Discussion Paper:(1)Case Study of Decision Making and Urban Planning:Loop Flip Errors and Phase Transitions in Local Potential Approximation in Non-trivial Loops(2024), (2)Case Study of Spatio-Temporal Approaches To Syndrome Errors in Decision Making with Applications Such as Bit Phase Flip Errors and Methods Discussion(2024)

Bonacich Centrality and Trembling Hand for Searching for Pairwise Stability against Fact Check: Game Theoretic Analysis in Non-Complete Information Game Environment

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Abstract: This paper is a note for discussion and organization of computational experiments and methods.This note examines a new game-theoretic model for analyzing strategic interactions among news providers in the context of fake news dissemination and fact-checking efforts, integrating Bonacich centrality and Trembling Hand Perfect Equilibrium (THPE) concepts are integrated to explore how information providers, modeled as players in a network game with imperfect information, make strategic decisions regarding the publication of news and the verification of information. Bonacich centrality is employed to quantify the relative influence of each information provider in the network, highlighting the role of central nodes in shaping the flow of information; THPE accounts for the possibility of suboptimal behavior due to errors and uncertainty and to explain the strategies employed by information providers Used for. The analysis reveals the conditions under which a network reaches a pairwise stable state where two providers cannot mutually benefit from a change in strategy. The model highlights the complex dynamics of news diffusion and the key factors that influence the effectiveness of fact-checking initiatives. The experimental design of this paper should contribute to policymakers, media organizations, and fact-checkers striving to combat the spread of fake news and promote the dissemination of accurate information.

Keywords: Game Theory, Bonacich Centrality, Trembling Hand Perfect Equilibrium, Fake News, Fact-Checking, Network Analysis, Strategic Interactions, News Providers, Information Asymmetry, Pairwise Stability

1. Introduction

This paper is a note for discussion and organization of computational experiments and methods. In game-theoretic network analysis, the combination of Bonacich centrality and trembling hand perfect equilibrium (THPE) plays a crucial role in capturing the dynamic relationship between the influence and strategic behavior of news providers in informational and digital health contexts. This approach allows us to identify key players in the dissemination of health information, understand the impact of their actions on the network as a whole, and develop strategies to effectively curb fake news and misinformation and promote accurate health information.

By quantifying the relative influence that news providers have within a network, Bonacich Centrality reveals which providers play the most important role in the flow of health information. This information is essential in developing strategies for spreading accurate information through the most influential providers in health awareness campaigns and public health initiatives. It can also help identify the risk of spreading misinformation by influential providers and plan

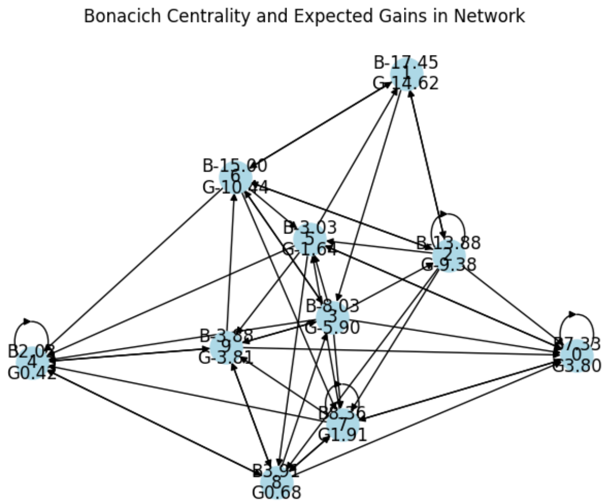

Fig. 1: Bonacich Centrality and Expected Gains in Network

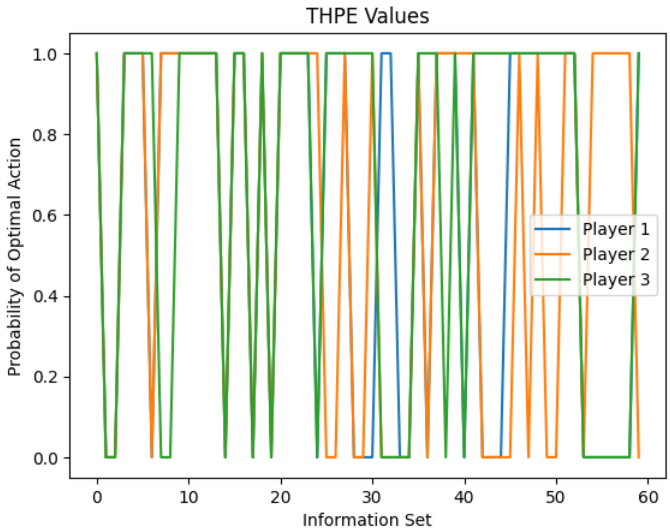

Fig. 2: Selection influence probability of shaking hands (error probability)

interventions to address it.

On the other hand, THPE considers the strategic actions that news providers may take and analyzes how each action affects their position within the network and the actions of other providers. In particular, by considering the possibility of taking non-optimal actions with minute probability, we can model real-world situations of uncertainty and imperfect information. This allows us to better understand the incentives for news providers to spread fake news or conduct fact-checking, and to develop more effective strategies to promote or discourage these behaviors.

Thus, the game-theoretic approach of combining Bonacich centrality and THPE provides important insights for protecting and promoting informational and digital health. Understanding the strategic behavior of news providers and their relative influence within networks will enable us to design more effective policies, programs, and interventions to improve the quality of health information and prevent the spread of misinformation. This paper focuses on non-complete information unfolding form games between news providers in the context of fake news and fact-checking, and proposes a new approach in the analysis of such networks. In particular, we analyze the interaction between the strategic behavior of news providers and their influence in the network using a model that combines Bonacich centrality and trembling hand perfect equilibrium (THPE). The study aims to provide new perspectives in understanding the role of news providers in propagating information and suppressing misinformation.

The issue of fake news and fact-checking has become an important challenge in today's society. In particular, the speed and scope of information dissemination has expanded dramatically with the spread of social media, making it imperative to develop effective methods to minimize the impact of misinformation. This study examines a method for modeling and analyzing the strategic decisions that news providers make in the process of selecting and publishing information, using a game theory framework.

Bonacich centrality is a measure of the relative influence of individual nodes in a network, which indicates the magnitude of the news provider's influence. On the other hand, trembling hand perfect equilibrium (THPE) is an equilibrium concept in game theory that takes into account the possibility of players taking non-optimal actions with minute probability. By combining these two concepts, this research can model and analyze, in a more realistic way, the strategic decisions that news providers make when choosing actions such as publishing information or conducting fact-checking.

Specifically, we reveal the strategic interactions among news providers and their outcomes in a non-complete information deployment game defined in the context of fake news and fact-checking through a redefinition of expected gains that takes into account Bonacich centrality, the derivation of THPEs, and the analysis of network-wide equilibrium states This will help to identify strategies for suppressing misinformation. By doing so, we aim to contribute to the development of strategic approaches to curb misinformation.

Our approach is unique in that it considers how the strategic behavior of news providers affects their relative influence within the network. We consider this analysis as a method that provides a new perspective when dealing with the issues of fake news and fact-checking, and that deepens our understanding of the propagation of information and the suppression of misinformation.

The framework of this study is designed to formalize the complex decision-making process faced by news providers and to analyze how their strategic interactions affect patterns of information propagation. In particular, we aim to capture the dynamic relationship between news providers' influence and their strategic behavior by combining Bonacich centrality and trembling hand perfect equilibrium (THPE). This is an important step toward understanding the role of news providers in information dissemination and misinformation suppression strategies.

The debate about fake news and fact-checking highlights the issues of credibility and transparency in the public debate arena. The actions of news providers, the originators of information, are key to addressing this issue. This study uses a theoretical framework to analyze the various strategic choices that news providers can take and their impact on the dissemination and reception of information. Through this approach, we aim to provide strategic insights to curb the spread of fake news and maximize the effectiveness of fact-checking.

In addition, this research will model the interaction between news providers as a network, allowing for a more detailed understanding of information propagation patterns. Through this network model, the impact of specific news providers and information flows on the network as a whole can be analyzed to identify effective approaches to combat the spread of fake news and misinformation.

In the context of informational and digital health, the framework of this study is of particular importance. The dynamics of fake news and fact-checking must be considered from a public health perspective, as the quality and accessibility of information has a direct impact on individuals' health awareness and behavior. The spread of misinformation about health information can cause unnecessary anxiety and promote risky health behaviors. Therefore, understanding the strategic actions of news providers and their information dissemination is essential to protect informational health and improve the quality of health information in the digital environment.

Insights from this study can help in the design of digital health promotion programs and strategies to improve health information literacy. Understanding how information propagates within a network will enable the development of strategies to maximize the accuracy and impact of health information. Maximizing the effectiveness of fake news suppression and fact-checking will also help increase the effectiveness of health promotion efforts through digital media.

Furthermore, this research will contribute to the development of public health policies that take into account the impact of digital media on individual health attitudes and behaviors. Understanding patterns of information dissemination will facilitate the development of policies and programs that promote equitable access to health information and the positive contributions of digital media to health.

Overall, this study provides insight into the dissemination and acceptance of health information, highlighting the role of news providers in the protection and promotion of informational and digital health. We hope that this will improve the quality of health information and contribute to improving public health in the digital environment.

2. Discussion:Investigation of Methods Introducing Pairwise Stability

When considering the context of fake news and fact-checking in a game of incomplete information among news providers, combining pairwise stability and network analysis requires first defining the framework of the model. Below outlines a general approach, but the actual model needs adjustments based on the research objectives and available data.

2.1 Model Construction

(1) **Network Definition:**
Define the network among news providers as a graph $G = (V, E)$, where V is the set of nodes (news providers) and E is the set of edges (relationships among news providers).

(2) **Utility Functions:**
Define the utility function U_i for each news provider i. This function may depend on multiple factors such as relationships with other news providers and the accuracy of information.

(3) **Pairwise Stability Condition:**
For any pair i and j, if both their utilities improve by having a relationship $(i, j) \in E$, then this relationship is considered pairwise stable. Mathematically, if there does not exist E' such that $U_i(E') > U_i(E)$ and $U_j(E') > U_j(E)$, then E is pairwise stable.

2.2 Computation Process

(1) **Network Initialization:**
Set the initial network state $G_0 = (V, E_0)$. This is constructed based on known relationships among news providers and initial patterns of information exchange.

(2) **Evaluation of Utility Functions:**
Calculate the utility $U_i(E_0)$ for each news provider in the initial state. This involves considering factors like the accuracy of news, influence, and reach.

(3) **Search for Pairwise Improvements:**
Evaluate whether the utility function improves for all possible additions or removals of edges. Specifically, calculate $U_i(E')$ and $U_j(E')$ for the new network $G' = (V, E')$ obtained by adding or removing the edge (i, j), and determine if there is an improvement.

