Discussion Paper:(1)Case Study of Decision Making and Urban Planning:Loop Flip Errors and Phase Transitions in Local Potential Approximation in Non-trivial Loops(2024), (2)Case Study of Spatio-Temporal Approaches To Syndrome Errors in Decision Making with Applications Such as Bit Phase Flip Errors and Methods Discussion(2024)

# Note:Information Distribution among Bidders and Valuation Games in Repeated Dilemmas: Theoretical Framework for Cocktail Party Effects Affecting the Nash Equilibrium of Information Auctions Associated with Valuation Rules in Non-Perfect Information Game

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**Abstract:** This paper summarizes the discussion of a hypothetical computational experiment. This paper scopes against methods for modeling and analyzing information asymmetries among bidders in information auctions and how these asymmetries are affected by cocktail party effects. In particular, we explore how the diffusion of information through network relationships among bidders affects the outcome of the auction, in particular the Nash equilibrium. Our model begins with a setup in which bidders each bid based on their valuation with respect to the value of their own information, and models the flow of information by introducing an adjacency matrix representing the network connections among bidders. By taking into account cocktail party effects, we mimic the process by which a bidder receives information from another bidder and analyze how valuations are adjusted as a result. By deriving expected gain functions and Nash equilibria, this study quantitatively assesses the impact of different network structures on auction outcomes. This approach provides insights into auction design and information policy, and is considered with respect to a new framework for understanding information asymmetry and the role of networks. By integrating bidder networks into the model, we examine the process of information diffusion among bidders and its subsequent effects on auction strategies and Nash equilibria.

**Keywords:** Incomplete Information Games, Information Asymmetry, Auction Theory, Cocktail Party Effect, Bidder Networks, Nash Equilibrium, Information Diffusion, Auction Outcomes, Network Analysis, Social Influence in Auctions

## 1. Introduction

This paper also continues and summarizes the discussion of the hypothetical computational experiments. In this note, we analyze information asymmetry in auctions and its effect on the outcome. We consider simulations that intend to address the value of misinformation bias from the perspective of informational and digital health, as well as preventive measures against the lack of information judgment in society as a whole due to the excess of rate.

Fig.1 the impact of social influence on bidder behavior in a second-price auction, labeled as the "cocktail party effect". This effect may be analogous to the echo chamber effect in the context of information dissemination, where the echo chamber effect refers to situations in which beliefs are amplified or reinforced by communication and repetition inside a closed system and insulated from rebuttal. Base Valuation (Blue Dots) In the context of information spread, this could

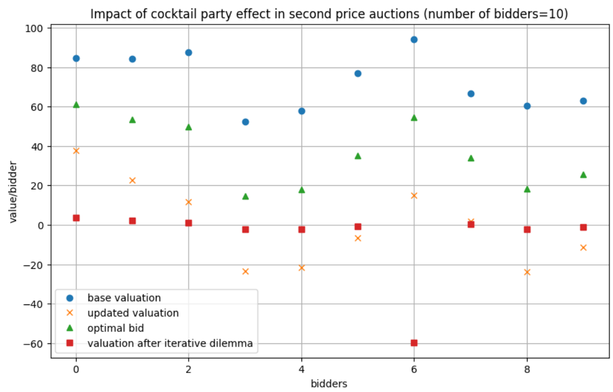


Fig. 1: Impact of Cocktail Party Effect in Second Price Auctions

represent initial individual opinions or beliefs before being influenced by the echo chamber. Risk: Individuals have a diverse range of beliefs that are not yet subject to group influence.

### Updated Valuation (Red Crosses)

This could symbolize how individual beliefs change after being exposed to the echo chamber, where opinions may become more extreme or polarized due to social influence. Risk: Information within an echo chamber can become distorted, leading to a departure from objective facts or initial positions.

### Optimal Bid (Green Triangles)

This could represent the "rationally adjusted" position one might take after considering the influence of the echo chamber, aiming to find a balance between personal beliefs and group opinions. Risk: Even when adjusting for the echo chamber effect, there's a risk of not fully escaping its influence or underestimating its impact on one's beliefs.

### Valuation after Iterative Dilemma (Red Squares)

This might indicate a further adjustment after multiple iterations of influence and learning, perhaps an individual's attempt to reach a more stable belief system. Risk: Over time, the echo chamber's influence could lead to increasingly entrenched positions, making it difficult for individuals to adjust their beliefs in light of new information.

From a risk management perspective, understanding these dynamics is critical.

### Diversity of Opinions

Encouraging a range of perspectives can mitigate the risk of the echo chamber effect by preventing any single viewpoint from becoming disproportionately influential.

### Critical Thinking

Promoting critical thinking skills can help individuals to evaluate information more objectively, rather than being swayed by the group.

### Open Communication Channels

Ensuring that communication channels remain open to outside information can help prevent the echo chamber from becoming too insular.

### Continuous Learning

Encouraging continuous learning and adjustment can help mitigate the risks associated with the iterative dilemma of increasingly entrenched beliefs.

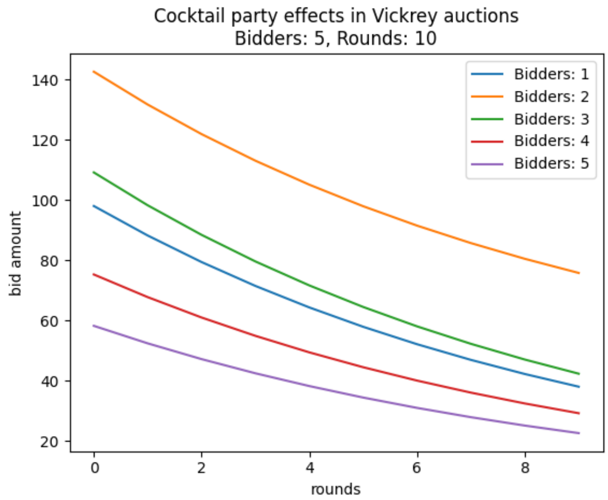


Fig. 2: Cocktail party effects in Vickrey auctions

In any system where information spread is important, such as markets, social networks, or even

Fig.2 shows bid amounts over several rounds in a Vickrey auction setting with a fixed number of bidders (5) and rounds (10). Each line represents the bid amount trajectory of a single bidder across these rounds. The trend shows that all bidders decrease their bid amounts as the rounds progress. This is labeled as the "Cocktail party effects in Vickrey auctions".

