# Enquiry:Game theory for Informational Health in Incomplete Information Games, The Role of Bonacich Centrality and Peer Effects on Tight-Trigger and Grim-Trigger Strategies

Yasuko Kawahata [†]

Faculty of Sociology, Department of Media Sociology, Rikkyo University, 3-34-1 Nishi-Ikebukuro,Toshima-ku, Tokyo, 171-8501, JAPAN.

ykawahata@rikkyo.ac.jp

**Abstract:** This paper also organizes the hypotheses and discussions for the computational experiments.This Note is a note for discussion and organization of computational experiments and methods.This note examines a new game-theoretic model for analyzing strategic interactions among news providers in the context of fake news dissemination and fact-checking efforts, integrating Bonacich centrality and Trembling Hand Perfect Equilibrium (THPE) concepts are integrated to explore how information providers, modeled as players in a network game with imperfect information, make strategic decisions regarding the publication of news and the verification of information. Bonacich centrality is employed to quantify the relative influence of each information provider in the network, highlighting the role of central nodes in shaping the flow of information; THPE accounts for the possibility of suboptimal behavior due to errors and uncertainty and to explain the strategies employed by information providers Used for. The analysis reveals the conditions under which a network reaches a pairwise stable state where two providers cannot mutually benefit from a change in strategy. The model highlights the complex dynamics of news diffusion and the key factors that influence the effectiveness of fact-checking initiatives. The experimental design of this paper should contribute to policymakers, media organizations, and fact-checkers striving to combat the spread of fake news and promote the dissemination of accurate information. It will also examine strategic responses to non-cooperative behavior and their impact on the network as a whole through the implementation of tight-trigger and grim-trigger strategies.

**Keywords:** Fact-checking, Peer Effects, Bonacich Centrality, Game Theory, Non-Complete Information Games, Non-Cooperative Games, Network Analysis, Informational Health, Digital Health, Shaking Hands Perfect Equilibrium, Agent Interaction

## 1. Introduction

This paper also organizes the hypotheses and discussions for the computational experiments.This Note is a note for discussion and organization of computational experiments and methods. In game-theoretic network analysis, the combination of Bonacich centrality and trembling hand perfect equilibrium (THPE) plays a crucial role in capturing the dynamic relationship between the influence and strategic behavior of news providers in informational and digital health contexts. This approach allows us to identify key players in the dissemination of health information, understand the impact of their actions on the network as a whole, and develop strategies to effectively curb fake news and misinformation and promote accurate health information.

In the introduction to this study, we introduced an approach that combines game theory and network analysis to
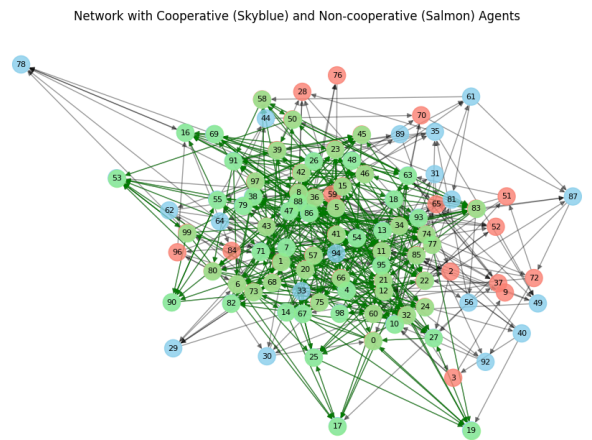


Fig. 1: Network with Cooperative (Skyblue) and Non-cooperative (Salmon) Agents
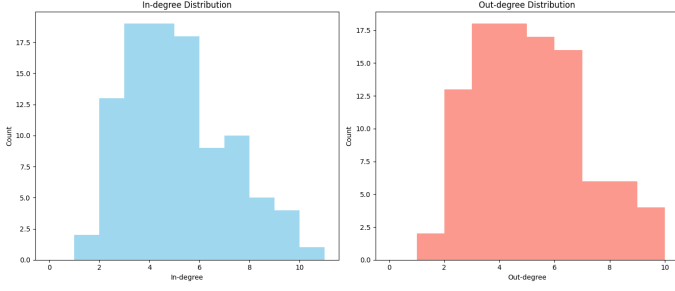
Fig. 2: In-degree, Out-degree

analyze strategic interactions among news providers in the context of fake news and fact checking. This approach uses Bonacich centrality and the perfect equilibrium of trembling hands (THPE) to explore the dynamics of information distribution and its impact on informational and digital health. In addition, by introducing grim-trigger and tight-trigger strategies, we consider strategic responses to non-cooperative behavior and their impact on the network as a whole.

In this context, building a network model among agents is an important step to better understand the interactions of non-cooperative and cooperative agents. through the analysis of In-degree and Out-degree distributions, the sources of agents' influence and and their role in the network. The extraction of k-core networks also allows us to identify important communities and clusters within the network and to identify bases of information propagation and trust.

The implementation of this model provides a foundation for understanding how agents' behavior in the distribution and reception of information affects phenomena such as the spread of fake news and the effectiveness of fact-checking, among others. By analyzing the impact of the behavior of non-cooperative and cooperative agents on the overall health of the network, we aim to contribute to the development of strategies that support informational and digital health.

Through the construction and analysis of such a network model, we will be able to take steps toward developing new insights and strategies for informational health. Specific computational processes and model implementations will use tools such as Python's NetworkX library and R's igraph package, which will enable the construction and analysis of networks based on real data. Through these analyses, we will seek to gain a better understanding of informational health and public digital health in the digital information environment and how to address challenges related to fake news and fact-checking.

In the introduction to this study, we propose an approach that combines game theory and network analysis to analyze strategic interactions among news providers in the context of fake news and fact checking. With this approach, we explore the dynamics of information distribution and its impact on informational and digital health, aiming to better understand

the interactions between agents of noncooperative and cooperative behavior. Using Bonacich centrality and Trembling Hand Perfect Equilibrium (THPE), we quantify information distribution and its impact on the dynamics of interactions between agents through the analysis of peer effects.

Furthermore, by building a network model among agents and generating In-degree and Out-degree distributions of non-cooperative and cooperative agents, as well as k-core network graphs, the project reveals the agents' roles in the network and the sources of their influence This allows us to identify information propagation and trust bases. In this way, we aim to identify bases of information propagation and trust and contribute to the formulation of strategies to support informational and digital health.

The analysis of peer effects is particularly important for understanding chain reactions of behavior among agents and their impact on each other. We will provide a detailed definition and computational process of peer effects and analyze their impact on information diffusion and credibility by quantitatively capturing the chain reaction of behavior among news providers. This analysis allows us to take steps toward developing new insights and strategies for information integrity.

## By introducing peer effects into the analysis of the digital information environment

By introducing peer effects into the analysis of the digital information environment, it is possible to better understand the effects of fact-checking and fake news pollution on informational health and public digital health. This section describes the expected contributions of this approach through a theoretical framework of analysis using peer effects, specific mathematical formulas, and examples of the computational process.

Peer effects in the digital information environment refer to the impact of the actions of individual news providers and users on others. In particular, it is important to analyze how the fact-checking behavior of one agent or the spread of fake news affects the behavior of other agents in the network. This influence alters the dynamics of information trust, acceptability, and diffusion, which in turn affects the overall health of the digital information environment.

The following formula is used to calculate the peer effect

$$PE_{ij} = \Delta P_{j|i}$$

where $PE_{ij}$ represents the peer effect of agent $i$s action on agent $j$, and $\Delta P_{j|i}$ indicates how much the action of agent $i$ changes the probability of agent $j$s action.

As an example of the computational process, we consider the peer effect on agent $B$ when agent $A$ takes action to spread fake news. Suppose that originally the probability of $B$ shar-

ing truthful information was 0.8, but after $A$ spread fake news, the probability of $B$ sharing fake news increased to 0.4. In this case, the peer effect can be computed as

$$PE_{AB} = 0.4 - 0.2 = 0.2$$

This calculation allows us to quantify the impact of fake news diffusion in $A$ on the behavior of $B$. This peer effect propagates to other agents in the network, affecting the credibility and soundness of the information.

Through the analysis of the peer effect, it is possible to identify how positive actions of fact-checking propagate within the network and potentially reduce the spread of fake news. It also allows us to understand how the spread of fake news can affect the behavior of other agents in the network and potentially undermine their informational health. This analysis provides insight for developing strategies to maximize the effectiveness of fact-checking and minimize the spread of fake news.