(4) **Determination of Stable States:**
The goal is to find a state where no pairwise improvements exist, meaning there are no edges where utility improves simultaneously by addition or removal. This state is considered a pairwise stable network.

2.3 Considerations

This approach requires modeling complex factors such as network dynamics and asymmetry of information.

Defining utility functions and selecting parameters significantly impact the predictive accuracy of the model. It is essential to define them appropriately based on actual data and prior research.

When considering the elements that can be expected by introducing the concept of pairwise stability into the game of incomplete information among news providers in the context of fake news and fact-checking, a network analysis perspective can be valuable. Pairwise stability refers to situations where, in a given state, no pair of actors can simultaneously achieve a better outcome by changing their relationship.

2.4 Expected Elements from Network Analysis

(1) **Understanding Patterns of Information Propagation:** Analyzing the network structure among news providers can reveal patterns of how information and fake news propagate. The concept of pairwise stability helps understand which links are stable and which ones are more prone to variation when capturing the dynamics of information propagation.

(2) **Identifying Strategic Positions of Information Providers:** Identifying the positions of information providers within the network (such as centrality or roles as bridges) can help strategize to maximize the suppression of fake news and the effectiveness of fact-checking. Pairwise stability may help identify which relationships are strategically significant.

(3) **Community Structure and Misinformation Diffusion:** Network analysis can reveal the community structure of information providers. From the perspective of pairwise stability, one can evaluate strategies for the flow of information and suppression of misinformation both within and outside these communities.

(4) **Asymmetry of Information and Network Influence:** Network analysis considering pairwise stability can help understand how the asymmetry of information influences strategic interactions among news providers. It allows for considering how the types and amounts of information possessed by specific providers affect their relationships and strategies within the network.

(5) **Analysis of Dynamic Networks:** The relationships among information providers change over time. Incorporating the concept of pairwise stability into network analysis enables tracking these dynamic changes and analyzing how the strategies of information providers evolve.

Introducing network analysis into the framework allows for a deeper understanding of interactions among news providers within the context of fake news and the development of new strategic approaches in combating fake news. However, implementing this approach requires advanced network analysis techniques and detailed data on the behavior of news providers.

2.5 Pairwise Stability and Trembling Hand Perfect Equilibrium (THPE) Incorporated Network Analysis

When designing the formulas and computational process for network analysis incorporating pairwise stability and Trembling Hand Perfect Equilibrium (THPE) in the context of games among news providers considering fake news and fact-checking, which is also addressed as a challenge in the preceding paragraph, it is necessary to consider the extensive form structure of the game and the incomplete information of each player. Below, we outline a general framework for analyzing such games.

The calculation of Trembling Hand Perfect Equilibrium (THPE) with pairwise stable trembling hands in an incomplete information extensive-form game is difficult to establish a single "generic" calculation process due to its highly advanced nature. However, here we provide an overview of the basic framework using several steps and formulas.

Definition of Game Extensive Form and Information Sets

The extensive form of the game is usually represented by a game tree. Each node n represents a decision point in the game, and each edge represents possible actions. The information set I_i is a set of decision nodes that player i cannot distinguish.

Definition of Player Beliefs and Strategies

Let $\beta_i(n|I_i)$ denote the probability that player i believes they are at node n in information set I_i. Player i's strategy σ_i specifies the probability distribution of actions in each information set.

Calculation of Expected Payoffs

The expected payoff for action a of player i in information set I_i is the weighted average of payoffs over all possible outcomes of that action. The expected payoff is calculated as follows:

$$E[U_i(\sigma)] = \sum_{n \in I_i} \beta_i(n|I_i) \sum_{a \in A(n)} \sigma_i(a|I_i) U_i(n,a)$$

Here, $A(n)$ is the set of possible actions at node n, and $U_i(n,a)$ is the payoff at node n when action a is taken.

Examination of Pairwise Stability

To confirm pairwise stability, it is necessary to ensure that for any two players i and j, their expected payoffs do not simultaneously improve by changing their strategies. This can be confirmed by examining all pairs of strategies (σ_i, σ_j) for all players.

Calculation of THPE

In Trembling Hand Perfect Equilibrium, each player may take suboptimal actions with a small probability ϵ. In THPE, it is necessary to confirm that all players' strategies are optimal responses to other players' strategies and that equilibrium is maintained even with the inclusion of suboptimal actions with a small probability.

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3. Discussion:Definition of the Game Tree

To explain the specific calculation process for the game tree and Trembling Hand Perfect Equilibrium (THPE) in an incomplete information extensive-form game, here we provide a step-by-step approach using a simplified example. In this example, we consider a game with two players (A and B). Player A acts first, followed by Player B, who does not know exactly what Player A chose (incomplete information).

First, represent the extensive form of the game using a game tree. Consider the following simple game tree:

(1) Player A can choose action a_1 or a_2.

(2) Player B, without knowing Player A's choice, chooses action b_1 or b_2.

Definition of Information Sets

Player B's information set I_B includes B's decision nodes when Player A chooses a_1 or a_2.

Definition of Player Beliefs

Player B believes that Player A chooses a_1 with probability β and a_2 with probability $1-\beta$.

Definition of Strategies

Let σ_A and σ_B denote the strategies of Player A and B, respectively. These are probability distributions over actions in information sets.

Calculation of Expected Payoffs

The expected payoffs for Player A and B can be calculated as follows:

$$E[U_A(\sigma_A, \sigma_B)] = \beta \cdot U_A(a_1, \sigma_B(I_B)) + (1-\beta) \cdot U_A(a_2, \sigma_B(I_B))$$
$$E[U_B(\sigma_A, \sigma_B)] = \beta \cdot U_B(\sigma_A(I_A), b_1) + (1-\beta) \cdot U_B(\sigma_A(I_A), b_2)$$

Here, U_A and U_B are the payoff functions for Player A and B, respectively, for specific combinations of actions.

Confirmation of THPE

To find THPE, confirm that each player's strategy is an optimal response to the strategies of the other players. This includes considering the possibility of players taking suboptimal actions with a small probability (trembling hand).

> Confirm that the strategies of Player A and B are optimal responses to each other's strategies.
>
> Confirm that these strategies remain optimal even if players take suboptimal actions with a small probability ϵ.

To demonstrate the calculation process for THPE in an incomplete information extensive-form game considering pairwise stability, specific contexts of the game are required. However, as a general guideline, the following steps are presented for deriving pairwise stable THPE. This extends the framework of the simplified game mentioned earlier.

Expansion of Game Tree and Information Sets

In the game tree, include all possible game states that occur after each action by Player A and B. Expand Player B's information set to reflect the uncertainty about Player A's actions.

Definition of Expanded Beliefs and Strategies

Update Player B's beliefs to probabilities for each of Player A's possible actions. These beliefs are updated based on the signals or observations Player B might receive. Define the strategies for Player A and B based on the expanded game tree.

Recalculation of Expected Payoffs

Recalculate the expected payoffs for each player based on the expanded beliefs and strategies. The calculation of expected payoffs is performed for each node and information set in the new game tree.

Confirmation of Pairwise Improvement

Considering pairwise stability, confirm that there is no opportunity for both players' expected payoffs to simultaneously improve by changing one player's strategy in any player pair (i, j). This confirmation is done for all information sets and possible belief updates.

Derivation of THPE and Integration with Pairwise Stability

In THPE, considering the possibility of players taking suboptimal actions with a small probability, derive a set of strategies that maintains pairwise stability under this condition. This process involves finding strategies that maximize the expected payoffs for each player and ensuring that they are not improved by pairwise improvement.

To illustrate the specific calculation process for pairwise stable THPE, specific parameters of the game (players' payoff functions, possible actions, information sets, etc.) are required. In a general form, it typically involves solving optimization problems like the following:

Find strategies that maximize the expected payoff for each player:

$$\max_{\sigma_i} E[U_i(\sigma_i, \sigma_{-i})]$$

Confirm that there is no opportunity for both players' expected payoffs to simultaneously improve by changing one player's strategy for all player pairs (i, j).

This process is often performed numerically for specific games using methods such as linear programming or iterative optimization algorithms.

3.1 Discussion:Expanding Game Tree and Information Sets in an Incomplete Information Extensive-Form Game

Game Setup

Consider a game with two players, A and B. Player A acts first and can choose either "Left (L)" or "Right (R)." Next, player B acts, but does not know what action player A chose. Player B can choose either "Up (U)" or "Down (D)." Player B has an information set I_B regarding player A's choice, but this information is incomplete.

Expansion of Game Tree and Information Sets

The game tree is expanded as follows:

(1) Player A chooses "Left (L)" or "Right (R)."

(2) Player B, based on incomplete information about player A's choice, chooses "Up (U)" or "Down (D)."

Player B's information set I_B represents a state where player A has chosen either "Left (L)" or "Right (R)," but it cannot distinguish which one.

Definition of Expanded Beliefs and Strategies

Player B's beliefs are defined by the probability β_L that player A chooses "Left (L)" and the probability $\beta_R = 1 - \beta_L$ that player A chooses "Right (R)."

The strategies for player A and player B are defined by the probability distributions of their respective action choices. Player A's strategy is $\sigma_A(L)$ and $\sigma_A(R) = 1 - \sigma_A(L)$, while player B's strategy is the probability of choosing "Up (U)" and "Down (D)" based on information set I_B, denoted as $\sigma_B(U|I_B)$ and $\sigma_B(D|I_B) = 1 - \sigma_B(U|I_B)$.

Example Calculation of Player B's Expected Payoff

$$E[U_B] = \beta_L\left(\sigma_B(U|I_B)U_B(L,U) + \sigma_B(D|I_B)U_B(L,D)\right)$$
$$+\beta_R\left(\sigma_B(U|I_B)U_B(R,U) + \sigma_B(D|I_B)U_B(R,D)\right)$$

In this equation, player B's expected payoff is calculated using player B's beliefs β_L and β_R about player A's choice of "Left (L)" or "Right (R)" and player B's strategy σ_B of choosing "Up (U)" or "Down (D)" at each information set. $U_B(L,U)$, $U_B(L,D)$, $U_B(R,U)$, and $U_B(R,D)$ represent the payoffs for specific combinations of actions.

In this step, expected payoffs are calculated using player strategies and beliefs, and optimal strategies are selected based on the results. The specifics of the equations may vary depending on the scenario and payoff structure of the game.

By examining the potential for pairwise improvement between players, we confirm that a set of strategies is pairwise stable. This means that no two players can simultaneously improve their expected payoffs by cooperating to change their strategies.

