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Entanglement in Trembling Hand Perfect Equilibrium: Game Theory of Fake News and Fact-Checking, with Applications to Sequential Move-Order Games in Non-Complete Information Sequential Games

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Abstract: This paper is mainly a text on game theory: organizing methods for computational experiments applying "perfect equilibrium with trembling hands".This note analyzes the spread of fake news and fact-checking in the context of non-perfect information games and sequential move-order games, and explores the strategic interactions among informants using non-optimal choice ("shaking hands") and perturbed perfect equilibrium (Perturbed Perfect Equilibrium) methods. Examine. We model the strategic decision-making process between fake news providers and fact-checkers and propose a game-theoretic approach that takes into account information quality and social consequences. We analyze the dynamics of fake news diffusion in a non-complete information game setting that includes the possibility that the information provider makes a non-optimal choice ("shaking hands") with minute probability. By applying the concept of perturbed perfect equilibrium, we consider possible strategies and micro-error probabilities taken by informants, identify the resulting equilibria, and provide new insights into fake news and its countermeasures. This approach organizes thinking to curb the spread of fake news and maintain the health of the information environment.

Keywords: Game Theory, Prisoner's Dilemma, Fake news, Fact-checking, Perturbed Complete Equilibrium, Incomplete Information Games, Sequential Move-Order Games, Strategic Interaction, Informants, Trembling Hands, Nash Equilibrium, Pareto Optimal

1. Introduction

This paper is mainly a text on game theory: organizing methods for computational experiments applying "perfect equilibrium with trembling hands". This note analyzes the mechanism of fake news diffusion in the context of non-perfect information games by modeling the strategic interaction between news providers as a sequential move-order game. In particular, we explore the strategic dynamics of fake news and fact-checking, using the concept of perturbed perfect equilibrium (Perturbed Perfect Equilibrium) to take into account situations where players make non-optimal choices with minute probabilities. The proliferation of fake news and its impact on society has attracted much attention in recent years. Fake news can disrupt public opinion formation by intentionally spreading inaccurate information, including misinformation and bias. To address this issue, it is essential for informational health to understand how fake news spreads and how fact-checking works against it.

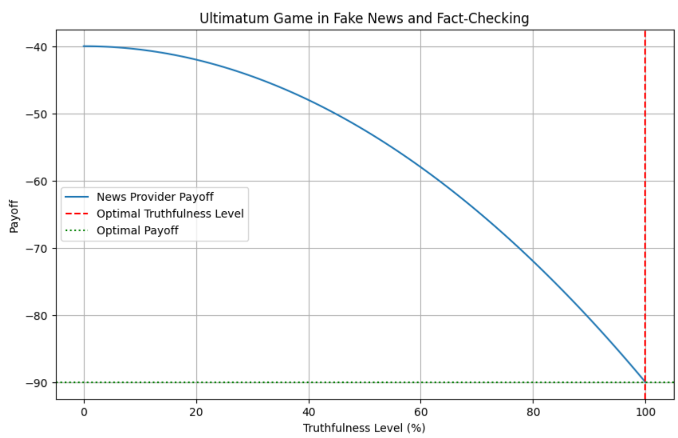

Fig. 1: Ultimatum Game in Fake News and Fact-Checking

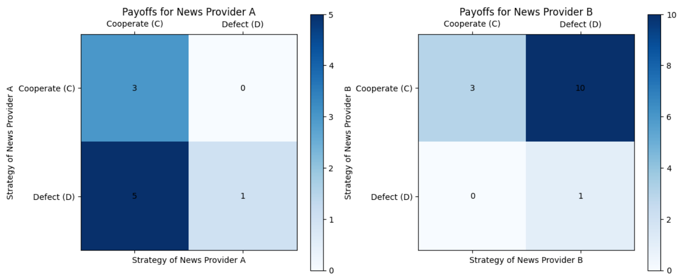

Fig. 2: Payoffs Matrix for News Provider A-B

Theoretical Explanation (Basics of Game Theory)

Game theory is a mathematical framework for analyzing the decision-making process in situations where multiple decision makers (players) interact with each other. Players choose strategies that maximize their own gain. A game is defined by its rules, its players, its set of strategies, and its gain for a combination of strategies.

Non-Complete Information Games and Sequential Turns Games

In non-perfect information games, players do not have complete information about all elements of the game, especially the types and choices of other players. Sequential turn games are games in which players take actions in a certain order, with each player making decisions based on the previous player's choices. This allows information to be revealed as the game progresses, and thus expectations and inferences play an important role in strategic decision making.

The Concept of Perturbed Perfect Equilibrium

Perturbed perfect equilibrium is an equilibrium concept that takes into account the possibility that players make non-optimal choices with minute probability in sequential move games. This concept is introduced to model contingent errors and uncertainty in players' actions. In a perturbation perfect equilibrium, a player's strategy must be the optimal response to other players' strategies and possible perturbations.

Modeling Fake News Diffusion

In this study, we consider a sequential move-order game in which news providers choose between truth and fake news. Each news provider chooses a strategy that takes into account the choices of other providers and the public response. Using a perturbed perfect equilibrium framework, we analyze these strategic interactions, including the possibility of news providers providing incorrect and inaccurate information. This theoretical approach is expected to contribute to a better understanding of the mechanisms of fake news dissemination and the effects of fact-checking, as well as to the formulation of strategies and policies to maintain information quality.

Prior Research

Prior research has analyzed the interaction between news providers in the context of fake news and fact-checking using a game theory framework. In particular, we applied the Cournot competition model and the ultimate game to investigate how news providers determine their strategies with respect to the "quality" and "truthfulness" of information.

Discussion of the interconnectedness perspective in the Cournot model indicates that news providers compete for high-quality information to increase public interest, but this comes at a cost. The profit function includes an interconnectivity parameter that indicates the sensitivity of the news provider's response to the quality of its competitors' information. Each news provider sought the level of information quality that maximized its own profit and derived a Nash equilibrium.

In the ultimate game in the incomplete information game, the news provider (the proposer) makes a decision regarding the "truthfulness" of the information, and the public (the responder) accepts or rejects the information. Respondents do not have perfect knowledge of the truthfulness of the information provided. The proposer chooses the optimal "truthiness" level, and the responder accepts or rejects the information.

Given the interdependence in the perfect information game in the ultimate game, the proposer makes a suggestion regarding the "quality" of the information. The responder chooses to accept or reject this proposal, and if accepted, both parties benefit based on the proposed information quality. When interconnectivity was high, the proposer was more sensitive to the expectations of the responder and tended to provide higher quality information.

Overall, this document provides insight into how news providers strategize with respect to information quality and truthfulness and how this affects the public's interests. It also provided considerations for developing strategies and policies to address fake news and fact-checking issues.

The final section discusses the full information game and the non-cooperation game in the context of fake news and fact-checking, as well as interdependence (entanglement) considerations in the prisoner's dilemma. Scenarios in which news providers A and B must choose between providing accurate information (cooperation) or spreading fake news (betrayal) are also considered.

In this game setting, we assume that news providers A and B have a "cooperation (C)" or "betrayal (D)" strategy and adapt the typical pay table of a prisoner's dilemma to the context of fake news. Interdependence refers to a situation in which one player's choices cause large fluctuations in the other's payments. In this game, a Nash equilibrium is reached when both players choose to betray, but it is in the best social interest for both to cooperate (Pareto optimal).

In the context of incomplete information games, players do not have complete information about their opponent's choices or intentions. Expected payoffs are computed taking into account the uncertainty associated with the player's beliefs and the opponent's strategy. In this scenario, we have mathematically represented the interdependence and outlined the computational process by which players determine the optimal strategy for their opponent's choices.

Through this analysis, we gained a better understanding of the strategic choices and interactions among news providers in fake news and fact-checking problems and their impact on social welfare.

In the context of a non-complete information game and a non-cooperative game, organizing media tendencies in terms of sequential move number games, perturbed perfect equilibrium, and trembling hands in this paper helps us understand the strategic interactions and information asymmetries among information providers. Below we show how to analyze media tendencies using this framework.

Perspectives on Sequential Move-Taking Games

The sequential move-order game captures the sequential decision-making process by which media disseminate information. The framework takes into account that the actions of prior media influence the choices of subsequent media.

Information precedence: When one media outlet precedes another in disseminating information through scoops or independent investigative reporting, the reaction of other media outlets to that information is determined sequentially. Competition and imitation: When some media outlets publish attention-grabbing information, others may follow suit by publishing similar information in an attempt to increase viewership and clicks. Verification and correction of information: If earlier information is inaccurate, subsequent media may play a role in verifying and correcting the information through fact-checking.

Perturbed Perfect Equilibrium Perspective

Perturbed perfect equilibrium is an equilibrium concept that takes into account the minute probability that the media may make an error (shaking hands) in providing information. This perspective is suitable for analyzing the risk of media misinformation and its consequences.

