providers in the context of fake news and fact-checking and their impact on the public. Specifically, it can analyze how much truthfulness each news provider should offer and how that affects the strategies of competing firms.

16. Applications of Game Theory and Inter-Connectivity

16.1 Incorporating Interconnectivity in the Prisoner's Dilemma

When considering "interconnectivity" or mutual interdependence in the context of the Prisoner's Dilemma, it is necessary to express how the strategies of players influence each other through mathematical formulas. Let's explore the formulation and computational process of a model that incorporates this concept into the framework of the Prisoner's Dilemma.

16.2 Basic Setting of the Prisoner's Dilemma

The Prisoner's Dilemma is usually represented by a payoff table like this:

	Cooperate (C)	Defect (D)
Cooperate (C)	(R, R)	(S, T)
Defect (D)	(T, S)	(P, P)

Here, - R represents "Reward" when both cooperate, which is the payoff. - S represents "Sucker's Payoff" when one cooperates and the other defects. - T represents "Temptation" when one defects and the other cooperates. - P represents "Punishment" when both defect.

16.3 Introduction of Interconnectivity

To consider interconnectivity, additional parameters are introduced to represent how the strategies of players affect each other's expected payoffs. To express this interdependence, interaction terms are added to the payoff functions.

For example, the expected payoff function for Player A would look like this:

$$E[\Pi_A] = p_A \cdot (U_A(C, C) + \alpha \cdot U_B(C, D)) + (1 - p_A) \cdot (U_A(D, C) + \alpha \cdot U_B(D, D)) \quad (7)$$

Here, p_A is the probability of Player A cooperating. α is the interaction parameter indicating the degree to which others' choices affect one's own payoffs. $U_A(X, Y)$ and $U_B(X, Y)$ are the payoff functions for Player A and B, respectively, where X and Y represent the choices of Player A and B (Cooperate C or Defect D).

A similar expected payoff function can be defined for Player B.

16.4 Computational Process

To find the Nash equilibrium, it is necessary to find the strategies that maximize the expected payoffs of each player. This corresponds to solving for probabilities p_A and p_B that maximize each player's expected payoff function.

1. Differentiate the expected payoff functions of Player A and B with respect to p_A and p_B, respectively. 2. Set the derivatives to zero and solve for p_A and p_B. 3. The obtained values p_A^* and p_B^* represent the Nash equilibrium.

16.5 Example

Using actual parameters of the Prisoner's Dilemma and the interaction parameter α, specific numerical values are substituted, and calculations are performed. For example, assuming $R = 3$, $S = 0$, $T = 5$, $P = 1$, and $\alpha = 0.5$, you can follow the steps mentioned above to find the Nash equilibrium.

Through this computational process, you can analyze how interdependence influences strategic choices. Higher interconnectivity would mean that players' choices are significantly influenced by the choices of others.

16.6 Incorporating Interconnectivity in Cournot model

When considering *"interconnectivity"* in the Cournot model, which means taking into account the interdependence among firms, it is necessary to express how the gains of each firm depend on the production of other firms. In the Cournot model, firms usually compete in a market where the market price depends on the total production of their own and competing firms. To emphasize interdependence, a parameter that indicates how strongly the actions of one firm affect the others can be introduced into the profit functions of firms.

16.7 Basic Setup of the Cournot Model

In the Cournot model, firms A and B compete in the market, and the gains of each firm are defined as follows:

$$\text{Profit of Firm A: } \Pi_A = P(Q)q_A - C(q_A)$$
$$\text{Profit of Firm B: } \Pi_B = P(Q)q_B - C(q_B)$$

Here,

q_A and q_B represent the production quantities of firms A and B, respectively.

$Q = q_A + q_B$ is the total market production.

$P(Q)$ is the market price, which is a function of the production quantity Q.

$C(q)$ represents production costs and is a function of the production quantity q.

16.8 Introduction of Interconnectivity

To incorporate interdependence into the model, a parameter β is introduced into the profit functions of firms, indicating the degree to which a firm's profit depends on the production of other firms.

$$\text{Modified Profit of Firm A: } \Pi_A = P(Q)q_A - C(q_A) + \beta(q_B - q_A)$$
$$\text{Modified Profit of Firm B: } \Pi_B = P(Q)q_B - C(q_B) + \beta(q_A - q_B)$$

16.9 Maximization of Profit Functions

Each firm seeks production quantities q_A^* and q_B^* that maximize its profit function by setting the first derivatives of profit functions with respect to q_A and q_B equal to zero.

$$\frac{\partial \Pi_A}{\partial q_A} = 0$$
$$\frac{\partial \Pi_B}{\partial q_B} = 0$$

16.10 Derivation of Nash Equilibrium

Solving these equations yields the optimal production quantities q_A^* and q_B^* at the Nash equilibrium.