Confirmation of Pairwise Improvement in Computational Experiments

In this process, we consider whether player i can improve their expected payoff by changing their strategy, which then induces player j to also change their strategy. This evaluation is conducted for all pairs of players (i, j) and all combinations of strategies σ_i and σ_j.

To demonstrate the absence of pairwise improvement, the following condition must be satisfied:

$$\forall \sigma_i', \sigma_j', \quad E[U_i(\sigma_i', \sigma_{-i})] \leq E[U_i(\sigma_i, \sigma_{-i})]$$
$$\text{or} \quad E[U_j(\sigma_i, \sigma_j')] \leq E[U_j(\sigma_i, \sigma_j)]$$

Here, σ_i' and σ_j' represent the new strategies that players i and j might choose. σ_{-i} denotes the combination of strategies chosen by all players except player i.

This condition indicates that if player i changes their strategy to σ_i', this change either decreases player j's expected payoff or does not improve player i's expected payoff even if player j changes their strategy to σ_j'.

Computational Example

To illustrate this, let's consider a simple game with only player A and player B. Suppose player A takes strategy σ_A and player B takes strategy σ_B. If player A changes to a new strategy σ_A', player B's expected payoff is calculated as:

$$E[U_B(\sigma_A', \sigma_B)]$$

If player B responds to this change by switching to strategy σ_B', player A's expected payoff becomes:

$$E[U_A(\sigma_A', \sigma_B')]$$

For pairwise improvement to be absent, the following condition must hold:

$$E[U_B(\sigma_A', \sigma_B)] \leq E[U_B(\sigma_A, \sigma_B)]$$

or

$$E[U_A(\sigma_A', \sigma_B')] \leq E[U_A(\sigma_A, \sigma_B)]$$

This condition implies that even if players A and B cooperate to change their strategies, at least one player's expected payoff does not improve.

Such considerations of pairwise improvement must be conducted for all pairs of players and possible changes in strategies. In practice, this process depends on the specific rules and payoff structures of the game and is often executed using numerical methods or game theory.

4. Discussion:Modeling Overview

To model the imperfect-information extensive-form game among news providers in the context of fake news and fact-checking, and to explain the process of constructing its network in detail, we present specific computational procedures and formulas below.

Game Modeling

Player Definitions

Model news providers as players. For example, define the set of news providers as $P = \{p_1, p_2, \ldots, p_n\}$.

Action Definitions

Define the possible actions for each news provider p_i as $A_i = \{a_{i1}, a_{i2}, \ldots, a_{im}\}$. For instance, a_{i1} could be "publish fake news" and a_{i2} could be "conduct fact-checking".

Payoff Functions

Define payoffs for each action combination. For instance, if player p_i takes action a_{ik} and the other players take actions from set A_{-i}, denote the payoff as $U_i(a_{ik}, A_{-i})$. Payoff functions are defined based on factors like influence, credibility, and reach.

Network Construction

Node Definitions

Represent each player p_i in the network as a node. The set of nodes V in the network $G = (V, E)$ corresponds to the set of news providers P.

Edge Definitions

Represent interactions between players as edges. An edge (p_i, p_j) indicates interaction between players p_i and p_j. The weight w_{ij} of an edge represents the strength or magnitude of the interaction. The set of edges E contains all interactions.

Construction of the Adjacency Matrix of the Network

Construct the adjacency matrix A of the network. Each element a_{ij} of A corresponds to the weight w_{ij} of the edge from player p_i to p_j. If there is no edge, set $a_{ij} = 0$.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

Calculation of Interaction Strength

The strength of interaction between players is determined by the weights of the edges. The total influence of a specific player p_i on others is calculated by summing the elements in the ith row of the adjacency matrix.

$$\text{Influence}_i = \sum_{j=1}^{n} a_{ij}$$

This model and computational process define the game among news providers and its network, providing a foundation for analyzing interactions and influence among players. Further modeling and calculations may be required depending on specific game scenarios and network structures.

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Modeling Organization 2

We describe the computational process for incorporating the Bonacich Power and Trembling-Hand Perfect Equilibrium (THPE) into the network of news providers in the context of imperfect-information extensive-form games involving fake news and fact-checking.

Introduction of Bonacich Power

Bonacich Power is a measure to quantify the influence of players within a network. The Bonacich Power index B_i is calculated as follows:

$$B_i = \sum_{j=1}^{n} a_{ij} + \alpha \sum_{j=1}^{n} \sum_{k=1}^{n} a_{ij} a_{jk} + \alpha^2 \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} a_{ij} a_{jk} a_{kl} + \cdots$$

Here, a_{ij} represents the influence (edge weight) from player i to j, α is the decay parameter (usually a positive value less than 1), and n is the total number of players.

Bonacich Power takes into account not only direct influence but also indirect influence (such as influence through friends of friends). The decay parameter α adjusts the importance of indirect influence.

Introduction of Trembling-Hand Perfect Equilibrium (THPE)

To derive THPE, it is necessary to ensure that each player's strategy accounts for trembling hand (the selection of suboptimal actions with a small probability). The specific computational process is as follows:

1. **Definition of Probability Mixed Strategies**: Define strategy $\sigma_i(a)$ for each action a of player i, representing the probability of selecting action a.

2. **Calculation of Expected Payoffs**: Calculate the expected payoff $E[U_i(a, \sigma_{-i})]$ for player i's action a. This is the expected value of the payoff for action a given the strategies σ_{-i} of the other players.

$$E[U_i(a, \sigma_{-i})] = \sum_{\sigma_{-i}} p(\sigma_{-i}) U_i(a, \sigma_{-i})$$

Here, $p(\sigma_{-i})$ is the probability of the combination of strategies of the other players.

Consideration of Trembling Hand

Consider that each player takes suboptimal actions with a small probability ϵ. The optimal strategy includes the optimal response to the strategies of other players, while also considering the possibility of taking suboptimal actions with a small probability.

Derivation of THPE

Ensure that each player's strategy, being the optimal response to the strategies of other players, does not break the equilibrium even with the inclusion of suboptimal actions with a small probability.

$$\forall i, \forall a \in A_i, \quad E[U_i(a, \sigma_{-i})] \geq E[U_i(a', \sigma_{-i})] - \epsilon$$

Here, a' represents all other possible actions of player i.

This computational process needs to be adapted based on the specific rules, payoff structures, and network topology of the game. In practice, these calculations are often complex and numerical solution methods or simulations are commonly used.

Modeling Organization 3

In the integration of network analysis, we analyze the interaction between news providers and the influence of their strategic decisions using the concepts of Bonacich Power and Trembling-Hand Perfect Equilibrium (THPE). While we can outline a general framework for this process, the specific calculations depend on the particular settings of the game.

Calculation of Influence Using Bonacich Power

The Bonacich Power B_i of each news provider i is calculated as described earlier. This indicates the relative influence that news providers hold within the network.

Decision of Strategies Using THPE

For each news provider i, considering the strategies of other news providers σ_{-i}, we determine the optimal strategy σ_i^* that maximizes the expected payoff $E[U_i(\sigma_i, \sigma_{-i})]$.

Integration of Network Analysis

Association between Strategies and Bonacich Power

We evaluate how the strategy choices of each news provider affect their Bonacich Power. For example, if a news provider with high Bonacich Power chooses to spread fake news, we analyze the impact it has on the overall network.

Adaptation of Strategies Based on THPE

News providers select optimal strategies based on THPE, taking into account the strategies of other providers and their own position in the network (Bonacich Power). This may include the possibility of taking suboptimal actions with a small probability.

Integration of Strategies

Taking into consideration the optimal strategies and Bonacich Power of all news providers, we analyze the dynamics of the entire network. This allows us to evaluate the impact of specific actions on the entire network, such as the spread of fake news or the effectiveness of fact-checking.

Specific Calculation Example

When calculating the change in expected payoff based on the interaction between news providers i and j:

$$\Delta E[U_i] = B_i \cdot \sum_{j \neq i} \sigma_j^* \cdot U_{ij}(\sigma_i^*, \sigma_j^*)$$

Here, $\Delta E[U_i]$ is the change in expected payoff for news provider i, B_i is the Bonacich Power of news provider i, σ_j^* is the optimal strategy of news provider j, and $U_{ij}(\sigma_i^*, \sigma_j^*)$ is the change in payoff resulting from that combination of strategies.

This computational process provides the foundation for understanding the impact of interactions between news providers and their strategic decisions on the entire network.

Modeling Organization 4: Consideration of Payoffs

In modeling the game, we present a detailed computational process for constructing an imperfect-information extensive-form game among news providers in the context of fake news and fact-checking. Here, we model the strategic decisions among news providers and the associated payoffs.

Definition of Players

We define the set of news providers as $P = \{p_1, p_2, \ldots, p_n\}$, where n is the total number of news providers.

Definition of Actions

Each news provider p_i can take a set of actions defined as $A_i = \{a_{i1}, a_{i2}, \ldots, a_{im}\}$. Examples of actions include a_{i1} for "publishing true news", a_{i2} for "publishing fake news", and a_{i3} for "fact-checking other providers' news".

Definition of Payoff Functions

We set up a function $U_i(a_{ik}, A_{-i})$ to define the payoff for each news provider p_i given their action a_{ik} and the combination of actions by other players A_{-i}. Payoffs are determined based on factors such as the influence of the news, reliability, and reach.

Calculation of Expected Payoff for Actions

The expected payoff for a news provider p_i taking action a_{ik} is calculated based on the probability distribution of actions by other news providers. The expected payoff $E[U_i(a_{ik})]$ is expressed as:

$$E[U_i(a_{ik})] = \sum_{A_{-i}} P(A_{-i}) \cdot U_i(a_{ik}, A_{-i})$$

Here, $P(A_{-i})$ represents the probability of the combination of actions by other players A_{-i}.

Optimization of Strategies

The optimal strategy for a news provider p_i is determined by selecting the action a_{ik}^* that maximizes the expected payoff:

$$a_{ik}^* = \arg \max_{a_{ik} \in A_i} E[U_i(a_{ik})]$$

This process is carried out for all news providers to find each provider's optimal strategy.

In actual calculations, it's crucial to determine how the probability distribution of other players' actions $P(A_{-i})$ is established. This often involves modeling based on players' beliefs and the incompleteness of information.

The definition of the payoff function $U_i(a_{ik}, A_{-i})$ depends on the specific context of the game and the objectives of news providers. Designing the payoff function appropriately determines the realism and effectiveness of the game model.

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Modeling Organization 5: Consideration of Payoffs

In the context of fake news and fact-checking, we construct a game among news providers as a network. This network consists of news providers (nodes) and their interactions (edges). Below, we provide a detailed explanation of the network construction process.

Definition of Nodes

News providers are represented as nodes in the network. Assuming there are n news providers, we denote the set of nodes as $V = \{v_1, v_2, \ldots, v_n\}$.