Risk of misinformation

The risk of the media disseminating misinformation can be attributed to the pressure to disseminate information in a hurry, lack of confirmation, etc. Maintaining credibility The media must adopt strategies to ensure the accuracy of information to avoid misinformation and maintain credibility. Information Correction Process When misinformation is disseminated, the announcement of corrections or corrections is a critical component of media credibility.

The Trembling Hand Perspective

The shaking hands concept indicates the potential for the media to make unintentional wrong choices and reflects the uncertainty in the media's handling of information.

Spread of misinformation

The unintentional spread of misinformation by the media can create confusion in society and undermine media credibility. Importance of verification mechanisms: To minimize misinformation, media need to strengthen their information verification mechanisms. Reaction and Countermeasures The media must react quickly to misinformation and provide corrective information to mitigate the impact and restore credibility.

The above analysis shows that media need to develop strategies that take into account the sequential decision-making process, the risk of misinformation, and the potential for the spread of misinformation when disseminating information. It also suggests that it is important to maintain media credibility and improve the quality of information in society by strengthening the processes of fact-checking and information correction.

In the context of noncomplete information and noncooperative games, organizing media tendencies in terms of sequential move-order games, perturbed perfect equilibrium, and trembling hands helps us understand strategic interactions and information asymmetries among information providers. Below, we show how to analyze media tendencies using this framework, The above analysis shows that media need to develop strategies that take into account the sequential decision-making process, the risk of misinformation, and the potential for the spread of misinformation when disseminating information. It also suggests once again the importance of maintaining media credibility and enhancing the quality of information in society by strengthening the processes of fact-checking and information correction.

2. Discussion:Computational Experimental Design: Non-Complete Information Game Analysis of Fake News and Fact-Checking

Objective

In this proposal, we propose a computational experiment on the non-perfect information game of fake news and fact-checking, applying the Cournot model. In particular, the objective is to analyze the competition among news providers regarding the quality of information and its impact on the public interest. The computational experiment is designed to embody the theoretical framework of non-perfect information games between news providers in the context of fake news and fact-checking and to elucidate the dynamics of strategic interactions using the concept of trembling hand theory: perturbed perfect equilibrium. This section details the objectives, methodology, and expected results of the computational experiments.

Objectives of the Computational Experiments

The Cournot model is often used to analyze oligopolistic markets, but this project applies this model to a non-perfect information game between news providers competing on the quality of information. Understanding how each news provider's strategy affects the public interest and how the prisoner's dilemma arises is the goal of this study.

1. Model validation of fake news diffusion: to assess how accurately the proposed game-theoretic model captures fake news diffusion and fact-checking behavior.

2. Analysis of strategic interactions: to explore how strategic interactions among news providers affect information quality and public perception.

3. Identifying perturbed perfect equilibria: identify equilibrium strategies in situations where players make non-optimal choices with minute probabilities and propose countermeasures to fake news proliferation.

Players: news providers A and B Strategies: level of information quality q_A and q_B Gain Functions: News provider A: q_A and news provider B: q_B News provider A: $\Pi_A(q_A, q_B) = P(Q)q_A C(q_A)$ News provider B: $\Pi_B(q_A, q_B) = P(Q)q_B C(q_B)$ where $Q = q_A + q_B$ is the total market-wide information quality, $P(Q)$ is the public's evaluation of information quality, and $C(q)$ is the cost of providing information quality.

Expected Gain Function

News provider A: $E[\Pi_A] = P(E[Q])q_A C(q_A)$ News provider B: $E[\Pi_B] = P(E[Q])q_B C(q_B)$ where $E[Q]$ is the expected value of information quality for the entire market.

Research Methods

Maximize the gain function, Derive the information quality levels q_A and q_B at which each news provider maximizes its own expected gain function. Compute Nash Equilibrium, Compute the Nash equilibrium from the optimal response functions of both news providers to obtain the quality level of information. 3. Analysis of Pareto Optimum and Prisoner's Dilemma: We examine the prisoner's dilemma situation by comparing the situation where each news provider cooperates to provide high quality information and the strategy when they do not cooperate.

Simulation model construction: We construct a sequential move-order game that includes the choices of the news providers (reporting the truth, spreading fake news, conducting fact-checking, etc.) and the public's responses to these choices (trust, suspicion, conducting fact-checking, etc.).

Parameter setting: Set parameters such as the probability of public reaction to the news provider's choices, the risk of fake news spread, the effect of fact-checking, etc. Apply numerical algorithms: Apply numerical methods such as backward induction and fixed point algorithms to find an equilibrium strategy. Sensitivity analysis: Evaluate the model's response to different values of parameters to verify the robustness of the results.

Expected Results

Based on the model's predictions, we show the strategic choices news providers make regarding the spread of fake news and the resulting changes in public perception.

We identify equilibrium strategies when micro-perturbations are taken into account and propose strategies and policies that are effective in reducing fake news proliferation.Use simulation results to provide insight into the public response to fake news and the effectiveness of fact-checking. Theoretical description underlying this paper (selected excerpts)

Theory of Non-Complete Information Games

In non-perfect information games, players are uncertain about some elements of the game (e.g., the choices and intentions of other players). In this context, news providers do not have perfect information about whether other providers will spread fake news or provide truthful information.

Application of Perturbed Perfect Equilibrium

Perturbed perfect equilibrium considers the possibility that each player makes a non-optimal choice with minute probability. Applying this concept allows the model to incorporate accidental errors and unintended choices that occur in the real world.

Game Dynamics and Information Flow

In sequential turn games, as the game progresses, players gain new information based on the choices made by the player with the previous turn. This information flow influences player expectations and strategy choices.

Strategies for Controlling the Spread of Fake News

Findings obtained through computational experiments can be used to develop strategies and policies to control the spread of fake news and preserve the quality of information. For example, there may be ways to increase the effectiveness of fact-checking and policies that provide news providers with incentives to provide high-quality information. policies that give news providers incentives to provide high-quality information, and so on.

Through this computational experiment and theoretical exposition, we aim to develop a comprehensive understanding of information flows and their quality in the context of fake news and fact-checking, and to contribute to solving real-world problems.

3. Discussion:Previous Work:Expected Payoff Function and Entanglement

When modeling the issue of fake news and fact-checking as a complete information game in extensive form and analyzing it using the structure of the prisoner's dilemma, we consider a scenario where media agents (for example, news providers A and B) choose whether to spread fake news or provide accurate information. In the framework of a non-cooperative game, each agent seeks to maximize their own payoff, influenced by the choices of other agents.

Game Setup

Players: News providers A and B

Strategies: Spread fake news (F), Provide accurate information (T)

Payoffs: The impact of news dissemination (increase in viewership, decrease in credibility, etc.)

Payoff Function

The payoff function represents the combination of strategies that news providers can take and the corresponding payoffs. Using a typical payoff structure from the prisoner's dilemma, we can set it up as follows:

If both spread fake news (F, F), both credibility decreases, and the payoff is the lowest (e.g., A's payoff = 1, B's payoff = 1).

If one spreads fake news (F) and the other provides accurate information (T), the one who spreads fake news gains a higher payoff, but the one who provides accurate information sees a decrease in their payoff (e.g., A's payoff = 3, B's payoff = 0).

If both provide accurate information (T, T), credibility is maintained, and the payoff is moderate (e.g., A's payoff = 2, B's payoff = 2).

Expected Payoff Function and Entanglement

The expected payoff function is used by players to calculate the expected payoff of their strategy against the strategies of other players. In this context, "entanglement" refers to how closely related the choices of players are, indicating how one's choice significantly affects the other.

The expected payoff function is expressed as follows:

Expected payoff for news provider A: $E[U_A] = p_F \cdot U_A(F, s_B) + p_T \cdot U_A(T, s_B)$

Expected payoff for news provider B: $E[U_B] = p_F \cdot U_B(s_A, F) + p_T \cdot U_B(s_A, T)$

Here, p_F and p_T represent the probabilities of choosing the strategy to spread fake news and to provide accurate information, respectively. s_A and s_B are the strategies chosen by news providers A and B, respectively.

Calculation Process

(1) Set up the payoff functions for each news provider.

(2) Calculate the payoffs for all combinations of strategies.

(3) Calculate the expected payoff for each news provider and choose the optimal strategy.

(4) Verify if the chosen strategy forms a Nash equilibrium.

Through this analysis, we can understand the incentives for news providers to spread fake news and its impact, as well as the importance of fact-checking. Moreover, by using the structure of the prisoner's dilemma, insights can be gained into the design of strategies and policies that encourage cooperative outcomes even in non-cooperative situations.

In the context of fake news and fact-checking, when considering complete information games and cooperative games in extensive form and taking into account entanglement (interdependence) in the prisoner's dilemma problem, we will detail the payoff functions and expected payoff functions. In this scenario, news providers A and B choose whether to spread fake news (betrayal D) or provide accurate information (cooperation C).

Game Setup

Players: News providers A and B

Strategies: Provide accurate information (cooperation C), Spread fake news (betrayal D)

Payoff Functions

We apply the payoff table of the prisoner's dilemma to the context of fake news.