16.11 Case Study

As an example with specific numerical values, consider a situation with a market price function $P(Q) = a - bQ$ (where a and b are constants), a production cost function $C(q) = cq$ (where c is a constant), and an interdependence parameter β.

(1) By taking the partial derivative of Firm A's profit function with respect to q_A and setting it to zero, the following equation is obtained:

$$a - 2bq_A - bq_B - c + \beta = 0$$

(2) Similarly, for Firm B, the following equation is obtained:

$$a - 2bq_B - bq_A - c - \beta = 0$$

(3) These equations can be solved to find q_A^* and q_B^*.

Through this calculation process, the analysis can reveal how interdependence affects the determination of firms' production quantities. The higher the interdependence, the more firms may consider the actions of competing firms in their decision-making process.

17. Interconnectivity in Cournot Model for News Providers

In the context of fake news and fact-checking, considering a complete information game among news providers and seeking interconnectivity in the Cournot model, news providers will compete for the "quality" of information. In this scenario, higher-quality information increases the public interest but also comes with a cost. Interconnectivity will illustrate how news providers respond to the information quality of their competitors.

17.1 Model Setup

Two news providers: A and B.

Quality of information provided by A and B: q_A and q_B, respectively.

Public interest depends on total information quality: $Q = q_A + q_B$.

Market price for information quality ($P(Q)$) and cost to provide information quality ($C(q)$) are considered.

17.2 Profit Functions

The profit functions for news providers are as follows:

$$\begin{aligned}
\text{Profit for news provider A:} \quad & \Pi_A(q_A, q_B) \\
&= P(Q)q_A - C(q_A) + \beta(q_B - q_A) \\
\text{Profit for news provider B:} \quad & \Pi_B(q_A, q_B) \\
&= P(Q)q_B - C(q_B) + \beta(q_A - q_B)
\end{aligned}$$

Here, β is a parameter representing interconnectivity, indicating how sensitively news providers respond to the information quality of their competitors.

17.3 Calculation Process

(1) **Maximization of Profit Functions**: Each news provider seeks the optimal information quality q_A^* and q_B^* that maximize their respective profit functions.

$$\begin{aligned}
\frac{\partial \Pi_A}{\partial q_A} &= P'(Q)q_A + P(Q) - C'(q_A) - \beta = 0 \\
\frac{\partial \Pi_B}{\partial q_B} &= P'(Q)q_B + P(Q) - C'(q_B) + \beta = 0
\end{aligned}$$

(2) **Derivation of Nash Equilibrium**: Solving the above partial differential equations with respect to q_A and q_B yields the optimal information quality levels q_A^* and q_B^* at the Nash equilibrium.

17.4 Example

Consider a market price function $P(Q) = a - bQ$ (where a and b are positive constants) and a cost function for information quality $C(q) = cq^2$ (where c is a positive constant). In this case, the equations for maximizing profit are as follows:

$$\begin{aligned}
a - b(q_A + q_B) - 2cq_A - \beta = 0 \\
a - b(q_A + q_B) - 2cq_B + \beta = 0
\end{aligned}$$

Solving these equations with respect to q_A and q_B allows us to determine the levels of information quality at the Nash equilibrium. Through this calculation, we can understand how news providers determine the level of information quality and how it affects the public interest in the context of fake news and fact-checking. It is assumed that a higher interconnectivity parameter β makes news providers more sensitive to the information quality levels of their competitors, with this sensitivity having a significant impact on their profits.

18. Non-Full Information Game between News Providers in Ultimatum Game

In the context of fake news and fact-checking, considering a non-full information game among news providers within the framework of the Ultimatum Game, the game is set up as follows:

18.1 Game Setting

(1) Player 1 (proposer) possesses information regarding the "truthfulness" of news and decides to what extent to disclose it. This "truthfulness" represents the degree to which the information is either fake news or fact-checked news.

(2) Player 2 (responder) does not have complete knowledge of the truthfulness of the information provided by Player 1 but decides whether to accept or reject it.