Definition of Edges

Interactions among news providers are represented as edges. An edge (v_i, v_j) indicates the flow of influence or information from news provider v_i to news provider v_j. Edges can be directed or undirected, where directed edges indicate the direction of influence.

Assignment of Edge Weights

Edges may be assigned weights to represent the strength or importance of interactions among news providers. Let w_{ij} denote the weight of the edge (v_i, v_j). Weights are determined based on factors such as the reliability of information, the scope of influence, and the frequency of interactions.

Construction of Adjacency Matrix

The network can be represented using an adjacency matrix A. A is an $n \times n$ matrix, where the element a_{ij} corresponds to the weight w_{ij} of the edge from node v_i to node v_j. If there is no edge, $a_{ij} = 0$.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

Calculation of Interaction Strength

The total influence from news provider v_i to other news providers is calculated as the sum of elements in the ith row of the adjacency matrix:

$$\text{Total Influence from } v_i = \sum_{j=1}^{n} a_{ij}$$

Similarly, the total influence to news provider v_i from other news providers is calculated as the sum of elements in the ith column of the adjacency matrix:

$$\text{Total Influence to } v_i = \sum_{j=1}^{n} a_{ji}$$

The construction and analysis of the network heavily depend on actual data of interactions among news providers. In the absence of data, edges and weights may be assigned based on expert opinions or hypotheses. The selection and assignment of edge weights involve various metrics to quantify the qualitative aspects of interactions. Careful consideration is required as the choice of weights significantly influences the results of network analysis.

Modeling Organization 6: Consideration of Payoffs

In the context of fake news and fact-checking, we introduce the calculation of Bonacich centrality into the network of news providers. Bonacich centrality is used to measure the influence of each node (in this case, each news provider) in the network.

Calculation of Bonacich Centrality

Bonacich centrality is computed using the adjacency matrix of the network. The adjacency matrix A represents connections between nodes (news providers) in the network, where the element a_{ij} of the matrix represents the weight (or 1 if present, 0 if not) of the edge from node i to node j.

The Bonacich centrality B_i for node i is calculated using the following formula:

$$B_i = \sum_{j=1}^{n} a_{ij} + \alpha \sum_{j=1}^{n} \sum_{k=1}^{n} a_{ij} a_{jk} + \alpha^2 \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} a_{ij} a_{jk} a_{kl} + \cdots$$

Here,

a_{ij} is the element of the adjacency matrix A representing the weight of the edge from node i to node j.

α is a damping factor that controls the influence of indirect connections (usually between 0 and 1).

n is the total number of nodes (news providers) in the network.

Calculation Process

Creation of Adjacency Matrix

Create the adjacency matrix A of the network. The size of this matrix is $n \times n$, where n is the number of nodes in the network.

Calculation of Bonacich Centrality

Direct Influence: For each node i, calculate the sum of elements in the ith row of the adjacency matrix: $\sum_{j=1}^{n} a_{ij}$.

Indirect Influence: Incorporate the influence of indirect connections using α. Square the adjacency matrix (e.g., $A^2, A^3, \ldots$) and add up the sum of elements in the ith row at each step, weighted by the corresponding power of α.

Iteration over All Nodes

Repeat the above calculation for all nodes in the network to determine the Bonacich centrality of each news provider.

Suppose the adjacency matrix is given as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

And, let the damping factor α be 0.5. Then, the Bonacich centrality B_1 for node 1 can be calculated as follows:

$$B_1 = \sum_{j=1}^{4} a_{1j} + 0.5 \sum_{j=1}^{4} \sum_{k=1}^{4} a_{1j} a_{jk} + 0.5^2 \sum_{j=1}^{4} \sum_{k=1}^{4} \sum_{l=1}^{4} a_{1j} a_{jk} a_{kl} + \cdots$$

Compute this to obtain the Bonacich centrality of node 1. Perform similar calculations for all nodes to determine the Bonacich centrality of each news provider.

Modeling Organization 7: Consideration of Payoffs

We derive the Trembling Hand Perfect Equilibrium (THPE), which is an equilibrium concept that takes into account the possibility of players taking suboptimal actions with a small probability. Below, we illustrate the process of calculating THPE in the context of an imperfect-information extensive-form game among news providers regarding fake news and fact-checking.

Derivation of Trembling Hand Perfect Equilibrium

Definition of Probability Mixed Strategies

Define probability mixed strategies $\sigma_i(a_{ik})$ for each news provider i, representing the probability of taking action a_{ik}. For all action sets A_i, it holds that $\sum_{a_{ik} \in A_i} \sigma_i(a_{ik}) = 1$.

Calculation of Expected Payoffs

Compute the expected payoff $E[U_i(a_{ik}, \sigma_{-i})]$ for each news provider i with action a_{ik}. Here, σ_{-i} represents the combination of strategies of all other news providers.

$$E[U_i(a_{ik}, \sigma_{-i})] = \sum_{\sigma_{-i}} P(\sigma_{-i}) \cdot U_i(a_{ik}, \sigma_{-i})$$

Here, $P(\sigma_{-i})$ is the probability of the combination of strategies of other news providers.

Derivation of Best Response Strategies

For each news provider i, given the strategies σ_{-i} of all other news providers, derive the best response strategy σ_i^* that maximizes the expected payoff.

$$\sigma_i^* = \arg\max_{\sigma_i} E[U_i(\sigma_i, \sigma_{-i})]$$

Consideration of Trembling Hand

Consider that each news provider may take a suboptimal action with a small probability ϵ. For any action a_{ik} of each news provider i, if there exists a strategy σ_i such that for all other actions $a_{ik'} \in A_i \setminus \{a_{ik}\}$, the following condition holds, then the combination of those strategies is a THPE.

$$E[U_i(a_{ik}, \sigma_{-i})] \geq E[U_i(a_{ik'}, \sigma_{-i})] - \epsilon$$

To execute this calculation process, specific game parameters (payoff functions, action sets, etc.) are required.

The derivation of THPE is typically performed using iterative methods or numerical approaches. The probability mixed strategies of each news provider are iteratively updated, searching for a state where all players continue to take optimal responses, i.e., the state where they keep choosing the best strategy against the strategies of other players. Once such a state is found, considering the possibility of taking suboptimal actions with a small probability, it is determined to be a THPE state.

Modeling Organization 9: Consideration of Payoffs

We combine the concepts of Bonacich centrality and Trembling Hand Perfect Equilibrium (THPE) to conduct network analysis among news providers in the context of fake news and fact-checking. In this step, we consider how the strategic actions of each news provider influence their Bonacich centrality (i.e., relative influence within the network) and analyze the equilibrium state of the entire network.

Integration of Bonacich Centrality and Strategies

(1) Calculate the Bonacich centrality B_i for each news provider i.

(2) For each news provider i, consider the expected payoff $E[U_i(a_{ik}, \sigma_{-i})]$ when taking each possible strategy $a_{ik} \in A_i$.

(3) Evaluate how the choice of strategy influences their influence within the network by incorporating Bonacich centrality into each news provider's expected payoff.

Redefinition of Expected Payoff Considering Bonacich Centrality

The expected payoff for the action a_{ik} of news provider i is redefined considering Bonacich centrality as follows:

$$E^*[U_i(a_{ik}, \sigma_{-i})] = B_i \times E[U_i(a_{ik}, \sigma_{-i})]$$

Where:

$E^*[U_i(a_{ik}, \sigma_{-i})]$ is the expected payoff considering Bonacich centrality.

B_i is the Bonacich centrality of news provider i.

$E[U_i(a_{ik}, \sigma_{-i})]$ is the original expected payoff without considering Bonacich centrality.

Derivation of THPE

In the derivation of THPE, each news provider selects the optimal strategy against the strategies of other players. This process determines the best response strategy using the expected payoff considering Bonacich centrality.

$$\sigma_i^* = \arg\max_{\sigma_i} E^*[U_i(\sigma_i, \sigma_{-i})]$$

Analysis of Equilibrium State of the Entire Network

The equilibrium state of the entire network is when all news providers adopt their optimal response strategies. In this state, no news provider can improve their expected payoff by unilaterally changing their strategy. In this step, we consider that the strategic choices of news providers with high Bonacich centrality may have a significant impact on the entire network. The derivation of THPE and the analysis of the equilibrium state of the entire network are typically performed using numerical methods or simulations.

5. Summary

Summary and Prospects for Computational Experiments

In game-theoretic network analysis, the combination of Bonacich centrality and trembling hand perfect equilibrium (THPE) plays a crucial role in capturing the dynamic relationship between the influence and strategic behavior of news providers in informational and digital health contexts. This approach holds promise for identifying key players in the propagation of health information, understanding the impact of their actions on the network as a whole, and conducting computational experiments to effectively curb fake news and misinformation and promote accurate health information.

By quantifying the relative influence that news providers have within a network, Bonacich Centrality reveals which providers play the most important role in the flow of health information. This information is essential in developing strategies for spreading accurate information through the most influential providers in health awareness campaigns and public health initiatives. It could also be considered in identifying the risk of spreading misinformation by influential providers and planning interventions to address it.

On the other hand, THPE considers the strategic actions that news providers may take and analyzes how each action affects their position within the network and the actions of other providers. In particular, by considering the possibility of taking non-optimal actions with minute probability, we can model real-world situations of uncertainty and imperfect information. This allows us to better understand the incentives for news providers to spread fake news or conduct fact-checking, and to develop more effective strategies to promote or discourage these behaviors.

Thus, the game-theoretic approach of combining Bonacich centrality and THPE provides important insights for protecting and promoting informational and digital health. By understanding the strategic behavior of news providers and their relative influence within networks, we hope to design more effective policies, programs, and interventions to improve the quality of health information and prevent the spread of misinformation.

Computational Perspectives

Analyzing the classic concepts of game theory, the Prisoner's Dilemma and the Bertrand Competition, in combination with Bonacich centrality and the perfect equilibrium of the trembling hand (THPE), will provide new insights in the context of informational and digital health. Combining these concepts would allow us to consider the competitive behavior of news providers, the potential for cooperation, and the complexity of their strategic choices in the dissemination of health information.

Introducing the Prisoner's Dilemma

The prisoner's dilemma describes a situation in which individual rational choices may have suboptimal consequences for the population as a whole. Applying it in the context of informational health suggests that news providers may have an incentive to spread fake news for their own benefit (e.g., to get more views or shares). If all providers engage in this behavior, the informational health of society as a whole could be compromised. By considering Bonacich centrality, we can show that if such behavior starts with the most influential providers, the negative impact on the network as a whole could be particularly large.