A \B	Cooperation (C)	Betrayal (D)
Cooperation (C)	(3, 3)	(0, 5)
Betrayal (D)	(5, 0)	(1, 1)

$U_A(C, C) = 3, U_B(C, C) = 3$: The payoff for both when cooperating

$U_A(D, C) = 5, U_B(C, D) = 0$: The payoff for A betraying while B cooperates

$U_A(C, D) = 0, U_B(D, C) = 5$: The payoff for A cooperating while B betrays

$U_A(D, D) = 1, U_B(D, D) = 1$: The payoff for both when betraying

Analysis of Entanglement

Entanglement arises from A's choices affecting B's payoffs and vice versa, creating the prisoner's dilemma.

Calculation Process

(1) **Deriving Nash Equilibrium:** Assuming each player knows the other's choice and chooses to maximize their payoff, the Nash equilibrium in this case would be both choosing betrayal (D, D).

(2) **Identifying Pareto Optimality:** The state where both players cooperate (C, C) maximizes social welfare, making it Pareto optimal.

(3) **Effects of Entanglement:** Due to the interdependence of the players' strategies, if one chooses to cooperate and the other betrays, the cooperator receives the lowest payoff. This entanglement creates a prisoner's dilemma situation where individual optimal strategies differ from the collective optimal solution.

Mathematical Representation of Entanglement

Entanglement is represented mathematically by the fact that the payoff functions of players A and B depend on the other's strategy. Specifically, A's payoff function U_A changes based on B's choice of C or D, and similarly, B's payoff function U_B changes based on A's choice.

Through this analysis, we can understand the strategic choices and interactions of news providers in the context of fake news and fact-checking, and how it affects social welfare. Especially, when individual optimal strategies are not optimal for the group as a whole, the importance of social cooperation and coordination mechanisms becomes evident.

To detail entanglement with mathematical formulas and calculation processes, we use an example where fake news and fact-checking are considered in the context of complete information and cooperative games within the prisoner's dilemma framework. News providers A and B choose whether to spread fake news (betrayal D) or provide accurate information (cooperation C).

Payoff Table

A \B	Cooperation (C)	Betrayal (D)
Cooperation (C)	(3, 3)	(0, 5)
Betrayal (D)	(5, 0)	(1, 1)

Definition of Payoff Functions

A cooperates, B cooperates: $U_A(C, C) = 3, U_B(C, C) = 3$

A betrays, B cooperates: $U_A(D, C) = 5, U_B(C, D) = 0$

A cooperates, B betrays: $U_A(C, D) = 0, U_B(D, C) = 5$

A betrays, B betrays: $U_A(D, D) = 1, U_B(D, D) = 1$

Mathematical Formulation of Entanglement

Entanglement is the state where A's choices directly affect B's payoffs and vice versa. This interdependence is mathematically represented.

Calculation Process

(1) **Deriving Nash Equilibrium:** If A's choice affects B's payoff, consider B's optimal response. If B chooses C, A's optimal choice is D ($U_A(D, C) > U_A(C, C)$). Similarly, if B's choice affects A's payoff, consider A's optimal response. If A chooses C, B's optimal choice is D ($U_B(D, C) > U_B(C, C)$). Therefore, the Nash equilibrium is (D, D).

(2) **Identifying Pareto Optimality:** If both players choose C, social welfare is maximized ($U_A(C, C) + U_B(C, C) = 6$), making it Pareto optimal.

(3) **Effects of Entanglement:** Entanglement becomes apparent when one player chooses C and the other chooses D. This choice results in the cooperator receiving the lowest payoff ($U_A(C, D) = 0$, $U_B(C, D) = 0$). The effect of entanglement can be mathematically represented by the change in expected payoff for B when A chooses C, and similarly for A when B chooses C.

Through these formulas and calculation processes, we can understand the strategic interactions and the impact of entanglement among news providers in the context of fake news and fact-checking. High entanglement means players' choices are strongly dependent on others, and individual optimal strategies may differ from the collective optimal solution.

In the context of fake news and fact-checking, consider a situation combining incomplete information games with cooperative games and develop payoff functions and expected payoff functions incorporating entanglement (interdependence) within the framework of the prisoner's dilemma. In this scenario, news providers (A and B) decide whether to spread fake news or provide truthful information, without having complete information about the other's choices or reliability.

Game Setup

Players: News providers A and B

Strategies: Provide truthful information (Cooperation C), Spread fake news (Betrayal D)

Setting up the Payoff Table

The typical payoff table for the prisoner's dilemma is adjusted to include elements of incomplete information as follows:

	Cooperation (C)	Betrayal (D)
Cooperation (C)	(3, 3)	(0, 5)
Betrayal (D)	(5, 0)	(1, 1)

Payoff Functions

The payoff functions for each player are as follows:

$$U_A(C, C) = 3p_A$$
$$U_A(D, C) = 5p_A$$
$$U_A(C, D) = 0p_A$$
$$U_A(D, D) = 1p_A$$

Here, p_A represents the potential cost or risk for player A when spreading fake news, and similarly, p_B is set for player B. This cost could include the loss of credibility if the fake news is exposed or the potential damage to reputation from fact-checking.

Expected Payoff Functions

In incomplete information games, the expected payoff function is calculated considering the player's beliefs and the uncertainty regarding the strategies of other players.

$$E[U_A(C)] = q_B \cdot (3p_A) + (1 - q_B) \cdot (0p_A)$$
$$E[U_A(D)] = q_B \cdot (5p_A) + (1 - q_B) \cdot (1p_A)$$

Here, q_B is the probability that A believes B will cooperate.

Considering Entanglement

Entanglement means that the choice of player A affects the expected payoff of player B and vice versa. To express this mathematically, define the expected payoff function for player B similarly and calculate how A's strategy affects B's expected payoff.

Calculation Process

(1) Derive the optimal strategy for player B based on A's strategy to maximize B's expected payoff.

(2) Derive the optimal strategy for player A considering B's optimal response to maximize A's expected payoff.

(3) Analyze the entanglement by considering how the strategies of A and B affect each other and assess the effects of interdependence.

Through this example, we can understand the strategic interactions between news providers in the context of fake news and fact-checking and how entanglement under incomplete information affects strategic choices. It's particularly important to consider how each player's choice impacts the expected payoff of others.

4. Discussion:Analyzing Fake News and Fact-Checking in Incomplete Information Games

Considering the text above, incomplete information games can arise due to factors such as lack of communication. This paper discusses the application of sequential move games, perturbed perfect equilibrium, and trembling hand theory in the context of media reporting challenges.

4.1 Game Setup

4.2 Players and Actions

Players: News providers A (Leader) and B (Follower).

Actions: Each player can choose "Spread fake news" (F) or "Provide accurate information" (T).

4.3 Perturbation (Trembling Hand)

Players consider the possibility of making a suboptimal choice with a small probability ε.

4.4 Expected Payoff Functions

The expected payoff functions for Player 1 and Player 2, denoted as U_1 and U_2, are based on their actions. For example, $U_1(A, B)$ represents the payoff for Player 1 choosing A while Player 2 chooses B.

4.5 Calculation Process

4.6 Identifying Pure Strategy Nash Equilibrium

First, the pure strategy Nash equilibrium is identified without considering the perturbation.

4.7 Calculating the Effect of Perturbation

The expected payoff functions are recalculated considering the perturbation. For example, for Player 1 choosing action A:

$$U_1(A) = (1 - \varepsilon)U_1(A, B) + \varepsilon U_1(B, B)$$

4.8 Deriving Perturbed Perfect Equilibrium

The optimal response strategies are recalculated considering the perturbation, using backward induction to derive the PPE. This analysis provides insights into strategic behavior in addressing fake news dissemination, highlighting the importance of considering trembling hands in strategic decision-making.

Given the passage above, a lack of communication among other factors can lead to the occurrence of games with incomplete information. Considering the challenges associated with media reporting, this analysis method requires a detailed examination of sequential move games, perturbed perfect equilibrium, and the introduction of trembling hand theory. Perturbed Perfect Equilibrium (PPE) in sequential move games accounts for the possibility that players may make a suboptimal choice with a minute probability at each stage of the game. This concept is employed in analyzing the strategic decision-making of players in sequential games, particularly in games with complete information. Here, the mathematical representation and calculation process of PPE in a specific sequential move game are explained.

Game Setup

(1) Players: Player 1 (Leader) and Player 2 (Follower)

(2) Actions: Both Player 1 and Player 2 can choose Action A or Action B.

(3) Perturbation (Trembling Hand): It is considered that each player might make a suboptimal choice between Actions A or B with a tiny probability ε.

Expected Payoff Functions

The expected payoff functions for Player 1 and Player 2 are denoted as U_1 and U_2, respectively, representing the payoffs for Actions A and B. For instance, $U_1(A, B)$ signifies the payoff for Player 1 choosing A while Player 2 chooses B.

Calculation Process

(1) **Identifying Pure Strategy Nash Equilibrium**: Initially, the game's pure strategy Nash equilibrium is identified without considering perturbation. This involves calculating the expected payoffs for each player choosing their optimal actions and finding the combination of actions that maximizes these payoffs.

