Discussion Paper:(1)Case Study of Decision Making and Urban Planning:Loop Flip Errors and Phase Transitions in Local Potential Approximation in Non-trivial Loops(2024), (2)Case Study of Spatio-Temporal Approaches To Syndrome Errors in Decision Making with Applications Such as Bit Phase Flip Errors and Methods Discussion(2024)

# Note:The Effects of Filter Bubbles and Cocktail Party Effects on the Diffusion and Valuation of Perceived Errors and Reliability Dynamics in Information Maximizing Expected in Sealed-Bid Auctions Non-Perfect Information Games: From First-Price to Second-Price Models

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**Abstract:** This paper summarizes the discussion of a hypothetical computational experiment.We propose a hypothetical model to simulate the effects of filter bubbles and cocktail party effects on the trading of misinformation in the context of information auctions. In particular, we focus on situations in which inappropriate information is traded at high prices and the resulting social and economic consequences. The model hypothesizes how the quality of information and its valuation can be distorted by filter bubbles and how the cocktail party effect affects participants' decisions. Furthermore, we apply the Meek method to redistribute bids based on misinformation and explore the feasibility of healthier information markets. The model provides new insights into information valuation and reallocation mechanisms, and provides insights into the impact of filter bubbles and cocktail party effects on information markets. We develop a theoretical model that analyzes the optimal bidding strategy employed by bidders to maximize their expected utility, taking into account the private valuation of information and the probability of winning. In the first-price auction model, bidders adjust their bids by estimating the behavior of their competitors. In the second price auction, on the other hand, the dominant strategy emerges as bidding one's true valuation, unaffected by the bidding behavior of others. The existence of filter bubbles adds further complexity because bidders' valuations can be influenced by biased information flows, leading to suboptimal bidding and market inefficiencies. Our findings consider the disparities among auction types dealing with information asymmetries and the pivotal role of market design in facilitating fair and efficient information transactions.

**Keywords:** Incomplete Information Games, Defense Counsel Matching Games, Shielded Bid Auctions, Equivalent Bidding Strategies,Repetitive Dilemma Game, First-Price Auction, Second-Price Auction, Game Theory, Nash Equilibrium, Information Markets, Filter Bubbles, Bidding Strategies, Expected Utility, Uncertainty, Private Value, Stochastic Approach, External Information Operations, Time-Varying

## 1. Introduction

We apply incomplete information game theory to explore the strategic behavior of bidders in the context of shielded bid auctions.In particular, we focus on the mechanism by which bidders determine their bids based on their private value and the resulting derivation of Nash equilibria.Bidders' gains are defined by the difference between the valuation and the winning bid price for an item, and through an equivalent bidding strategy, each bidder determines the bid amount that maximizes its own gains.

In this process, we examine the effects of external infor-

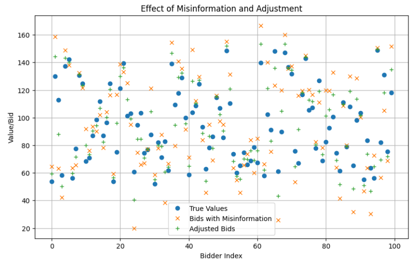


Fig. 1: Effect of Misinformation and Adjustment

mation manipulation and changes in valuations over time on bidders' beliefs and strategies. Nash equilibrium calculations are used to analyze the stable state reached by the market when bidders act on the basis of each other's optimal strategies. The aim of this study is to improve our understanding of uncertainty in auction theory and to support decision making in actual auction design and policy making. This text focuses on defense matching games and second-order information auction games in information markets. In particular, it uses a probabilistic approach to analyze market instability and dynamics in perfect and non-perfect information scenarios. The text describes defense matching strategies, the value and cost of information, and ways to deal with market instability. It also discusses auction game theory and how the reliability of information affects market competition and the value of information.

We also provide a theoretical analysis of the defense matching game and the second-order information auction game in the information market. In particular, we use a stochastic approach to model and analyze instability and market dynamics in perfect and non-perfect information scenarios. In the defense matching game, we will examine how information reliability is valued and affects market dynamics through the interaction between defense counsel and the information market. We will also analyze the impact of information credibility and its increasing value on intra-market competition in auction games. The study will explore the impact of Bonacich centrality, trembling hand perfect equilibrium (THPE), entanglement, and peer effects on market participants' strategies and their contribution to knowledge sharing and trust building in information markets. The results of this study will also add insights into understanding the impact of information uncertainty on markets and promoting efficient information sharing and market stability.

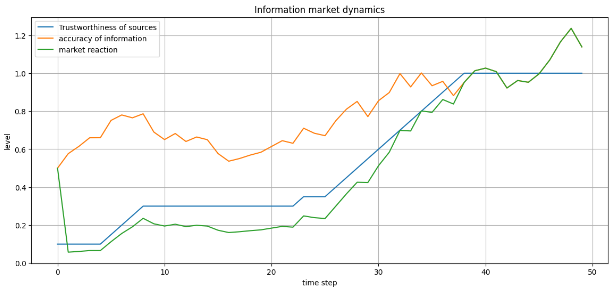


Fig. 2: 'Information market dynamics

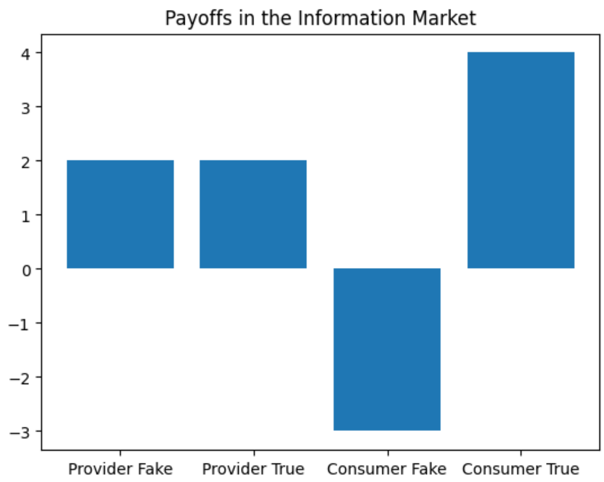


Fig. 3: Payoffs in the Information Market

## 2. The Effects of Filter Bubbles and Cocktail Party Effects on the Diffusion and Valuation of Misinformation in Information Auctions

We also examine the filter bubble and cocktail party effect in information auctions with respect to an analysis to simulate the impact of the filter bubble on the diffusion of misinformation and the evaluation of value. The filter bubble refers to a condition in which information is filtered based on personal interests and beliefs, resulting in increased exposure to specific information and perspectives. The cocktail party effect, on the other hand, refers to the ability to direct attention to specific information or voices among a lot of noise, and in this context refers to the phenomenon of paying attention to specific information or trends in a noisy information environment and making behavioral decisions based on them.

## 2.1 Filter Bubble and Cocktail Party Effect

This analytical approach analyzes how bidders' individual valuations of information are affected by filter bubbles and how the cocktail party effect affects bidders' bidding strategies amidst the noise and chaos during auctions. In particular, we examine how the quality of information and its soundness affect bidders' valuations and bids, and we express the impact of filter bubbles and cocktail party effects on information valuations and bidding strategies through a mathematical model.

The concept of a Nash equilibrium in a shielded bid auction is defined as each bidder predicts the strategies of other bidders and chooses his optimal strategy based on them. In an auction model in a non-complete information game, the bidder must take uncertainty into account. This includes uncertainty about the valuations of other bidders and changes in valuations due to external manipulation of information. Bidders adopt strategies to maximize their own expected gains in light of this uncertainty, reaching Nash equilibrium in the process. By incorporating the effects of time variation and external information into the model, the dynamics of real-world auctions and information markets can be captured more accurately. Factors such as filter bubbles and information manipulation affect bidders' beliefs and valuations, and strategies that take these into account are necessary.

We also analyzed theoretically the impact of temporal variation and associated cognitive errors in information markets on information choices and decisions. In particular, we explored the effects of the introduction of the Grim Trigger Strategy, the Tight Trigger Strategy, and the Trembling Hand Perfect Equilibrium (THPE) on market dynamics and how Bonacich centrality and peer effects can be enhanced or suppressed by these strategies. The analysis showed that information choices and decisions under uncertainty caused by cognitive errors and misplaced "shortest cost paths" can have serious consequences for the health and efficiency of information markets. While the Grim Trigger and Tight Trigger strategies provide incentives for information providers to focus on accuracy and build long-term trusting relationships, the THPE concept emphasizes the importance of allowing for small unintentional errors. The study also examined the magnitude of the impact of the behavior of highly Bonacich-centric players on the overall market and the role that peer effects play in the acceptance and diffusion of information.