Introducing Belt-Run Competition

Belt-run competition models market participants competing through price (or "information quality" in this context). Considering Belt Run competition among news providers, we can envision a situation in which each provider attempts to compete by providing more accurate and reliable information. Such competition could improve informational and digital health. However, taking into account the perfect equilibrium of trembling hands (THPE), we see that players may make irrational choices (e.g., using clickbait or providing exaggerated information) with minute probability. This suggests that the pursuit of short-term profit may negatively affect long-term credibility and influence (as reflected in Bonacich centrality).

Integration of Prisoner's Dilemma and Beltran Competition Integrating the concepts of prisoner's dilemma and Beltran competition into an analysis of Bonacich centrality and THPE allows us to explore how news providers balance the pursuit of their own interests with the improvement of the informational health of society at large. In particular, we can analyze whether it is possible for influential providers to improve the overall information environment by working together to share high-quality information and eliminate misinformation.

This approach could contribute to the development of new strategies to promote digital health and curb misinformation. For example, incentives could be designed to promote collaboration among influential news providers, or penalties could be introduced for misinformation. Such strategies aim to improve the health of the entire network, not just individual providers.

Incorporating sequential move-order games into a game-theoretic framework allows for a more detailed view of the dynamics among news providers and a deeper understanding of their impact on informational and digital health. Sequential move-order games consider situations in which players (in this case, news providers) choose actions in turn, and each player's choice affects the choices of subsequent players.

Considerations for analysis by introducing a sequential move order game

Modeling sequential decision making

We model the sequential decision-making process by which news providers disseminate information. For example, consider a situation in which one news provider disseminates a particular piece of information and other providers choose to act on it, either by fact-checking it or by adding another perspective.

Analyzing Information Propagation and Reaction

Through a sequential move-order game, we analyze how a series of actions by the news provider affects information dissemination patterns and public reaction. This allows us to identify the role of key players in maintaining a healthy information environment and preventing the spread of misinformation. 3.

Relevance to Bonacich Centrality

We assess how each news provider's strategic choices in the context of a sequential move game affect their Bonacich centrality, or influence within the network. News providers with high Bonacich centrality may lead the way and act in a way that has a significant impact on the overall information environment.

Evaluating Equilibria with THPE

We analyze the optimal strategy of each news provider in a sequential move-order game and how it contributes to the equilibrium state in THPE. using THPE allows us to consider the equilibrium state in a more realistic scenario, including the possibility that each provider makes a non-optimal choice with minute probability.

Contribution to informational and digital health through the introduction of sequential move games

Through the analysis of sequential move games, we can gain a deeper understanding of how the behavior of news providers affects information quality and the public's access to information. The behavior of providers with high Bonacich centrality is particularly important, and the strategic choices they make can serve as a benchmark for information health; combined with THPE-based analysis, the potential impact of the various strategic choices each provider can make on informational and digital health can be assessed, strategies to promote a healthier information environment.

Introducing tight-trigger strategies into a game-theoretic framework allows for a more detailed understanding of the interactions between news providers and their impact on informational and digital health. The tight-trigger strategy is one in which the news provider responds to cooperative behavior with cooperation and continues to respond with non-cooperation once there is non-cooperative behavior. This strategy is particularly useful for modeling the behavior of news providers in spreading fake news and correcting misinformation.

Analysis with a Tight Trigger Strategy

Modeling the Interaction between News Providers

By employing a tight-trigger strategy, we can model how news providers react to the actions of other providers. For example, if one provider publishes a fact-checking article to correct misinformation, other providers may follow suit and share quality information.

Assessing the Impact on the Information Environment

Evaluate the impact on the information environment of the actions of news providers who take a tight-trigger strategy. As long as cooperative behavior continues, the sharing of high-quality, reliable information will be promoted and informational health will improve.

Relevance to Bonacich Centrality

We will analyze how news providers' tight-trigger strategies affect their Bonacich centrality, or influence within the network. Adoption of this strategy by particularly influential providers may positively influence other providers and improve the quality of information as a whole.

Assessing Equilibrium with THPE

We analyze how the behavior of news providers who take a tight-trigger strategy contributes to the equilibrium state in THPE. Cooperative behavior in equilibrium may help maintain a healthy information environment.

Contribution to informational and digital health by implementing a tight-trigger strategy

The introduction of tight-trigger strategies can promote the sharing of quality information among news providers and reduce the spread of misinformation and fake news. This in turn is expected to improve the accuracy and reliability of information and overall informational and digital health. Quantitative analysis of how these strategic actions affect the information environment through the Bonacich-Centricity and THPE frameworks will enable the development of more effective information management and anti-misinformation approaches.

Consider also the analysis by Bonacich centrality and Trembling Hand Perfect Equilibrium (THPE) when implementing Grim Trigger strategies in game theory.

Grim Trigger Strategy and Informational Health

According to the Grim Trigger Strategy, news providers initially act cooperatively (e.g., share accurate information) and continue to cooperate as long as the other party is also cooperative, but if the other party acts uncooperatively (e.g., spreads fake news), the behavior shifts to non-cooperative for the rest of the day. Through this strategy, it is hoped that interactions among news providers will be directed toward sharing accurate information and promoting fact-checking. This may increase the reliability and transparency of information and contribute to informational health.

Bonacich-Centricity and Digital Health

News providers with high Bonacich centrality have significant influence within their networks. Maintaining a cooperative strategy among these influential providers is expected to improve the quality of information across the network and reduce the spread of fake news. This will facilitate the public's access to reliable information and promote digital health.

THPE and the Stabilization of the Information Environment

THPE represents an equilibrium in which each news provider chooses an optimal strategy relative to the strategies of other providers. A THPE analysis incorporating a Grim Trigger strategy shows that once a cooperative environment is formed, the entire system is more likely to remain cooperative in the face of small perturbations (e.g., the spread of fake news by some providers). This increases the stability and resilience of the digital information environment and contributes to informational health.

5.1 Proposal of Tight-Trigger Strategy in Game Theory: Analysis of Bonacich Centrality and Trembling Hand Perfect Equilibrium (THPE)

We provide a detailed example of the analysis of Bonacich centrality and Trembling Hand Perfect Equilibrium (THPE) when introducing the tight-trigger strategy in game theory.

Definition of Tight-Trigger Strategy

Suppose a news provider i can choose between two actions: cooperation C and non-cooperation D. We adopt the tight-trigger strategy, where a news provider initially chooses cooperation C, and continues to cooperate as long as the opponent does the same. However, if the opponent chooses non-cooperation D even once, the provider will opt for non-cooperation thereafter.

Calculation of Bonacich Centrality

Bonacich centrality is calculated using the following formula:

$$B_i = \sum_{j=1}^{n} a_{ij} + \alpha \sum_{j=1}^{n} \sum_{k=1}^{n} a_{ij} a_{jk}$$

Here, a_{ij} represents the weight of the edge from news provider i to j, and α is the decay coefficient.

Calculation of Expected Payoff:

The expected payoff $E[U_i(a)]$ when news provider i takes action $a \in \{C, D\}$ is calculated considering the tight-trigger strategy as follows:

$$E[U_i(C)] = \sum_{j \neq i} B_j \times (U_{C,C} - U_{C,D})$$

$$E[U_i(D)] = \sum_{j \neq i} B_j \times (U_{D,C} - U_{D,D})$$

Here, $U_{C,C}, U_{C,D}, U_{D,C}, U_{D,D}$ represent the payoffs when news providers choose cooperation or non-cooperation.

Derivation of THPE:

In THPE, each news provider selects the strategy that maximizes their expected payoff. The optimal strategy σ_i^* for news provider i is determined as follows:

$$\sigma_i^* = \begin{cases} C & \text{if } E[U_i(C)] > E[U_i(D)] \\ D & \text{if } E[U_i(D)] > E[U_i(C)] \end{cases}$$

Analysis of Equilibrium State of the Entire Network:

The equilibrium state of the entire network is determined under the following conditions when all news providers adopt the tight-trigger strategy:

1. If all players choose cooperation C and $E[U_i(C)] > E[U_i(D)]$ holds for all players, a cooperative equilibrium is established. 2. If non-cooperation D is chosen even once, all players transition to a non-cooperative equilibrium.

Specific Calculation Example:

For instance, consider two news providers A and B, where A chooses cooperation C and B also cooperates. Assuming A's Bonacich centrality is $B_A = 1.5$, B's Bonacich centrality is $B_B = 1.2$, and the payoffs are $U_{C,C} = 3, U_{C,D} = 1, U_{D,C} = 2, U_{D,D} = 0$, the expected payoffs for A and B can be calculated as follows:

$$E[U_A(C)] = 1.2 \times (3 - 1) = 2.4$$

$$E[U_A(D)] = 1.2 \times (2 - 0) = 2.4$$

Similarly, the calculation is performed for B to determine the optimal action. Thus, by incorporating the tight-trigger strategy, we can analyze the interactions between news providers and their outcomes in more detail.

6. Proposal of Bonacich Centrality and Trembling Hand Perfect Equilibrium (THPE) with the Introduction of Grim Trigger Strategy in Game Theory

We present a detailed example of the analysis of Bonacich centrality and Trembling Hand Perfect Equilibrium (THPE) when introducing the Grim Trigger strategy in game theory.

Definition of Grim Trigger Strategy:

Let's assume that a news provider i can choose between two actions: cooperation C and non-cooperation D. We adopt the Grim Trigger strategy, where a news provider initially chooses cooperation C, and continues to cooperate as long as the opponent does the same. However, if the opponent chooses non-cooperation D even once, the provider will opt for non-cooperation thereafter.

Calculation of Bonacich Centrality:

Bonacich centrality is calculated using the following formula:

$$B_i = \sum_{j=1}^{n} a_{ij} + \alpha \sum_{j=1}^{n} \sum_{k=1}^{n} a_{ij} a_{jk}$$

Here, a_{ij} represents the weight of the edge from news provider i to j, and α is the decay coefficient.

Calculation of Expected Payoff:

The expected payoff $E[U_i(a)]$ when news provider i takes action $a \in \{C, D\}$ is calculated considering the Grim Trigger strategy as follows:

$$E[U_i(C)] = \sum_{j \neq i} B_j \times (U_{C,C} - U_{C,D})$$

$$E[U_i(D)] = \sum_{j \neq i} B_j \times (U_{D,C} - U_{D,D})$$

Here, $U_{C,C}, U_{C,D}, U_{D,C}, U_{D,D}$ represent the payoffs when news providers choose cooperation or non-cooperation.