(2) **Calculating the Impact of Perturbation**: The expected payoff functions are recalculated, taking into account the possibility that each player might make a suboptimal choice with a small probability ε. For example, if Player 1 chooses Action A, the expected payoff considering perturbation would be $U_1(A) = (1-\varepsilon)U_1(A, B) + \varepsilon U_1(B, B)$.

(3) **Deriving Perturbed Perfect Equilibrium**: With perturbation taken into account, the optimal response strategies for Player 1 and Player 2 are recalculated. The optimal response for Player 2 to Player 1's choice is determined, followed by Player 1 choosing the optimal action based on that response. This process is conducted using backward induction. If the strategy combination derived through this calculation remains the optimal response strategy for both players despite the small perturbation, it is considered a PPE.

Through this calculation process, the strategic decision-making of players in sequential move games can be analyzed, taking into account the trembling hand (perturbation). PPE offers a more realistic equilibrium concept by including the possibility of suboptimal choices by players at each stage of the game.

When considering the issue of fake news dissemination and its normalization within the context of incomplete information games and extensive form games, the Perturbed Perfect Equilibrium (PPE) derives an equilibrium by accounting for the possibility that each player (in this case, news providers) makes an incorrect choice (trembling hand) with a small probability in games where players make sequential decisions and can observe the choices of other players. Here, we discuss the process of news providers choosing and publishing information, calculating the expected payoff for each choice, and finding the final perturbed perfect equilibrium.

Game Setup

Players: News providers A and B

Actions: Each news provider chooses to "verify" or "not verify" the information.

Information: News providers do not have complete information about the accuracy of the information (incomplete information game).

Extensive Form: The game proceeds in stages, with news provider A choosing an action first, followed by news provider B.

Expected Payoff Functions

Let the expected payoff for news provider A be U_A and for news provider B be U_B. Let p be the probability that the information is accurate, and $1-p$ be the probability that it is inaccurate. Let C be the cost of verifying information, R be the payoff for providing accurate information, and L be the loss for disseminating inaccurate information.

The expected payoff for news provider A can be expressed as:

When verifying information: $U_A = p(R-C) + (1-p)(-L-C)$

When not verifying information: $U_A = pR+(1-p)(-L)$

The expected payoff for news provider B can also be calculated similarly but depends on the choice of news provider A.

Introduction of the Trembling Hand

Consider that news providers may make an incorrect choice (e.g., disseminate inaccurate information without verification) with a tiny probability ε.

Calculation Process

(1) **Identifying Pure Strategy Equilibria**: Find the pure strategy equilibrium for each news provider. This involves calculating the expected payoff for news providers A and B when they choose either to “verify” or “not verify” and determining the optimal choice.

(2) **Considering Perturbation**: Next, consider the possibility that news providers make an incorrect choice with a small probability ε. This may subtly alter the optimal strategies at each stage.

(3) **Recalculating Optimal Responses**: Recalculate the optimal response strategies for each news provider, taking the “trembling hand” into account.

(4) **Deriving Perturbed Perfect Equilibrium**: If the optimal response for all news providers does not change despite the tiny “tremble,” that strategy combination is considered a perturbed perfect equilibrium.

Through this process, we can analyze the strategic interaction between news providers in addressing the issue of fake news dissemination and normalization and derive more robust strategies for maintaining information quality.

When considering the "Trembling Hand Perfect Equilibrium" in the context of incomplete information games and extensive form games among news providers dealing with fake news and fact-checking, the expected payoff functions, the formula for the trembling hand, and their calculation processes are as follows:

In the Case of Incomplete Information Games

In incomplete information games, players do not have complete information about the types or intentions of other players. In this context, the possibility of news providers inadvertently spreading fake news is modeled as the "trembling hand."

Expected Payoff Function

The expected payoff function for player i is represented as a weighted average of payoffs depending on the strategies and types of other players.

$$U_i(s_i, s_{-i}, \theta_i) = \sum_{\theta_{-i} \in \Theta_{-i}} \sum_{s_{-i} \in S_{-i}} \pi_i(s_i, s_{-i}, \theta_i, \theta_{-i}) \cdot p(\theta_{-i}) \cdot p(s_{-i}|\theta_{-i})$$

Here, π_i is the payoff function, θ_i and θ_{-i} are the types of player i and other players respectively, S_{-i} is the strategy set of other players, and Θ_{-i} is the set of types of other players.

Trembling Hand Formula

The trembling hand models the player making an unintended strategy choice with a small probability ε.

$$s_i^{\varepsilon} = (1 - \varepsilon)s_i^* + \varepsilon \Delta S_i$$

Here, s_i^* is the pure strategy equilibrium of player i, and ΔS_i is a uniform probability distribution within S_i.

Calculation Process

(1) Identify the pure strategy equilibrium for each player.

(2) Introduce a small tremble ε to the pure strategy equilibrium to calculate the trembling hand strategy.

(3) Recalculate the expected payoff for each player using the trembling hand strategy.

(4) If the trembling hand strategy remains the optimal response for all players, that strategy profile is considered the trembling hand perfect equilibrium.

In the Case of Extensive Form Games (Complete Information Games)

In extensive form games, decisions in the game are sequenced over time, and each player makes decisions after observing the choices of previous players. In this context, the dissemination of fake news and the response of fact-checking are modeled as sequenced decisions.

Expected Payoff Function

In extensive form games, each player’s expected payoff depends on the decisions at each node of the game tree and the subsequent responses of other players.

$$U_i(s_i, s_{-i}) = \sum_{h \in H} \pi_i(s_i(h), s_{-i}(h)) \cdot p(h)$$

Here, H represents all possible histories on the game tree, h is a specific history, and π_i is the payoff function.

Trembling Hand Formula

The concept of the trembling hand takes into account the possibility that layers might choose different actions at each decision node with a small probability.

The trembling hand strategy at each node is primarily calculated using backward induction.

Calculation Process

(1) Define the structure of the game using a game tree.

(2) Use backward induction to identify the optimal strategy at each decision node.

(3) Introduce a small tremble at each node and recalculate the optimal strategy considering its impact.

(4) If the trembling hand strategy remains the optimal response at all nodes, that strategy profile is considered the trembling hand perfect equilibrium.

Through this approach, it becomes possible to analyze the dynamics of the dissemination of fake news and the response of fact-checking in more detail and to develop strategies that take into account the risk of misinformation.

5. Discussion:Trembling Hand Perfect Equilibrium in Incomplete Information and Extensive Form Games Among News Providers

In the Case of Incomplete Information Games

In incomplete information games, players lack complete information about the types or intentions of other players. The possibility of news providers inadvertently spreading fake news is modeled as the "trembling hand."

Expected Payoff Function

The expected payoff function for player i is represented as a weighted average of payoffs, dependent on the strategies and types of other players:

$$U_i(s_i, s_{-i}, \theta_i) = \sum_{\theta_{-i} \in \Theta_{-i}} \sum_{s_{-i} \in S_{-i}} \pi_i(s_i, s_{-i}, \theta_i, \theta_{-i}) \cdot p(\theta_{-i}) \cdot p(s_{-i} | \theta_{-i}) \tag{1}$$

Here, π_i is the payoff function, θ_i and θ_{-i} represent the types of player i and other players, respectively, S_{-i} is the strategy set of other players, and Θ_{-i} is the set of types of other players.

Trembling Hand Formula

The trembling hand models the player making an unintended strategy choice with a small probability ε:

$$s_i^{\varepsilon} = (1 - \varepsilon) s_i^* + \varepsilon \Delta S_i \tag{2}$$

Here, s_i^* is the pure strategy equilibrium of player i, and ΔS_i is a uniform probability distribution within S_i.

Calculation Process

Identify the pure strategy equilibrium for each player, Introduce a small tremble ε to the pure strategy equilibrium to calculate the trembling hand strategy. Recalculate the expected payoff for each player using the trembling hand strategy. If the trembling hand strategy remains the optimal response for all players, that strategy profile is considered the trembling hand perfect equilibrium.

In the Case of Extensive Form Games (Complete Information Games)

In extensive form games, decisions are sequenced over time, and each player makes decisions after observing the choices of previous players. The dissemination of fake news and the response of fact-checking are modeled as sequenced decisions.

Expected Payoff Function

In extensive form games, each player's expected payoff depends on the decisions at each node of the game tree and the subsequent responses of other players:

$$U_i(s_i, s_{-i}) = \sum_{h \in H} \pi_i(s_i(h), s_{-i}(h)) \cdot p(h) \tag{3}$$

Here, H represents all possible histories on the game tree, h is a specific history, and π_i is the payoff function.

Trembling Hand Formula

The concept of the trembling hand takes into account the possibility that players might choose different actions at each decision node with a small probability. The trembling hand strategy at each node is primarily calculated using backward induction.

1. Define the structure of the game using a game tree.

2. Use backward induction to identify the optimal strategy at each decision node.

3. Introduce a small tremble at each node and recalculate the optimal strategy considering its impact.

4. If the trembling hand strategy remains the optimal response at all nodes, that strategy profile is considered the trembling hand perfect equilibrium.