(3) If Player 2 accepts the information, both players receive rewards based on the truthfulness of the information. If Player 2 rejects the information, neither player gains any rewards (or receives a basic reward).

18.2 Mathematical Setting

q: The level of truthfulness of the information provided by Player 1 (taking values between 0 and 1, where 1 represents completely truthful information).

$R(q)$: Rewards obtained by Player 1 and Player 2 based on the level of truthfulness q.

18.3 Utility Functions

Player 1's utility: $U_1 = R(q) \cdot s$, where s is 1 if Player 2 accepts the information and 0 if Player 2 rejects it.

Player 2's utility: $U_2 = R(q) \cdot (1 - s)$.

18.4 Calculation Process

(1) **Player 1's Strategy**: Player 1 determines the level of "truthfulness" q^* that makes it most likely for Player 2 to accept. This is determined considering the trade-off between Player 2's acceptance threshold and Player 1's profit maximization.

(2) **Player 2's Expected Utility**: Player 2 decides whether to accept the provided information based on the level of truthfulness q. Player 2 assesses based on the expected value $E[q]$ of q and accepts the information if it is above a certain threshold q_{th}.

(3) **Derivation of Interconnectivity**: The interconnectivity between Player 1's strategy q^* and Player 2's acceptance threshold q_{th} can be determined by solving the optimization problems of Player 2's expected utility and Player 1's utility function.

18.5 Case Study

Assuming that the level of truthfulness q of the information provided by Player 1 follows the reward function $R(q) = aq - bq^2$ (where a and b are positive constants), Player 1 selects q^* with the highest likelihood of being accepted by Player 2. Player 2 accepts the provided information if it exceeds a threshold q_{th}. This threshold is based on Player 2's expectations and experiences regarding the truthfulness of the information. Interconnectivity arises as Player 1 predicts Player 2's response and selects the optimal q^* based on that prediction.

Through this analysis, we can gain insights into how information provisioning strategies are formed in the context of fake news and fact-checking, and how they impact the public interest. It is suggested that this is because it simplifies the scenario's solutions due to being a non-full information game.

19. Full information game between news providers in Ultimatum Game

In the context of fake news and fact-checking, considering a complete information game among news providers in the framework of the Ultimatum Game and seeking interconnectivity, the game would be set up as follows:

19.1 Game Setup

News provider A (proposer) makes a proposal regarding the "quality" of certain information (resource).

News provider B (responder) chooses whether to accept or reject the proposal.

If the proposal is accepted, both parties benefit based on the proposed information quality.

If the proposal is rejected, neither party benefits (or receives a basic benefit).

19.2 Mathematical Modeling

Let q represent the quality of the proposed information, with $0 \leq q \leq 1$. $U_A(q)$ is the utility function for proposer A, and $U_B(q)$ is the utility function for responder B. Proposer A gains a profit of aq based on the proposed information quality q, while responder B gains a profit of $(1 - q)b$ (where a and b are positive constants representing the value of information quality).

19.3 Considering Interconnectivity

We introduce the concept of the minimum acceptable quality q_{min} for responder B. This reflects the minimum level of information quality that B is willing to accept and incorporates B's evaluation and expectations regarding information quality. We introduce a parameter β to represent interconnectivity, where a larger value indicates that proposer A is more sensitive to responder B's minimum acceptable quality.

19.4 Calculation Process

19.4.1 Derivation of Optimal Proposal by Proposer A

Proposer A selects a q that maximizes their own profit aq but must exceed the minimum quality q_{min} that responder B is willing to accept. If we define $U_A(q) = aq - \beta|q - q_{min}|$ (where β represents interconnectivity), proposer A chooses q^* that maximizes this utility.

19.4.2 Determination of Responder B's Minimum Acceptable Quality

Responder B decides whether to accept or reject the proposal based on their own profit $(1 - q)b$ and their expectation of q_{min} for information quality. Responder B's utility function is defined as $U_B(q) = (1-q)b-\gamma(q_{min}-q)$, where γ represents the level of dissatisfaction felt by B when deviating from the minimum acceptable quality.

19.4.3 Derivation of Nash Equilibrium

By simultaneously considering the strategies of proposer A and responder B, we seek the Nash equilibrium where both parties' benefits are maximized. To achieve this, we need to find q^* and q_{min} that maximize $U_A(q)$ and $U_B(q)$.