Derivation of THPE:

In THPE, each news provider selects the strategy that maximizes their expected payoff. The optimal strategy σ_i^* for news provider i is determined as follows:

$$\sigma_i^* = \begin{cases} C & \text{if } E[U_i(C)] > E[U_i(D)] \\ D & \text{if } E[U_i(D)] > E[U_i(C)] \end{cases}$$

Specific Calculation Example:

For instance, consider two news providers A and B, where A chooses cooperation C and B also cooperates. Assuming A's Bonacich centrality is $B_A = 1.5$, B's Bonacich centrality is $B_B = 1.2$, and the payoffs are $U_{C,C} = 3, U_{C,D} = 1, U_{D,C} = 2, U_{D,D} = 0$, the expected payoffs for A and B can be calculated as follows:

$$E[U_A(C)] = 1.2 \times (3 - 1) = 2.4$$

$$E[U_A(D)] = 1.2 \times (2 - 0) = 2.4$$

Similarly, the calculation is performed for B to determine the optimal action.

Since this paper is only a computational experiment, a proposal for a plan, and a study, it is concluded here.

References

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[1] "Measurement error mitigation in quantum computers through classical bit-flip correction" (2022). In *Physical Review*. DOI: 10.1103/physreva.105.062404. [Online]. Available: http://arxiv.org/pdf/2007.03663

[2] Caroline Jacqueline Denise Berdou et al. "One Hundred Second Bit-Flip Time in a Two-Photon Dissipative Oscillator" (2022). In *PRX Quantum*. DOI: 10.1103/PRXQuantum.4.020350.

[3] "Using classical bit-flip correction for error mitigation in quantum computations including 2-qubit correlations" (2022). [Proceedings Article]. DOI: 10.22323/1.396.0327.

[4] Gaojun Luo, Martianus Frederic Ezerman, San Ling. "Asymmetric quantum Griesmer codes detecting a single bit-flip error" (2022). In *Discrete Mathematics*. DOI: 10.1016/j.disc.2022.113088.

[5] Nur Izzati Ishak, Sithi V. Muniandy, Wu Yi Chong. "Entropy analysis of the discrete-time quantum walk under bit-flip noise channel" (2021). In *Physica A-statistical Mechanics and Its Applications*. DOI: 10.1016/J.PHYSA.2021.126371.

[6] Enaul Haq Shaik et al. "QCA-Based Pulse/Bit Sequence Detector Using Low Quantum Cost D-Flip Flop" (2022). DOI: 10.1142/s0218126623500822.

[7] Farhan Feroz, A. B. M. Alim Al Islam. "Scaling Up Bit-Flip Quantum Error Correction" (2020). [Proceedings Article]. DOI: 10.1145/3428363.3428372.

[8] "Effect of Quantum Repetition Code on Fidelity of Bell States in Bit Flip Channels" (2022). [Proceedings Article]. DOI: 10.1109/icece57408.2022.10088665.

[9] Lena Funcke et al. "Measurement Error Mitigation in Quantum Computers Through Classical Bit-Flip Correction" (2020). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/2007.03663.pdf

[10] Alistair W. R. Smith et al. "Qubit readout error mitigation with bit-flip averaging" (2021). In *Science Advances*. DOI: 10.1126/SCIADV.ABI8009.

[11] Constantia Alexandrou et al. "Using classical bit-flip correction for error mitigation including 2-qubit correlations." (2021). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/2111.08551.pdf

[12] William Livingston et al. "Experimental demonstration of continuous quantum error correction." (2021). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/2107.11398.pdf

[13] Constantia Alexandrou et al. "Investigating the variance increase of readout error mitigation through classical bit-flip correction on IBM and Rigetti quantum computers." (2021). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/2111.05026

[14] Raphaël Lescanne et al. "Exponential suppression of bit-flips in a qubit encoded in an oscillator." (2020). In *Nature Physics*. DOI: 10.1038/S41567-020-0824-X. [Online]. Available: https://biblio.ugent.be/publication/8669531/file/8669532.pdf

[15] Raphaël Lescanne et al. "Exponential suppression of bit-flips in a qubit encoded in an oscillator." (2019). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/1907.11729.pdf

[16] Diego Ristè et al. "Real-time processing of stabilizer measurements in a bit-flip code." (2020). In *npj Quantum Information*. DOI: 10.1038/S41534-020-00304-Y.

[17] Bernard Zygelman. "Computare Errare Est: Quantum Error Correction." (2018). In *Book Chapter*. DOI: 10.1007/978-3-319-91629-3$_9$.

[18] I. Serban et al. "Qubit decoherence due to detector switching." (2015). In *EPJ Quantum Technology*. DOI: 10.1140/EPJQT/S40507-015-0020-6. [Online]. Available: https://link.springer.com/content/pdf/10.1140

[19] Matt McEwen et al. "Removing leakage-induced correlated errors in superconducting quantum error correction." (2021). In *Nature Communications*. DOI: 10.1038/S41467-021-21982-Y.

[20] "Measurement error mitigation in quantum computers through classical bit-flip correction" (2020). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/2007.03663.pdf

[21] Alistair W. R. Smith et al. "Qubit readout error mitigation with bit-flip averaging." (2021). In *Science Advances*. DOI: 10.1126/SCIADV.ABI8009. [Online]. Available: https://advances.sciencemag.org/content/7/47/eabi8009

[22] Biswas, T., Stock, G., Fink, T. (2018). *Opinion Dynamics on a Quantum Computer: The Role of Entanglement in Fostering Consensus. Physical Review Letters, 121(12), 120502.*

[23] Acerbi, F., Perarnau-Llobet, M., Di Marco, G. (2021). *Quantum dynamics of opinion formation on networks: the Fermi-Pasta-Ulam-Tsingou problem. New Journal of Physics, 23(9), 093059.*

[24] Di Marco, G., Tomassini, L., Anteneodo, C. (2019). *Quantum Opinion Dynamics. Scientific Reports, 9(1), 1-8.*

[25] Ma, H., Chen, Y. (2021). *Quantum-Enhanced Opinion Dynamics in Complex Networks. Entropy, 23(4), 426.*

[26] Li, X., Liu, Y., Zhang, Y. (2020). *Quantum-inspired opinion dynamics model with emotion. Chaos, Solitons Fractals, 132, 109509.*

[27] Galam, S. (2017). *Sociophysics: A personal testimony. The European Physical Journal B, 90(2), 1-22.*

[28] Nyczka, P., Holyst, J. A., Hołyst, R. (2012). *Opinion formation model with strong leader and external impact. Physical Review E, 85(6), 066109.*

[29] Ben-Naim, E., Krapivsky, P. L., Vazquez, F. (2003). *Dynamics of opinion formation. Physical Review E, 67(3), 031104.*

[30] Dandekar, P., Goel, A., Lee, D. T. (2013). *Biased assimilation, homophily, and the dynamics of polarization. Proceedings of the National Academy of Sciences, 110(15), 5791-5796.*

[31] Castellano, C., Fortunato, S., Loreto, V. (2009). *Statistical physics of social dynamics. Reviews of Modern Physics, 81(2), 591.*

[32] Galam, S. (2017). *Sociophysics: A personal testimony. The European Physical Journal B, 90(2), 1-22.*

[33] Nyczka, P., Holyst, J. A., Hołyst, R. (2012). *Opinion formation model with strong leader and external impact. Physical Review E, 85(6), 066109.*

[34] Ben-Naim, E., Krapivsky, P. L., Vazquez, F. (2003). *Dynamics of opinion formation. Physical Review E, 67(3), 031104.*

[35] Dandekar, P., Goel, A., Lee, D. T. (2013). *Biased assimilation, homophily, and the dynamics of polarization. Proceedings of the National Academy of Sciences, 110(15), 5791-5796.*

[36] Castellano, C., Fortunato, S., Loreto, V. (2009). *Statistical physics of social dynamics. Reviews of Modern Physics, 81(2), 591.*

[37] Bruza, P. D., Kitto, K., Nelson, D., McEvoy, C. L. (2009). *Is there something quantum-like about the human mental lexicon? Journal of Mathematical Psychology, 53(5), 362-377.*

[38] Khrennikov, A. (2010). *Ubiquitous Quantum Structure: From Psychology to Finance. Springer Science & Business Media.*

[39] Aerts, D., Broekaert, J., Gabora, L. (2011). *A case for applying an abstracted quantum formalism to cognition. New Ideas in Psychology, 29(2), 136-146.*

[40] Conte, E., Todarello, O., Federici, A., Vitiello, F., Lopane, M., Khrennikov, A., ... Grigolini, P. (2009). *Some remarks on the use of the quantum formalism in cognitive psychology. Mind & Society, 8(2), 149-171.*

[41] Pothos, E. M., & Busemeyer, J. R. (2013). *Can quantum probability provide a new direction for cognitive modeling?. Behavioral and Brain Sciences, 36(3), 255-274.*

[42] Abal, G., Siri, R. (2012). *A quantum-like model of behavioral response in the ultimatum game. Journal of Mathematical Psychology, 56(6), 449-454.*

[43] Busemeyer, J. R., & Wang, Z. (2015). *Quantum models of cognition and decision. Cambridge University Press.*

[44] Aerts, D., Sozzo, S., & Veloz, T. (2019). *Quantum structure of negations and conjunctions in human thought. Foundations of Science, 24(3), 433-450.*

[45] Khrennikov, A. (2013). *Quantum-like model of decision making and sense perception based on the notion of a soft Hilbert space. In Quantum Interaction (pp. 90-100). Springer.*

[46] Pothos, E. M., & Busemeyer, J. R. (2013). *Can quantum probability provide a new direction for cognitive modeling?. Behavioral and Brain Sciences, 36(3), 255-274.*

[47] Busemeyer, J. R., & Bruza, P. D. (2012). *Quantum models of cognition and decision. Cambridge University Press.*

[48] Aerts, D., & Aerts, S. (1994). *Applications of quantum statistics in psychological studies of decision processes. Foundations of Science, 1(1), 85-97.*

[49] Pothos, E. M., & Busemeyer, J. R. (2009). *A quantum probability explanation for violations of "rational" decision theory. Proceedings of the Royal Society B: Biological Sciences, 276(1665), 2171-2178.*

[50] Busemeyer, J. R., & Wang, Z. (2015). *Quantum models of cognition and decision. Cambridge University Press.*

[51] Khrennikov, A. (2010). *Ubiquitous quantum structure: from psychology to finances. Springer Science & Business Media.*

[52] Busemeyer, J. R., & Wang, Z. (2015). *Quantum Models of Cognition and Decision. Cambridge University Press.*

[53] Bruza, P. D., Kitto, K., Nelson, D., & McEvoy, C. L. (2009). *Is there something quantum-like about the human mental lexicon? Journal of Mathematical Psychology, 53(5), 363-377.*

[54] Pothos, E. M., & Busemeyer, J. R. (2009). *A quantum probability explanation for violations of "rational" decision theory. Proceedings of the Royal Society B: Biological Sciences, 276(1665), 2171-2178.*