Thus, peer effects analysis provides an effective approach for promoting informational health and public digital health in the digital information environment. Combining network analysis with game theory allows us to better understand the dynamics of information distribution and reception and to explore new strategies to address challenges related to fake news and fact-checking.

When considering peer effects using Bonacich centrality in game theory, the general formula and computational process is assumed to be expressed as follows.

## Bonacich centrality

The Bonacich centrality $B_i$ of a node $i$ in the network is defined using the adjacency matrix $A$ as follows

$$B_i = \sum_{j=1}^{n} a_{ij} + \alpha \sum_{j=1}^{n}\sum_{k=1}^{n} a_{ij}a_{jk} + \alpha^2 \sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} a_{ij}a_{jk}a_{kl} + \cdots$$

where $a_{ij}$ is the edge weight from node $i$ to node $j$, $\alpha$ is the attenuation factor, and $n$ is the total number of nodes in the network.

## Peer Effects

The peer effect $PE_{ij}$ that the action of news provider $i$ has on the action choices of other news providers $j$ is caused by the change in the probability of the action taken by $j$ due to the action of $i$.

$$PE_{ij} = P_{j|i}^{\text{after}} - P_{j}^{\text{before}}$$

where $P_{j|i}^{\text{after}}$ is the probability that $j$ takes a particular action after $i$ takes a particular action and $P_{j}^{\text{before}}$ is the basic probability that $j$ takes that action regardless of the action of $i$.

We construct the adjacency matrix $A$ of the network. For each node (news provider), assign a set of actions and a base probability of taking those actions. Calculate the centrality of each news provider using the Bonacich centrality formula above.

For each news provider $i$, compute the change that would occur in the action choice probabilities of the other news providers $j$ if they took a particular action. The change in the action probability of $j$ due to the action of $i$ is calculated as the peer effect $PE_{ij}$.

The above calculations are performed for all news provider pairs to analyze the peer effect for the entire network. We pay particular attention to the impact of the behavior of news providers with high Bonacich centrality on the peer effect.

We analyze how the probability that other news providers in the network believe and spread fake news changes when the news provider $B$ takes action to spread fake news. If the Bonacich centrality of $A$ is high, then the actions of $A$ may have a large peer effect on many news providers, including $B$. Through this analysis, we can consider the importance of fact-checking and measures to prevent the spread of fake news.

By combining game theory and network analysis in this way, we expect to understand the dynamics of information propagation and peer effects, which will be useful for maintaining a healthy digital information environment.

Finally, the introduction of grim-trigger and tight-trigger strategies into the game-theoretic framework provides a detailed understanding of the interactions among news providers and their impact on informational and digital health. The tight-trigger strategy is one in which the news provider responds to the other's cooperation with cooperation and continues to respond with non-cooperation once there is non-cooperation. This allows us to model how news providers respond to fake news proliferation and misinformation correction, and continued cooperation improves information quality and deepens our consideration of the health of the information environment.

## 2. Discussion: Analysis by Introducing Tight Trigger Strategies and Peer Effects in Bonacich Centrality

Introducing tight-trigger strategies into a game-theoretic framework allows for a more detailed understanding of the interactions between news providers and their impact on informational and digital health. The tight-trigger strategy is a strategy in which news providers respond to cooperative behavior by cooperating with their counterparts and continue to respond with non-cooperation once there is non-cooperative

behavior. This strategy is particularly useful for modeling the behavior of news providers in spreading fake news and correcting misinformation.

## Modeling the Interaction between News Providers

By employing a tight-trigger strategy, we can model how news providers react to the actions of other providers. For example, if one provider publishes a fact-checking article to correct misinformation, other providers may follow suit and share quality information.

## Assessing the Impact on the Information Environment

Evaluate the impact on the information environment of the actions of news providers who take a tight-trigger strategy. As long as cooperative behavior continues, the sharing of high-quality, reliable information will be promoted and informational health will improve.

## Relevance to Bonacich Centrality

We will analyze how news providers' tight-trigger strategies affect their Bonacich centrality, or influence within the network. Adoption of this strategy by particularly influential providers may positively influence other providers and improve the quality of information as a whole.

## Assessing Equilibrium with THPE

We analyze how the behavior of news providers who take a tight-trigger strategy contributes to the equilibrium state in THPE. Cooperative behavior in equilibrium may help maintain a healthy information environment.

The introduction of tight-trigger strategies can promote the sharing of quality information among news providers and reduce the spread of misinformation and fake news. This in turn is expected to improve the accuracy and reliability of information and overall informational and digital health. Quantitative analysis of how these strategic actions affect the information environment through the Bonacich-Centricity and THPE frameworks will enable the development of more effective information management and anti-misinformation approaches.

## Definition of Tight-Trigger Strategy

Let's assume there are two possible actions for a news provider $i$: cooperation $C$ and non-cooperation $D$, and that the Tight-Trigger strategy is adopted. In this strategy, the news provider initially chooses cooperation $C$ and continues to cooperate as long as the opponent also cooperates. However, if the opponent chooses non-cooperation $D$ even once, the news provider will choose non-cooperation thereafter.

## Calculation of Bonacich Centrality

Bonacich centrality is calculated using the following formula:

$$B_i = \sum_{j=1}^{n} a_{ij} + \alpha \sum_{j=1}^{n} \sum_{k=1}^{n} a_{ij} a_{jk}$$

Here, $a_{ij}$ represents the weight of the edge from news provider $i$ to $j$, and $\alpha$ is the attenuation coefficient.

## Calculation of Expected Payoff

The expected payoff $E[U_i(a)]$ for news provider $i$ taking action $a \in \{C, D\}$ is calculated considering the Tight-Trigger strategy as follows:

$$E[U_i(C)] = \sum_{j \neq i} B_j \times (U_{C,C} - U_{C,D})$$

$$E[U_i(D)] = \sum_{j \neq i} B_j \times (U_{D,C} - U_{D,D})$$

Here, $U_{C,C}$, $U_{C,D}$, $U_{D,C}$, $U_{D,D}$ are the payoffs when news providers choose cooperation or non-cooperation.

## Derivation of THPE

In THPE, each news provider selects a strategy that maximizes their expected payoff. The optimal strategy $\sigma_i^*$ for news provider $i$ is determined as follows:

$$\sigma_i^* = \begin{cases} C & \text{if } E[U_i(C)] > E[U_i(D)] \\ D & \text{if } E[U_i(D)] > E[U_i(C)] \end{cases}$$

If all news providers adopt the Tight-Trigger strategy, the equilibrium state of the entire network is determined by the following conditions:

(1) If all players choose cooperation $C$ and $E[U_i(C)] > E[U_i(D)]$ holds for all players, a cooperative equilibrium is established.

(2) If non-cooperation $D$ is chosen even once, all players transition to a non-cooperative equilibrium.

For example, let's consider two news providers, $A$ and $B$, where $A$ chooses cooperation $C$ and $B$ also cooperates. Assuming $A$'s Bonacich centrality is $B_A = 1.5$, $B$'s Bonacich centrality is $B_B = 1.2$, and the payoffs are $U_{C,C} = 3$, $U_{C,D} = 1$, $U_{D,C} = 2$, $U_{D,D} = 0$, the expected payoffs for $A$ and $B$ can be calculated as follows:

$$E[U_A(C)] = 1.2 \times (3 - 1) = 2.4$$
$$E[U_A(D)] = 1.2 \times (2 - 0) = 2.4$$

Similarly, we calculate for $B$ and determine which action is optimal. Thus, incorporating the Tight-Trigger strategy into the model allows for a more detailed analysis of the interactions between news providers and their outcomes.

Let's consider the analysis of Bonacich centrality and Trembling Hand Perfect Equilibrium (THPE) when introducing the Grim Trigger strategy in game theory.

## Definition of Grim Trigger Strategy

Let's assume there are two possible actions for a news provider $i$: cooperation $C$ and non-cooperation $D$, and that the Grim Trigger strategy is adopted. In this strategy, the news provider initially chooses cooperation $C$ and continues to cooperate as long as the opponent also cooperates. However, if the opponent chooses non-cooperation $D$ even once, the news provider will choose non-cooperation thereafter.