In filter bubbles, the phenomenon in which information is customized based on an individual's past behavior and preferences. This effect makes bidders more exposed to information that is consistent with their beliefs and interests, and reduces their exposure to diverse and opposing information. The cocktail party effect is the ability to focus on specific voices or information in a noisy environment. In the context of an auction, it indicates that a bidder may overreact to the actions of other bidders or specific information and deviate from his or her original valuation. In the expected gain intended here, the profit a bidder expects to gain from bidding. The model analyzes how expected gains vary with information quality, filter bubbles, and cocktail party effects.

This study provides a theoretical framework for understanding the impact of filter bubbles and cocktail party effects in the context of information auctions and provides insight into how misinformation may affect valuations and bidding strategies. It also discusses the impact of information soundness on market dynamics. We analyze a bidder's strategy in a shielded bid auction by applying non-perfect information game theory in the context of first-price and second-price auctions. In first-price auctions, the bidder must guess the behavior of other bidders and adjust his bid to maximize his expected gain. In the second-price auction, on the other hand, the optimal strategy is for the bidders to bid their own true valuation. The document also mentions that information distortions caused by filter bubbles can affect the bidders' valuations, which can lead to market inefficiencies. The document also describes the process by which bidders update their own beliefs based on auction results and calculate their expected gain in the next auction. It also discusses how the Meek method can be applied to information auction games to optimize the distribution of correct information.

In first-price and second-price auctions, these auction formats require different approaches to the bidder's strategy. In first-price auctions, bidders typically use shading (the strategy of bidding less than one's true valuation), whereas in second-price auctions there is an incentive to bid the true valuation The Meek method is a redistribution method usually used in the context of elections, with an excess number of votes redistributes votes from candidates to runners-up, leading to fairer results. This document applies this concept to the context of information auctions to explore the proper allocation of information. Through these analyses, we can understand the strategic behavior of the bidders in auction games under non-perfect information and its impact on the market. We also discuss the impact of filter bubbles on bidders' valuations and how market efficiency can be improved through the proper allocation of information.

In this thesis, we consider four initial hypotheses concerning information commerce. In particular, we delve into the scenarios of misperceptions over time and the spread of misinformation within the market.

(1) A dilemma where a market with high reliability loses trust as a result of spreading misinformation.

(2) A case where a market considered unreliable actually has a high rate of correct information and gradually gains trust from the market.

(3) A scenario where one is swayed by incorrect information midway, leading to a dilemma.

(4) A situation where one is intentionally swayed by incorrect information midway and is incited by others to enter a dilemma state.

This scenario poses a highly complex problem within the context of information markets and filter bubbles. The process of determining the truthfulness of information may change over time or based on the reliability of specific sources. Below, we present theoretical explanations and considerations based on the proposed scenarios.

**(1) The Dilemma of a Reliable Market Losing Trust**

If a reliable source spreads incorrect information, it may lose its reliability, potentially damaging the overall credibility of the market. This dilemma highlights the difficulty for information providers in balancing accuracy with promptness.

*Consideration Points*

Verifying information requires time and resources, and especially for urgent information, the verification process may not keep up.

Once a source loses trust, it is difficult to recover, potentially having a long-term impact on consumers' information selection biases.

**(2) An Unreliable Market Gaining Trust**

A market considered unreliable may gradually gain trust by continuously providing accurate information over time. This process can overcome consumers' confirmation biases and prompt a reevaluation of the information source.

*Consideration Points*

Information markets are dynamic, and should be evaluated based on the current quality of information provision, not just past reputation.

Trust can be fostered by increasing transparency from information providers and acknowledging and correcting mistakes.

**(3) Being Swayed by Incorrect Information**

A market or individual may make decisions based on incorrect information midway, leading to a dilemma. This scenario illustrates how the initial acceptance of information can affect subsequent perceptions and decision-making.

*Consideration Points*

The initial acceptance of misinformation can inhibit the acceptance of accurate information in later stages.

Acceptance or rejection of information is often based on individuals' existing beliefs and values, potentially exacerbating the dilemma.

**(4) Being Intentionally Swayed by Incorrect Information**

There may be situations where incorrect information is intentionally spread by specific actors, leading markets or individuals into a dilemma. Such scenarios are often seen as strategies in information warfare or propaganda.

*Consideration Points*

Information manipulation can be used to advance specific agendas, threatening the health of information markets.

It's crucial for consumers and market participants to consider the motivations and backgrounds of information sources, making critical thinking indispensable.

*Additional Consideration Ideas*

Information Overload: A situation where an abundance of information makes it difficult for consumers to judge, facilitating the spread of misinformation.

Echo Chambers: A situation where only similar opinions or information circulate, reinforcing incorrect information by excluding different perspectives.

Lack of Digital Literacy: When consumers lack the skills to judge the veracity of information, they are more susceptible to being influenced by misinformation.

In each scenario, accurately assessing the truthfulness of information and maintaining a healthy information market requires the education and literacy improvement of both information providers and consumers, strengthening the information verification process, and access to diverse information sources.

By considering misperceptions and recognition errors over time as "mistakes in the shortest cost path" and applying this to the repeated prisoner's dilemma model, interesting insights can be gained, especially in situations where random occurrences or certain groups continue to overestimate that information as fact.

**Mistakes in the Shortest Cost Path**

The "shortest cost path" refers to the perceived path of least effort or cost when acquiring information. Mistakes in this path can lead to the easy spread of incorrect information or misperceptions. These misrecognitions can occur randomly or may be maintained intentionally or unconsciously by specific groups.

**The Model of Repeated Prisoner's Dilemma**

The repeated prisoner's dilemma is a model in game theory where participants make the same choices repeatedly. Each participant chooses between cooperation (sharing the truth) or betrayal (spreading misinformation), with rewards based on their choices. In this model, betrayal may provide benefits in the short term, but cooperation leads to mutual benefits in the long term.

**Scenario Analysis**

(1) Random Recognition Errors: Random recognition errors can cause temporary information confusion, potentially negatively impacting individual decision-making.

(2) Overconfidence by Specific Groups: Overconfidence in certain information by specific groups can promote the spread of misinformation within the group, giving betrayal an advantage over cooperation.

In conclusion, mistakes in the "shortest cost path" that lead to recognition errors can be considered within the context of the repeated prisoner's dilemma in information markets. Both random errors and overconfidence can promote betrayal actions (spread of misinformation) in the short term, but there are incentives to shift to cooperative actions (sharing of truth) in the long term. This dynamic change offers important insights into maintaining the health and efficiency of information markets.

The analysis of recognition errors and mistakes in the "shortest cost path" using submodular and Metzler functions in the context of repeated prisoner's dilemma models allows for an exploration of changes in information ownership rates and market interactions.

**Application of Submodular Functions**

Submodular functions are used to model the increase in utility when new information is added to the information set. In the information market, they are suitable for assessing how the addition of specific information (true or fake news) changes the utility of the market.

*Examples of Consideration*

Random Misrecognition: In the case of random misrecognition, submodular functions make the impact of information addition on market utility irregular. The addition of incorrect information to the market might temporarily increase utility, but it could harm the market's health in the long term.

Misrecognition Based on Overconfidence: If a group overestimates certain information as factual, their strategy of adding information might consistently have a positive or negative impact on market utility. Using submodular functions, we can evaluate how the addition of information based on overconfidence affects the market.

**Application of Metzler Functions**

Metzler functions are used to model the impact of information dissemination on the interactions among market participants. These functions allow for the analysis of how the spread of new information affects the relationships with other information and the overall market interactions.

*Examples of Consideration*

Changes in Interaction Due to Random Misrecognition: Random misrecognition can destabilize interactions among market participants. Using Metzler functions, we can analyze how random information dissemination affects market interactions. Increased uncertainty may reduce market efficiency.

Changes in Interaction Due to Overconfidence: If a group's overconfidence in certain information makes it easier for that information to spread throughout the market, Metzler functions can be used to assess the long-term impact of such information dissemination on market interactions. Information dissemination based on overconfidence could either improve market efficiency or harm market health.

**Conclusion**

Analyzing changes in information ownership rates and market interactions due to recognition errors and mistakes in the "shortest cost path" using submodular and Metzler functions provides insights into information market dynamics. Random misrecognition and misrecognition based on overconfidence affect the market differently, and these impacts are significant for the health and efficiency of information markets.