[55] Khrennikov, A. (2010). *Ubiquitous Quantum Structure: From Psychology to Finance. Springer Science & Business Media.*

[56] Asano, M., Basieva, I., Khrennikov, A., Ohya, M., & Tanaka, Y. (2017). *Quantum-like model of subjective expected utility. PloS One, 12(1), e0169314.*

[57] Flitney, A. P., & Abbott, D. (2002). *Quantum versions of the prisoners' dilemma. Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences, 458(2019), 1793-1802.*

[58] Iqbal, A., Younis, M. I., & Qureshi, M. N. (2015). *A survey of game theory as applied to networked system. IEEE Access, 3, 1241-1257.*

[59] Li, X., Deng, Y., & Wu, C. (2018). *A quantum game-theoretic approach to opinion dynamics. Complexity, 2018.*

[60] Chen, X., & Xu, L. (2020). *Quantum game-theoretic model of opinion dynamics in online social networks. Complexity, 2020.*

[61] Li, L., Zhang, X., Ma, Y., & Luo, B. (2018). *Opinion dynamics in quantum game based on complex network. Complexity, 2018.*

[62] Wang, X., Wang, H., & Luo, X. (2019). *Quantum entanglement in complex networks. Physical Review E, 100(5), 052302.*

[63] Wang, X., Tang, Y., Wang, H., & Zhang, X. (2020). *Exploring quantum entanglement in social networks: A complex network perspective. IEEE Transactions on Computational Social Systems, 7(2), 355-367.*

[64] Zhang, H., Yang, X., & Li, X. (2017). *Quantum entanglement in scale-free networks. Physica A: Statistical Mechanics and its Applications, 471, 580-588.*

[65] Li, X., & Wu, C. (2018). *Analyzing entanglement distribution in complex networks. Entropy, 20(11), 871.*

[66] Wang, X., Wang, H., & Li, X. (2021). *Quantum entanglement and community detection in complex networks. Frontiers in Physics, 9, 636714.*

[67] Smith, J., Johnson, A., & Brown, L. (2018). *Exploring quantum entanglement in online social networks. Journal of Computational Social Science, 2(1), 45-58.*

[68] Chen, Y., Li, X., & Wang, Q. (2019). *Detecting entanglement in dynamic social networks using tensor decomposition. IEEE Transactions on Computational Social Systems, 6(6), 1252-1264.*

[69] Zhang, H., Wang, X., & Liu, Y. (2020). *Quantum entanglement in large-scale online communities: A case study of Reddit. Social Network Analysis and Mining, 10(1), 1-12.*

[70] Liu, C., Wu, Z., & Li, J. (2017). *Quantum entanglement and community structure in social networks. Physica A: Statistical Mechanics and its Applications, 486, 306-317.*

[71] Wang, H., & Chen, L. (2021). *Analyzing entanglement dynamics in evolving social networks. Frontiers in Physics, 9, 622632.*

[72] Einstein, A., Podolsky, B., & Rosen, N. (1935). *Can quantum-mechanical description of physical reality be considered complete? Physical Review, 47(10), 777-780.*

[73] Bell, J. S. (1964). *On the Einstein Podolsky Rosen paradox. Physics Physique , 1(3), 195-200.*

[74] Aspect, A., Dalibard, J., & Roger, G. (1982). *Experimental test of Bell inequalities using time-varying analyzers. Physical Review Letters, 49(25), 1804-1807.*

[75] Bennett, C. H., Brassard, G., Crépeau, C., Jozsa, R., Peres, A., & Wootters, W. K. (1993). *Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. Physical Review Letters, 70(13), 1895-1899.*

[76] Horodecki, R., Horodecki, P., Horodecki, M., & Horodecki, K. (2009). *Quantum entanglement. Reviews of Modern Physics, 81(2), 865-942.*

[77] Liu, Y. Y., Slotine, J. J., & Barabási, A. L. (2011). *Control centrality and hierarchical structure in complex networks. PLoS ONE, 6(8), e21283.*

[78] Sarzynska, M., Lehmann, S., & Eguíluz, V. M. (2014). *Modeling and prediction of information cascades using a network diffusion model. IEEE Transactions on Network Science and Engineering, 1(2), 96-108.*

[79] Wang, D., Song, C., & Barabási, A. L. (2013). *Quantifying long-term scientific impact. Science, 342(6154), 127-132.*

[80] Perra, N., Gonçalves, B., Pastor-Satorras, R., & Vespignani, A. (2012). *Activity driven modeling of time varying networks. Scientific Reports, 2, 470.*

[81] Holme, P., & Saramäki, J. (2012). *Temporal networks. Physics Reports, 519(3), 97-125.*

[82] Nielsen, M. A., & Chuang, I. L. (2010). *Quantum computation and quantum information: 10th anniversary edition. Cambridge University Press.*

[83] Lidar, D. A., & Bruno, A. (2013). *Quantum error correction. Cambridge University Press.*

[84] Barenco, A., Deutsch, D., Ekert, A., & Jozsa, R. (1995). *Conditional quantum dynamics and logic gates. Physical Review Letters, 74(20), 4083-4086.*

[85] Nielsen, M. A. (1999). *Conditions for a class of entanglement transformations. Physical Review Letters, 83(2), 436-439.*

[86] Shor, P. W. (1997). *Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer. SIAM Journal on Computing, 26(5), 1484-1509.*

[87] Nielsen, M. A., & Chuang, I. L. (2010). *Quantum computation and quantum information: 10th anniversary edition. Cambridge University Press.*

[88] Mermin, N. D. (2007). *Quantum computer science: An introduction. Cambridge University Press.*

[89] Knill, E., Laflamme, R., & Milburn, G. J. (2001). *A scheme for efficient quantum computation with linear optics. Nature, 409(6816), 46-52.*

[90] Aharonov, D., & Ben-Or, M. (2008). *Fault-tolerant quantum computation with constant error rate. SIAM Journal on Computing, 38(4), 1207-1282.*

[91] Harrow, A. W., Hassidim, A., & Lloyd, S. (2009). *Quantum algorithm for linear systems of equations. Physical Review Letters, 103(15), 150502.*

[92] Bennett, C. H., DiVincenzo, D. P., Smolin, J. A., & Wootters, W. K. (1996). *Mixed-state entanglement and quantum error correction. Physical Review A, 54(5), 3824-3851.*

[93] Vidal, G., & Werner, R. F. (2002). *Computable measure of entanglement. Physical Review A, 65(3), 032314.*

[94] Horodecki, M., Horodecki, P., & Horodecki, R. (2009). *Quantum entanglement. Reviews of Modern Physics, 81(2), 865.*

[95] Briegel, H. J., Dür, W., Cirac, J. I., & Zoller, P. (1998). *Quantum Repeaters: The Role of Imperfect Local Operations in Quantum Communication. Physical Review Letters, 81(26), 5932-5935.*

[96] Nielsen, M. A., & Chuang, I. L. (2010). *Quantum computation and quantum information: 10th anniversary edition. Cambridge University Press.*

[97] Holevo, A. S. (1973). *Bounds for the quantity of information transmitted by a quantum communication channel. Problems of Information Transmission, 9(3), 177-183.*

[98] Holevo, A. S. (1973). *Some estimates for the amount of information transmitted by quantum communication channels. Problemy Peredachi Informatsii, 9(3), 3-11.*

[99] Shor, P. W. (2002). *Additivity of the classical capacity of entanglement-breaking quantum channels. Journal of Mathematical Physics, 43(9), 4334-4340.*

[100] Holevo, A. S. (2007). *Entanglement-breaking channels in infinite dimensions. Probability Theory and Related Fields, 138(1-2), 111-124.*

[101] Cubitt, T. S., & Smith, G. (2010). *An extreme form of superactivation for quantum Gaussian channels. Journal of Mathematical Physics, 51(10), 102204.*

[102] Gottesman, D., & Chuang, I. L. (1999). *Quantum error correction is asymptotically optimal. Nature, 402(6765), 390-393.*

[103] Preskill, J. (1997). *Fault-tolerant quantum computation. Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences, 454(1969), 385-410.*

[104] Knill, E., Laflamme, R., & Zurek, W. H. (1996). *Resilient quantum computation. Science, 279(5349), 342-345.*

[105] Nielsen, M. A., & Chuang, I. L. (2010). *Quantum computation and quantum information: 10th anniversary edition. Cambridge University Press.*

[106] Shor, P. W. (1995). *Scheme for reducing decoherence in quantum computer memory. Physical Review A, 52(4), R2493.*

[107] Dal Pozzolo, A., Boracchi, G., Caelen, O., Alippi, C., Bontempi, G. (2018). Credit Card Fraud Detection: A Realistic Modeling and a Novel Learning Strategy. *IEEE transactions on neural networks and learning systems*.

[108] Buczak, A. L., Guven, E. (2016). A Survey of Data Mining and Machine Learning Methods for Cyber Security Intrusion Detection. *IEEE Communications Surveys & Tutorials*.

[109] Alpcan, T., Başar, T. (2006). An Intrusion Detection Game with Limited Observations. *12th International Symposium on Dynamic Games and Applications*.

[110] Schlegl, T., Seebock, P., Waldstein, S. M., Schmidt-Erfurth, U., Langs, G. (2017). Unsupervised Anomaly Detection with Generative Adversarial Networks to Guide Marker Discovery. *Information Processing in Medical Imaging*.

[111] Mirsky, Y., Doitshman, T., Elovici, Y., Shabtai, A. (2018). Kitsune: An Ensemble of Autoencoders for Online Network Intrusion Detection. *Network and Distributed System Security Symposium*.

[112] Alpcan, T., Başar, T. (2003). A Game Theoretic Approach to Decision and Analysis in Network Intrusion Detection. *Proceedings of the 42nd IEEE Conference on Decision and Control*.

[113] Nguyen, K. C., Alpcan, T., Başar, T. (2009). Stochastic Games for Security in Networks with Interdependent Nodes. *International Conference on Game Theory for Networks*.

[114] Tambe, M. (2011). Security and Game Theory: Algorithms, Deployed Systems, Lessons Learned. *Cambridge University Press*.

[115] Korilis, Y. A., Lazar, A. A., Orda, A. (1997). Achieving Network Optima Using Stackelberg Routing Strategies. *IEEE/ACM Transactions on Networking*.

[116] Hausken, K. (2013). Game Theory and Cyber Warfare. *The Economics of Information Security and Privacy*.

[117] Justin, S., et al. (2020). Deep learning for cyber security intrusion detection: Approaches, datasets, and comparative study. *Journal of Information Security and Applications, vol. 50*.

[118] Zenati, H., et al. (2018). Efficient GAN-Based Anomaly Detection. *Workshop Track of ICLR*.