Through the analysis of this game, we can understand how the interaction and interdependence among news providers in the context of fake news and fact-checking influence the determination of information quality. Particularly, in cases with high interconnectivity, proposers are likely to be sensitive to the expectations of responders, resulting in a tendency to provide higher-quality information.

20. Superadditivity and Convexity to the Prisoner's Dilemma Problem, Setting of Cournot Model

In the context of fake news and fact-checking, let's consider the framework of both complete information games and cooperative games and explore the application of the concepts of superadditivity and convexity to the prisoner's dilemma problem, along with some example scenarios. In this case, we model how news providers behave towards the market (viewers) and how this behavior impacts the spread of fake news.

20.1 Game Setup

Players: News providers A and B. Strategies: Provide true news (C: cooperate) or provide fake news (D: deceive).

Let's apply Cournot's model to the context of fake news and fact-checking, and combine the concepts of superadditivity and convexity with the prisoner's dilemma problem. Here, we model how news providers determine the quality of information and how it affects the spread of fake news.

20.2 Basic Setting of Cournot Model

In the Cournot model, each firm (in this case, news provider) optimally reacts to the quantity (in this case, the quality of information) of other firms and maximizes its profit. Let's assume there are two news providers, A and B, and they represent the "quality" of information they provide with quantities q_A and q_B, respectively.

20.3 Payoff Functions

The profit of news providers depends on the quality of the information they provide and the reaction of the market (viewers). The payoff functions are expressed as follows:

$\Pi_A(q_A, q_B) = aq_A bq_A^2 cq_A q_B \quad \Pi_B(q_A, q_B) = aq_B bq_B^2 cq_B q_A$

Here, a, b, and c are positive constants, where a represents the viewers' basic reaction to the quality of information, b represents the diminishing effect on revenue as the quality of information increases, and c represents the negative impact of a competitor's information quality on one's own revenue.

20.4 Best Response Functions

The quality of information q_A that maximizes the profit of news provider A is obtained by setting the derivative of Π_A with respect to q_A equal to zero:

$$\frac{\partial \Pi_A}{\partial q_A} = a2bq_A cq_B = 0$$

Solving this, we get:

$$q_A^{=} \frac{acq_B}{2b}$$

Similarly, for news provider B:

$$q_B^{=} \frac{acq_A}{2b}$$

20.5 Nash Equilibrium

By solving the best response functions of news providers A and B as a system of equations, we can determine the values of q_A and q_B for the Nash equilibrium.

$$q_A^{=} \frac{ac\frac{acq_A}{2b}}{2b}$$

$$q_B^{=} \frac{ac\frac{acq_B}{2b}}{2b}$$

Solving these equations yields the values of q_A and q_B at the Nash equilibrium.

20.6 Consideration of Superadditivity and Convexity

While the direct application of the concepts of superadditivity and convexity may be challenging in this context, they become important when considering the overall societal welfare. For instance, if both news providers cooperating and providing true news results in the most socially desirable outcome, this cooperation satisfies the condition of superadditivity. Also, if the societal welfare increases as more news providers provide true information, then the condition of convexity is met.

20.7 Calculation Process

1. Setting Up Payoff Functions: Consider the market's reactions to each news provider's actions to set up the payoff functions. 2. Calculating Nash Equilibrium: Each player selects the strategy that maximizes their own payoff while considering the actions of others. In the Cournot model, this involves seeking strategies that optimally respond to the opponent's actions.

21. Complete Information Games, Cournot's model and examine the prisoner's dilemma problem

In the context of fake news and fact-checking, we will consider a non-cooperative game in the framework of complete information games. We will apply the concepts of superadditivity and convexity to Cournot's model and examine the prisoner's dilemma problem. In this scenario, multiple news providers (players) exist, and each must choose between spreading fake news or providing news based on facts.

21.1 Game Setup

21.2 Players and Strategies

Players: News providers A and B.

Strategies: Provide fake news (F) or provide news based on facts (T).

21.3 Payoff Table

We will consider a payoff table based on the typical payoff table of the prisoner's dilemma:

$A \backslash B$	T	F
T	(3,3)	(0,5)
F	(5,0)	(1,1)

Here, (3,3) represents the payoff when both providers provide news based on facts, (0,5) represents the payoff when A provides news based on facts and B provides fake news, (5,0) represents the opposite case, and (1,1) represents the payoff when both providers provide fake news.