Considering misperceptions and recognition errors over time as "mistakes in the shortest cost path" within the framework of repeated prisoner's dilemma models and auction games is a valuable approach to understanding the dynamics in information markets.

**Misperceptions and Recognition Errors in Repeated Prisoner's Dilemma**

In a repeated prisoner's dilemma, players (information providers or recipients) choose between cooperation (providing or accepting accurate information) or betrayal (providing fake news or crediting it) in each round. As time progresses, players adjust their strategies based on the past actions of others, pursuing optimal response strategies. However, when misperceptions and recognition errors arise due to "mistakes in the shortest cost path," players may fail to accurately assess the intentions or reliability of others, leading to suboptimal strategies that were believed to be optimal.

**Information Reliability and Value in Auction Games**

In the context of auction games, the higher the reliability of information, the more valuable it is considered. Players compete to acquire high-reliability information, prepared to pay prices commensurate with the value of the information. This process leads to an increase in demand for reliable information in the information market, resulting in an auction game state.

**Impact of Recognition Errors on Auction Games**

When recognition errors occur, players may misjudge the true value of information, leading to overbidding due to overvaluation or missing valuable information due to undervaluation. Especially when "mistakes in the shortest cost path" are present, players risk paying a higher price than the actual value due to underestimation of information acquisition costs. Additionally, if a group overestimates certain information as factual, excessive demand for that information can arise, leading to market price distortions.

**Theoretical Considerations**

- Information Asymmetry: Misperceptions and "mistakes in the shortest cost path" lead to information asymmetry. Differences in the quality and quantity of information available between information providers and receivers can result in inaccurate information being traded as valuable.
- Market Instability: Actions based on recognition errors can cause market instability, leading to short-term price fluctuations and potential long-term declines in market credibility.
- Self-Reinforcing Cycle: Overestimation of certain information by the market can lead to a self-reinforcing cycle, where other players also overvalue the information, potentially leading to bubble formation or collapse.

Recognition errors and "mistakes in the shortest cost path" over time complicate the dynamics of repeated prisoner's dilemma and auction games in information markets, affecting players' strategies, market stability, and information value assessment mechanisms, with significant implications for overall market efficiency and health.

The introduction of the grim trigger strategy in an information market, where players' choices and judgments fluctuate over time, leading to uncertainty, significantly impacts the interactions among players. The grim trigger strategy entails that if a player ever takes a betraying action, they will continue to betray indefinitely. This strategy shifts players' focus from pursuing short-term gains to maintaining long-term cooperative relationships. Incorporating Bonacich centrality, Trembling Hand Perfect Equilibrium (THPE), entanglement, and peer effects into this context provides further insights.

**Grim Trigger Strategy and the Information Market**

The implementation of a grim trigger strategy implies severe consequences for players in the information market who disseminate incorrect information even once, leading to a permanent loss of trust from others. Consequently, players become more cautious in assessing the accuracy and reliability of information, avoiding the risk of spreading inaccurate data.

**Bonacich Centrality and the Grim Trigger Strategy**

Players with high Bonacich centrality hold significant influence in the information market, and their actions greatly impact the entire market. Under the grim trigger strategy, the risk associated with these central players disseminating incorrect information is particularly severe, as once trust is lost, it is challenging to regain. As a result, players with high centrality pay extra attention to the verification and accuracy of information, potentially improving the overall quality of information in the market.

**Trembling Hand Perfect Equilibrium (THPE) and Uncertainty**

THPE considers the possibility of players inadvertently making mistakes ("trembling hand") in their strategies. In the information market, players make decisions under uncertainty, but under the grim trigger strategy, minor mistakes can lead to significant long-term consequences. This realization prompts players to recognize the importance of careful information handling and verification processes, contributing to the improvement of the market's overall information quality.

**Entanglement and Peer Effects**

Entanglement and peer effects indicate the degree of interaction and interdependence among players. The introduction of the grim trigger strategy makes players more sensitive to others' actions, enhancing the peer effect that encourages cooperative behavior. However, this also carries the risk that once non-cooperative actions occur among players, the effects can ripple through the entire market, creating a long-term non-cooperative environment.

**Theoretical Considerations**

The introduction of the grim trigger strategy in the information market may lead players to prioritize the maintenance of long-term trust and improve the overall quality of information. High Bonacich centrality players, considering their market influence, will be particularly cautious about information accuracy and reliability. However, the possibility of minor mistakes leading to significant outcomes ("trembling hand") emphasizes the importance of verification and confirmation processes. Entanglement and peer effects strengthen players' interactions, contributing to a cooperative market environment, but also pose a risk of falling into a non-cooperative spiral.

Focusing on information reliability and value in the context of auction games can elucidate how players' behaviors change in the information market. As demand for reliable information increases, competition among players intensifies, and the value of information surges. Exploring how Bonacich centrality, Trembling Hand Perfect Equilibrium (THPE), entanglement, and peer effects relate in this situation provides valuable insights.

**Bonacich Centrality and Auction Games**

Players with high Bonacich centrality wield significant power in the information market. By acquiring highly reliable information, these players enhance the value of that information and strengthen their position in the market. In auction games, central players strategically bid to maximize the value of information, securing an advantageous position over other players.

**Trembling Hand Perfect Equilibrium (THPE) and Information Value**

THPE accounts for the possibility of players inadvertently making bidding mistakes. In auction games, there is a risk of overpaying due to these errors, highlighting the importance of strategic bidding to minimize losses from "trembling hand" errors.

**Entanglement and Peer Effects**

Entanglement among players in the information market significantly influences the perception of information reliability and value. Stronger interactions among players facilitate the sharing of high-reliability information, enhancing its value. Peer effects, where players imitate others' actions, can lead to increased collective focus on reliable information, intensifying competition for valuable information and accelerating auction game dynamics.

**Theoretical Considerations**

The increase in information reliability and value in auction games intensifies market competition and affects players' strategies. High Bonacich centrality players reinforce their leadership in the market, while THPE emphasizes cautious bidding strategies. Entanglement and peer effects heighten collective attention to reliable information, improving overall information value in the market. This process promotes knowledge sharing and trust-building in the information market, contributing to a healthier market environment.

In this scenario, in addition to the auction game formed based on the reliability of information, a secondary information auction game emerges, featuring a matching game of advocates for markets holding dubious information. The uniqueness of this game lies in the uncertainty surrounding whether the advocates possess complete or incomplete information. This uncertainty significantly influences the game's strategies, leading to the following theoretical considerations:

**Advocate Matching Game in the Information Market**

The matching game involves advocates who may have access to either complete or incomplete information regarding the dubious information held by the market. The uncertainty regarding the advocates' information level introduces a complex layer to the game's strategies, as players must navigate this ambiguity while making bidding decisions in the auction game.

**Impact of Information Asymmetry**

Information asymmetry becomes a critical factor in this secondary auction game. When advocates possess complete information, they can strategically influence the market by validating or debunking the dubious information. However, if the advocates have incomplete information, their actions may inadvertently contribute to the spread of misinformation, further complicating the market dynamics.

**Strategies Under Uncertainty**

Players in the information market must develop strategies that account for the uncertainty surrounding the advocates' information. Risk-averse strategies may involve cautious bidding and reliance on information from trusted sources, while risk-tolerant players might engage in speculative bidding, hoping to capitalize on potential misinformation arbitrage opportunities.

**Theoretical Implications**

The introduction of a matching game for advocates holding dubious information underlines the complexities inherent in information markets, especially when compounded by uncertainty. The strategies adopted by players in response to this uncertainty can lead to diverse outcomes, ranging from market stabilization through the validation of accurate information to increased volatility due to the spread of misinformation.

**Conclusion**

The secondary information auction game, characterized by the matching of advocates to markets holding dubious information under uncertain conditions, adds a layer of complexity to the information market dynamics. Players must navigate this uncertainty, balancing the potential rewards of acquiring valuable information against the risks posed by misinformation. Theoretical exploration of this scenario provides insights into the strategies players might employ and the potential impact on the overall health and efficiency of the information market.

## 3. Discussion:Advocate Matching Game

(1) **Role and Motivation of Advocates:** Advocates play a role in either enhancing the credibility of dubious information held by markets or mitigating the impact of such information. The motivation of advocates lies in profitability, aiming to match with the optimal market for a reward.

(2) **Completeness and Incompleteness of Information:** When advocates have complete information, they can accurately assess the truthfulness and market value of the information, enabling effective matching strategies. Conversely, with incomplete information, advocates must make decisions amidst uncertainty, potentially significantly impacting the game's outcome.