## Calculation of Bonacich Centrality

Bonacich centrality is calculated using the following formula:

$$B_i = \sum_{j=1}^{n} a_{ij} + \alpha \sum_{j=1}^{n} \sum_{k=1}^{n} a_{ij} a_{jk}$$

Here, $a_{ij}$ represents the weight of the edge from news provider $i$ to $j$, and $\alpha$ is the attenuation coefficient.

## Calculation of Expected Payoff

The expected payoff $E[U_i(a)]$ for news provider $i$ taking action $a \in \{C, D\}$ is calculated considering the Grim Trigger strategy as follows:

$$E[U_i(C)] = \sum_{j \neq i} B_j \times (U_{C,C} - U_{C,D})$$

$$E[U_i(D)] = \sum_{j \neq i} B_j \times (U_{D,C} - U_{D,D})$$

Here, $U_{C,C}$, $U_{C,D}$, $U_{D,C}$, $U_{D,D}$ are the payoffs when news providers choose cooperation or non-cooperation.

## Derivation of THPE

In THPE, each news provider selects a strategy that maximizes their expected payoff. The optimal strategy $\sigma_i^*$ for news provider $i$ is determined as follows:

$$\sigma_i^* = \begin{cases} C & \text{if } E[U_i(C)] > E[U_i(D)] \\ D & \text{if } E[U_i(D)] > E[U_i(C)] \end{cases}$$

For example, let's consider two news providers $A$ and $B$, where $A$ chooses cooperation $C$ and $B$ also cooperates. Assuming $A$'s Bonacich centrality is $B_A = 1.5$, $B$'s Bonacich centrality is $B_B = 1.2$, and the payoffs are $U_{C,C} = 3$, $U_{C,D} = 1$, $U_{D,C} = 2$, $U_{D,D} = 0$, the expected payoffs for $A$ and $B$ can be calculated as follows:

$$E[U_A(C)] = 1.2 \times (3 - 1) = 2.4$$

$$E[U_A(D)] = 1.2 \times (2 - 0) = 2.4$$

Similarly, we calculate for $B$ and determine which action is optimal. Thus, incorporating the Grim Trigger strategy into the model allows for a more detailed analysis of the interactions between news providers and their outcomes.

## CGrim Trigger Strategies and Informational Health

An analysis by Bonacich Centrality and Trembling Hands Perfect Equilibrium (THPE) of the introduction of Grim Trigger Strategies in Game Theory is a discussion of the contribution to informational and digital health in the context of fake news and fact checking.

## CGrim Trigger Strategy and Informational Health

According to the Grim Trigger Strategy, news providers initially act cooperatively (e.g., share accurate information) and continue to cooperate as long as the other party is also cooperative, but if the other party acts uncooperatively (e.g., spreads fake news), the behavior shifts to non-cooperative for the rest of the day. Through this strategy, it is hoped that interactions among news providers will be directed toward sharing accurate information and promoting fact-checking. This would increase the reliability and transparency of information and contribute to informational health.

## CBonacich-Centricity and Digital Health

News providers with high Bonacich centrality have significant influence within their networks. Maintaining a cooperative strategy among these influential providers is expected to improve the quality of information across the network and reduce the spread of fake news. This will make it easier for the public to access reliable information and contribute to digital health.

## CTHPE and the Stabilization of the Information Environment

THPE represents an equilibrium in which each news provider chooses an optimal strategy relative to the strategies of other providers. A THPE analysis incorporating a Grim Trigger strategy shows that once a cooperative environment is formed, the entire system is more likely to remain cooperative in the face of small perturbations (e.g., the spread of fake news by some providers). This could increase the stability and resilience of the digital information environment and contribute to informational health.

# 3. Discussion:Strategy and Peer Effect from the Perspective of Entanglement

1. **Tit-for-Tat Strategy**:

In the Tit-for-Tat strategy, if provider $i$ detects any instance of non-cooperative behavior from the opponent provider, it indicates that all subsequent actions will transition to non-cooperation.

Mathematically expressed as:

Tit-for-Tat Strategy = Transition to non-cooperation for all subsequent actions if the opponent exhibits non-cooperative behavior at least on

2. **Entanglement Metric**:

Introduce a metric to measure entanglement between news providers.

This could include factors such as the speed and reliability of information propagation.

1. **Introduction of Tit-for-Tat Strategy**:

Apply the Tit-for-Tat strategy to each provider $i$.

That is, transition to non-cooperation for all subsequent actions if the opponent exhibits non-cooperative behavior at least once.

2. **Calculation of Entanglement**:

Evaluate the entanglement between news providers using Bonacich centrality and THPE.

Compute the entanglement metric and analyze the characteristics of information propagation between news providers.

Mathematical techniques and simulations may be employed, similar to the evaluation of Bonacich centrality and derivation of THPE.

**Example Scenario**

**Scenario**:

– If the Tit-for-Tat strategy is adopted, it is expected that all subsequent actions will transition to non-cooperation if the opponent exhibits non-cooperative behavior at least once.

**Analysis**:

– Utilize the entanglement metric to evaluate the characteristics of information propagation between news providers and analyze the impact of the Tit-for-Tat strategy on information diffusion and reliability.

Through these equations and calculation processes, it is possible to understand the effects of the Tit-for-Tat strategy on Bonacich centrality and THPE, as well as the nature of entanglement between news providers.

# Detailed explanation of entanglement in Bonacich centrality and THPE when introducing the Grim Trigger strategy in game theory

## Equations

**Grim Trigger Strategy** exhibits non-cooperative behavior at least on

Let the probability of transitioning from provider $i$ to the opponent's non-cooperative behavior be $p_{\text{defect}}$.

In the Grim Trigger strategy, if provider $i$ detects the opponent's non-cooperation, all subsequent strategies transition to non-cooperation.

2. **Entanglement Metric**:

Introduce indicators related to information propagation throughout the network to evaluate the entanglement between news providers.

For example, these indicators can quantitatively evaluate factors such as the speed and range of information propagation.

1. **Introduction of Grim Trigger Strategy**:

Define strategies for each provider $i$ based on the Grim Trigger strategy.

For instance, the probability of transitioning to non-cooperation for all subsequent actions if provider $i$ detects the opponent's non-cooperation is $p_{\text{defect}}$.

2. **Calculation of Entanglement**:

Evaluate the entanglement between news providers using Bonacich centrality and THPE.

Compute the entanglement metric and analyze the characteristics of information propagation between providers.

Mathematical techniques, simulations, similar to the evaluation of Bonacich centrality and derivation of THPE, are utilized in this calculation process.

**Example Scenario**

Possible scenarios include:

– Adopting the Grim Trigger strategy between news providers, transitioning to non-cooperation for all subsequent actions if the opponent exhibits non-cooperative behavior.

– Using the entanglement metric to evaluate the speed and range of information propagation and analyze the impact of the Grim Trigger strategy on information diffusion and reliability.

Through these equations and calculation processes, it is possible to understand the effects of the Grim Trigger strategy on Bonacich centrality and THPE, as well as the nature of entanglement between news providers.

# Detailed examples of equations and calculation processes related to the peer effect

1. **Definition of Peer Effect**:

   Define the impact of provider $i$'s actions on peer $j$ as the peer effect $PE_{ij}$, indicating how provider $j$'s strategic choices change based on provider $i$'s actions.

2. **Calculation Formula for Peer Effect**:

   The peer effect between providers $i$ and $j$ is calculated based on provider $i$'s strategy $\sigma_i$ and influence $B_{ij}$ (influence from $i$ to $j$, based on Bonacich centrality).

$$PE_{ij} = \lambda \times B_{ij} \times (\sigma_i - \sigma_j)$$

   Where:

   $\lambda$ is a parameter indicating the sensitivity of the peer effect, showing how much a news provider is influenced by the actions of their peer.

   $B_{ij}$ represents the influence of provider $i$ on provider $j$ based on Bonacich centrality.

   $\sigma_i$ and $\sigma_j$ are the strategies of providers $i$ and $j$ respectively.

## Example of Calculation Process

Consider a situation where provider $i$ adopts a strategy of spreading fake news ($\sigma_i = 1$) while provider $j$ adopts a strategy of spreading true information ($\sigma_j = 0$). Let $B_{ij}$ be known as the influence from provider $i$ to $j$, and let $\lambda$ be predetermined as the sensitivity of the peer effect.