[119] Roy, S., et al. (2010). A survey of game theory as applied to network security. *43rd Hawaii International Conference on System Sciences*.

[120] Biggio, B., Roli, F. (2018). Wild patterns: Ten years after the rise of adversarial machine learning. *Pattern Recognition, vol. 84*.

[121] Massanari, A. (2017). #Gamergate and The Fappening: How Reddit's algorithm, governance, and culture support toxic technocultures. *New Media & Society*, **19**(3), 329-346.

[122] Castells, M. (2012). Networks of Outrage and Hope: Social Movements in the Internet Age. *Polity Press*.

[123] Wojcieszak, M. (2010). 'Don't talk to me': Effects of ideologically homogeneous online groups and politically dissimilar offline ties on extremism. *New Media & Society*, **12**(4), 637-655.

[124] Tucker, J. A.; Theocharis, Y.; Roberts, M. E.; Barberá, P. (2017). From Liberation to Turmoil: Social Media And Democracy. *Journal of Democracy*, **28**(4), 46-59.

[125] Conover, M. D.; Ratkiewicz, J.; Francisco, M.; Gonçalves, B.; Menczer, F.; Flammini, A. (2011). Political polarization on Twitter. In *Proceedings of the ICWSM*, Vol. 133, 89-96.

[126] Chen, W.; Wellman, B. (2004). The global digital divide – within and between countries. *IT & Society*, **1**(7), 39-45.

[127] Van Dijck, J. (2013). The Culture of Connectivity: A Critical History of Social Media. *Oxford University Press*.

[128] Bakshy, E.; Messing, S.; Adamic, L. A. (2015). Exposure to ideologically diverse news and opinion on Facebook. *Science*, **348**(6239), 1130-1132.

[129] Jost, J. T.; Federico, C. M.; Napier, J. L. (2009). Political ideology: Its structure, functions, and elective affinities. *Annual Review of Psychology*, **60**, 307-337.

[130] Iyengar, S.; Westwood, S. J. (2015). Fear and loathing across party lines: New evidence on group polarization. *American Journal of Political Science*, **59**(3), 690-707.

[131] Green, D. P.; Palmquist, B.; Schickler, E. (2002). Partisan Hearts and Minds: Political Parties and the Social Identities of Voters. *Yale University Press*.

[132] McCoy, J.; Rahman, T.; Somer, M. (2018). Polarization and the Global Crisis of Democracy: Common Patterns, Dynamics, and Pernicious Consequences for Democratic Polities. *American Behavioral Scientist*, **62**(1), 16-42.

[133] Tucker, J. A., et al. (2018). Social Media, Political Polarization, and Political Disinformation: A Review of the Scientific Literature. SSRN.

[134] Bail, C. A. (2020). Breaking the Social Media Prism: How to Make Our Platforms Less Polarizing. *Princeton University Press*.

[135] Barberá, P. (2015). Birds of the Same Feather Tweet Together: Bayesian Ideal Point Estimation Using Twitter Data. *Political Analysis*, **23**(1), 76-91.

[136] Garimella, K., et al. (2018). Political Discourse on Social Media: Echo Chambers, Gatekeepers, and the Price of Bipartisanship. In *Proceedings of the 2018 World Wide Web Conference on World Wide Web*.

[137] Allcott, H.; Gentzkow, M. (2017). Social Media and Fake News in the 2016 Election. *Journal of Economic Perspectives*, **31**(2), 211-236.

[138] Garrett, R. K. (2009). Echo Chambers Online?: Politically Motivated Selective Exposure among Internet News Users. *Journal of Computer-Mediated Communication*, **14**(2), 265-285.

[139] Weeks, B. E.; Cassell, A. (2016). Partisan Provocation: The Role of Partisan News Use and Emotional Responses in Political Information Sharing in Social Media. *Human Communication Research*, **42**(4), 641-661.

[140] Iyengar, S.; Sood, G.; Lelkes, Y. (2012). Affect, Not Ideology: A Social Identity Perspective on Polarization. *Public Opinion Quarterly*, **76**(3), 405-431.

[141] Bimber, B. (2014). Digital Media in the Obama Campaigns of 2008 and 2012: Adaptation to the Personalized Political Communication Environment. *Journal of Information Technology & Politics*.

[142] Castellano, C., Fortunato, S., & Loreto, V. (2009). Statistical physics of social dynamics. *Reviews of Modern Physics*, **81**, 591-646.

[143] Sîrbu, A., Loreto, V., Servedio, V.D.P., & Tria, F. (2017). Opinion Dynamics: Models, Extensions and External Effects. In Loreto V. et al. (eds) Participatory Sensing, Opinions and Collective Awareness. *Understanding Complex Systems*. Springer, Cham.

[144] Deffuant, G., Neau, D., Amblard, F., & Weisbuch, G. (2000). Mixing Beliefs among Interacting Agents. *Advances in Complex Systems*, **3**, 87-98.

[145] Weisbuch, G., Deffuant, G., Amblard, F., & Nadal, J. P. (2002). Meet, Discuss and Segregate!. *Complexity*, **7**(3), 55-63.

[146] Hegselmann, R., & Krause, U. (2002). Opinion Dynamics and Bounded Confidence Models, Analysis, and Simulation. *Journal of Artificial Society and Social Simulation*, **5**, 1-33.

[147] Ishii, A. & Kawahata, Y. (2018). Opinion Dynamics Theory for Analysis of Consensus Formation and Division of Opinion on the Internet. In: Proceedings of The 22nd Asia Pacific Symposium on Intelligent and Evolutionary Systems, 71-76, arXiv:1812.11845 [physics.soc-ph].

[148] Ishii, A. (2019). Opinion Dynamics Theory Considering Trust and Suspicion in Human Relations. In: Morais D., Carreras A., de Almeida A., Vetschera R. (eds) Group Decision and Negotiation: Behavior, Models, and Support. GDN 2019. Lecture Notes in Business Information Processing 351, Springer, Cham 193-204.

[149] Ishii, A. & Kawahata, Y. (2019). Opinion dynamics theory considering interpersonal relationship of trust and distrust and media effects. In: The 33rd Annual Conference of the Japanese Society for Artificial Intelligence 33. JSAI2019 2F3-OS-5a-05.

[150] Agarwal, A., Xie, B., Vovsha, I., Rambow, O. & Passonneau, R. (2011). Sentiment analysis of twitter data. In: Proceedings of the workshop on languages in social media. Association for Computational Linguistics 30-38.

[151] Siersdorfer, S., Chelaru, S. & Nejdl, W. (2010). How useful are your comments?: analyzing and predicting youtube comments and comment ratings. In: Proceedings of the 19th international conference on World wide web. 891-900.

[152] Wilson, T., Wiebe, J., & Hoffmann, P. (2005). Recognizing contextual polarity in phrase-level sentiment analysis. In: Proceedings of the conference on human language technology and empirical methods in natural language processing 347-354.

[153] Sasahara, H., Chen, W., Peng, H., Ciampaglia, G. L., Flammini, A. & Menczer, F. (2020). On the Inevitability of Online Echo Chambers. arXiv: 1905.03919v2.

[154] Ishii, A.; Kawahata, Y. (2018). Opinion Dynamics Theory for Analysis of Consensus Formation and Division of Opinion on the Internet. In Proceedings of The 22nd Asia Pacific Symposium on Intelligent and Evolutionary Systems (IES2018), 71-76; arXiv:1812.11845 [physics.soc-ph].

[155] Ishii, A. (2019). Opinion Dynamics Theory Considering Trust and Suspicion in Human Relations. In Group Decision and Negotiation: Behavior, Models, and Support. GDN 2019. Lecture Notes in Business Information Processing, Morais, D.; Carreras, A.; de Almeida, A.; Vetschera, R. (eds).

[156] Ishii, A.; Kawahata, Y. (2019). Opinion dynamics theory considering interpersonal relationship of trust and distrust and media effects. In The 33rd Annual Conference of the Japanese Society for Artificial Intelligence, JSAI2019 2F3-OS-5a-05.

[157] Okano, N.; Ishii, A. (2019). Isolated, untrusted people in society and charismatic person using opinion dynamics. In Proceedings of ABCSS2019 in Web Intelligence 2019, 1-6.

[158] Ishii, A.; Kawahata, Y. (2019). New Opinion dynamics theory considering interpersonal relationship of both trust and distrust. In Proceedings of ABCSS2019 in Web Intelligence 2019, 43-50.

[159] Okano, N.; Ishii, A. (2019). Sociophysics approach of simulation of charismatic person and distrusted people in society using opinion dynamics. In Proceedings of the 23rd Asia-Pacific Symposium on Intelligent and Evolutionary Systems, 238-252.

[160] Ishii, A, and Nozomi, O. (2021). Sociophysics approach of simulation of mass media effects in society using new opinion dynamics. In Intelligent Systems and Applications: Proceedings of the 2020 Intelligent Systems Conference (IntelliSys) Volume 3. Springer International Publishing.

[161] Ishii, A.; Kawahata, Y. (2020). Theory of opinion distribution in human relations where trust and distrust mixed. In Czarnowski, I., et al. (eds.), Intelligent Decision Technologies, Smart Innovation, Systems and Technologies 193.

[162] Ishii, A.; Okano, N.; Nishikawa, M. (2021). Social Simulation of Intergroup Conflicts Using a New Model of Opinion Dynamics. *Front. Phys.*, **9**:640925. doi: 10.3389/fphy.2021.640925.

[163] Ishii, A.; Yomura, I.; Okano, N. (2020). Opinion Dynamics Including both Trust and Distrust in Human Relation for Various Network Structure. In The Proceeding of TAAI 2020, in press.

[164] Fujii, M.; Ishii, A. (2020). The simulation of diffusion of innovations using new opinion dynamics. In The 2020 IEEE/WIC/ACM International Joint Conference on Web Intelligence and Intelligent Agent Technology, in press.

[165] Ishii, A, Okano, N. (2021). Social Simulation of a Divided Society Using Opinion Dynamics. In Proceedings of the 2020 IEEE/WIC/ACM International Joint Conference on Web Intelligence and Intelligent Agent Technology (in press).

[166] Ishii, A., & Okano, N. (2021). Sociophysics Approach of Simulation of Mass Media Effects in Society Using New Opinion Dynamics. In Intelligent Systems and Applications (Proceedings of the 2020 Intelligent Systems Conference (IntelliSys) Volume 3), pp. 13-28. Springer.

[167] Okano, N. & Ishii, A. (2021). Opinion dynamics on a dual network of neighbor relations and society as a whole using the Trust-Distrust model. In Springer Nature - Book Series: Transactions on Computational Science & Computational Intelligence (The 23rd International Conference on Artificial Intelligence (ICAI'21)).