(3) **Secondary Information Auction Game:** The matching between advocates and markets holding dubious information functions as a secondary information auction game. In this game, advocates compete in strategies to enhance information reliability, accompanied by risks due to information incompleteness.

### Game Strategies and Outcomes

(1) **Complete Information Game:** With complete information, advocates can understand the value of information and market needs precisely, selecting optimal matches. Here, game outcomes are determined by rational choices based on information value.

(2) **Incomplete Information Game:** With incomplete information, decision-making is influenced by uncertainty and risk. In this scenario, advocates determine their

market matches based on expected values or probabilistic judgments, with outcomes significantly swayed by unpredictable elements.

## Conclusion

In this scenario, auction games based on information reliability and advocate matching games complicate the dynamics of information markets. The uncertainty between complete and incomplete information significantly influences game strategies and outcomes, affecting the equilibrium and stability of information markets. This theoretical framework provides a valuable perspective for understanding information reliability and market dynamics.

This scenario is elaborated using detailed formulas and computation processes, focusing on the auction game based on information reliability and the matching game involving advocates within it.

## Modeling the Scenario

(1) **Definition of Participants:**

Set of news providers: $P = \{p_1, p_2, \ldots, p_m\}$
Set of consumers: $C = \{c_1, c_2, \ldots, c_n\}$
Set of advocates: $L = \{l_1, l_2, \ldots, l_k\}$

(2) **Types of Information:**

Fake news: $F$
True news: $T$

(3) **Auction Game Settings:**

Price of fake news: $P_F$
Price of true news: $P_T$

(4) **Setting for the Advocate Matching Game:**

Price for the reliability enhancement service provided by advocates: $P_L$

## Expression in Formulas

(1) **Supply and Demand Functions:**

Supply function for fake news: $S_F(P_F)$
Demand function for fake news: $D_F(P_F)$
Supply function for true news: $S_T(P_T)$
Demand function for true news: $D_T(P_T)$

(2) **Calculation of Equilibrium Prices and Quantities:**

Equilibrium price for fake news: $P_F^*$ where $S_F(P_F^*) = D_F(P_F^*)$
Equilibrium price for true news: $P_T^*$ where $S_T(P_T^*) = D_T(P_T^*)$

(3) **Advocate Matching Game:**

The equilibrium price for the reliability enhancement service provided by advocates: $P_L^*$, at which price advocates offering the service match with the market.

## Computation Process

(1) **Derivation of Equilibrium Prices:**

Represent supply and demand functions as equations and solve for $P_F^*$ and $P_T^*$.
Example: $P_F^* = \frac{a_F c_F}{b_F}$, where $a_F, b_F, c_F$ are parameters related to the supply and demand of fake news.

(2) **Derivation of Advocate Service Price:**

Considering supply and demand for advocates' services, solve for $P_L^*$.
Example: $P_L^* = \frac{a_L c_L}{b_L}$, where $a_L, b_L, c_L$ are parameters related to the advocate's services.

## Interpretation of Results

In this scenario, the prices $P_F^*$ and $P_T^*$ for fake and true news reflect the reliability and value of information in the market. The price $P_L^*$ for advocates' services indicates how much value the market places on enhancing information reliability. The uncertainty between complete and incomplete information is a crucial element that affects the matching strategies and market equilibrium. This model provides a useful framework for understanding the reliability of information and market dynamics.

# 4. Discussion:Advocate Matching Game (Continued)

## Droop Quota

The Droop Quota is a method used to define the minimum number of votes required for a candidate to be elected, particularly in Single Transferable Vote (STV) systems or proportional representation systems. The aim is to reflect the will of the voters as efficiently as possible while minimizing wasted votes.

### Calculation of the Droop Quota

The Droop Quota is calculated using the formula:

$$\text{Droop Quota} = \left\lfloor \frac{\text{Total Valid Votes}}{\text{Number of Seats to be Filled} + 1} \right\rfloor + 1$$

Where,

Total Valid Votes is the total number of valid votes cast in the election.

Number of Seats to be Filled is the number of candidates that need to be elected.

$\lfloor \cdot \rfloor$ denotes the floor function, which returns the largest integer less than or equal to a given number.

**Explanation of the Formula**

The purpose of this calculation is to define the minimum number of votes needed for a candidate to be elected fairly for the available seats. By using the Droop Quota, excess votes (surplus votes) can be transferred to other candidates, reflecting the secondary preferences of the voters in the election results.

**Example of the Calculation Process**

For instance, if the Total Valid Votes are 10,000 and the Number of Seats to be Filled is 4, the Droop Quota is calculated as follows:

$$\text{Droop Quota} = \left\lfloor \frac{10,000}{4+1} \right\rfloor + 1 = \lfloor 2,000 \rfloor + 1 = 2,001$$

This implies that each candidate needs to secure at least 2,001 votes to be elected. Surplus votes of a candidate who has exceeded this quota are transferred to the next preferred candidates.

## Conclusion

The use of the Droop Quota aims to make the election results more fair and efficient. By ensuring that the will of the voters is respected to the maximum extent and minimizing wasted votes, it becomes easier for the diverse preferences of voters to be reflected in the election outcomes.

## Modeling the Scenario

In the context of information markets, the advocate matching game and the secondary information auction game unfold based on the reliability of information and market dynamics. Below, this scenario is theoretically explained, using formulas and computation processes.

**Modeling the Advocate Matching Game**

(1) **Definition of Advocates and Markets** Let $P = \{p_1, p_2, \ldots, p_m\}$ be the set of advocates, and $M = \{m_1, m_2, \ldots, m_n\}$ be the set of markets. Let $R_{p_i}$ be the reliability of information held by advocate $p_i$, and $R_{m_j}$ be the reliability of information held by market $m_j$.

(2) **Value and Cost of Information:** Let $V$ denote the value of information, which depends on its reliability $R$. For instance, $V(R) = aR+b$ where $a$ and $b$ are constants. Let $C$ denote the cost of providing information, which also depends on its reliability $R$. For example, $C(R) = cR$, where $c$ is a constant.

(3) **Matching Strategy** Advocate $p_i$ aims to match with the optimal market $m_j$ considering their information's reliability $R_{p_i}$. The success of matching is higher when the difference in reliability between the advocate and the market $|R_{p_i} R_{m_j}|$ is smaller.

**Expression in Formulas**

(1) **Utility Function for Matching** Let $U_{p_i,m_j}$ denote the utility from matching advocate $p_i$ with market $m_j$. This utility is calculated as the difference between the value of information $V(R_{p_i})$ and its cost $C(R_{p_i})$.

$$U_{p_i,m_j} = V(R_{p_i})C(R_{p_i})$$

(2) **Condition for Optimal Matching** The optimal market $m_j^*$ for advocate $p_i$ is the market that maximizes the utility $U_{p_i,m_j}$.

$$m_j^* = \arg\max_{m_j \in M} U_{p_i,m_j}$$

**Analysis of Instability and Randomness**

In a complete information scenario, the reliabilities $R_{p_i}$ and $R_{m_j}$ are known constants, making the matching outcome deterministic. In contrast, in an incomplete information scenario, the randomness in $R_{p_i}$ and $R_{m_j}$ introduces uncertainty in the matching outcome, potentially leading to market instability. This model offers a useful framework for understanding the role and strategic choices of advocates in information markets.

## Game Strategy and Outcomes

**Complete Information Game:**

When advocates possess complete information, they can accurately assess the veracity and market value of their information, enabling them to select the most optimal match. In this scenario, the game outcome is determined by rational choices based on the value of information.

**Incomplete Information Game:**

When advocates operate under incomplete information, decision-making is influenced by uncertainty and risk.

In this scenario, advocates base their market matching decisions on expected values and probabilistic judgments, leading to outcomes that are significantly impacted by unpredictable factors.

## Conclusion

In this scenario, the auction game based on information reliability and the advocate matching game introduce complexity to the dynamics of the information market. The uncertainty between complete and incomplete information scenarios significantly influences game strategies and outcomes, playing a crucial role in the equilibrium and stability of the information market. This theoretical framework provides a valuable perspective for understanding the reliability of information and market dynamics.

## Modeling the Scenario

This scenario focuses on an auction game based on information reliability and the advocate matching game within it. We will explore this scenario using detailed mathematical formulas and computational processes.