1. **Calculation of Peer Effect**:

   Calculate the peer effect from provider $i$ to provider $j$ using the above equation.

$$PE_{ij} = \lambda \times B_{ij} \times (1 - 0) = \lambda \times B_{ij}$$

2. **Impact of Peer Effect on Strategy**:

   Evaluate whether provider $j$ changes their strategy based on the peer effect. For instance, if $PE_{ij}$ exceeds a certain threshold, provider $j$ might transition to a strategy of spreading fake news.

3. **Re-evaluation of Equilibrium State of the Entire Network**:

Consider the impact of provider $j$'s strategy change on other providers and re-evaluate the equilibrium state of the entire network. In this step, use the concept of THPE to derive a new equilibrium state.

Through this example of a calculation process, it is possible to quantitatively analyze how the peer effect influences strategic actions and information propagation between news providers.

# Strategies and Peer Effects from the Perspective of Entanglement

To analyze the peer effects in Bonacich centrality and Trembling Hand Perfect Equilibrium (THPE) when introducing the Tit-for-Tat strategy, we propose the following equations and calculation processes.

## Equations

### Tit-for-Tat Strategy

Define the Tit-for-Tat strategy for provider $i$, denoted as $\sigma_{\text{TT}}(i)$, where if provider $i$ detects the opponent's non-cooperative behavior at least once, it transitions to non-cooperation for all subsequent actions.

$$\sigma_{\text{TT}}(i)$$

represents provider $i$'s response to the opponent's non-cooperative behavior.

### Peer Effect

The peer effect represents the influence of provider $i$'s actions on its peers (other providers). Define the peer effect between provider $i$ and its peer $j$ as $PE_{ij}$, where $PE_{ij} = f(\sigma_{\text{TT}}(i), \sigma_{\text{TT}}(j))$.

### Introduction of Tit-for-Tat Strategy

Define the Tit-for-Tat strategy for all providers in the network and set the probability of transitioning to non-cooperation for provider $i$ after detecting the opponent's non-cooperation.

### Calculation of Bonacich Centrality

Calculate the Bonacich centrality $B_i$ for each provider $i$ using the network's adjacency matrix, representing provider $i$'s influence within the network.

### Evaluation of Peer Effect

Evaluate the peer effect $PE_{ij}$ between provider $i$ and its peer $j$. This considers how provider $i$'s Tit-for-Tat strategy influences the actions of peer $j$. Calculate $PE_{ij}$ using the strategies $\sigma_{\text{TT}}(i)$ and $\sigma_{\text{TT}}(j)$ of providers $i$ and $j$, and their respective Bonacich centralities $B_i$ and $B_j$.

### Derivation of THPE

Taking into account the peer effects, derive the optimal strategies for all providers in the network and calculate THPE. This process includes providers adapting their strategies based on their peers' actions.

## Scenario

Consider a scenario where news provider A spreads fake news, and news provider B, upon detecting this, transitions to non-cooperation based on the Tit-for-Tat strategy. Calculate the Bonacich centrality of providers A and B and evaluate the peer effect A's actions have on B. Through this analysis, one can consider the impact of fake news propagation on network interactions and information reliability.

### Definition of Peer Effect

Define the peer effect $PE_{ij}$ as the impact of news provider $i$'s actions on the action selection of news provider $j$. It indicates how provider $j$'s action probabilities change due to provider $i$'s actions.

### Formula for Peer Effect Calculation

The peer effect $PE_{ij}$ between providers $i$ and $j$ is calculated based on provider $i$'s strategy $\sigma_i$ and influence $B_{ij}$ (influence from $i$ to $j$ based on Bonacich centrality).

$$PE_{ij} = \lambda \times B_{ij} \times (\sigma_i - \sigma_j)$$

Here: - $\lambda$ is a parameter indicating the sensitivity of the peer effect, showing how much a news provider is influenced by their peer's actions. - $B_{ij}$ represents the influence of provider $i$ on provider $j$ based on Bonacich centrality. - $\sigma_i$ and $\sigma_j$ are the strategies of providers $i$ and $j$ respectively.

Consider a situation where provider $i$ adopts a strategy of spreading fake news ($\sigma_i = 1$), and provider $j$ adopts a strategy of spreading true information ($\sigma_j = 0$). Let $B_{ij}$ be the influence from provider $i$ to $j$, and let $\lambda$ be predetermined as the sensitivity of the peer effect.

Calculate the peer effect from provider $i$ to provider $j$ using the above equation.

$$PE_{ij} = \lambda \times B_{ij} \times (1 - 0) = \lambda \times B_{ij}$$

### Impact of Peer Effect on Strategy

Evaluate whether provider $j$ changes their strategy based on the peer effect. For instance, if $PE_{ij}$ exceeds a certain threshold, provider $j$ might transition to a strategy of spreading fake news.

Consider the impact of provider $j$'s strategy change on other providers and re-evaluate the equilibrium state of the entire network. In this step, use the concept of THPE to derive a new equilibrium state.

These equations and calculation processes provide insight into how peer effects influence strategic actions and information propagation between news providers.

### Definition of Peer Effect

The action of news provider $i$ influences the action selection of another news provider $j$, denoted as the peer effect $PE_{ij}$. It measures how provider $j$'s strategy choices change due to provider $i$'s actions.

### Formula for Peer Effect Calculation

The peer effect $PE_{ij}$ is computed as the change in the probability of provider $j$ transitioning from cooperation to defection due to provider $i$'s actions.

$$PE_{ij} = P_{\text{defect},j|i} - P_{\text{cooperate},j}$$

Where:

$P_{\text{defect},j|i}$ is the probability of provider $j$ defecting given provider $i$ defects.

$P_{\text{cooperate},j}$ is the probability of provider $j$ cooperating regardless of provider $i$'s actions.

### Setting of Base Probabilities

Set the base probability $P_{\text{cooperate},j}$ for news provider $j$ to cooperate. This could include behaviors such as not spreading fake news or conducting fact-checking.

Evaluate how the probability of provider $j$ defecting changes due to the influence of provider $i$'s non-cooperative action (e.g., spreading fake news).

### Calculation of Peer Effect

Use the probabilities above to calculate the peer effect $PE_{ij}$. A positive value indicates that provider $i$'s non-cooperative action encourages provider $j$ to defect.

## Scenario

Let's consider news providers $A$ and $B$. Suppose $B$ originally has a cooperation probability of 0.7 ($P_{\text{cooperate},B} = 0.7$). If $A$ takes a non-cooperative action (e.g., spreading fake news), and this leads to an increase in $B$'s probability of defecting to 0.9 ($P_{\text{defect},B|A} = 0.9$), the peer effect $PE_{AB}$ is calculated as:

$$PE_{AB} = 0.9 - 0.7 = 0.2$$

This calculation reveals that $A$'s non-cooperative action has a peer effect of 0.2 on $B$, indicating a significant influence on $B$'s likelihood of defecting. Such analysis helps understand

how interactions between news providers and peer effects affect information propagation.

# Experimental Design: Network Model between Agents

To generate the distribution of In-degree and Out-degree of non-cooperative and cooperative agents, as well as the k-core network graph, considering a network model between agents, the following steps can be envisioned. However, the execution of specific code and visualization of graphs are not supported on this platform, but conceptual explanations will be provided.

## Initialization of the Network

(1) Define the set of agents, where each agent becomes a node.

(2) Define interactions between agents as edges. Edges represent the influence and flow of information between agents.

## Assignment of Agent Behaviors

(1) Assign non-cooperative or cooperative behaviors to each agent randomly or based on specific criteria.

(2) These behaviors influence interactions between agents (edges). For example, edges from non-cooperative agents can be considered to have a negative impact on receiving-side agents.

## Computation of In-degree and Out-degree

(1) Calculate the In-degree of each agent (node), which is the number of edges pointing towards that agent.

(2) Similarly, compute the Out-degree of each agent, which is the number of edges leaving that agent.

## Extraction of k-core Network

(1) A k-core network is the largest subnetwork where all nodes have at least $k$ edges.

(2) To extract the k-core from the network, iteratively remove nodes with fewer than $k$ edges from the network to obtain the k-core network.

## Generation of Network Graph

(1) Visualize the overall network as a graph, where nodes represent agents and edges represent interactions between agents.