21.4 Cournot Model Setup

In applying the Cournot model to the context of fake news and fact-checking, we consider the concepts of superadditivity and convexity. We will discuss the equations and calculations when examining the prisoner's dilemma problem. In this scenario, we treat the "quality" of information provided by news providers as a quantity and determine how much "news based on facts" they choose to provide.

21.5 Basic Cournot Model Settings

There are two news providers (firms), each determining the quality of information, represented as q_A and q_B.

The "quality" of information corresponds to reducing the provision of fake news and increasing the provision of news based on facts.

21.6 Payoff Functions

The profit of news providers depends on the quality of information and can be modeled as follows:

$$\text{News Provider A's Profit: } \Pi_A(q_A, q_B) = aq_A - \frac{1}{2}bq_A^2 - cq_Aq_B$$

$$\text{News Provider B's Profit: } \Pi_B(q_A, q_B) = aq_B - \frac{1}{2}bq_B^2 - cq_Bq_A$$

Here, a represents the fundamental impact of information quality on profit, b represents the diminishing effect on profit as information quality increases, and c represents the negative impact of a competitor's information quality on one's own profit.

21.7 Best Response Functions

21.8 News Provider A's Best Response Function

News Provider A's best response function is determined by differentiating Π_A with respect to q_A and setting it equal to zero:

$$\frac{\partial \Pi_A}{\partial q_A} = a - bq_A - cq_B = 0$$

Solving for q_A, we obtain the optimal information quality for News Provider A:

$$q_A^* = \frac{a - cq_B}{b}$$

21.9 News Provider B's Best Response Function

Similarly, for News Provider B:

$$q_B^* = \frac{a - cq_A}{b}$$

21.10 Nash Equilibrium

By solving the best response functions of News Providers A and B as a system of equations, we can determine the Nash equilibrium. In this case, the system of equations is as follows:

$$q_A^* = \frac{a - c\frac{a-cq_A^*}{b}}{b}$$

$$q_B^* = \frac{a - c\frac{a-cq_B^*}{b}}{b}$$

Solving these equations yields the values of q_A^* and q_B^* at the Nash equilibrium.

21.11 Consideration of Superadditivity and Convexity

In this scenario, while superadditivity and convexity are not directly applied, they are relevant in the sense that both providers offering news based on facts (high q_A and q_B) leads to socially desirable outcomes. Reducing the provision of fake news and increasing the provision of news based on facts is expected to improve overall societal trust and information quality.

21.12 Verification of Superadditivity and Convexity

21.13 Superadditivity

In this context, superadditivity implies that the total or societal gains obtained by players cooperating (providing news based on facts) are greater than the gains obtained by not cooperating (providing fake news). This concept is similar to the situation in the prisoner's dilemma where cooperation benefits the overall interest but individual incentives promote non-cooperation.

21.14 Convexity

Convexity is not directly applicable in this context, but it is related to the idea that increasing the number of news providers offering news based on facts enhances societal trust and leads to an increase in overall benefits.

21.15 Calculating Payoffs

Compute the payoffs corresponding to the choices of each player based on the payoff table.

21.16 Determining Optimal Strategies

In the prisoner's dilemma, each player chooses a strategy (in this case, providing fake news) that maximizes their payoff without considering the opponent's choice. However, in the context of a cooperative game, players must consider the overall benefit (improving societal trust) when choosing their strategies.

21.17 Identifying Nash Equilibrium

In this case, (F, F) constitutes the Nash equilibrium, which is different from the socially desirable outcome (T, T).

22. Complete Information Game and a Non-Cooperative g Game, Cournot's model and examine the prisoner's dilemma problem

In the context of fake news and fact-checking, we consider the scenario of a complete information game and a non-cooperative game, incorporating entanglement (mutual dependence) using the Prisoner's Dilemma. This example involves two news providers, A and B, who must choose between spreading fake news (betrayal) or providing accurate information (cooperation).

22.1 Game Setup

Players: News providers A and B

Strategies: Provide accurate information (Cooperation: C) or spread fake news (Betrayal: D)

22.2 Payoff Table

We adapt the typical Prisoner's Dilemma payoff table to the context of fake news as follows:

	Cooperation (C)	Betrayal (D)
Cooperation (C)	$(3, 3)$	$(0, 5)$
Betrayal (D)	$(5, 0)$	$(1, 1)$

Here, the first element in each pair represents Player A's payoff, and the second element represents Player B's payoff.