This approach allows for a more detailed analysis of the dynamics of the dissemination of fake news and the response of fact-checking, enabling the development of strategies that take into account the risk of misinformation.

When considering the Trembling Hand Perfect Equilibrium in the context of fake news and fact-checking, where the excessive spread of fake news and the emergence of filter bubbles are treated as incomplete information games and sequential games, the analysis using trembling hand perfect equilibrium becomes complex. In this game, the news provider (Player 1) chooses a strategy to disseminate true or fake news, and the reader (Player 2) decides whether to believe the news or be skeptical and fact-check it. In incomplete information games, players do not have complete information about the other player's type (for example, whether they are a reliable news provider).

5.1 Expected Payoff Function

The expected payoff of the news provider depends on the type of news they disseminate (true or fake) and the reader's response (believe or fact-check). The reader's expected payoff depends on how they process the provided news (believe or doubt) and the truthfulness of the news.

News provider's expected payoff: $U_{NP}(s_{NP}, s_R, t_{NP})$

Reader's expected payoff: $U_R(s_{NP}, s_R, t_{NP})$

Where s_{NP} is the strategy of the news provider, s_R is the strategy of the reader, and t_{NP} is the type of the news provider (reliability).

5.2 Trembling Hand Formula

The trembling hand perfect equilibrium considers the possibility that players make errors (tremble) in their strategy selection with a small probability. This error indicates the possibility of accidentally choosing a different strategy.

Probability of trembling hand: $\varepsilon(s_i \neq s_i^)$

Construct the Game Tree

As a sequential game, build a game tree that includes all possible moves and information sets. Identify Strategy Profiles, Identify all possible strategy profiles for the news provider and the reader. Introduce Trembling Hand, Introduce a small trembling hand probability ε for each strategy. Calculate Optimal Responses, Calculate the optimal response for each player, considering the strategies of other players and the trembling hand probability. Verify Equilibrium, If the optimal responses of all players match and do not change even with an error probability due to the trembling hand, that strategy profile is considered as a trembling hand perfect equilibrium.

Through this analysis, we can understand the strategic interaction between news providers and readers regarding the spread of fake news and filter bubbles, and the equilibrium state resulting from it. This approach may provide useful insights for strategizing and policy-making to address fake news.

In the context of fake news and fact-checking, when analyzing the excessive spread of fake news and its normalization as an incomplete information game and a sequential game, the expected payoff functions considering the trembling hand perfect equilibrium and their calculation process are as follows, News providers (players) choose strategies based on the accuracy of information (true or fake). Meanwhile, recipients (other players) decide whether to believe the provided information or to fact-check it. In this context, incomplete information arises from the uncertainty players have about each other's strategies.

5.3 Introduction of Trembling Hand Perfect Equilibrium

Players may unintentionally choose the wrong strategy (e.g., treating true information as fake). This "trembling hand" is modeled by assuming a small probability of error in strategy selection.

5.4 Expected Payoff Function

The expected payoff function of the news provider depends on the chosen strategy (providing true or fake information) and the recipient's response (believing the information or conducting a fact check).

News provider's expected payoff function: $U_N(s_N, s_R) = \sum_{s_R} \pi_N(s_N, s_R) \times p_R(s_R)$

Recipient's expected payoff function: $U_R(s_N, s_R) = \sum_{s_N} \pi_R(s_N, s_R) \times p_N(s_N)$

Where s_N is the strategy of the news provider, s_R is the strategy of the recipient, π_N and π_R are the payoff functions of the news provider and the recipient, respectively, and $p_R(s_R)$ and $p_N(s_N)$ are the probabilities that the recipient and the news provider choose specific strategies, respectively.

5.5 Trembling Hand Formula

The trembling hand probability is denoted by ε and represents the possibility of choosing the wrong strategy.

Probability of trembling hand: $p(s_i \neq s_i^) = \varepsilon$

Define Strategy Sets

Identify the possible strategy sets for each player. Determine Payoff Functions, Determine the payoff for each strategy combination for the players.

Introduce Trembling Hand Probability

Introduce a small error probability ε for each strategy. Calculate Optimal Response Strategies, Calculate the optimal response strategies for each player, taking into account the trembling hand probability. Identify Equilibrium: Identify the strategy combination when all players are taking optimal response strategies, and verify if it meets the trembling hand perfect equilibrium.

Through this analysis, we can understand the strategic responses of players to the spread of fake news and the filter bubble phenomenon, and the stability of the resulting equilibrium.

When addressing the issue of excessive spread of fake news within the context of fake news and fact-checking as an incomplete information game, considering the Nash Equilibrium conditions and Pareto Optimality conditions using

Trembling Hand Perfect Equilibrium is essential. The conditions are as follows:

5.6 Nash Equilibrium Conditions

In incomplete information games, Nash Equilibrium occurs when each player selects the optimal strategy based on their information set, and no player can unilaterally change their strategy to increase their payoff.

> Nash Equilibrium formula: $\forall i, U_i(s_i^{*}, s_{-i}^{*}) \geq U_i(s_i, s_{-i}^{*})$, where s_i is player i's Nash Equilibrium strategy, and s_{-i} is the combination of other players' Nash Equilibrium strategies.

5.7 Pareto Optimality Conditions

Pareto Optimality is a state where it's impossible to change strategies to improve one player's payoff without reducing the payoff of another player.

> Pareto Optimality formula: $\nexists(s_i', s_{-i}')$ such that $\forall i, U_i(s_i', s_{-i}') \geq U_i(s_i, s_{-i})$ and $\exists j, U_j(s_j', s_{-j}') > U_j(s_j, s_{-j})$

Introduce Strategies and Trembling Hand

Consider each player's set of strategies and the possibility of a trembling hand, where a different strategy might be chosen with a small probability. Identify Nash Equilibrium, Find the state where each player maximizes their payoff, and no player can unilaterally change their strategy to improve their payoff, given the strategies of others (Nash Equilibrium). Verify Pareto Optimality, Check if there are no strategy changes that can simultaneously improve the payoffs of all players.

Through this analysis, strategic insights can be gained to address the problem of fake news spread. In particular, understanding the normalization of information under filter bubbles and the importance of fact-checking is deepened.

When dealing with the excessive spread and normalization of fake news in the context of fake news and fact-checking as a complete information game, it is crucial to consider the Nash Equilibrium conditions and Pareto Optimality conditions using Trembling Hand Perfect Equilibrium. In complete information games, all players are aware of other players' payoff functions and possible strategies, but the possibility of players unintentionally choosing the wrong strategy (trembling hand) is considered.

5.8 Nash Equilibrium Conditions

In Nash Equilibrium, all players choose the best strategy for themselves, and given the strategies of other players, no one can unilaterally change their strategy to improve their payoff. The consideration of trembling hand examines whether the pure strategy Nash Equilibrium persists even in the presence of a small probability of choosing other strategies.

> Nash Equilibrium formula: $\forall i, \forall s_i' \neq s_i, U_i(s_i, s_{-i}) \geq U_i(s_i', s_{-i})$

Where U_i is the payoff function of player i, s_i is player i's strategy, and s_{-i} is the combination of strategies of all other players.

5.9 Pareto Optimality Conditions

Pareto Optimality refers to a situation where it's impossible to improve one player's payoff without harming another player's payoff. This condition identifies socially desirable strategy combinations.

> Pareto Optimality formula: $\nexists(s_i', s_{-i}')$ such that $\forall i, U_i(s_i', s_{-i}') \geq U_i(s_i, s_{-i})$ and $\exists j, U_j(s_j', s_{-j}') > U_j(s_j, s_{-j})$

Define Strategy Sets

Define the set of strategies available to players. In this case, strategies might include spreading fake news, conducting fact-checks, or doing nothing. Set Up Payoff Functions: Define the payoffs for each combination of strategies for players. Spreading fake news might bring short-term benefits but could lead to long-term loss of credibility.

Introduce Trembling Hand

Introduce a small probability of a 'trembling hand' for each player's strategy and identify all pure strategy equilibria. Identify Nash Equilibrium, Look for a state where all players choose the best strategy, and no one can unilaterally change their strategy to improve their payoff given the strategies of others. Verify Pareto Optimality, Examine if there's another combination of strategies that can simultaneously improve the payoffs for all players.

This analysis allows for strategic insights to prevent excessive fake news spread and maintain information quality.

In the context of fake news and fact-checking, considering the spread of excessive fake news and its normalization as an incomplete information game and a sequential cooperative game, the expected payoff functions are defined based on the set of actions available to news providers (players). In this game, players choose whether to spread fake news, conduct fact-checking, or do nothing. Due to the nature of incomplete information, players cannot fully know the type or intentions of other players, and their choices influence each other.