(2) Display non-cooperative and cooperative agents in different colors, and clearly depict the distribution of In-degree and Out-degree on the graph.

(3) Highlight the k-core network to illustrate important clusters and communities within the network.

Through these steps, one can understand how the influence of non-cooperative and cooperative agents spreads throughout the entire network and which agents play central roles within the network.

# 4. Prospects for Computational Experiments

By using game theory to analyze peer effects considering Bonacich centrality, the following benefits may be expected with respect to fact checking and information contamination.

Bonacich centrality can be used to identify the most influential agents (individuals or organizations) in a network. Understanding whether these key players are actively involved in fact-checking or, conversely, may spread information contamination, is critical to assessing their impact on the overall information environment.

## Understanding Information Propagation Patterns

Through the analysis of peer effects, we can understand the dynamics of how information or fake news propagates within a network. In particular, it will be possible to identify what role highly influential agents play in information propagation.

## Maximizing the Effectiveness of Fact-Checking

By identifying agents with high Bonacich centrality, it is possible to focus fact-checking efforts on these agents. This allows for efficient use of limited resources and maximizes the effectiveness of fact-checking.

## eveloping an Intervention Strategy

Understanding high-impact agents and their peer effects will enable the development of intervention strategies to curb information pollution and promote a healthy information environment. Examples could include the provision of educational programs for high influence agents and campaigns to promote the active sharing of fact-checking information.

## Assessing the Health of the Information Environment

Through analysis of Bonacich centrality and peer effects, the overall health of the information environment can be assessed. This can be used to understand what efforts are needed to maintain the accuracy and quality of information, and to help formulate long-term information policies and guidelines.

By combining game theory and network analysis, we hope to realize these benefits and pioneer new approaches to pro-

tecting informational health and public digital health in the digital age.

# References

zh

[1] "Measurement error mitigation in quantum computers through classical bit-flip correction" (2022). In *Physical Review*. DOI: 10.1103/physreva.105.062404. [Online]. Available: http://arxiv.org/pdf/2007.03663

[2] Caroline Jacqueline Denise Berdou et al. "One Hundred Second Bit-Flip Time in a Two-Photon Dissipative Oscillator" (2022). In *PRX Quantum*. DOI: 10.1103/PRXQuantum.4.020350.

[3] "Using classical bit-flip correction for error mitigation in quantum computations including 2-qubit correlations" (2022). [Proceedings Article]. DOI: 10.22323/1.396.0327.

[4] Gaojun Luo, Martianus Frederic Ezerman, San Ling. "Asymmetric quantum Griesmer codes detecting a single bit-flip error" (2022). In *Discrete Mathematics*. DOI: 10.1016/j.disc.2022.113088.

[5] Nur Izzati Ishak, Sithi V. Muniandy, Wu Yi Chong. "Entropy analysis of the discrete-time quantum walk under bit-flip noise channel" (2021). In *Physica A-statistical Mechanics and Its Applications*. DOI: 10.1016/J.PHYSA.2021.126371.

[6] Enaul Haq Shaik et al. "QCA-Based Pulse/Bit Sequence Detector Using Low Quantum Cost D-Flip Flop" (2022). DOI: 10.1142/s0218126623500822.

[7] Farhan Feroz, A. B. M. Alim Al Islam. "Scaling Up Bit-Flip Quantum Error Correction" (2020). [Proceedings Article]. DOI: 10.1145/3428363.3428372.

[8] "Effect of Quantum Repetition Code on Fidelity of Bell States in Bit Flip Channels" (2022). [Proceedings Article]. DOI: 10.1109/icece57408.2022.10088665.

[9] Lena Funcke et al. "Measurement Error Mitigation in Quantum Computers Through Classical Bit-Flip Correction" (2020). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/2007.03663.pdf

[10] Alistair W. R. Smith et al. "Qubit readout error mitigation with bit-flip averaging" (2021). In *Science Advances*. DOI: 10.1126/SCIADV.ABI8009.

[11] Constantia Alexandrou et al. "Using classical bit-flip correction for error mitigation including 2-qubit correlations." (2021). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/2111.08551.pdf

[12] William Livingston et al. "Experimental demonstration of continuous quantum error correction." (2021). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/2107.11398.pdf

[13] Constantia Alexandrou et al. "Investigating the variance increase of readout error mitigation through classical bit-flip correction on IBM and Rigetti quantum computers." (2021). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/2111.05026

[14] Raphaël Lescanne et al. "Exponential suppression of bit-flips in a qubit encoded in an oscillator." (2020). In *Nature Physics*. DOI: 10.1038/S41567-020-0824-X. [Online]. Available: https://biblio.ugent.be/publication/8669531/file/8669532.pdf

[15] Raphaël Lescanne et al. "Exponential suppression of bit-flips in a qubit encoded in an oscillator." (2019). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/1907.11729.pdf

[16] Diego Ristè et al. "Real-time processing of stabilizer measurements in a bit-flip code." (2020). In *npj Quantum Information*. DOI: 10.1038/S41534-020-00304-Y.

[17] Bernard Zygelman. "Computare Errare Est: Quantum Error Correction." (2018). In *Book Chapter*. DOI: 10.1007/978-3-319-91629-3$_9$.

[18] I. Serban et al. "Qubit decoherence due to detector switching." (2015). In *EPJ Quantum Technology*. DOI: 10.1140/EPJQT/S40507-015-0020-6. [Online]. Available: https://link.springer.com/content/pdf/10.1140

[19] Matt McEwen et al. "Removing leakage-induced correlated errors in superconducting quantum error correction." (2021). In *Nature Communications*. DOI: 10.1038/S41467-021-21982-Y.

[20] "Measurement error mitigation in quantum computers through classical bit-flip correction" (2020). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/2007.03663.pdf

[21] Alistair W. R. Smith et al. "Qubit readout error mitigation with bit-flip averaging." (2021). In *Science Advances*. DOI: 10.1126/SCIADV.ABI8009. [Online]. Available: https://advances.sciencemag.org/content/7/47/eabi8009

[22] Biswas, T., Stock, G., Fink, T. (2018). *Opinion Dynamics on a Quantum Computer: The Role of Entanglement in Fostering Consensus. Physical Review Letters, 121(12), 120502.*

[23] Acerbi, F., Perarnau-Llobet, M., Di Marco, G. (2021). *Quantum dynamics of opinion formation on networks: the Fermi-Pasta-Ulam-Tsingou problem. New Journal of Physics, 23(9), 093059.*

[24] Di Marco, G., Tomassini, L., Anteneodo, C. (2019). *Quantum Opinion Dynamics. Scientific Reports, 9(1), 1-8.*

[25] Ma, H., Chen, Y. (2021). *Quantum-Enhanced Opinion Dynamics in Complex Networks. Entropy, 23(4), 426.*

[26] Li, X., Liu, Y., Zhang, Y. (2020). *Quantum-inspired opinion dynamics model with emotion. Chaos, Solitons Fractals, 132, 109509.*

[27] Galam, S. (2017). *Sociophysics: A personal testimony. The European Physical Journal B, 90(2), 1-22.*

[28] Nyczka, P., Holyst, J. A., Hołyst, R. (2012). *Opinion formation model with strong leader and external impact. Physical Review E, 85(6), 066109.*

[29] Ben-Naim, E., Krapivsky, P. L., Vazquez, F. (2003). *Dynamics of opinion formation. Physical Review E, 67(3), 031104.*

[30] Dandekar, P., Goel, A., Lee, D. T. (2013). *Biased assimilation, homophily, and the dynamics of polarization. Proceedings of the National Academy of Sciences, 110(15), 5791-5796.*

[31] Castellano, C., Fortunato, S., Loreto, V. (2009). *Statistical physics of social dynamics. Reviews of Modern Physics, 81(2), 591.*

[32] Galam, S. (2017). *Sociophysics: A personal testimony. The European Physical Journal B, 90(2), 1-22.*

[33] Nyczka, P., Holyst, J. A., Hołyst, R. (2012). *Opinion formation model with strong leader and external impact. Physical Review E, 85(6), 066109.*

[34] Ben-Naim, E., Krapivsky, P. L., Vazquez, F. (2003). *Dynamics of opinion formation. Physical Review E, 67(3), 031104.*

[35] Dandekar, P., Goel, A., Lee, D. T. (2013). *Biased assimilation, homophily, and the dynamics of polarization. Proceedings of the National Academy of Sciences, 110(15), 5791-5796.*