### Modeling the Advocate Matching Game

(a) **Participants' Definition:**

Set of news providers: $P = \{p_1, p_2, \ldots, p_m\}$
Set of consumers: $C = \{c_1, c_2, \ldots, c_n\}$
Set of advocates: $L = \{l_1, l_2, \ldots, l_k\}$

(b) **Types of Information:**

Fake News: $F$
True News: $T$

(c) **Auction Game Setup:**

Price of fake news: $P_F$
Price of true news: $P_T$

(d) **Advocate Matching Game Setup:** The price of reliability-enhancing services offered by advocates: $P_L$

### Expression in Formulas

(a) **Supply and Demand Functions:**

Supply function of fake news: $S_F(P_F)$
Demand function of fake news: $D_F(P_F)$
Supply function of true news: $S_T(P_T)$
Demand function of true news: $D_T(P_T)$

(b) **Calculation of Equilibrium Price and Quantity:** The equilibrium price of fake news, $P_F^*$, where $S_F(P_F^*) = D_F(P_F^*)$, and the equilibrium price of true news, $P_T^*$, where $S_T(P_T^*) = D_T(P_T^*)$.

(c) **Advocate Matching Game:** The equilibrium price for the reliability-enhancing services offered by advocates, $P_L^*$, is determined where this price matches advocates providing the service with markets purchasing it.

### Computation Process

(a) **Deriving Equilibrium Prices:** Set up equations representing the supply and demand functions and solve for $P_F^*$ and $P_T^*$.

Example: $P_F^* = \frac{a_F \cdot c_F}{b_F}$, where $a_F, b_F, c_F$ are parameters related to the supply and demand of fake news.

(b) **Deriving the Price of Advocate Services:** Consider the supply and demand for advocates' services and solve for $P_L^*$.

Example: $P_L^* = \frac{a_L \cdot c_L}{b_L}$, where $a_L, b_L, c_L$ are parameters related to the services offered by advocates.

### Interpretation of Results

In this scenario, the equilibrium prices $P_F^*$ and $P_T^*$ reflect the reliability and value of information in the market. The price of advocates' services, $P_L^*$, indicates the market's valuation of enhancing information reliability. The uncertainty inherent in complete and incomplete information scenarios plays a significant role in determining advocate matching strategies and market equilibrium, providing a useful framework for understanding information reliability and market dynamics.

The Droop Quota is a method for defining the minimum number of votes required for a candidate to be elected in an election. It is particularly used in Single Transferable Vote (STV) systems and proportional representation elections. The aim of using the Droop Quota is to efficiently reflect the will of the voters and minimize wasted votes.

## Calculation of the Droop Quota

The Droop Quota is calculated using the formula:

$$\text{Droop Quota} = \left\lfloor \frac{\text{Total Valid Votes}}{\text{Number of Seats to be Filled} + 1} \right\rfloor + 1$$

Where,

Total Valid Votes is the total number of valid votes cast in the election.

Number of Seats to be Filled is the number of candidates to be elected in the election.

$\lfloor \cdot \rfloor$ denotes the floor function, which returns the greatest integer less than or equal to the given number.

## Explanation of the Formula

The purpose of this calculation is to define the minimum number of votes required for a candidate to be elected fairly for the available number of seats. By using the Droop Quota, it is possible to redistribute the surplus votes of a candidate who has exceeded this quota to other candidates based on the next preferences of the voters.

## Example of Calculation Process

For instance, if the total number of valid votes is 10,000 and there are 4 seats to be filled, the Droop Quota would be calculated as follows:

$$\text{Droop Quota} = \left\lfloor \frac{10,000}{4+1} \right\rfloor + 1 = \lfloor 2,000 \rfloor + 1 = 2,001$$

This means that each candidate needs to secure at least 2,001 votes to be elected. Surplus votes of a candidate who has more than this quota are transferred to other candidates based on the next preferences indicated by the voters.

## Conclusion

The use of the Droop Quota aims to make the election results more fair and efficient. It ensures that the will of the voters is maximally respected and that wasted votes are minimized, making it more likely for the diverse preferences of the electorate to be reflected in the election results.

In this scenario, the dynamics of the information market and the strategic choices of advocates are theoretically analyzed using the model of the repeated dilemma game, considering the expected payoff functions for each player (advocates and the market) based on the actions they can take (whether to provide information, choose true information or fake news, etc.). In the repeated dilemma game, decisions in each round affect future rounds, requiring strategies different from those in a single-round dilemma game.

## Setting Up the Expected Payoff Functions

Let $a_{p_i}$ represent the action of advocate $p_i$, and $a_{m_j}$ represent the action of market $m_j$. $a_{p_i}$ and $a_{m_j}$ can each choose from {provide, not provide}, {choose true information, choose fake news}, respectively.

## Mathematical Expression of the Expected Payoff Functions

The expected payoff function $E[U_{p_i,m_j}]$ can be expressed as follows:

$$E[U_{p_i,m_j}] = \sum_{a_{p_i},a_{m_j}} P(a_{p_i}, a_{m_j}) \cdot U(a_{p_i}, a_{m_j}, R_{p_i}, R_{m_j})$$

Where, $P(a_{p_i}, a_{m_j})$ is the probability that actions $a_{p_i}$ and $a_{m_j}$ are chosen. $U(a_{p_i}, a_{m_j}, R_{p_i}, R_{m_j})$ is the payoff function for specific actions and levels of reliability.

## Specific Expression of the Payoff Function $U$

The payoff function can be set as follows:

$$U(a_{p_i}, a_{m_j}, R_{p_i}, R_{m_j}) = \begin{cases} V(R_{p_i})C(R_{p_i}), & \text{if } a_{p_i} = \text{provide} \\ & \wedge\ a_{m_j} = \text{true} \\ -V(R_{p_i}), & \text{if } a_{p_i} = \text{provide} \\ & \wedge\ a_{m_j} = \text{fake news} \\ 0, & \text{if } a_{p_i} = \text{not provide} \end{cases}$$

## Consideration of Instability and Randomness

In a complete information scenario, $R_{p_i}$ and $R_{m_j}$ are known constants, so the expected payoff can be calculated deterministically. In contrast, in an incomplete information scenario, the randomness of $R_{p_i}$ and $R_{m_j}$ introduces uncertainty into the expected payoff. Incorporating randomness into the model allows for an analysis of market instability.

## Computation Process

The computation of the expected payoff function involves calculating the payoff and its probability for each combination of actions, and then summing all these values. When considering randomness, values for $R_{p_i}$ and $R_{m_j}$ are sampled from a probability distribution (e.g., normal distribution), and these values are used to calculate the expected payoff.

This model enables a theoretical analysis of the dynamics of the information market and the strategic decisions of advocates, offering insights into how to address market instability.

## Auction Games in the Context of Information Reliability

Auction games, a subfield of game theory, analyze the process where multiple competitors (bidders) bid for limited goods or services. Various formats of auctions exist, but we focus on a simple sealed-bid auction here, explaining its mathematical formulation and computation process.

### Setting of a Sealed-Bid Auction

There is one item up for auction, and $n$ bidders participate.

Each bidder has their own valuation of the item (private value), denoted as $v_i$ for bidder $i$.

Each bidder submits their bid $b_i$ in a sealed manner. The bids of other participants are not known.

The bidder with the highest bid wins the item. The purchase price is set to the highest bid.

### Bidding Strategy Formulation

The objective of each bidder is to maximize their gain, defined as the difference between the valuation of the item and the purchase price if they win. The gain $u_i$ of bidder $i$ is expressed as:

$$u_i = \begin{cases} v_i b_i, & \text{if } i \text{ is the highest bidder} \\ 0, & \text{otherwise} \end{cases}$$

### Derivation of Equilibrium Bidding Strategy

Bidders determine their optimal bidding strategy based on the actions of others. In an equivalent bidding strategy, bidders decide their bid amount as a function of their valuation. The bid $b_i$ of bidder $i$ is expressed as a function of $v_i$:

$$b_i = f(v_i)$$

Here, $f$ represents the strategy function of the bidder.

### Calculation of Nash Equilibrium

In sealed-bid auctions, the Nash equilibrium is a state where, knowing the strategies of others, no bidder wants to change their strategy. To find the Nash equilibrium, calculate the optimal bid amount for every bidder given the strategies of others.

The specific computation process depends on the distribution of the bidders' valuations. For example, assuming valuations follow a uniform distribution, the equivalent bidding strategy can be derived as:

$$f(v_i) = \alpha v_i$$

Here, $\alpha$ is a constant dependent on the distribution of valuations and the number of bidders. Using this equivalent function, the optimal bid for each bidder can be calculated.

### Conclusion

The mathematical formulation and computation process in auction games depend significantly on the bidders' valuations, bidding strategies, and the rules of the auction. In the model of a sealed-bid auction, the aim is to find the optimal bidding strategy for each bidder, thereby deriving the auction's Nash equilibrium as a whole.