In the context of risk management for information spread within an echo chamber, the graph can be interpreted metaphorically. Here's how the depicted trends might relate to the dynamics of an echo chamber.

### Descending Bids Over Rounds

This could represent the diminishing strength of an initial opinion or piece of information as it's repeatedly shared within an echo chamber. It may indicate a dilution effect where the original message loses its impact over time due to constant repetition or challenges within the chamber.

### Different Rates of Decline

The varying slopes for each bidder suggest that not all individuals within an echo chamber adjust their views at the same rate. Some may be more resistant to change than others, which is common in social groups where some individuals hold onto beliefs more tightly.

### Risk Management Implications

### Monitoring Change Over Time

Just as the bids change over rounds, opinions in an echo chamber may evolve. It's important to monitor these changes

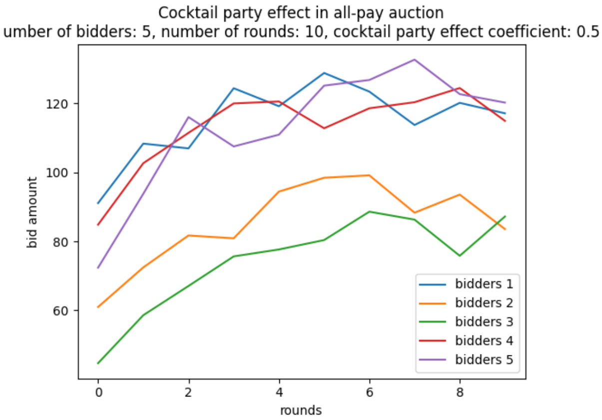


Fig. 3: Cocktail party effect in all-pay auction

to manage the risk of misinformation or the hardening of extreme views.

#### Encouraging Diversity

The varied trajectories suggest the benefit of having a diversity of opinions within a group to prevent homogenization of thought. Understanding Group Dynamics Just as each bidder's strategy adjusts over time in response to others, understanding the dynamics within an echo chamber can help manage the risks of groupthink and polarization.

#### Strategic Adjustments

As the bidders in the auction adapt their strategies based on others' bids, individuals within an echo chamber might also adjust their beliefs in response to social cues. Recognizing these adjustments is crucial for countering the risks of misinformation.

To manage risks associated with echo chambers effectively, one should: Promote fact-checking and verification to maintain a baseline of truth. Encourage exposure to diverse viewpoints to counteract the homogenizing effects. Facilitate open dialogue and constructive debate to help refine and moderate opinions. Be vigilant about the sources of information that feed into the echo chamber to prevent the spread of false narratives.

Fig.3 shows the bid amounts of 5 bidders across 10 rounds in an all-pay auction with the cocktail party effect, where the coefficient is set to 0.5. Unlike a traditional auction, in an all-pay auction, all bidders must pay their bid regardless of whether they win, which can mirror the way individuals 'pay' with their time or reputation when contributing to an echo chamber.

In terms of risk management for information spread within echo chambers, the graph can provide several insights.

#### Volatile Bid Trajectories

The bid amounts do not follow a consistent trend; instead, they show fluctuations over the rounds. This volatility might represent the variable nature of information within an echo chamber, where opinions and the spread of information can fluctuate significantly based on external factors or internal dynamics.

#### Cocktail Party Effect

The coefficient suggests that bidders are influenced by the average bid. In an echo chamber, individuals might similarly adjust their opinions to align with the perceived average position of the group, which could lead to group conformity over time.

#### Bid Convergence and Divergence

Some bidders' amounts converge or diverge as the rounds progress, which may reflect how certain views become more similar or more polarized within an echo chamber. It's important to manage the risk of extreme polarization, which can lead to misinformation or radicalization.

#### Risk Management Implications

#### Monitoring and Intervention

Just as bid strategies need monitoring and potentially adjustment in auctions, information spread within echo chambers requires constant monitoring and, if necessary, intervention to prevent misinformation.

#### Encouraging Independent Thinking

To counteract conformity, encouraging independent thinking and skepticism can help individuals critically assess information rather than following the crowd.

#### Diversity of Perspectives

The introduction of diverse viewpoints can disrupt the echo chamber effect, preventing any single narrative from dominating and thus reducing risks.

The result provides a visual representation of how individuals' contributions (or bids) to a discussion (or auction) may evolve when influenced by others around them. It illustrates the importance of understanding individual and collective behavior dynamics when managing the spread of information within any group, especially in an echo chamber where the risk of misinformation is heightened.

In particular, we model a situation in which there is information asymmetry among bidders, and organize ideas to examine how the auction outcome changes when some bidders have more advantageous information than others. We explore how this information asymmetry is affected by cocktail party effects, and by introducing network relationships among bidders, we examine how information diffuses through the network. We also use this model to organize with respect to how network structure affects the cocktail party effect and the diffusion of misinformation through changes in Nash equilibrium.

Our analysis begins in a setting where there are N bidders with respect to some fictional mixed information and each bidders has a valuation based on private information. This valuation can vary under information asymmetry and cocktail party effects. The network relationship between the bidders is represented by an adjacency matrix, which shows how the expected gain function and valuation adjustment in this model are defined. We also discuss how the bidder's probability of winning a bid is calculated.

It is possible to quantitatively analyze how information asymmetries, cocktail party effects, and network relationships among bidders affect the outcome of an auction. This approach provides new insights into auction theory and seeks to better understand the impact of information diffusion on economic decision making.

We will also focus on information asymmetries among bidders in shield bid auctions, and in particular analyze the impact of cocktail party effects and repetition dilemmas on bidders' strategies. Shield bid auctions, in which bidders submit their bids in secret and the highest bidder wins, are found in many real-world bidding environments. The strategic significance of this study is emphasized by modeling the effects of the cocktail party effect, in which bidders adjust their own valuations based on information gathered from other participants, and the strategic interactions that develop over multiple auction rounds, i.e., the effects of repeated dilemmas.

Specifically, we use a mathematical model to formulate the process of updating a bidder's beliefs and the associated strategy adjustment in order to find the optimal bidding strategy to maximize the bidder's expected gain. This process involves updating information via cocktail party effects, evaluating expected gains in future rounds given an iterative dilemma, and deriving a Nash equilibrium that synthesizes these factors.