5.10 Definition of Expected Payoff Functions

The expected payoff functions are defined based on the combinations of actions available to news providers and the potential outcomes those actions could lead to. Let S_i be the strategy set of player i, a_i be the action of player i, and θ_i be the type of player i, with S_{-i} and a_{-i} being the strategy set and actions of other players, respectively. The expected payoff function for player i is represented as:

$$U_i(a_i, a_{-i}, \theta_i) = \sum_{\theta_{-i}} p(\theta_{-i}|\theta_i) \cdot u_i(a_i, a_{-i}, \theta_i, \theta_{-i})$$

Where u_i is the payoff function of player i, and $p(\theta_{-i}|\theta_i)$ is the belief (conditional probability) that player i has about other players' types θ_{-i}.

In the context of fake news and fact-checking, when analyzing the problem of excessive spread of fake news as an incomplete information game using Trembling Hand Perfect Equilibrium, the conditions for Nash Equilibrium and Pareto Optimality are as follows:

Nash Equilibrium Conditions

In incomplete information games, Nash Equilibrium is a state where each player chooses their optimal strategy based on their information set, and no player can unilaterally change their strategy to increase their payoff.

Nash Equilibrium formula: $\forall i, U_i(s_i^{;}s^{)}_{-i} \geq U_i(s_i, s^{)}_{-i}$, where s_i is player i's Nash Equilibrium strategy, and s_{-i} is the combination of other players' Nash Equilibrium strategies.

Pareto Optimality Conditions

Pareto Optimality is a state where a change in strategy that can improve the payoff for one player does not decrease the payoff for any other player.

Pareto Optimality formula: $\nexists(s_i', s_{-i}')$ such that $\forall i, U_i(s_i', s_{-i}') \geq U_i(s_i, s_{-i})$ and $\exists j, U_j(s_j', s_{-j}') > U_j(s_j, s_{-j})$

Calculation Process

Define Information Sets

Define the set of information each player possesses. In this context, players might have limited information about the veracity of fake news or whether other players will perform fact-checking. Set Up Payoff Functions, Establish payoff functions for each strategy, such as spreading fake news, conducting fact-checks, or doing nothing. Payoffs could be based on the impact of fake news, the effectiveness of fact-checks, and the loss of social credit.

Introduce Strategies and Trembling Hand

Consider each player's set of strategies and the possibility of a trembling hand, where a different strategy might be chosen with a small probability.

Identify Nash Equilibrium, Find the state where each player maximizes their payoff, and no player can unilaterally change their strategy to improve their payoff, given the strategies of others (Nash Equilibrium).

Verify Pareto Optimality, heck if there are no strategy changes that can simultaneously improve the payoffs of all players.

Through this analysis, strategic insights can be gained to address the problem of fake news spread. In particular, understanding the normalization of information under filter bubbles and the importance of fact-checking is deepened.

In the context of fake news and fact-checking, when the excessive spread of fake news is treated as a complete information game and analyzed using Trembling Hand Perfect Equilibrium, it is crucial to consider Nash Equilibrium conditions and Pareto Optimality conditions. In a complete information game, all players are aware of other players' payoff functions and possible strategies but consider the possibility of players unintentionally choosing the wrong strategy (trembling hand).

Nash Equilibrium Conditions

In Nash Equilibrium, all players choose the optimal strategy for themselves, and given the strategies of other players, no one can unilaterally change their strategy to improve their payoff. The consideration of trembling hand examines whether the pure strategy Nash Equilibrium persists even in situations with a small probability of choosing other strategies.

Nash Equilibrium formula: $\forall i, \forall s_i' \neq s_i, U_i(s_i, s_{-i}) \geq U_i(s_i', s_{-i})$

Where U_i is player i's payoff function, s_i is player i's strategy, and s_{-i} is the combination of strategies of all other players.

Pareto Optimality Conditions

Pareto Optimality refers to a situation where it's impossible to improve one player's payoff without harming another player's payoff. This condition is used to identify socially desirable strategy combinations.

Pareto Optimality formula: $\nexists(s_i', s_{-i}')$ such that $\forall i, U_i(s_i', s_{-i}') \geq U_i(s_i, s_{-i})$ and $\exists j, U_j(s_j', s_{-j}') > U_j(s_j, s_{-j})$

Calculation Process

Define Strategy Sets

Define the set of strategies available to players. In this case, strategies might include spreading fake news, conducting fact-checks, or doing nothing.

Set Up Payoff Functions

Define the payoffs for each combination of strategies for players. Spreading fake news might bring short-term benefits but could lead to long-term loss of credibility.

Introduce Trembling Hand

Introduce a small probability of a 'trembling hand' for each player's strategy and identify all pure strategy equilibria.

Identify Nash Equilibrium

Look for a state where all players choose the best strategy, and no one can unilaterally change their strategy to improve their payoff given the strategies of others.

Verify Pareto Optimality

Examine if there's another combination of strategies that can simultaneously improve the payoffs for all players.

This analysis allows for strategic insights to prevent excessive fake news spread and maintain information quality.

When considering the issue of excessive spread and normalization of fake news in the context of fake news and fact-checking as an incomplete information game and a sequential cooperative game, the expected payoff functions are defined based on the set of actions that news providers (players) may take. In this game, players choose whether to spread fake news, conduct fact-checking, or do nothing. Due to the nature of incomplete information, players cannot fully know the type or intentions of other players, and their choices influence each other.

Definition of Expected Payoff Functions

The expected payoff functions are defined based on the combinations of actions that news providers can take and the potential outcomes those actions may lead to. Let S_i be the strategy set of player i, a_i be the action of player i, and θ_i be the type of player i, with S_{-i} and a_{-i} being the strategy set and actions of other players, respectively. The expected payoff function for player i is represented as:

$U_i(a_i, a_{-i}, \theta_i) = \sum_{\theta_{-i}} p(\theta_{-i}|\theta_i) \cdot u_i(a_i, a_{-i}, \theta_i, \theta_{-i})$

Where u_i is player i's payoff function, and $p(\theta_{-i}|\theta_i)$ is the belief (conditional probability) that player i has about other players' types θ_{-i}.

Calculation Process

Identifying the Strategy Set

Determine all possible strategies that players can take (e.g., spreading fake news, conducting fact-checks, doing nothing). Defining Payoff Functions, Define the payoffs for each combination of strategies for players. These payoffs are determined considering the risks and rewards of spreading fake news, the effectiveness of fact-checking, and the impact on public opinion and information quality. Updating Beliefs, layers update their beliefs based on the actions of other players and past actions, and use these to calculate expected payoffs. Choosing Optimal Strategies, Each player selects the strategy that maximizes their expected payoff. This choice depends on the strategies and beliefs of other players. Searching for Nash Equilibrium, A combination of strategies forms a Nash Equilibrium when all players simultaneously choose optimal strategies. In this equilibrium, given the strategies of others, no player can unilaterally change their strategy to increase their payoff.

Evaluating Pareto Optimality, Assess whether it's possible to simultaneously improve the payoffs of all players, and if possible, consider that combination of strategies as Pareto optimal.

This modeling allows for the analysis of strategic interactions regarding the spread of fake news and the effectiveness of fact-checking, leading to optimal strategies to curb the excessive spread of fake news and improve information quality.

When modeling the issue of excessive fake news spread and its normalization in the context of fake news and fact-checking as an incomplete information game, considering Nash Equilibrium and Pareto Optimal conditions is crucial for understanding strategic interactions and information asymmetry. Below is a general description of Nash Equilibrium and Pareto Optimal conditions and the calculation process in this context.

Nash Equilibrium Conditions

In Nash Equilibrium, each news provider (player) chooses the strategy that maximizes their payoff, given the strategies of other players. In an incomplete information game, each player does not have complete information about the types or choices of others but selects strategies based on expectations of possible actions and beliefs of others.

Formulas and Calculation Process

Setting Player's Payoff Functions

$U_i(s_i, s_{-i}, \theta_i, \theta_{-i})$ where s_i is player i's strategy, s_{-i} is the strategy of other players, θ_i is player i's type, and θ_{-i} is the type of other players.

Calculating Expected Payoff

Players calculate their expected payoff $E[U_i]$ based on the probability distribution of other types and beliefs about other players' strategies.

Choosing Optimal Strategies

Each player selects their strategy s_i that maximizes their expected payoff, based on given beliefs and the strategies of other players.

Verifying Nash Equilibrium

The strategy profile $(s'_1 s'_2 ..., s'_n)$ constitutes a Nash Equilibrium if the chosen strategy is the best response for all players.

Pareto Optimal Conditions

Pareto Optimality refers to a state where one player's situation can be improved without causing loss to another. The goal is to find a combination of strategies that suppresses the spread of fake news while maximizing the overall quality of information.

Formulas and Calculation Process

um of All Players' Payoffs

Consider the sum of all players' payoffs, $\sum_{i=1}^{n} U_i(s_i, s_{-i}, \theta_i, \theta_{-i})$.

Searching for Improvable Strategies

Review all possible combinations of strategies to check if there's a strategy that can improve one player's payoff without harming others.

Identifying Pareto Optimal

The combination of strategies that cannot improve all players' payoffs without harming others is considered Pareto optimal.

This approach allows for the analysis of strategic interactions in incomplete information games in the context of fake news and fact-checking, and to derive strategies that maximize information quality while preventing the excessive spread of fake news.