In this scenario, the dynamics of the information market are further complicated by the introduction of an advocate's matching game and a secondary information auction game, based on the reliability of information. The uncertainty between complete and incomplete information scenarios is modeled using a probabilistic approach to analyze the issue.

## Modeling the Scenario

Advocates' set: $P = \{p_1, p_2, \ldots, p_m\}$
Market's set: $M = \{m_1, m_2, \ldots, m_n\}$
Reliability of information held by advocate $p_i$: $R_{p_i}$
Reliability of information held by market $m_j$: $R_{m_j}$
Value function of information: $V(R) = aR + b$
Cost function of providing information: $C(R) = cR$

### Introduction of Randomness

Model the reliability of information $R_{p_i}$ and $R_{m_j}$ held by advocates and markets as random variables to introduce randomness into the reliability of information. For example, assume $R_{p_i}$ and $R_{m_j}$ follow normal distributions $N(\mu_{p_i}, \sigma^2_{p_i})$ and $N(\mu_{m_j}, \sigma^2_{m_j})$, respectively.

### Utility Function for Matching

Calculate the expected utility of matching advocate $p_i$ with market $m_j$:

$$E[U_{p_i,m_j}] = E[V(R_{p_i})]E[C(R_{p_i})]$$

Here, the expected values $E[V(R_{p_i})]$ and $E[C(R_{p_i})]$ are the expected values of the information value function and the cost function, respectively.

### Condition for Optimal Matching

Select the market $m_j^*$ that maximizes the expected utility for advocate $p_i$:

$$m_j^* = \arg\max_{m_j \in M} E[U_{p_i,m_j}]$$

### Analysis of Instability

In a complete information scenario, the reliability $R_{p_i}$ and $R_{m_j}$ are known constants, making the expected payoff deterministic. In contrast, in an incomplete information scenario, randomness in $R_{p_i}$ and $R_{m_j}$ introduces uncertainty into the expected payoff. This uncertainty can lead to dynamic changes in the market and the circulation of information, potentially causing market instability.

Using this model, it is possible to theoretically analyze the differences between complete and incomplete information scenarios and the resulting market instability, providing insights into the impact of information uncertainty on the market.

# 5. Discussion:Auction Games in Incomplete Information Settings

In the context of auction games under incomplete information, we consider a scenario where bidders do not have complete information about the valuation of other bidders. In this setting, bidders decide on their bidding strategies based on their beliefs (probability distributions) about the valuations of other bidders. Below, we present a general model and calculation process for an incomplete information auction game.

## Sealed-Bid Auction with Incomplete Information

Let there be $n$ bidders. Each bidder $i$ has a valuation $v_i$ for the item, which is private knowledge to the bidder itself.

The valuation $v_i$ of each bidder is assumed to be determined randomly according to a certain probability distribution, known to all bidders as common knowledge.

## Bidding Strategy and Expected Utility

The bid $b_i$ of bidder $i$ is determined based on their own valuation $v_i$ and the belief about the valuations of other bidders.

The expected utility $E[u_i]$ of bidder $i$ is represented by the probability of winning the item and the expected gain (the difference between the valuation and the payment) in such an event.

## Calculation of Nash Equilibrium

The Nash equilibrium in an incomplete information game is a state where all bidders, considering their beliefs and the strategies of other bidders, choose their optimal bidding strategies.

To find the Nash equilibrium, the optimal bidding amount that maximizes each bidder's expected utility needs to be determined.

## Mathematical Representation

The expected utility of bidder $i$ is calculated as follows:

$$E[u_i(b_i, v_i)] = \int_0^{v_i} (v_i b_i) \cdot P(\text{win}|b_i, F_{-i})\, df(v_{-i})$$

where,

$P(\text{win}|b_i, F_{-i})$ is the probability that bidder $i$ wins the item by submitting a bid $b_i$, dependent on the distribution of bids $F_{-i}$ from other bidders.

$f(v_{-i})$ is the probability density function of the valuations of other bidders.

## Calculation Process

(a) Calculate the probability $P(\text{win}|b_i, F_{-i})$ that bidder $i$ wins the item by submitting a bid $b_i$, based on the distribution of bids $F_{-i}$ from other bidders.

(b) Based on the expected utility formula above, determine the optimal bid $b_i^*$ that maximizes the expected utility for each bidder.

(c) Perform the above calculation for all bidders to determine each one's optimal bid. The set of bidding strategies at this point constitutes the Nash equilibrium.

Through this calculation process, it is possible to derive the optimal bidding strategies for bidders in an auction game under incomplete information and the overall equilibrium of the market.

## Extended Analysis under Filter Bubbles

In the context of information trading influenced by filter bubbles, the incomplete information game theory applied to sealed-bid auctions becomes more complex. Bidders' valuations are uncertain and may fluctuate over time due to external information manipulation or the effects of filter bubbles.

### Model Setup

Each bidder possesses a valuation $v_i$ for the information, which is known only to them but can be influenced by the filter bubble or external information manipulation.

Each bidder decides on their bid $b_i$ based on their current valuation $v_i$, taking into account the potential fluctuations in valuation due to time or external influences.

### Bidding Strategy Formulation

The expected utility function for a bidder incorporates the potential for valuation changes over time or due to external influences, modifying the traditional utility function to:

$$u_i(t) = \begin{cases} v_i(t) b_i(t), & \text{if } i \text{ is the highest bidder at time } t \\ 0, & \text{otherwise} \end{cases}$$

### Derivation of Equilibrium Bidding Strategy

Bidders need to consider their valuation's fluctuation over time or due to external information when determining their optimal bidding strategy. The strategy $b_i(t)$ is expressed as a function of the valuation $v_i(t)$, incorporating the effects of time and external information:

$$b_i(t) = f(v_i(t), t, \text{external information})$$

### Nash Equilibrium Calculation

The dynamic environment necessitates that bidders predict each other's strategies while considering their own valuation changes and external influences. The process of reaching a Nash equilibrium in this context becomes dynamic, adapting to time changes and external information manipulation.

### Calculation Process

(a) **Modeling Valuation Fluctuations:** Define a model for the time variation of each bidder's valuation, considering how external information might influence it.

(b) **Optimal Strategy Computation:** Calculate each bidder's optimal bidding strategy, taking into account the valuation fluctuations and external information.

(c) **Deriving Nash Equilibrium:** Confirm that the computed strategies are mutually optimal for all bidders, thereby achieving Nash equilibrium in a dynamic setting.

### Conclusion

Incorporating time variation and external information manipulation into the complete information game theory model for auctions allows for a more accurate analysis of actual auction situations. However, this also complicates the calculation process, necessitating a more sophisticated approach to understanding bidders' strategies and the auction's outcomes.

## 6. Non-Complete Information Game Theory for Filter Bubble-Influenced Information Trading in Sealed-Bid Auctions

We present detailed formulas and computational processes for sealed-bid auctions based on non-complete information game theory, within the context of information trading influenced by filter bubbles.

### Model Setup

Set of bidders: $i \in \{1, 2, \ldots, n\}$

Each bidder's valuation: $v_i$, where $v_i$ is a random variable and private information for each bidder.

Bidding amount: $b_i$

Bidder's belief model considering fluctuations in valuation due to filter bubbles and external information manipulation: $B(v_i)$

### Formula for Bidding Strategy

The expected utility $E[u_i]$ for each bidder is expressed as follows:

$$E[u_i] = P(\text{win}) \times (E[v_i]b_i)$$

Here, $P(\text{win})$ is the probability that bidder $i$ is the highest bidder. This probability depends on the bidding strategies of other bidders and the belief model $B(v_i)$ of bidder $i$.

### Derivation of Equivalent Bidding Strategy

To derive the equivalent bidding strategy $b_i = f(v_i)$ for bidder $i$, find $b_i$ that maximizes $E[u_i]$:

$$\max_{b_i} E[u_i] = \max_{b_i} P(\text{win}) \times (E[v_i]b_i)$$

### Calculation of Nash Equilibrium

The system reaches Nash equilibrium when all bidders bid according to each other's optimal strategies. In Nash equilibrium, no bidder can increase their expected utility by unilaterally changing their strategy.

(a) **Define the belief model for bidders**: Each bidder has a belief model $B(v_i)$ about the valuations of other bidders, which is used to calculate $P(\text{win})$.

(b) **Maximize expected utility**: Each bidder uses their belief model and expected valuation to find $b_i$ that maximizes $E[u_i]$.