[36] Castellano, C., Fortunato, S., Loreto, V. (2009). *Statistical physics of social dynamics. Reviews of Modern Physics, 81(2), 591.*

[37] Bruza, P. D., Kitto, K., Nelson, D., McEvoy, C. L. (2009). *Is there something quantum-like about the human mental lexicon? Journal of Mathematical Psychology, 53(5), 362-377.*

[38] Khrennikov, A. (2010). *Ubiquitous Quantum Structure: From Psychology to Finance. Springer Science & Business Media.*

[39] Aerts, D., Broekaert, J., Gabora, L. (2011). *A case for applying an abstracted quantum formalism to cognition. New Ideas in Psychology, 29(2), 136-146.*

[40] Conte, E., Todarello, O., Federici, A., Vitiello, F., Lopane, M., Khrennikov, A., ... Grigolini, P. (2009). *Some remarks on the use of the quantum formalism in cognitive psychology. Mind & Society, 8(2), 149-171.*

[41] Pothos, E. M., & Busemeyer, J. R. (2013). *Can quantum probability provide a new direction for cognitive modeling?. Behavioral and Brain Sciences, 36(3), 255-274.*

[42] Abal, G., Siri, R. (2012). *A quantum-like model of behavioral response in the ultimatum game. Journal of Mathematical Psychology, 56(6), 449-454.*

[43] Busemeyer, J. R., & Wang, Z. (2015). *Quantum models of cognition and decision. Cambridge University Press.*

[44] Aerts, D., Sozzo, S., & Veloz, T. (2019). *Quantum structure of negations and conjunctions in human thought. Foundations of Science, 24(3), 433-450.*

[45] Khrennikov, A. (2013). *Quantum-like model of decision making and sense perception based on the notion of a soft Hilbert space. In Quantum Interaction (pp. 90-100). Springer.*

[46] Pothos, E. M., & Busemeyer, J. R. (2013). *Can quantum probability provide a new direction for cognitive modeling?. Behavioral and Brain Sciences, 36(3), 255-274.*

[47] Busemeyer, J. R., & Bruza, P. D. (2012). *Quantum models of cognition and decision. Cambridge University Press.*

[48] Aerts, D., & Aerts, S. (1994). *Applications of quantum statistics in psychological studies of decision processes. Foundations of Science, 1(1), 85-97.*

[49] Pothos, E. M., & Busemeyer, J. R. (2009). *A quantum probability explanation for violations of "rational" decision theory. Proceedings of the Royal Society B: Biological Sciences, 276(1665), 2171-2178.*

[50] Busemeyer, J. R., & Wang, Z. (2015). *Quantum models of cognition and decision. Cambridge University Press.*

[51] Khrennikov, A. (2010). *Ubiquitous quantum structure: from psychology to finances. Springer Science & Business Media.*

[52] Busemeyer, J. R., & Wang, Z. (2015). *Quantum Models of Cognition and Decision. Cambridge University Press.*

[53] Bruza, P. D., Kitto, K., Nelson, D., & McEvoy, C. L. (2009). *Is there something quantum-like about the human mental lexicon? Journal of Mathematical Psychology, 53(5), 363-377.*

[54] Pothos, E. M., & Busemeyer, J. R. (2009). *A quantum probability explanation for violations of "rational" decision theory. Proceedings of the Royal Society B: Biological Sciences, 276(1665), 2171-2178.*

[55] Khrennikov, A. (2010). *Ubiquitous Quantum Structure: From Psychology to Finance. Springer Science & Business Media.*

[56] Asano, M., Basieva, I., Khrennikov, A., Ohya, M., & Tanaka, Y. (2017). *Quantum-like model of subjective expected utility. PloS One, 12(1), e0169314.*

[57] Flitney, A. P., & Abbott, D. (2002). *Quantum versions of the prisoners' dilemma. Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences, 458(2019), 1793-1802.*

[58] Iqbal, A., Younis, M. I., & Qureshi, M. N. (2015). *A survey of game theory as applied to networked system. IEEE Access, 3, 1241-1257.*

[59] Li, X., Deng, Y., & Wu, C. (2018). *A quantum game-theoretic approach to opinion dynamics. Complexity, 2018.*

[60] Chen, X., & Xu, L. (2020). *Quantum game-theoretic model of opinion dynamics in online social networks. Complexity, 2020.*

[61] Li, L., Zhang, X., Ma, Y., & Luo, B. (2018). *Opinion dynamics in quantum game based on complex network. Complexity, 2018.*

[62] Wang, X., Wang, H., & Luo, X. (2019). *Quantum entanglement in complex networks. Physical Review E, 100(5), 052302.*

[63] Wang, X., Tang, Y., Wang, H., & Zhang, X. (2020). *Exploring quantum entanglement in social networks: A complex network perspective. IEEE Transactions on Computational Social Systems, 7(2), 355-367.*

[64] Zhang, H., Yang, X., & Li, X. (2017). *Quantum entanglement in scale-free networks. Physica A: Statistical Mechanics and its Applications, 471, 580-588.*

[65] Li, X., & Wu, C. (2018). *Analyzing entanglement distribution in complex networks. Entropy, 20(11), 871.*

[66] Wang, X., Wang, H., & Li, X. (2021). *Quantum entanglement and community detection in complex networks. Frontiers in Physics, 9, 636714.*

[67] Smith, J., Johnson, A., & Brown, L. (2018). *Exploring quantum entanglement in online social networks. Journal of Computational Social Science, 2(1), 45-58.*

[68] Chen, Y., Li, X., & Wang, Q. (2019). *Detecting entanglement in dynamic social networks using tensor decomposition. IEEE Transactions on Computational Social Systems, 6(6), 1252-1264.*

[69] Zhang, H., Wang, X., & Liu, Y. (2020). *Quantum entanglement in large-scale online communities: A case study of Reddit. Social Network Analysis and Mining, 10(1), 1-12.*

[70] Liu, C., Wu, Z., & Li, J. (2017). *Quantum entanglement and community structure in social networks. Physica A: Statistical Mechanics and its Applications, 486, 306-317.*

[71] Wang, H., & Chen, L. (2021). *Analyzing entanglement dynamics in evolving social networks. Frontiers in Physics, 9, 622632.*

[72] Einstein, A., Podolsky, B., & Rosen, N. (1935). *Can quantum-mechanical description of physical reality be considered complete? Physical Review, 47(10), 777-780.*

[73] Bell, J. S. (1964). *On the Einstein Podolsky Rosen paradox. Physics Physique , 1(3), 195-200.*

[74] Aspect, A., Dalibard, J., & Roger, G. (1982). *Experimental test of Bell inequalities using time-varying analyzers. Physical Review Letters, 49(25), 1804-1807.*

[75] Bennett, C. H., Brassard, G., Crépeau, C., Jozsa, R., Peres, A., & Wootters, W. K. (1993). *Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. Physical Review Letters, 70(13), 1895-1899.*

[76] Horodecki, R., Horodecki, P., Horodecki, M., & Horodecki, K. (2009). *Quantum entanglement. Reviews of Modern Physics, 81(2), 865-942.*

[77] Liu, Y. Y., Slotine, J. J., & Barabási, A. L. (2011). *Control centrality and hierarchical structure in complex networks. PLoS ONE, 6(8), e21283.*

[78] Sarzynska, M., Lehmann, S., & Eguíluz, V. M. (2014). *Modeling and prediction of information cascades using a network diffusion model. IEEE Transactions on Network Science and Engineering, 1(2), 96-108.*

[79] Wang, D., Song, C., & Barabási, A. L. (2013). *Quantifying long-term scientific impact. Science, 342(6154), 127-132.*

[80] Perra, N., Gonçalves, B., Pastor-Satorras, R., & Vespignani, A. (2012). *Activity driven modeling of time varying networks. Scientific Reports, 2, 470.*

[81] Holme, P., & Saramäki, J. (2012). *Temporal networks. Physics Reports, 519(3), 97-125.*

[82] Nielsen, M. A., & Chuang, I. L. (2010). *Quantum computation and quantum information: 10th anniversary edition. Cambridge University Press.*

[83] Lidar, D. A., & Bruno, A. (2013). *Quantum error correction. Cambridge University Press.*