22.3 Payoff Functions and Entanglement

Entanglement refers to situations where the choices of one player significantly affect the payoffs of the other player. In this game, choosing betrayal by one player leads to substantial variations in the other player's payoffs.

A cooperates, B cooperates: $U_A(C, C) = 3, \quad U_B(C, C) = 3$

A betrays, B cooperates: $U_A(D, C) = 5, \quad U_B(C, D) = 0$

A cooperates, B betrays: $U_A(C, D) = 0, \quad U_B(D, C) = 5$

A betrays, B betrays: $U_A(D, D) = 1, \quad U_B(D, D) = 1$

22.3.1 Identifying Nash Equilibrium

Assuming that each player knows the other player's choice perfectly and seeks to maximize their own payoffs, the choice of betrayal (D) by both players becomes the Nash equilibrium.

22.3.2 Determining Pareto Optimality

If both players cooperate (C), it results in the highest social benefit for both, making it a Pareto optimal state.

22.3.3 Analysis of Entanglement

Entanglement arises from the fact that the choices of Player A and Player B directly influence each other's payoffs. If one player chooses to cooperate while the other chooses betrayal, the cooperator receives the lowest payoff. This represents entanglement, where individual optimal strategies lead to a situation different from the collective optimal solution in the Prisoner's Dilemma.

22.3.4 Representation of Entanglement in Formulas

Entanglement is expressed mathematically by the fact that Player A's payoff function U_A changes based on B's choices (C or D), and similarly, Player B's payoff function U_B changes based on A's choices.

Through this analysis, we gain an understanding of the strategic choices and interactions between news providers in the context of fake news and fact-checking, and how they impact societal welfare.

23. Expected Payoff Functions by Considering Entanglement (Mutual dependence) in the Prisoner's Dilemma Problem

In the context of fake news and fact-checking, when combining incomplete information games with non-cooperative games, it is possible to develop payoff functions and expected payoff functions by considering entanglement (mutual dependence) in the Prisoner's Dilemma problem. In this scenario, news providers A and B must choose between spreading fake news (betrayal, D) and providing true information (cooperation, C). However, they do not possess complete information about each other's choices or intentions.

23.1 Game Setup

23.2 Players and Strategies

Players: News providers A and B

Strategies: Provide true information (Cooperation, C), spread fake news (Betrayal, D)

23.3 Payoff Functions

We adjust the payoff table of the Prisoner's Dilemma, taking into account elements of incomplete information.

$A \backslash B$	Cooperation (C)	Betrayal (D)
Cooperation (C)	$(3, 3)$	$(0, 5)$
Betrayal (D)	$(5, 0)$	$(1, 1)$

23.4 Expected Payoff Functions

In incomplete information games, expected payoffs are calculated considering uncertainties related to players' beliefs and their opponents' strategies.

$$\text{A's expected payoff: } E[U_A] = p \cdot U_A(C, B) + (1 - p) \cdot U_A(D, B)$$
$$\text{B's expected payoff: } E[U_B] = q \cdot U_B(A, C) + (1 - q) \cdot U_B(A, D)$$

Here, p represents the probability that Player B believes A will cooperate, and q represents the probability that Player A believes B will cooperate.

23.5 Analysis of Entanglement

Entanglement refers to situations where one player's strategy affects the expected payoffs of the other player. To express this mathematically, we analyze the changes in each player's expected payoffs based on the opponent's strategy.

23.6 Calculation of Player B's Expected Payoffs

$$E[U_B|A = C] = q \cdot U_B(C, C) + (1 - q) \cdot U_B(D, C)$$
$$E[U_B|A = D] = q \cdot U_B(C, D) + (1 - q) \cdot U_B(D, D)$$

23.7 Calculation of Player A's Expected Payoffs

$$E[U_A|B = C] = p \cdot U_A(C, C) + (1 - p) \cdot U_A(C, D)$$
$$E[U_A|B = D] = p \cdot U_A(D, C) + (1 - p) \cdot U_A(D, D)$$

23.8 Derivation of Optimal Strategies

Players A and B select strategies that maximize their respective expected payoffs. This process involves determining their optimal responses to the opponent's choice.

23.9 Analysis of the Effect of Entanglement

We analyze how A's strategy affects B's expected payoffs (and vice versa). For instance, we observe how B's expected payoffs change if A switches from cooperation to betrayal.