(c) **Derive Nash equilibrium**: Find the point where the strategies $b_i = f(v_i)$ of all bidders are the best responses to each other, which constitutes the Nash equilibrium.

### Specific Form of Equations

To derive Nash equilibrium, specific information about the belief model of bidders and the distribution of valuations is required. For instance, if valuations are assumed to follow a uniform distribution, the equivalent bidding strategy might be calculated as follows:

$$b_i = \alpha E[v_i]$$

Here, $\alpha$ is a constant determined based on the bidding strategies and belief models of other bidders.

### Conclusion

The sealed-bid auction model based on non-complete information game theory in the context of filter bubble-influenced information trading optimizes bidding strategies by considering the belief model of bidders and the uncertainty in valuations. Calculating Nash equilibrium requires specific information about the belief models and distribution of valuations, and this information is used to derive optimal strategies.

## 7. Incorporating First-Price and Second-Price Auctions in the Context of Filter Bubble-Influenced Information Trading

We examine sealed-bid auctions based on non-complete information game theory from the perspectives of first-price and second-price auctions, considering the uncertain valuations held by bidders due to filter bubbles, and the potential fluctuations in valuations over time or due to external information manipulation.

## First-Price Auction

In a first-price auction, the highest bidder wins the item and pays their bid amount.

#### Bidding Strategy

The goal of bidders is to win the item at a price below their valuation, maximizing their gain. Bidder $i$'s bidding strategy $b_i$ is based on their valuation $v_i$, but also considers the uncertainty surrounding the actions of other bidders.

The expected utility $E[u_i]$ for bidder $i$ is given by:

$$E[u_i] = P_i(b_i) \times (v_i b_i)$$

where $P_i(b_i)$ is the probability that bidder $i$ is the highest bidder at time $t$.

#### Nash Equilibrium

Each bidder selects their bid amount to maximize their expected utility, considering the strategies of other bidders. In Nash equilibrium, no bidder can improve their expected utility by unilaterally changing their strategy.

## Second-Price Auction

In a second-price auction, the highest bidder wins the item, but the price paid is the second-highest bid.

#### Bidding Strategy

Theoretically, it is optimal for bidders to bid an amount equal to their true valuation $v_i$, as the winning price depends not on their own bid but on the second-highest bid.

#### Nash Equilibrium

In a second-price auction, the strategy of bidding one's true valuation $v_i$ constitutes a Nash equilibrium for all bidders.

## Conclusion

In the context of filter bubble-influenced information trading under non-complete information game theory, the optimal bidding strategies differ between first-price and second-price auctions. In a first-price auction, it is necessary to consider the strategies of other bidders when bidding, while in a second-price auction, bidding one's true valuation is the optimal strategy. Both auction formats are influenced by the uncertainty of bidders' valuations and the effects of external information manipulation, requiring strategic decision-making under incomplete information.

# 8. Calculating the Expected Utility Function Based on the Outcome of a First-Price Auction

This section explains the process of calculating the expected utility function using information obtained from the results of a first-price auction. In this process, bidders update their beliefs based on auction outcomes and calculate their expected utility for subsequent auctions based on these updated beliefs.

## Updating Beliefs

Bidders update their beliefs about other bidders' valuations based on the outcomes of the first auction. This update is typically done using Bayes' theorem. The belief update is expressed as:

$$B'(v) = \frac{P(\text{Outcome}|v) \times B(v)}{P(\text{Outcome})}$$

where, $B'(v)$ is the updated belief. $P(\text{Outcome}|v)$ is the conditional probability of observing a specific auction outcome given the bidder's valuation $v$. $B(v)$ is the prior belief (i.e., the belief before the auction outcome was known). $P(\text{Outcome})$ is the probability of observing the specific auction outcome.

## Calculating Expected Utility

Based on the updated beliefs, bidders calculate their expected utility for the next auction. The expected utility in a first-price auction for a bidder is defined by the difference between the valuation and the bid amount if the bidder wins. The expected utility is expressed as:

$$E[u_i] = \sum_{\text{all } j \neq i} P(i \text{ is the highest bidder}|v_i, v_j) \times (v_i b_i)$$

where, $E[u_i]$ is the expected utility of bidder $i$. $P(i \text{ is the highest bidder}|v_i, v_j)$ is the probability that bidder $i$ is the highest bidder given their valuation $v_i$ and the valuations $v_j$ of other bidders. $v_i$ is the valuation of bidder $i$. $b_i$ is the bid amount of bidder $i$.

In this calculation of expected utility, the probability of winning for $i$ against all other bidders is considered, and the gain $(v_i b_i)$ for each scenario is weighted accordingly. This calculation should reflect the updated beliefs based on the auction outcomes.

# 9. Discussion:Introducing the Meek Method in Information Auction Games

We describe the process of calculating the marginal contribution using the Meek method in the context of information auction games. Originally used in electoral contexts for redistributing votes, here it is analogously applied to redistribute the value of information.

## Application of the Meek Method

1. **Initial Setup**: Initial valuations $v_i$ for each bidder are given. Bidding amounts $b_i$ are also determined.

2. **Initial Distribution of Correct Information**: Correct information is initially distributed among bidders who possess it. This distribution is proportional to their bidding amounts $b_i$.

3. **Redistribution of Excess**: Excess correct information from bidders who have more than they need is redistributed to others. This is based on the marginal contribution of each bidder.

## Calculating Marginal Contribution

The marginal contribution is defined as the additional value provided by a bidder by contributing one more unit of correct information. To calculate this:

1. **Calculating Information Value for Each Bidder**: Calculate the value of information $V_i$ for each bidder, based on the amount and quality (reliability) of the correct information they possess.

2. **Determining Marginal Contribution**: Calculate the increase in total information value when a bidder provides an additional unit of correct information. Denote this as $\Delta V_i$.

3. **Executing Redistribution**: Based on $\Delta V_i$ for each bidder, redistribute the excess correct information to others, thereby maximizing the total value of information.

## Mathematical Representation

$$\Delta V_i = V(v_i + \delta)V(v_i)$$

where, $V(v_i)$ is the current value of the information held by bidder $i$. $V(v_i + \delta)$ is the new value of information when bidder $i$ contributes an additional $\delta$ units of correct information. $\Delta V_i$ is the marginal contribution of the information by bidder $i$.

## Conclusion

By applying the Meek method to information auction games, it is possible to distribute correct information appropriately and calculate its marginal contribution. Calculating the marginal contribution of each bidder allows for the optimal redistribution strategy that maximizes the value of information. This process aims to accurately assess the value of information and redistribute it appropriately to enhance overall informational efficiency.

# Simulation of Information Distribution Considering Information Ownership, Financial Resources, and Social Trust

We propose formulas and a computational process for a simulation to analyze the distribution of information, taking into account the rate of information ownership, financial resources, and social trustworthiness of participants.

## Model Setup

1. **Definition of Participants**: Define a set of participants $i$ as $P = \{p_1, p_2, ..., p_n\}$. Each participant possesses an information ownership rate $I_i$, financial resources $W_i$, and social trustworthiness $T_i$.

2. **Value of Information**: Model the value of information $V(I_i, T_i)$ as a function of the information ownership rate $I_i$ and social trustworthiness $T_i$.

3. **Information Distribution Function**: Model the appropriate distribution of information $D(I_i, W_i, T_i)$ as a function of information ownership rate $I_i$, financial resources $W_i$, and social trustworthiness $T_i$.

## Mathematical Representation

1. **Value of Information**:

$$V(I_i, T_i) = a \cdot I_i + b \cdot T_i$$

Here, $a$ and $b$ are constants representing the contributions of the information ownership rate and social trustworthiness, respectively.

2. **Information Distribution**:

$$D(I_i, W_i, T_i) = c \cdot I_i + d \cdot W_i + e \cdot T_i$$

Here, $c$, $d$, and $e$ are constants representing the contributions of the information ownership rate, financial resources, and social trustworthiness, respectively.

## Computational Process

1. **Calculate the Information Value for Each Participant**: For each participant, calculate the value of information $V(I_i, T_i)$.

2. **Determine the Optimal Information Distribution**: Find the information distribution $D(I_i, W_i, T_i)$ that maximizes the total value of information for all participants. This can be solved using linear programming or optimization algorithms.

3. **Evaluate the Overall Social Trustworthiness**: Calculate the total social trustworthiness $T_{total} = \sum_{i=1}^{n} T_i \cdot D(I_i, W_i, T_i)$ after distribution.