The goal of this study is to reveal the complexity of information asymmetries and interactions in the formation of bidder strategies in shield bid auctions, and to provide strategic insights for auction designers and participants. This will connect insights on designing fairer and more efficient auctions to thinking about the routes of fake news diffusion and pathways of suppression to fake news by reading the information design framework backwards.

The model is premised on a setting where bidders possess private valuations influenced by asymmetric information and the cocktail party effect, which simulates the social dynamics of information exchange. Through rigorous mathematical formulations, we establish the expected utility function for each bidder, incorporating adjustments for valuation based on network-driven information flows and the intensity of the cocktail party effect. We model a situation of information asymmetry among bidders and analyze auction outcomes when certain bidders have advantageous information. We introduce different auction formats into the model, including first-price auctions, second-price auctions, and Vickrey auctions, compare the impact of cocktail party effects on each auction format, and finally consider the case where the auction is evaluated in terms of changes in Nash equilibrium.

## 2. Discussion:Cocktail Party Effects and the Diffusion of Misinformation through Changes in Nash Equilibrium

This paper also continues and summarizes the discussion of the hypothetical computational experiments. In this note, we analyze information asymmetry in auctions and its effect on the outcome. We consider simulations that intend to address the value of misinformation bias from the perspective of informational and digital health, as well as preventive measures against the lack of information judgment in society as a whole due to the excess of rate. In particular, this paper models a situation in which there is information asymmetry among bidders who invest in information, and organizes ideas to examine how auction outcomes change when some bidders have more favorable information than others. Explore how this information asymmetry is affected by the cocktail party effect due to the spread of noisy outer information, the spread of fake news, and other information scattering, and by introducing network relationships among bidders, how the network structure affects the cocktail party effect and the spread of misinformation By introducing network relationships among bidders, we have organized a method to evaluate how network structure affects cocktail party effects and the diffusion of misinformation through changes in Nash equilibrium.

Our analysis begins in a setting where there are N bidders with respect to some fictional mixed information and each bidders has a valuation based on private information. This valuation can vary under information asymmetry and cocktail party effects. The network relationship between the bidders is represented by an adjacency matrix, which shows how the expected gain function and valuation adjustment in this model are defined. We also touch on how the bidder's probability of winning a bid is calculated.

This paper takes a deep dive into information asymmetry among bidders and its strategic implications in the context of shielded bid auctions, specifically analyzing how cocktail party effects and repetition dilemmas affect bidders' behavior and auction outcomes. Shielded bid auctions are an auction format in which participants submit one sealed bid at a time, and the secretive nature of this process adds unique complexity to the bidders' strategies. Bidders must make their own bidding decisions based on limited information, and in doing so, they typically infer the potential strategies and valuations of other participants.

The cocktail party effect refers to the phenomenon in which a bidder overreacts to bits and pieces of information gathered from the words and actions of other bidders and adjusts his or her own evaluation and strategy accordingly. This effect causes the bidders' beliefs and strategies to be heavily influenced by the expected behavior of other bidders, independent of the actual progress of the auction. Repeated dilemmas, on the other hand, refer to strategic interactions that occur in situations where the same bidders participate in multiple auctions, creating complex dynamics in which the outcomes of past rounds affect future strategies.

In this study, we systematically analyze these phenomena using mathematical models to formulate the optimization problems faced by bidders. Specifically, we construct a model that takes into account the bidders' belief updating mechanism, information distortion due to cocktail party effects, and strategic interaction due to iterative dilemmas, with the goal of finding the optimal bidding strategy to maximize the bidders' expected gains. Through this model, we examine how bidders update their own beliefs, derive optimal bidding strategies, and how this affects the overall outcome of the auction.

In addition, we use the concept of Nash equilibrium to analyze the situation in which bidders mutually take optimal strategies and derive the equilibrium state of the auction. Through this analysis, we aim to provide insights for auction designers and participants to make more strategic decisions. Understanding the complexity of information asymmetries and strategic interactions in auctions is critical to the design and implementation of efficient and fair auction mechanisms. We would like to consider the implications of this research for the development of theoretical and practical knowledge in this area.

# 3. Discussion:Cocktail Party Effects and the Diffusion of Misinformation through Changes in Nash Equilibrium

We considers the mathematical model and calculation process that considers the cocktail party effect in imperfect information auctions. This model takes into account the situation where specific bidders have advantageous information compared to other bidders and analyzes the differences in strategies and outcomes in first-price auctions, second-price auctions, and Vickrey auctions.

Let $N$ be the set of bidders, with each bidder denoted by $i \in N$.

Let $v_i$ represent the true valuation of bidder $i$, and $b_i$ represent the bid amount of bidder $i$.

Assume that a specific bidder $k$ has advantageous information $I_k$ compared to other bidders.

## Modeling the Cocktail Party Effect

The cocktail party effect is modeled as the influence $\alpha_{ij}$ that bidder $i$ receives from the bidding strategy of others. It represents how much bidder $i$ is influenced by the actions of bidder $j$.

### First-Price Auction

(1) Expected utility of a bidder:

$$E[u_i] = (v_i - b_i) \times P(\text{win}_i | b_i, I_k)$$

where $P(\text{win}_i | b_i, I_k)$ is the probability of bidder $i$ winning the auction considering the bid amount $b_i$ and information $I_k$.

(2) Nash equilibrium: Bidders choose their bidding strategies to maximize their own expected utilities. The Nash equilibrium is a set of optimal strategies $b_i^*$ for all bidders.

### Second-Price Auction

(1) Expected utility of a bidder:

$$E[u_i] = (v_i - b_{\text{second}}) \times P(\text{win}_i | b_i, I_k)$$

where $b_{\text{second}}$ is the second-highest bid amount.

(2) Nash equilibrium: Bidders' strategy of bidding their true valuations $v_i$ becomes the Nash equilibrium.

### Vickrey Auction

(1) Expected utility of a bidder:

$$E[u_i] = (v_i - b_{\text{second}}) \times P(\text{win}_i | I_k)$$

In a Vickrey auction, the winner pays the second-highest bid amount, so the bid amount $b_i$ does not affect the expected utility.