By expressing entanglement mathematically and outlining the calculation process, we gain insights into the interaction between incomplete information games and non-cooperative games in the context of fake news and fact-checking within the framework of the Prisoner's Dilemma. In this scenario, news providers A and B choose between spreading fake news and providing true information, while acknowledging the uncertainty surrounding each other's choices and types (trusted or untrusted sources).

24. Summary

Situations that can occur in the real world are non-perfect information game situations. We propose an analysis of non-perfect information games on the topic of fake news and fact-checking. A non-perfect information game is a situation in which participants in the game do not have complete information about the types and choices of other participants (e.g., the quality of information provided by other news providers). This setting is very common in real-world media environments. Here we will focus on the challenges for the media (news providers) and the recipients (information receivers) and consider the Pareto optimum in terms of bias, missing information and poor communication.

The analysis of fake news and fact-checking in the non-perfect information game sheds light on the multiple challenges that arise between the media and the recipients. These challenges are closely related to bias, missing information, and poor communication. While taking these factors into account, we will discuss the state of the Pareto optimum.

24.1 Media (news providers) challenges

24.1.1 Information Bias

The media may intentionally or unintentionally provide information that reflects a particular viewpoint or ideology. This means that recipients are not informed about all sides of the story, resulting in information bias.

24.2 Missing Information

News providers may not have all relevant information to report. This may be due to limited sources, press time constraints, or lack of resources to dig deeper.

24.3 Barriers to Communication

The media may struggle to effectively communicate information to the public. This may be due to the use of jargon, failure to simplify complex information, or disagreement with the recipient's existing perceptions.

24.4 Recipient (recipient of information) challenges

24.4.1 Evaluating information sources

Recipients have difficulty evaluating the reliability of available information sources. This is due to the sheer volume of information and the obscurity of the information sources. 2.

24.4.2 Cognitive bias

Recipients tend to pay biased attention to information that is consistent with their existing beliefs and opinions. This increases the risk of unintentionally accepting fake news or misinformation.

24.4.3 Information overload

Information overload causes recipients to either miss important information or focus excessively on unimportant information.

24.4.4 Consideration of Pareto Optimum

The Pareto optimum refers to a situation in which one player's situation can be improved while the other is not harmed by it. When considering the Pareto optimum in the context of fake news and fact-checking, the following points should be considered

24.4.5 Mutual transparency and improved communication

Improving transparency and communication between the media and recipients is important to increase mutual understanding and reduce the impact of misinformation and fake news. This includes media clarifying the sources of information and their verification processes, and recipients using critical thinking in their consumption of information.

24.4.6 Promoting Information Literacy

Increasing recipients' information literacy will help them better assess the reliability of information sources and address cognitive biases.

The description of this computational experimental design aimed to analyze the non-perfect information game between news providers in the context of fake news and fact checking. The study applied the Cournot model to explore what strategies news providers adopt with respect to information quality and how these strategies affect the public interest.

24.4.7 The Media Approach

Media (news providers) treat information quality as a strategic variable. Each media outlet influences the quality of information in the market as a whole (Q through the quality of the information it provides (q_A and the quality of the information it receives (q_B. The media aims to maximize its own gain function Π and determines its strategy by considering the cost of providing information quality $C(q)$ and the public's evaluation of information quality $P(Q)$. Since the media act under a non-complete information game, they do not have complete information about the information quality of other media and choose their strategies based on their expected gain function$E[\Pi]$.

24.4.8 Recipient Concept

Recipients (viewers or readers) evaluate the information provided by the media based on its quality. $P(Q)$ represents the public's evaluation of the total quality of information in the market as a whole, which reflects the level of trust and satisfaction that recipients have with the quality of information. Recipients prefer high quality information, which in turn affects media gains.

24.5 Game Dynamics in Prisoner's Dilemma

In this computational experiment, competition among media regarding the quality of information may create a prisoner's dilemma situation. Each media outlet seeks to maximize its individual interests, but this does not necessarily coincide with the interests of the group as a whole, i.e., the public interest. The problem is that while it is in the best public interest for the media to cooperate to provide high quality information, there are incentives for individual media to maximize their profits by providing lower quality information.

24.6 Expected Impact

Through this study, we expect to gain insight into the optimal information provision strategies that media outlets should take and the potential for policy interventions to promote the provision of high quality information while curbing the spread of fake news. It is also hoped that an understanding of how strategic interactions among the media affect the public interest will lay the groundwork for developing more effective fact-checking approaches.

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