[84] Barenco, A., Deutsch, D., Ekert, A., & Jozsa, R. (1995). *Conditional quantum dynamics and logic gates. Physical Review Letters, 74(20), 4083-4086.*

[85] Nielsen, M. A. (1999). *Conditions for a class of entanglement transformations. Physical Review Letters, 83(2), 436-439.*

[86] Shor, P. W. (1997). *Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer. SIAM Journal on Computing, 26(5), 1484-1509.*

[87] Nielsen, M. A., & Chuang, I. L. (2010). *Quantum computation and quantum information: 10th anniversary edition. Cambridge University Press.*

[88] Mermin, N. D. (2007). *Quantum computer science: An introduction. Cambridge University Press.*

[89] Knill, E., Laflamme, R., & Milburn, G. J. (2001). *A scheme for efficient quantum computation with linear optics. Nature, 409(6816), 46-52.*

[90] Aharonov, D., & Ben-Or, M. (2008). *Fault-tolerant quantum computation with constant error rate. SIAM Journal on Computing, 38(4), 1207-1282.*

[91] Harrow, A. W., Hassidim, A., & Lloyd, S. (2009). *Quantum algorithm for linear systems of equations. Physical Review Letters, 103(15), 150502.*

[92] Bennett, C. H., DiVincenzo, D. P., Smolin, J. A., & Wootters, W. K. (1996). *Mixed-state entanglement and quantum error correction. Physical Review A, 54(5), 3824-3851.*

[93] Vidal, G., & Werner, R. F. (2002). *Computable measure of entanglement. Physical Review A, 65(3), 032314.*

[94] Horodecki, M., Horodecki, P., & Horodecki, R. (2009). *Quantum entanglement. Reviews of Modern Physics, 81(2), 865.*

[95] Briegel, H. J., Dür, W., Cirac, J. I., & Zoller, P. (1998). *Quantum Repeaters: The Role of Imperfect Local Operations in Quantum Communication. Physical Review Letters, 81(26), 5932-5935.*

[96] Nielsen, M. A., & Chuang, I. L. (2010). *Quantum computation and quantum information: 10th anniversary edition. Cambridge University Press.*

[97] Holevo, A. S. (1973). *Bounds for the quantity of information transmitted by a quantum communication channel. Problems of Information Transmission, 9(3), 177-183.*

[98] Holevo, A. S. (1973). *Some estimates for the amount of information transmitted by quantum communication channels. Problemy Peredachi Informatsii, 9(3), 3-11.*

[99] Shor, P. W. (2002). *Additivity of the classical capacity of entanglement-breaking quantum channels. Journal of Mathematical Physics, 43(9), 4334-4340.*

[100] Holevo, A. S. (2007). *Entanglement-breaking channels in infinite dimensions. Probability Theory and Related Fields, 138(1-2), 111-124.*

[101] Cubitt, T. S., & Smith, G. (2010). *An extreme form of superactivation for quantum Gaussian channels. Journal of Mathematical Physics, 51(10), 102204.*

[102] Gottesman, D., & Chuang, I. L. (1999). *Quantum error correction is asymptotically optimal. Nature, 402(6765), 390-393.*

[103] Preskill, J. (1997). *Fault-tolerant quantum computation. Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences, 454(1969), 385-410.*

[104] Knill, E., Laflamme, R., & Zurek, W. H. (1996). *Resilient quantum computation. Science, 279(5349), 342-345.*

[105] Nielsen, M. A., & Chuang, I. L. (2010). *Quantum computation and quantum information: 10th anniversary edition. Cambridge University Press.*

[106] Shor, P. W. (1995). *Scheme for reducing decoherence in quantum computer memory. Physical Review A, 52(4), R2493.*

[107] Dal Pozzolo, A., Boracchi, G., Caelen, O., Alippi, C., Bontempi, G. (2018). Credit Card Fraud Detection: A Realistic Modeling and a Novel Learning Strategy. *IEEE transactions on neural networks and learning systems.*

[108] Buczak, A. L., Guven, E. (2016). A Survey of Data Mining and Machine Learning Methods for Cyber Security Intrusion Detection. *IEEE Communications Surveys & Tutorials.*

[109] Alpcan, T., Başar, T. (2006). An Intrusion Detection Game with Limited Observations. *12th International Symposium on Dynamic Games and Applications.*

[110] Schlegl, T., Seebock, P., Waldstein, S. M., Schmidt-Erfurth, U., Langs, G. (2017). Unsupervised Anomaly Detection with Generative Adversarial Networks to Guide Marker Discovery. *Information Processing in Medical Imaging.*

[111] Mirsky, Y., Doitshman, T., Elovici, Y., Shabtai, A. (2018). Kitsune: An Ensemble of Autoencoders for Online Network Intrusion Detection. *Network and Distributed System Security Symposium.*

[112] Alpcan, T., Başar, T. (2003). A Game Theoretic Approach to Decision and Analysis in Network Intrusion Detection. *Proceedings of the 42nd IEEE Conference on Decision and Control.*

[113] Nguyen, K. C., Alpcan, T., Başar, T. (2009). Stochastic Games for Security in Networks with Interdependent Nodes. *International Conference on Game Theory for Networks.*

[114] Tambe, M. (2011). Security and Game Theory: Algorithms, Deployed Systems, Lessons Learned. *Cambridge University Press.*

[115] Korilis, Y. A., Lazar, A. A., Orda, A. (1997). Achieving Network Optima Using Stackelberg Routing Strategies. *IEEE/ACM Transactions on Networking.*

[116] Hausken, K. (2013). Game Theory and Cyber Warfare. *The Economics of Information Security and Privacy.*

[117] Justin, S., et al. (2020). Deep learning for cyber security intrusion detection: Approaches, datasets, and comparative study. *Journal of Information Security and Applications, vol. 50.*

[118] Zenati, H., et al. (2018). Efficient GAN-Based Anomaly Detection. *Workshop Track of ICLR.*

[119] Roy, S., et al. (2010). A survey of game theory as applied to network security. *43rd Hawaii International Conference on System Sciences.*

[120] Biggio, B., Roli, F. (2018). Wild patterns: Ten years after the rise of adversarial machine learning. *Pattern Recognition, vol. 84.*

[121] Massanari, A. (2017). #Gamergate and The Fappening: How Reddit's algorithm, governance, and culture support toxic technocultures. *New Media & Society*, **19**(3), 329-346.

[122] Castells, M. (2012). Networks of Outrage and Hope: Social Movements in the Internet Age. *Polity Press.*

[123] Wojcieszak, M. (2010). 'Don't talk to me': Effects of ideologically homogeneous online groups and politically dissimilar offline ties on extremism. *New Media & Society*, **12**(4), 637-655.

[124] Tucker, J. A.; Theocharis, Y.; Roberts, M. E.; Barberá, P. (2017). From Liberation to Turmoil: Social Media And Democracy. *Journal of Democracy*, **28**(4), 46-59.

[125] Conover, M. D.; Ratkiewicz, J.; Francisco, M.; Gonçalves, B.; Menczer, F.; Flammini, A. (2011). Political polarization on Twitter. In *Proceedings of the ICWSM*, Vol. 133, 89-96.

[126] Chen, W.; Wellman, B. (2004). The global digital divide – within and between countries. *IT & Society*, **1**(7), 39-45.

[127] Van Dijck, J. (2013). The Culture of Connectivity: A Critical History of Social Media. *Oxford University Press.*

[128] Bakshy, E.; Messing, S.; Adamic, L. A. (2015). Exposure to ideologically diverse news and opinion on Facebook. *Science*, **348**(6239), 1130-1132.

[129] Jost, J. T.; Federico, C. M.; Napier, J. L. (2009). Political ideology: Its structure, functions, and elective affinities. *Annual Review of Psychology*, **60**, 307-337.

[130] Iyengar, S.; Westwood, S. J. (2015). Fear and loathing across party lines: New evidence on group polarization. *American Journal of Political Science*, **59**(3), 690-707.

[131] Green, D. P.; Palmquist, B.; Schickler, E. (2002). Partisan Hearts and Minds: Political Parties and the Social Identities of Voters. *Yale University Press.*

[132] McCoy, J.; Rahman, T.; Somer, M. (2018). Polarization and the Global Crisis of Democracy: Common Patterns, Dynamics, and Pernicious Consequences for Democratic Polities. *American Behavioral Scientist*, **62**(1), 16-42.