(2) Nash equilibrium: Similarly, bidders' strategy of bidding their true valuations $v_i$ becomes the Nash equilibrium.

## Impact of the Cocktail Party Effect

The cocktail party effect can potentially alter the expected utilities and optimal strategies of bidders. Particularly in a first-price auction, the influence from other bidders' strategies might be significant, making the Nash equilibrium more complex and dynamic.

## Calculation Process

(1) Define the expected utility functions for bidders in each auction format.

(2) Modify the expected utility functions to include the cocktail party effect.

(3) Derive the optimal strategies for each bidder and find the Nash equilibrium.

(4) Conduct simulations with different auction formats and intensities of the cocktail party effect to compare and analyze the results.

This model allows for an understanding of the strategies and outcomes in different auction formats under imperfect information, as well as the impact of the cocktail party effect. Detailed analysis of the calculation process should be conducted through numerical analysis or simulations.

To model the asymmetry of information among bidders in the context of auctions considering the cocktail party effect, we develop detailed mathematical expressions and calculation processes as follows.

### Modeling the Cocktail Party Effect

The cocktail party effect is the phenomenon where bidders excessively react to the bidding behaviors or information of others. To model this effect, we introduce an updating function for the bidders' valuations.

(1) Basic valuation of bidders: Each bidder $i$ has a basic valuation $v_i$ for the object. This is based on the initial information each bidder possesses.

(2) Information update: Due to the cocktail party effect, bidders update their valuations based on the actions of others. The updated valuation $\tilde{v}_i$ for bidder $i$ is modeled as follows:

$$\tilde{v}_i = v_i + \sum_{j \neq i} \alpha_{ij}(b_j - v_i)$$

where $b_j$ is the bid amount of bidder $j$, and $\alpha_{ij}$ is a parameter indicating how much bidder $i$ is influenced by the information from bidder $j$.

### Expected Utility in First-Price Auction

(1) Expected utility of a bidder: The expected utility $E[u_i]$ for bidder $i$ is the gain from winning the auction minus the bid amount, using the updated valuation:

$$E[u_i] = (\tilde{v}_i - b_i) \times P(\text{win}_i | b_i)$$

where $P(\text{win}_i | b_i)$ is the probability of bidder $i$ winning the auction with the bid amount $b_i$.

### Impact of the Cocktail Party Effect

The cocktail party effect influences bidders' expected utilities and strategic choices. Especially, bidders need to adjust their bidding amounts considering the influence they receive from others' actions.

### Calculation Process

(1) Setting parameters: Set the values for $\alpha_{ij}$ to determine how much bidders are influenced by others' information.

(2) Calculating expected utility: Consider the updated valuation $\tilde{v}_i$ and each bidder's bidding strategy to calculate the expected utility.

(3) Deriving Nash equilibrium: Find the Nash equilibrium by identifying the bidding strategies that maximize each bidder's expected utility. Optimization techniques and game theory tools can be employed for this purpose.

To analyze the impact of the cocktail party effect in a first-price auction, we develop the following steps for mathematical expressions and calculation processes.

**Basic Valuation and Cocktail Party Effect**

Each bidder $i$ has a basic valuation $v_i$ for the object and updates their valuation based on bids observed from others. The update due to the cocktail party effect is modeled as follows:

$$\tilde{v}_i = v_i + \sum_{j \neq i} \alpha_{ij}(b_j - v_i)$$

Here, $\tilde{v}_i$ is the updated valuation, $b_j$ is the bid amount of bidder $j$, and $\alpha_{ij}$ indicates how much bidder $i$ is influenced by the bid of bidder $j$.

**Expected Utility in a First-Price Auction**

The expected utility for bidder $i$ in a first-price auction is expressed as:

$$E[u_i] = (\tilde{v}_i - b_i) \times P(\text{win}_i | b_i)$$

Here, $P(\text{win}_i | b_i)$ is the probability of bidder $i$ winning the auction with the bid amount $b_i$.

**Calculating Winning Probability**

The winning probability $P(\text{win}_i | b_i)$ depends on the bidding strategies of other bidders. A common approach is to assume that the other bidders' bids follow a certain probability distribution. For example, if we assume that the other bids are uniformly distributed, the winning probability can be expressed as:

$$P(\text{win}_i | b_i) = \prod_{j \neq i} P(b_j < b_i)$$

**Deriving Optimal Bidding Strategy**

Bidders choose their bid amounts to maximize their expected utilities. To achieve this, differentiate the expected utility function $E[u_i]$ with respect to $b_i$, and set the derivative to zero:

$$\frac{dE[u_i]}{db_i} = \frac{d}{db_i}\left[(\tilde{v}_i - b_i) \times P(\text{win}_i | b_i)\right] = 0$$

Solving this equation will yield the optimal bidding strategy $b_i^*$ for bidder $i$.

**Analyzing the Impact of the Cocktail Party Effect**

To analyze the impact of the cocktail party effect, calculate the optimal bidding strategy $b_i^*$ for different values of $\alpha_{ij}$ and compare how these parameters affect the bidders' strategies and the auction outcomes.

**Simulation and Analysis**

To understand the broader implications of the cocktail party effect in auction scenarios, conduct simulations under varying auction conditions and with different intensities of the cocktail party effect. By changing the parameters and analyzing the outcomes, insights can be gained into how information asymmetry and bidder behavior influence auction dynamics.

**Extending to Second-Price and Vickrey Auctions**

The analysis can be extended to second-price (Vickrey) auctions, where the winning bidder pays the second-highest bid. The cocktail party effect may influence bidding strategies differently in this context, as the optimal strategy in a Vickrey auction under standard conditions is to bid one's true valuation.

$$E[u_i] = (\tilde{v}_i - p_{\text{second}}) \times P(\text{win}_i | b_i)$$

Here, $p_{\text{second}}$ is the second-highest bid, and the expected utility calculation adjusts accordingly. The impact of the cocktail party effect in such auctions warrants a separate analysis, as the strategic considerations differ from first-price auctions.

**Modeling Repetitive Auctions**

In repetitive auction settings, bidders may adjust their strategies based on outcomes of previous auctions. This dynamic introduces a layer of complexity, as bidders must weigh historical performance and future expectations in their decision-making process.