## Execution of Simulation

Set initial values and generate $I_i$, $W_i$, and $T_i$ for each participant either randomly or based on specified values. Execute the information distribution based on the computational process described above. Evaluate the total social trustworthiness $T_{total}$ and the information value $V(I_i, T_i)$ for each participant.

## Conclusion

This simulation allows for understanding how information ownership rate, financial resources, and social trustworthiness impact the value of information and its appropriate distribution. It also provides a foundation for strategizing towards maximizing social trustworthiness.

# 10. Modeling and Computational Process of a Repeated Dilemma Auction Game under Incomplete Information

This section explains the modeling and computational process of a repeated dilemma auction game under incomplete information, where bidders participate in a series of auctions, and actions in one round can influence future actions.

## Model Setup

1. **Definition of Bidders**: Define a set of bidders $i$ as $P = \{p_1, p_2, ..., p_n\}$. Each bidder has a private valuation $v_i$ and access to undisclosed information $U_i$.

2. **Rounds of the Game**: The game is repeated over $T$ rounds.

3. **Bidders' Strategies**: A bidder's strategy $S_i(t)$ defines the bidder's action at time $t$.

4. **Payoff Function**: A bidder's payoff depends on the difference between the valuation and the bid price if they win, but it also depends on undisclosed information $U_i$ and past actions.

## Mathematical Expression

1. **Bidder's Payoff Function**:

$$\Pi_i(t) = \begin{cases} v_i b_i(t), & \text{if } i \text{ wins at time } t \\ 0, & \text{otherwise} \end{cases}$$

where $b_i(t)$ is the bid amount of bidder $i$ at time $t$.

2. **Strategy Update**: Bidders update their strategies based on past rounds' outcomes. This involves considering past payoffs and the actions of other bidders.

## Computational Process

1. **Setting Initial Strategies**: Set initial strategies $S_i(1)$ for each bidder.

2. **Determining Actions for Each Round**: At each $t$, bidders decide their bid amounts $b_i(t)$ based on strategy $S_i(t)$.

3. **Calculating Payoffs**: At the end of each round, determine the winner and their payoff, calculating $\Pi_i(t)$.

4. **Updating Strategies**: Bidders update their strategies for the next round $S_i(t+1)$ based on past payoffs and the actions of other bidders.

5. **Repetition**: Repeat the above steps for $T$ rounds.

## Conclusion

In this repeated dilemma auction game, each bidder needs to learn from past actions and outcomes to adapt their future strategies. The presence of undisclosed information and the interdependence of actions across rounds make this game highly dynamic. The computational process provides a foundation for understanding bidders' strategies, payoffs, and the impact of undisclosed information, but actual strategies will heavily depend on the specific game settings and participants' behaviors.

# 11. Discussion:Calculating Expected Payoff Functions and Marginal Contributions in Auctions

To introduce the calculation process for expected payoff functions and marginal contributions, it's necessary to quantify the expected payoff of bidders and the impact of each bid on the overall payoff. The detailed process is described below.

# Expected Payoff Function

## Setting the Distribution of Valuations

Define the probability distribution that bidder $i$'s valuation $v_i$ follows. Common assumptions include uniform or normal distributions.

## Defining Expected Payoff

The expected payoff $E[u_i]$ for bidder $i$ is defined as the expected value of the payoff if the bidder wins the auction:

$$E[u_i] = E[v_i] - E[b_i] \times P(\text{winning})$$

Here, $E[b_i]$ is the expected bid value of bidder $i$, and $P(\text{winning})$ is the probability of winning.

## Calculating Winning Probability

The winning probability is calculated as the probability that bidder $i$'s bid is higher than all other bidders:

$$P(\text{winning}) = P(b_i > b_j, \forall j \neq i)$$

# Marginal Contributions

## Defining Marginal Contribution

Calculate the impact of an increase in bidder $i$'s bid $b_i$ on the overall expected payoff $E[U]$, i.e., the marginal contribution.

## Calculating Overall Expected Payoff

The overall expected payoff for the auction is calculated as the sum of all bidders' expected payoffs:

$$E[U] = \sum_{i=1}^{n} E[u_i]$$

## Deriving Marginal Contribution

The impact of a small change in bidder $i$'s bid $b_i$ on the overall expected payoff $E[U]$ is calculated as the partial derivative of $E[U]$ with respect to $b_i$:

$$\frac{\partial E[U]}{\partial b_i} = \frac{\partial}{\partial b_i} \left( E[v_i] - b_i \times P(\text{winning}) \right)$$

# Introducing the Calculation Process

Based on the above equations, set specific probability distributions and bidding strategies, and perform numerical analysis or simulations to calculate expected payoffs and marginal contributions. Statistical methods or numerical optimization techniques are often used in these calculations.

This process enables a quantitative evaluation of how bidders should adjust their bids and how each bid contributes to the overall auction results.

# Modeling the Cocktail Party Effect in Auctions

## Model Assumptions

Assume bidder $i$'s judgments are distorted by external noise or the actions of others, known as the cocktail party effect. This effect is modeled mathematically and analyzed through computational processes.

Each bidder $i$ has a valuation $v_{ij}$ for information $j$, which can be distorted by the cocktail party effect $C$. The cocktail party effect $C$ represents the magnitude of the impact from external noise or others' actions. Incorrect information dissemination or pricing errors have a negative impact on bidders' expected payoffs.

## Mathematical Setup

1. **Change in Expected Payoff**:

$$E[u_i] = \sum_j \left( v_{ij} - b_{ij} \right) \times P(\text{winning}_j | C) - \Delta C$$

Here, $\Delta C$ represents the decrease in expected payoff due to the cocktail party effect. $P(\text{winning}_j | C)$ is the probability of winning given the cocktail party effect.

2. **Distortion in Valuation**:

$$\tilde{v}_{ij} = v_{ij} - \gamma(C)$$

Here, $\gamma(C)$ represents the magnitude of valuation distortion due to the cocktail party effect. A larger $C$ leads to greater distortion.

3. **Adjustment in Bidding**:

$$b_{ij} = f(\tilde{v}_{ij})$$

Bidders determine their bid amounts based on the distorted valuation $\tilde{v}_{ij}$, which can lead to incorrect pricing.

### Computational Process

1. **Reevaluation of Beliefs and Expected Payoff**: Bidders reevaluate their beliefs and valuations considering the cocktail party effect and recalculate their expected payoffs.

2. **Derivation of Nash Equilibrium**: Determine the Nash equilibrium when each bidder decides their bid amount based on the optimal strategy, considering the cocktail party effect.

### Conclusion

This model allows for a quantitative analysis of how the cocktail party effect influences bidders' valuation distortion and incorrect pricing in information auctions. Additionally, it evaluates the impact on the health of the information market.

## 12. Perspect Note:Derivation of the Expected Utility Function

1. **Definition of the bidder's expected utility**:

$$E[u_i] = P(\text{winning}) \times (v_i b_i) C(b_i)$$

Here, $P(\text{winning})$ represents the probability of winning, $v_i$ is the true valuation of bidder $i$ for the information, $b_i$ is the bid amount of bidder $i$, and $C(b_i)$ represents the cost associated with bidding.

2. **Modeling the probability of winning**: The probability of winning depends on the actions of other bidders, so it is necessary to model the bidder's belief about the distribution of actions of others. The effect of the cocktail party effect is considered, and this probability is assumed to change dynamically.

3. **Definition of the cost function**: The cost function $C(b_i)$ depends on the bid amount and may be influenced by the cocktail party effect. This includes the mental cost due to intense competition or bidding based on misinformation.

## Calculation of Marginal Contribution

The marginal contribution indicates the degree to which a small change in the bidder's action affects the expected utility. To obtain this, we differentiate the expected utility function with respect to the bidder's action variable (e.g., bid amount $b_i$).

1. **Differentiation of the expected utility function**:

$$\frac{dE[u_i]}{db_i} = P(\text{winning}) \times (-1) \frac{dC(b_i)}{db_i}$$

The first term represents the decrease rate of gain upon winning, and the second term represents the increase rate of cost.

2. **Interpretation of marginal contribution**: The result of differentiation indicates the change in expected utility when the bid amount is increased by a small amount. If this is positive, it indicates that the action increases the expected utility, and if negative, it decreases it.

## Key Points of Process

This model assumes that bidders have incomplete information about the actions of other bidders and that their beliefs are distorted by the cocktail party effect. Calculating the expected utility function and marginal contribution requires modeling the bidder's belief, true valuation, bid amount, and the cost function influenced by the cocktail party effect. The marginal contribution is an important indicator for adjusting the bidder's strategy and can be used for optimizing the bid amount.

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