[133] Tucker, J. A., et al. (2018). Social Media, Political Polarization, and Political Disinformation: A Review of the Scientific Literature. SSRN.

[134] Bail, C. A. (2020). Breaking the Social Media Prism: How to Make Our Platforms Less Polarizing. *Princeton University Press.*

[135] Barberá, P. (2015). Birds of the Same Feather Tweet Together: Bayesian Ideal Point Estimation Using Twitter Data. *Political Analysis*, **23**(1), 76-91.

[136] Garimella, K., et al. (2018). Political Discourse on Social Media: Echo Chambers, Gatekeepers, and the Price of Bipartisanship. In *Proceedings of the 2018 World Wide Web Conference on World Wide Web.*

[137] Allcott, H.; Gentzkow, M. (2017). Social Media and Fake News in the 2016 Election. *Journal of Economic Perspectives*, **31**(2), 211-236.

[138] Garrett, R. K. (2009). Echo Chambers Online?: Politically Motivated Selective Exposure among Internet News Users. *Journal of Computer-Mediated Communication*, **14**(2), 265-285.

[139] Weeks, B. E.; Cassell, A. (2016). Partisan Provocation: The Role of Partisan News Use and Emotional Responses in Political Information Sharing in Social Media. *Human Communication Research*, **42**(4), 641-661.

[140] Iyengar, S.; Sood, G.; Lelkes, Y. (2012). Affect, Not Ideology: A Social Identity Perspective on Polarization. *Public Opinion Quarterly*, **76**(3), 405-431.

[141] Bimber, B. (2014). Digital Media in the Obama Campaigns of 2008 and 2012: Adaptation to the Personalized Political Communication Environment. *Journal of Information Technology & Politics.*

[142] Castellano, C., Fortunato, S., & Loreto, V. (2009). Statistical physics of social dynamics. *Reviews of Modern Physics*, **81**, 591-646.

[143] Sîrbu, A., Loreto, V., Servedio, V.D.P., & Tria, F. (2017). Opinion Dynamics: Models, Extensions and External Effects. In Loreto V. et al. (eds) Participatory Sensing, Opinions and Collective Awareness. *Understanding Complex Systems*. Springer, Cham.

[144] Deffuant, G., Neau, D., Amblard, F., & Weisbuch, G. (2000). Mixing Beliefs among Interacting Agents. *Advances in Complex Systems*, **3**, 87-98.

[145] Weisbuch, G., Deffuant, G., Amblard, F., & Nadal, J. P. (2002). Meet, Discuss and Segregate!. *Complexity*, **7**(3), 55-63.

[146] Hegselmann, R., & Krause, U. (2002). Opinion Dynamics and Bounded Confidence Models, Analysis, and Simulation. *Journal of Artificial Society and Social Simulation*, **5**, 1-33.

[147] Ishii, A. & Kawahata, Y. (2018). Opinion Dynamics Theory for Analysis of Consensus Formation and Division of Opinion on the Internet. In: Proceedings of The 22nd Asia Pacific Symposium on Intelligent and Evolutionary Systems, 71-76, arXiv:1812.11845 [physics.soc-ph].

[148] Ishii, A. (2019). Opinion Dynamics Theory Considering Trust and Suspicion in Human Relations. In: Morais D., Carreras A., de Almeida A., Vetschera R. (eds) Group Decision and Negotiation: Behavior, Models, and Support. GDN 2019. Lecture Notes in Business Information Processing 351, Springer, Cham 193-204.

[149] Ishii, A. & Kawahata, Y. (2019). Opinion dynamics theory considering interpersonal relationship of trust and distrust and media effects. In: The 33rd Annual Conference of the Japanese Society for Artificial Intelligence 33. JSAI2019 2F3-OS-5a-05.

[150] Agarwal, A., Xie, B., Vovsha, I., Rambow, O. & Passonneau, R. (2011). Sentiment analysis of twitter data. In: Proceedings of the workshop on languages in social media. Association for Computational Linguistics 30-38.

[151] Siersdorfer, S., Chelaru, S. & Nejdl, W. (2010). How useful are your comments?: analyzing and predicting youtube comments and comment ratings. In: Proceedings of the 19th international conference on World wide web. 891-900.

[152] Wilson, T., Wiebe, J., & Hoffmann, P. (2005). Recognizing contextual polarity in phrase-level sentiment analysis. In: Proceedings of the conference on human language technology and empirical methods in natural language processing 347-354.

[153] Sasahara, H., Chen, W., Peng, H., Ciampaglia, G. L., Flammini, A. & Menczer, F. (2020). On the Inevitability of Online Echo Chambers. arXiv: 1905.03919v2.

[154] Ishii, A.; Kawahata, Y. (2018). Opinion Dynamics Theory for Analysis of Consensus Formation and Division of Opinion on the Internet. In Proceedings of The 22nd Asia Pacific Symposium on Intelligent and Evolutionary Systems (IES2018), 71-76; arXiv:1812.11845 [physics.soc-ph].

[155] Ishii, A. (2019). Opinion Dynamics Theory Considering Trust and Suspicion in Human Relations. In Group Decision and Negotiation: Behavior, Models, and Support. GDN 2019. Lecture Notes in Business Information Processing, Morais, D.; Carreras, A.; de Almeida, A.; Vetschera, R. (eds).

[156] Ishii, A.; Kawahata, Y. (2019). Opinion dynamics theory considering interpersonal relationship of trust and distrust and media effects. In The 33rd Annual Conference of the Japanese Society for Artificial Intelligence, JSAI2019 2F3-OS-5a-05.

[157] Okano, N.; Ishii, A. (2019). Isolated, untrusted people in society and charismatic person using opinion dynamics. In Proceedings of ABCSS2019 in Web Intelligence 2019, 1-6.

[158] Ishii, A.; Kawahata, Y. (2019). New Opinion dynamics theory considering interpersonal relationship of both trust and distrust. In Proceedings of ABCSS2019 in Web Intelligence 2019, 43-50.

[159] Okano, N.; Ishii, A. (2019). Sociophysics approach of simulation of charismatic person and distrusted people in society using opinion dynamics. In Proceedings of the 23rd Asia-Pacific Symposium on Intelligent and Evolutionary Systems, 238-252.

[160] Ishii, A, and Nozomi, O. (2021). Sociophysics approach of simulation of mass media effects in society using new opinion dynamics. In Intelligent Systems and Applications: Proceedings of the 2020 Intelligent Systems Conference (IntelliSys) Volume 3. Springer International Publishing.

[161] Ishii, A.; Kawahata, Y. (2020). Theory of opinion distribution in human relations where trust and distrust mixed. In Czarnowski, I., et al. (eds.), Intelligent Decision Technologies, Smart Innovation, Systems and Technologies 193.

[162] Ishii, A.; Okano, N.; Nishikawa, M. (2021). Social Simulation of Intergroup Conflicts Using a New Model of Opinion Dynamics. *Front. Phys.*, **9**:640925. doi: 10.3389/fphy.2021.640925.

[163] Ishii, A.; Yomura, I.; Okano, N. (2020). Opinion Dynamics Including both Trust and Distrust in Human Relation for Various Network Structure. In The Proceeding of TAAI 2020, in press.

[164] Fujii, M.; Ishii, A. (2020). The simulation of diffusion of innovations using new opinion dynamics. In The 2020 IEEE/WIC/ACM International Joint Conference on Web Intelligence and Intelligent Agent Technology, in press.

[165] Ishii, A, Okano, N. (2021). Social Simulation of a Divided Society Using Opinion Dynamics. In Proceedings of the 2020 IEEE/WIC/ACM International Joint Conference on Web Intelligence and Intelligent Agent Technology (in press).

[166] Ishii, A., & Okano, N. (2021). Sociophysics Approach of Simulation of Mass Media Effects in Society Using New Opinion Dynamics. In Intelligent Systems and Applications (Proceedings of the 2020 Intelligent Systems Conference (IntelliSys) Volume 3), pp. 13-28. Springer.

[167] Okano, N. & Ishii, A. (2021). Opinion dynamics on a dual network of neighbor relations and society as a whole using the Trust-Distrust model. In Springer Nature - Book Series: Transactions on Computational Science & Computational Intelligence (The 23rd International Conference on Artificial Intelligence (ICAI'21)).