**Addressing Complex Dynamics**

The cocktail party effect can lead to complex dynamics, especially when combined with other factors like bid shading, risk aversion, and auction format variations. Advanced modeling techniques, including simulations and agent-based models, may be required to fully capture these dynamics.

The mathematical models and calculation processes outlined provide a framework for analyzing the cocktail party effect in auction settings. By incorporating factors such as information asymmetry, bidder behavior, and auction dynamics, these models contribute to a deeper understanding of strategic interactions in auctions. Future research could further refine these models, explore empirical validations, and extend the analysis to more complex auction formats and settings.

# 4. Discussion:Modeling the Iterative Dilemma

The iterative dilemma illustrates the situation where bidders learn from the outcomes of past auctions and adapt their strategies for the future. The strategy of each bidder in each round is updated based on the results of previous rounds.

$$b_i^{(t+1)} = f(b_i^{(t)}, B_{-i}^{(t)}, \text{outcome}^{(t)})$$

where $b_i^{(t)}$ is the bid of bidder $i$ in round $t$, $B_{-i}^{(t)}$ is the set of bids from other bidders in round $t$, and outcome$^{(t)}$ is the result of round $t$.

1. **Initial Setup:** Set the initial bid and valuation for each bidder. 2. **Influence of the Cocktail Party Effect:** In each round, bidders adjust their bidding strategy based on the bids of other bidders. 3. **Update of the Iterative Dilemma:** Update the strategy of bidders based on the results of each round. 4. **Search for Nash Equilibrium:** Find the equilibrium state when each bidder chooses a strategy that maximizes their expected utility.

The computational process of modeling the iterative dilemma is explained in detail. In the iterative dilemma, each bidder must learn from past outcomes and adapt their future strategy. This process requires defining how a bidder's strategy changes based on past experiences.

### Definition of Bidder's Strategy

Let $s_i^{(t)}$ represent the strategy of bidder $i$ at time step $t$, indicating the bidder's action (e.g., bid amount). The initial strategy $s_i^{(0)}$ is set based on the bidder's initial beliefs and information.

### Learning from Past Outcomes

Bidders update their strategy based on the outcomes of past auctions. This process is represented by a general function as follows:

$$s_i^{(t+1)} = \phi(s_i^{(t)}, \text{history}^{(t)})$$

where $\phi$ is the strategy update function, and history$^{(t)}$ is the history of all auction outcomes up to time step $t$.

### An Example of a Strategy Update Function

An example of a strategy update function $\phi$ could be to increase the bid amount if winning and decrease it if losing. In this case, the function would be:

$$s_i^{(t+1)} = s_i^{(t)} + \delta \times (\text{win}_i^{(t)} - \text{lose}_i^{(t)})$$

where $\delta$ is a parameter to adjust the strategy, $\text{win}_i^{(t)}$ is a function that returns 1 if bidder $i$ wins at time step $t$ and 0 otherwise, and $\text{lose}_i^{(t)}$ is a function that returns 1 if the bidder loses and 0 otherwise.

#### Adaptability of the Strategy

The strategy of a bidder changes adaptively over the course of repetition. This adaptability can be incorporated into the strategy update function to allow for flexibility. For example, the influence of past outcomes can be varied over time steps, enabling a bidder to place more emphasis on recent results.

#### Iteration of the Computational Process

The computational process in this model is iterative. At each time step, bidders update their strategy and participate in the auction based on the new strategy. This process continues until a predefined condition (e.g., a certain number of time steps has elapsed) is met or until the strategies converge (stop changing).

## Discussion:Modeling the Effects of Cocktail Party Phenomenon and Iterative Dilemma in Double Auctions

To model the effects of the cocktail party phenomenon and the iterative dilemma in double auctions, it is necessary to consider the situation where multiple bidders and sellers exist, and to represent how each strategy influences each other. Here, we explain how the cocktail party effect influences the strategies of both bidders and sellers, and how the iterative dilemma is incorporated into the strategies.

Let the set of bidders be $B = \{b_1, b_2, ..., b_n\}$ and the set of sellers be $S = \{s_1, s_2, ..., s_m\}$. Each bidder $b_i$ has a valuation $v_{b_i}$ for the goods, and each seller $s_j$ has a minimum acceptable price $c_{s_j}$ for the goods. Due to the cocktail party effect, bidders and sellers are influenced by the actions of others and adjust their strategies accordingly.

### Modeling the Cocktail Party Effect

The adjustment of bidders' bid amounts and sellers' set prices, considering the cocktail party effect, can be modeled as follows:

1. Adjustment of bidders' bid amounts:

$$b_i^{(t+1)} = b_i^{(t)} + \alpha \sum_{k \neq i} \left( b_k^{(t)} - b_i^{(t)} \right)$$

Here, $b_i^{(t)}$ is the bid amount of bidder $i$ at time $t$, and $\alpha$ is a parameter indicating the sensitivity to the bid amounts of other bidders.

2. Adjustment of sellers' set prices:

$$c_j^{(t+1)} = c_j^{(t)} + \beta \sum_{l \neq j} \left( c_l^{(t)} - c_j^{(t)} \right)$$

Here, $c_j^{(t)}$ is the set price of seller $j$ at time $t$, and $\beta$ is a parameter indicating the sensitivity to the set prices of other sellers.

### Modeling the Iterative Dilemma

In the iterative dilemma, bidders and sellers learn from the results of past transactions and adjust their future strategies. This adaptation process can be represented as follows:

1. Strategy update based on past transaction results: Bidders and sellers record their victories and losses in past transactions and adjust their strategies based on these records. For example, a bidder could increase or decrease future bid amounts based on past successful bid amounts.

2. Definition of the strategy update function:

$$s_i^{(t+1)} = s_i^{(t)} + \gamma \left( \text{win}_i^{(t)} - \text{lose}_i^{(t)} \right)$$

Here, $s_i^{(t)}$ is the strategy of participant $i$ at time $t$, $\gamma$ is a parameter indicating the sensitivity of strategy updates, $\text{win}_i^{(t)}$ and $\text{lose}_i^{(t)}$ are indicators of victory and defeat, respectively.

### Computational Process

1. At each time step, bidders and sellers adjust their strategies considering the cocktail party effect. 2. After transactions are made, bidders and sellers update their strategies based on the outcomes. 3. This process is repeated until a specific condition is met or the strategies converge.