Exploring Nash Equilibrium and Pareto Optimality in the context of news providers in a complete information game within the realm of fake news and fact-checking is beneficial for understanding strategic decision-making regarding news quality and its impacts. In this scenario, each news provider makes strategic choices about whether to report the truth or disseminate fake news. In a complete information game, each player fully understands the payoff functions and available strategies of other players.

Deriving Nash Equilibrium

In Nash Equilibrium, each news provider selects their optimal strategy considering the strategies of other providers. In this state, no player can unilaterally change their strategy to improve their payoff.

Formulas

Let S_i be the strategy set for player (news provider) i, where $s_i \in S_i$ represents the strategies available to player i (reporting truth T or spreading fake news F). Let $\Pi_i(s_i, s_i)$ be the payoff function for player i, where s_i is the combination of strategies of all players other than i.

Nash Equilibrium satisfies the following condition: $\forall i, s_i^{=} \arg\max_{s_i \in S_i} \Pi_i(s_i, s_i^{)}$

Calculation Process

Define the payoff functions for each news provider. Calculate the payoffs for each player for all possible combinations of strategies. Look for the combination of strategies where no player can improve their payoff by changing their strategy unilaterally.

Identifying Pareto Optimality

Pareto optimality is a state where improving the situation of one player cannot be done without worsening the situation of another.

Formulas Let $(s_1^P, s_2^P, ..., s_n^P)$ be the Paretooptimal strategy combination.

Pareto optimality satisfies the following condition: $\forall i, \nexists(s'_1, s'_2, ..., s'_n), \Pi_i(s'_1, s'_2, ..., s'_n) > \Pi_i(s_1^P, s_2^P, ..., s_n^P)$ and $\forall j \neq i, \Pi_j(s'_1, s'_2, ..., s'_n) \geq \Pi_j(s_1^P, s_2^P, ..., s_n^P)$

Calculation Process

Calculate the payoffs for each news provider for all possible combinations of strategies. Look for the combination of strategies that improves the payoff of one player without worsening the payoffs of others.

This approach provides a framework to understand the strategic interactions among news providers and the potential impact of policies or regulations to prevent the spread of fake news.

Applying Nash Equilibrium and Pareto Optimality in the context of an incomplete information game among news providers in the realm of fake news and factchecking helps in understanding the competitive dynamics for news quality and reliability. Here, it is assumed that news providers make choices to provide highquality (truthful) or lowquality (inaccurate or fake) news.

Nash Equilibrium

Nash Equilibrium is a state where all players choose their strategies after considering the strategies of other players, resulting in a situation where no player can improve their payoff by unilaterally changing their strategy.

Formulas

Let S_A and S_B be the strategy sets for news providers A and B, respectively, with strategies to provide highquality news (H) or lowquality news (L). Let $\Pi_A(s_A, s_B)$ and $\Pi_B(s_A, s_B)$ be the payoff functions for news providers A and B, respectively.

Nash Equilibrium satisfies the following conditions: $s_A^{=}$ arg $\max_{s_A \in S_A} \Pi_A(s_A, s_B^{)}$ $s_B^{=}$ arg $\max_{s_B \in S_B} \Pi_B(s\text{'}_A s_B)$

Calculation Process

Define the payoff functions for each news provider. Find the best response strategy for one news provider assuming the strategy of the other is fixed. Identify the point where all news providers cannot improve their payoffs by changing their strategies unilaterally.

Pareto Optimality

Pareto Optimality is a state where the situation of one player can be improved without worsening the situation of another.

Formulas Let (s_A^P, s_B^P) be the Paretooptimal strategy combination.

Pareto Optimality satisfies the following conditions: For all $s_A \in S_A$ and $s_B \in S_B$, $\Pi_A(s_A^P, s_B^P) \geq \Pi_A(s_A, s_B)$ and $\Pi_B(s_A^P, s_B^P) \geq \Pi_B(s_A, s_B)$.

Calculation Process

Search among all possible combinations of strategies for the one where the payoffs for both news providers A and B are maximized. Identify the combination where improving the payoff of one news provider does not harm the other.

This framework provides insights into the incentives for news providers to offer highquality information and design policies to curb the spread of fake news.

When applying the concept of "Trembling Hand Perfect Equilibrium" in the context of an incomplete information game among news providers dealing with fake news and factchecking, scenarios including the possibility of news providers mistakenly disseminating inaccurate information ("trembling hand" errors) are considered. This concept helps in strategizing to maintain the quality and accuracy of information while accounting for the risk of unintentionally providing misinformation by news providers.

Formulas for Trembling Hand Perfect Equilibrium

Strategy Sets of News Providers

Let S_A and S_B be the strategy sets for news providers A and B. Each strategy represents a level of information accuracy.

Identification of Pure Strategy Equilibria

First, identify the pure strategy equilibria s_A and s_B. This is the combination of strategies where both news providers choose the optimal level of accuracy for each other.

Introduction of Trembling Hand

Introduce a "tremble" where each news provider selects an incorrect level of information with a small probability ε.

Definition of Expected Payoff Function

The expected payoff for news provider A is represented as follows:

$U_A(s_A, s_B) = \sum_{s_B \in S_B} \pi_A(s_A, s_B) \prod_B p_B(s_B)$

Here, $\pi_A(s_A, s_B)$ is the payoff for news provider A in the strategy combination (s_A, s_B), and $p_B(s_B)$ is the probability that news provider B chooses strategy s_B.

Calculation of Optimal Response Strategies

Calculate the optimal response strategies $s_A^{(}\varepsilon)$ and $s_B^{(}\varepsilon)$ for each news provider in the presence of "tremble".

Identification of Trembling Hand Perfect Equilibrium

If $s_A^{(}\varepsilon)$ and $s_B^{(}\varepsilon)$ remain unchanged even when a small probability ε of other strategies being selected, then $(s\text{'}_A s_B^{)}$ constitutes a trembling hand perfect equilibrium.

Calculation Process

Identify pure strategy equilibria, Introduce a small "tremble" to the pure strategies and recalculate the expected payoffs for each news provider. Find the optimal responses for each news provider in the presence of tremble, If the optimal responses for all news providers remain the original pure strategy equilibria even in the presence of a small "tremble", then the strategy combination is a trembling hand perfect equilibrium.

This application allows news providers to formulate more robust strategies to maintain the quality and accuracy of information by considering the possibility of errors. It can also be helpful in designing policies and regulations to maintain the quality of information provision in the context of fake news and factchecking.

Introducing "Trembling Hand Perfect Equilibrium" (THPE) in the context of an incomplete information game among news providers dealing with fake news and factchecking provides a valuable perspective for analyzing the behaviors and strategies of information providers. Below are detailed benefits, drawbacks, and expected theoretical supplements.

Benefits

Increased Robustness

THPE offers equilibria that are stable even against small errors or uncertainties. In the context of fake news, it helps model the risk of information providers inadvertently disseminating inaccurate information, aiding in the construction of robust strategies.

Improved Strategic Predictions

By considering the interactions among news providers, THPE helps predict which strategies are sustainable in the long term, enabling more effective countermeasures against fake news.

Reduced Misinformation Risk

Using THPE clarifies strategies to prevent the spread of misinformation, guiding how to maintain information quality while considering the risk of disseminating false information.

Drawbacks

Increased Complexity

Introducing THPE increases the complexity of the analysis. Modeling realworld behaviors of information providers requires considering many additional factors.

Practicality Issues

There is often a gap between theoretical models and the real world. The applicability of THPE to actual strategies of news providers needs empirical validation.

Information Asymmetry

In incomplete information games, not all players have all the information, complicating the analysis using THPE and making it challenging to determine if a particular strategy is indeed optimal.

Expected Theoretical Supplements

Addressing Information Asymmetry

Further developments in THPE in incomplete information games are expected to handle information asymmetry in more detail, allowing for the analysis of equilibria that take into account the uneven distribution of information among players.

Empirical Validation

Empirical validation of models using THPE is expected to assess the theory's practicality and applicability in the real world.

Consideration of Dynamics

Since the context of information provision changes over time, incorporating dynamic elements into THPE models is desired to analyze how strategies evolve over time.

Introducing THPE in the analysis of incomplete information games among news providers in the context of fake news and factchecking offers new insights but requires caution in its practicality and implementation. Further development and empirical validation are expected to deepen the understanding in this field.

6. Summary:Game Theory: The Promising Application of "Shaking Hands Perfect Equilibrium"

Applying the "shaking hands perfect equilibrium" to the non-perfect information game between news providers in the context of fake news and fact-checking allows for an analysis that takes into account the possibility that an information provider may inadvertently provide inaccurate information. This approach would be useful in developing strategies to maintain the quality and accuracy of information, taking into account the risk of unintentional spread of fake news by news providers.

The Concept of Perfect Equilibrium with Shaking Hands

The shaking hands perfect equilibrium is a concept of equilibrium that takes into account the possibility that each player in the game makes a non-optimal choice with minute probability. This concept can be used to verify whether a player's strategy is robust against small errors and uncertainties.