This model allows us to understand how the cocktail party effect influences the strategies of bidders and sellers and how the iterative dilemma contributes to the evolution of strategies.

In modeling the iterative dilemma, it is necessary to quantify the process by which players (bidders or sellers) learn from past experiences and adjust their future behavioral strategies accordingly. Here, we describe the evolution of strategies in such iterative games mathematically and explain the computational process.

### Modeling the Iterative Dilemma

1. **Strategy Definition:** Let $s_i$ represent the strategy of each player $i$, which is the set of strategic parameters (e.g., bid amount, price setting) determined based on the player's past experiences.

2. **Experience Update:** After each round, players update their 'experience' using the outcomes (win or lose) of that round. This experience represents the internal state or beliefs that influence future strategic decisions.

#### Mathematical Expression

Given the strategy $s_i^{(t)}$ of player $i$ at round $t$, the strategy $s_i^{(t+1)}$ for round $t+1$ is updated as follows:

$$s_i^{(t+1)} = s_i^{(t)} + \gamma \left( \text{Outcome}_i^{(t)} \cdot s_i^{(t)} \right)$$

where $\gamma$ is the learning rate (a parameter controlling the speed of strategy adjustment), and $\text{Outcome}_i^{(t)}$ is the adjustment based on the outcomes of round $t$.

#### Computational Process

1. **Setting Initial Conditions:** Set the initial strategy $s_i^{(0)}$ for each player $i$. This defines the initial actions of the players. The initial strategy can be set randomly or based on assumed strategic behaviors.

2. **Executing Rounds:** In round $t$, each player $i$ acts according to the strategy $s_i^{(t)}$. According to the rules of the game, determine the results of each player's actions (win, lose, draw, etc.).

3. **Evaluating Results and Learning:** For each player $i$, calculate the adjustment $\text{Outcome}_i^{(t)}$ based on the results of round $t$. For example, set a positive value for a win and a negative value for a loss. Update the strategy for the next round using the learning rate $\gamma$:

$$s_i^{(t+1)} = s_i^{(t)} + \gamma \left( \text{Outcome}_i^{(t)} \cdot s_i^{(t)} \right)$$

Here, $\text{Outcome}_i^{(t)} \cdot s_i^{(t)}$ represents the difference between the outcome of the round and the current strategy, and this difference is used to adjust the strategy.

4. **Checking for Convergence:** Repeat steps 2 and 3 until the strategy updates meet a certain criterion (e.g., the change in strategy falls below a certain threshold). Continue the calculation until a predetermined number of rounds have passed or until the strategies converge.

### Example Calculation

Here is a simple example to illustrate the computational process:

**Initial Strategies:** $s_1^{(0)} = 0.5$, $s_2^{(0)} = 0.5$ (initial strategies of players 1 and 2)

**Learning Rate:** $\gamma = 0.1$

**Round 1 Results:** Player 1 wins ($\text{Outcome}_1^{(1)} = 1$, $\text{Outcome}_2^{(1)} = -1$)

**Strategy Update for Player 1:**

$$s_1^{(1)} = 0.5 + 0.1 \times (1 \cdot 0.5) = 0.55$$

**Strategy Update for Player 2:**

$$s_2^{(1)} = 0.5 + 0.1 \times (-1 \cdot 0.5) = 0.45$$

## 5. Discussion:Modeling the Cocktail Party Effect and Iterative Dilemma in Common Value Auctions

To model the cocktail party effect and the iterative dilemma in common value auctions, we proceed with the following steps. In this setting, there exists a true value of the item (common value) for all bidders, but bidders have uncertainty about this true value. The cocktail party effect is modeled as a phenomenon where bidders update their valuation based on information gained from the actions and statements of other bidders.

### Model Assumptions

The true value of the item is $V$, but each bidder does not have complete information about $V$.

Each bidder $i$ has their own belief $v_i$ about $V$, which is an uncertain estimate of $V$.

Through the cocktail party effect, bidders can gain additional information about $V$ from the actions of other bidders.

1. **Bidders' Expected Utility Function:**

$$E[u_i] = (V - b_i) \times P(\text{winning} | b_i, \text{info}_i)$$

Here, $b_i$ is the bid amount of bidder $i$, and $\text{info}_i$ is the information held by bidder $i$, formed through the cocktail party effect.

2. **Information Update through the Cocktail Party Effect:**

$$\text{info}_i^{new} = f(\text{info}_i, \text{actions}_{-i})$$

Here, $\text{actions}_{-i}$ represents the information obtained from the actions of bidders other than $i$.

3. **Calculation of Nash Equilibrium:** Bidders choose their bid amounts $b_i$ to maximize their own expected utility. The combination of bid amounts at this point forms a Nash equilibrium.

### Computational Process

1. **Setting Initial Beliefs:** Initial beliefs $v_i^{(0)}$ about $V$ for each bidder are set.

2. **Information Update Loop:** Observe the actions of each bidder in round $t$ and update each bidder's information $\text{info}_i$. Based on the updated information, calculate a new belief $v_i^{(t+1)}$ for each bidder about $V$.

3. **Determination of Bidding Amounts and Calculation of Nash Equilibrium:** Based on the updated beliefs, calculate the bidding amount for each bidder that maximizes the expected utility. The combination of optimal strategies when all bidders adopt their best strategy is considered as Nash equilibrium.

4. **Consideration of the Iterative Dilemma:** Accumulate the results of Nash equilibria from each round and use them for long-term strategy adjustments. For maximizing long-term gains, bidders may accept short-term losses.

This model allows us to analyze the impact of the cocktail party effect in common value auctions and how the iterative dilemma influences bidders' strategies. This analysis provides a foundation for understanding how bidders form and adjust their optimal strategies amidst imperfect information.

# 6. Discussion:Detailed Explanation of the Computational Process

The computational process, considering the cocktail party effect and the iterative dilemma in common value auctions, is elaborated focusing on the information each bidder holds, their expected utility, and the derivation of Nash equilibrium.

**Setting Initial Beliefs**

For each bidder $i$, initial beliefs $v_i^{(0)}$ about the true value $V$ of the item are set. These beliefs could be formed based on public information about the item or past experiences.

**Information Update**

In each round $t$, bidders observe the actions $\text{actions}_{-i}^{(t)}$ of other bidders and update their own information $\text{info}_i^{(t)}$. This information update is represented by the following function:

$$\text{info}_i^{(t+1)} = f\left(\text{info}_i^{(t)}, \text{actions}_{-i}^{(t)}\right)$$

where $f$ is the information update function, representing how new information is incorporated based on the actions of other bidders.

**Belief Update**

After the information is updated, each bidder calculates a new belief $v_i^{(t+1)}$ as follows:

$$v_i^{(t+1)} = v_i^{(t)} + \lambda\left(\text{info}_i^{(t+1)} - v_i^{(t)}\right)$$

where $\lambda$ is a parameter that represents the speed of information update, ranging from 0 to 1.

**Calculation of Expected Utility and Bidding Amounts**

Based on the updated beliefs, the expected utility for each bidder is calculated as follows:

$$E[u_i] = (v_i^{(t+1)} - b_i) \times P(\text{winning}|b_i, \text{info}_i^{(t+1)})$$

where $P(\text{winning}|b_i, \text{info}_i^{(t+1)})$ is the probability of bidder $i$ winning the auction with bid $b_i$, depending on the actions of other bidders and market conditions.

Bidders will choose the bid amount $b_i$ that maximizes this expected utility.

**Derivation of Nash Equilibrium**

The combination of bid amounts when all bidders adopt their strategy that maximizes their own expected utility forms a Nash equilibrium. Nash equilibrium refers to a state where no bidder can improve their payoff by unilaterally changing their strategy, given the strategies of other bidders are fixed.

**Consideration of the Iterative Dilemma**

The iterative dilemma refers to a situation where bidders adopt non-cooperative strategies for temporary gains, at the expense of long-term benefits. To model this effect, consider the evolution of strategies over multiple rounds, allowing bidders to choose the optimal strategy from a long-term perspective.

Through this computational process, the impact of the cocktail party effect in common value auctions can be quantitatively assessed, and the influence of the iterative dilemma on bidders' strategies can be analyzed.

## Detailed Explanation of Information Update Process

The information update process in Next Step is elaborated here. This step models how each bidder incorporates new information from the actions of other bidders.

### Information Update Process

Each bidder $i$ observes the actions $\text{actions}_{-i}^{(t)}$ of other bidders in round $t$ and updates their own information $\text{info}_i^{(t)}$ to form new information $\text{info}_i^{(t+1)}$.

### Mathematical Formulation

The information update function is represented as follows:

$$\text{info}_i^{(t+1)} = f\left(\text{info}_i^{(t)}, \text{actions}_{-i}^{(t)}\right)$$

where, $\text{info}_i^{(t)}$ is the information set of bidder $i$ at round $t$, $\text{actions}_{-i}^{(t)}$ is the set of actions from bidders other than $i$ in round $t$, $f$ is the information update function, indicating how new information is integrated.

1. **Analysis of Other Bidders' Actions:** Bidder $i$ observes the actions $\text{actions}_{-i}^{(t)}$ of other bidders and analyzes how these actions affect their own information set. For example, if a bidder bids unexpectedly high, bidder $i$ may reassess the value of the item.

2. **Update of Information:** Bidder $i$ updates their own information set based on the observed actions. This process is formalized as the formation of new information $\text{info}_i^{(t+1)}$.

### Specific Example

For instance, consider the information update function $f$ takes the following form:

$$\text{info}_i^{(t+1)} = \text{info}_i^{(t)} + \sum_{j \neq i} w_{ij} \cdot \left(\text{action}_j^{(t)} - \text{info}_i^{(t)}\right)$$

where, $w_{ij}$ is the weight of the impact of bidder $j$'s action on bidder $i$'s information update, $\text{action}_j^{(t)}$ is the action of bidder $j$ in round $t$.

This equation averages the impact of other bidders' actions on bidder $i$'s information set, weighted by $w_{ij}$, which indicates how much bidder $i$ trusts the action of bidder $j$. Through this process, bidders learn new information from the actions of other bidders and update their information sets.

## Belief Update Process

Models the process by which each bidder updates their belief based on newly acquired information. This step formalizes how bidders incorporate new inferences from other bidders' actions and auction conditions into their beliefs and strategies.

#### Belief Update Process

Each bidder $i$ uses the newly acquired information $\text{info}_i^{(t+1)}$ to update their belief $\text{belief}_i^{(t)}$, forming a new belief $\text{belief}_i^{(t+1)}$ for the next round.

#### Mathematical Formulation

The belief update function is represented as follows:

$$\text{belief}_i^{(t+1)} = g\left(\text{belief}_i^{(t)}, \text{info}_i^{(t+1)}\right)$$

where, $\text{belief}_i^{(t)}$ is the belief of bidder $i$ at round $t$, $\text{info}_i^{(t+1)}$ is the new information acquired by bidder $i$, $g$ is the belief update function, indicating how new information is incorporated into beliefs.

1. **Evaluation of New Information:** Bidder $i$ evaluates the newly acquired information $\text{info}_i^{(t+1)}$ and analyzes its impact on their own belief.

2. **Update of Belief:** Based on the evaluated information, bidder $i$ updates their belief $\text{belief}_i^{(t)}$ to form a new belief $\text{belief}_i^{(t+1)}$.

#### Specific Example

Consider a belief update function $g$ that takes the following form:

$$\text{belief}_i^{(t+1)} = \text{belief}_i^{(t)} + \lambda \cdot \left(\text{info}_i^{(t+1)} - \text{belief}_i^{(t)}\right)$$

where, $\lambda$ is a parameter that represents the speed or adaptability of information update, indicating how quickly a bidder incorporates new information into their belief.

This equation shows the process of updating beliefs based on the difference between new information $\text{info}_i^{(t+1)}$ and the current belief $\text{belief}_i^{(t)}$. If the new information significantly differs from the current belief, the belief undergoes a larger change. Through this process, bidders incorporate new information into their beliefs, forming the basis for strategizing in the next round.

# 7. Perspect of Research:Deriving Nash Equilibrium Based on Updated Beliefs and Strategies

We derive the Nash Equilibrium, which is a state where all players choose their optimal strategies and no player can improve their payoff by unilaterally changing their strategy. This step involves finding an overall equilibrium state by each bidder considering the strategies of others and selecting their own optimal strategy.