Modeling a non-perfect information game between news providers

Player

News providers make a choice between providing truthful news or spreading fake news Strategy. Players' strategies relate to the accuracy of the information (true or fake) and how they handle the information (publish or not publish).

Gain Function

Each player's gain depends on the strategy chosen and the strategies of the other players. The gain is determined by the influence, credibility, and fact-checking results of the information.

Derivation of the perfect equilibrium of the trembling hand

Identification of pure-strategy Nash equilibria First, we identify the optimal pure-strategy Nash equilibrium for all players in the game they face.

Introducing the Trembling Hand

Next, we introduce into the game the possibility that a player makes a non-optimal choice (a trembling move) with minute probability. This allows us to verify that each player's strategy is robust to small errors. 3.

Recalculation of Expected Gain

Recalculate the expected gain of each player, taking into account the shaking hands. 4.

Checking for perfect equilibrium of the shaking hands If all players cannot improve their gains by changing their strategies in the presence of a trembling move, then the strategy combination is a perfect equilibrium for the trembling move.

Application of Analysis: Non-Complete Information Games

One of the causes of non-perfect information games is the lack of communication. Particularly in media coverage, information asymmetry between information providers (news media) and information receivers (viewers or readers) can cause non-perfect information game situations. Using the present analytical approach, we will organize media trends in terms of sequential move number games, perturbed perfect equilibrium, and trembling hands.

Perspectives on Sequential Move Number Games

The sequential move guard game is a model that takes into account the timing of the media's release of information and the reaction of the recipient to that information (e.g., accept, scandalize, or fact-check) in a stepwise manner. Considering media trends from this perspective, the media release one piece of information, observe the reaction of the recipient, and then decide their next course of action (e.g., provide further information, issue a correction, etc.) accordingly. This process can be viewed as a dynamic exchange of information between the media and the receiver, suggesting that poor communication can easily lead to misinterpretation of information.

Perturbed Perfect Equilibrium Perspective

Perturbed perfect equilibrium is a concept of equilibrium that takes into account the possibility that the media and receivers make non-optimal choices with minute probability (e.g., the media provides incorrect information, the receiver doubts the truth of the information). Analyzing media trends from this perspective reveals that even when media pay utmost attention to the accuracy of information, the possibility of misinformation is always present, and that recipients do not always accurately understand media information. This situation can create an environment in which poor communication and misunderstandings can easily spread.

Perspectives on the Shaking Hand

The perfect equilibrium of the trembling hand is an analysis that takes into account the possibility that the media and recipients make unintentional wrong choices (errors due to "trembling hands"). Looking at media trends from this perspective suggests that the spread of misunderstandings and misinformation between providers and receivers of information is often the result of unavoidable errors. Particularly in the digital age of high-speed information dissemination, such errors can quickly have far-reaching effects.

These analyses suggest that media trends include sequential interactions between the provision of information and the reaction of recipients, uncertainty about the accuracy of information, and the potential for the spread of misinformation due to errors. Poor communication makes these problems more likely and requires strategic efforts to maintain information quality.

Application of Analysis: Complete Information Games

In perfect information games, analysis of media tendencies using the concepts of sequential turn-taking games, perturbed perfect equilibrium, and trembling hands can help reveal the dynamics of the process of information provision and its reception. In perfect information games, all players (in this context, the media and their receivers) are assumed to have complete knowledge of all aspects of the game (e.g., possible strategies and gains of other players).

Perspectives on Sequential Turns Games

In sequential move-order games, players take actions at each stage of the game in turn. Given the media's propensity in this context, the media, when publishing information, anticipate the reaction from the recipients of that information and plan their next actions (e.g., provide additional information, make corrections, etc.) accordingly. Under perfect information, the media is assumed to be able to accurately predict the receiver's reaction and, based on this, selects the optimal information delivery strategy.

Perturbed Perfect Equilibrium Perspective

Perturbed perfect equilibrium is a concept of equilibrium that takes into account the possibility that players make non-optimal choices with minute probability. In the context of the media, this "perturbation" can be thought of as the possibility that the media unintentionally publishes incorrect information or that the receiver mistakenly misinterprets the information. Considering perturbation perfect equilibrium within the framework of the perfect information game provides insight into how carefully the media should handle information and how recipients should verify information.

The Trembling Hand Perspective

The trembling hand concept indicates the potential for players to make unintentional wrong choices when selecting strategies. In the case of the media, this represents the risk of misinformation or the risk that the receiver cannot correctly judge the truth or falsity of information. Considering the perfect equilibrium of the trembling hand in the perfect information game underscores how the media should build systems to ensure the accuracy of information (fact-checking, source verification, etc.) and the importance of the receiver critically evaluating information.

These analytical methods indicate that media tendencies include strategic decision-making in the process of providing information, the risk of misinformation and how to counteract it, and the importance of the receiver's ability to evaluate information. The complete information game framework is a theoretical model, and because of the uncertainty and asymmetric information present in real media environments, these analyses may serve as a guide for media strategies, but they do not fully explain all phenomena.

Application of Analysis: Incomplete Information Games and Cooperative Games In the context of non-perfect information and cooperative games, analyzing media trends using the concepts of sequential turn-taking games, perturbed perfect equilibrium, and trembling moves can provide insight into the dynamics of information sharing and cooperation. In non-perfect information games, we consider situations in which players (in this case, media officials and informants) do not have complete information about the types and choices of other players. In cooperative games, this means that players may cooperate to achieve a common goal.

Perspectives on Sequential Turns Games

In a sequential move order game, we consider a process in which media parties take turns disclosing or sharing information. Each party determines its own strategy based on the choices of the previous player and may cooperate to improve the accuracy and reliability of the information. In the context of non-perfect information, each media stakeholder is not fully aware of the sources and intentions of the other stakeholders, so they build cooperative relationships by sharing reliable information and ensuring the accuracy of information through fact-checking.

Perturbed Perfect Equilibrium Perspective

In perturbed perfect equilibrium, we consider the possibility that media actors make non-optimal choices (e.g., share incorrect information) with minute probability. Analyzing media tendencies from this perspective reveals the risk of error in the information sharing process and the importance of cooperative mechanisms (e.g., mutual fact-checking and verification of information) to address it. Cooperative mechanisms to ensure the accuracy of information among media stakeholders can be a robust response to perturbations.

The Trembling Hands Perspective

The shaking hands concept represents the risk of unintentional misinformation by media actors. In the context of non-perfect information and cooperative games, it underscores the importance of sharing and cross-validating information among stakeholders to minimize this risk. A cooperative approach that takes into account the risk of shaky hands can contribute to improving overall media credibility and information quality.

Considering the media trends from this analysis using a non-perfect information and cooperative game framework, the importance of cooperation in sharing and verifying information, robust measures against unintended errors, and collaboration among media stakeholders to provide reliable information are key elements. This suggests a strategic approach to prevent the spread of fake news and maintain the credibility and accuracy of the public information space.

Application of Analysis: Non-Complete Information Games and Non-Cooperative Games Analyzing media tendencies in the framework of non-complete information and non-cooperative games, using the concepts of sequential turn of the hand games, perturbed perfect equilibrium, and trembling hands, provides insight into the competitive interactions and information asymmetries among media actors.

Perspectives on Sequential Move Number Games

The sequential move order game considers the process by which media actors (players) disseminate information in turn. Under non-perfect information, each player does not fully know about the strategies and sources of information of the other players. In this context, media personnel observe the behavior of others and strategically determine the timing and content of their own information dissemination. In a competitive environment, we may see strategies such as moving for scoops or quickly following up on information reported by other media outlets.

Perspectives on Perturbed Perfect Equilibrium

Perturbed perfect equilibrium is an equilibrium concept that takes into account the possibility that players make non-optimal choices with minute probability. In the media context, this can represent the risk of reporting incorrect information or using misleading headlines. Analyzing media trends from a perturbed perfect equilibrium perspective identifies scenarios that may sacrifice accuracy in the face of competition and strategies to minimize such risks.

The Trembling Hand Perspective

The trembling hand concept indicates the potential for a player to make an unintentional wrong choice. For media professionals, this could mean the risk of misinformation or the misdelivery of information due to editorial errors. In the context of the non-cooperative game, strategies for self-protection in the information war against competitors and for correcting misinformation become important, taking into account the risk of the trembling hand.

Organizing Comprehensive Media Trends

Analyzing media trends using the framework of non-perfect information and non-cooperative games reveals the following points

Information Wars

Media professionals strategically disseminate information in order to get the scoop in the information competition with other media.

Risk of misinformation

The emphasis on breaking news in competition increases the risk of misinformation.

Self-protection measures

Taking into account the risk of shaky hands, it is important to strengthen mechanisms to correct misinformation and internal checks to ensure the accuracy of information. Balance with public interest: Even in a competitive media environment, care must be taken not to compromise the accuracy of information and the public interest.

This analysis provides insight into how the media should balance the accuracy of information with breaking news and what strategies should be employed to minimize misinformation.

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