## Definition of Nash Equilibrium

The Nash Equilibrium in an auction, denoted as $\mathbf{b}^* = (b_1^*, b_2^*, ..., b_N^*)$, is the combination of bid amounts for all bidders $i$ that satisfies the following condition for each bidder:

$$b_i^* = \arg\max_{b_i} E[u_i(b_i, \mathbf{b}_{-i}^*)]$$

where $\mathbf{b}_{-i}^*$ represents the combination of optimal bidding strategies of all bidders except bidder $i$.

## Computational Process

1. **Derivation of Optimal Response Functions for Each Bidder:** For each bidder $i$, derive the optimal response function representing the optimal bid amount $b_i^*$ given the strategies $\mathbf{b}_{-i}$ of other bidders.

$$R_i(\mathbf{b}_{-i}) = \arg\max_{b_i} E[u_i(b_i, \mathbf{b}_{-i})]$$

2. **Setting Up Simultaneous Equations:** Use the optimal response functions of all bidders to set up simultaneous equations.

$$b_i^* = R_i(\mathbf{b}_{-i}^*), \quad \forall i$$

3. **Derivation of Nash Equilibrium:** Solve the above simultaneous equations to derive the Nash Equilibrium $\mathbf{b}^*$ for the strategies of all bidders.

## Specific Computational Methods

**Numerical Solution:** In many cases, the simultaneous equations are difficult to solve analytically, so numerical methods (e.g., fixed-point iteration or Newton's method) are used to find an approximate solution.

**Simulation:** Using simulation, each bidder can try different strategies and search for the optimal response to the strategies of other bidders. By repeating the simulation, we can approach an equilibrium state.

## Conclusion

Deriving Nash Equilibrium demands more sophisticated mathematical and numerical methods as the auction model and bidders' strategies become more complex. In practice, it is often difficult to obtain a complete analytical solution, and it is common to rely on approximate solutions or numerical methods. Through this process, auction designers and participants can gain deep insights into how different strategies interact and affect the final outcomes of the auction.

To solve the optimization problem faced by bidders, we explain a method using the assumption that the bidders' beliefs follow a normal distribution. In this case, the bidder's belief $\text{belief}_i^{(t+1)}$ is modeled as a normal distribution $N(\mu, \sigma^2)$ with mean $\mu$ and standard deviation $\sigma$. Bidders attempt to optimize their bid amounts based on this distribution.

**Definition of Expected Utility**

Bidder $i$'s expected utility $E[u_i]$ is defined based on their own bid amount $b_i$ and the distribution of other bidders' bid amounts. The expected utility is expressed as follows:

$$E[u_i] = \int_v (v - b_i) \cdot P(\text{winning}|b_i, v) \cdot f(v|\mu, \sigma^2) dv$$

where $P(\text{winning}|b_i, v)$ is the probability that bidder $i$ wins the auction by bidding $b_i$ for a value $v$, and $f(v|\mu, \sigma^2)$ is the probability density function of the normal distribution that the bidder's belief follows.

**Setting Up the Optimization Problem**

The objective function that bidder $i$ seeks to maximize is the above-defined expected utility $E[u_i]$. The optimization problem is expressed as:

$$\max_{b_i} E[u_i]$$

**Derivation of the Solution**

To solve this optimization problem, we first differentiate $E[u_i]$ with respect to $b_i$ and set the derivative equal to 0 to find the optimal bid amount $b_i^*$.

$$\frac{dE[u_i]}{db_i} = 0$$

The $b_i^*$ that satisfies this derivative equation is the optimal bid amount for bidder $i$.

**Numerical Optimization**

In many cases, the integral and differentiation mentioned above are difficult to solve analytically, so numerical optimization methods (e.g., Newton's method or gradient descent) are used to approximate $b_i^*$.

## Numerical Example

As a specific numerical example, suppose the bidder's belief follows a normal distribution with $\mu = 100$ and $\sigma = 15$. Using the above computational process along with specific numerical values, the optimal bid amount $b_i^*$ is numerically determined.

1. Select a numerical optimization method (e.g., Newton's method). 2. Set an initial value $b_i^{(0)}$ and iteratively update $b_i$ to satisfy $\frac{dE[u_i]}{db_i} = 0$. 3. The $b_i$ at the point where the convergence criterion (e.g., the absolute value of the change is below a certain threshold) is met is considered the optimal bid amount $b_i^*$.

Through this process, bidders can quantitatively determine the optimal bidding strategy considering their beliefs and market conditions.

When considering the repeated dilemma, bidders need to take into account not only the current round's payoff but also future rounds' payoffs. Therefore, the calculation of a bidder's expected utility models the discounted sum of future payoffs in addition to the direct payoff of a single round. Here, we explain the computational process for calculating expected utility considering the repeated dilemma.

**Modeling Future Payoffs**

Bidder $i$'s future payoffs are modeled as follows:

$$E[u_i^{future}] = \sum_{t=1}^{T} \delta^t \cdot E[u_i^{(t)}]$$

where $E[u_i^{(t)}]$ is the expected utility of bidder $i$ in future round $t$, $\delta$ is the discount rate for future payoffs ($0 < \delta < 1$), and $T$ is the total number of future rounds considered.

**Redefining Current Round's Expected Utility**

The expected utility for the current round is redefined as the sum of the direct utility and the discounted sum of future payoffs.

$$E[u_i^{current}] = E[u_i] + E[u_i^{future}]$$

where $E[u_i]$ is the direct expected utility in the current round.

**Redefining the Optimization Problem**

Bidders choose their bid amount $b_i$ to maximize the current expected utility $E[u_i^{current}]$. The optimization problem is thus expressed as:

$$\max_{b_i} E[u_i^{current}]$$

**Derivation of the Solution**

To find the optimal bid amount $b_i^*$, differentiate $E[u_i^{current}]$ with respect to $b_i$ and set it equal to 0.

$$\frac{dE[u_i^{current}]}{db_i} = 0$$

**Numerical Optimization**

Often, it is challenging to solve the above differential equation analytically, so numerical optimization methods are used to find $b_i^*$.

## Numerical Example

Assuming a discount rate $\delta = 0.9$, a total of $T = 5$ future rounds, a direct expected utility for the current round $E[u_i] = 50$, and an expected utility of $E[u_i^{(t)}] = 40$ for each future round, the total future payoffs are calculated as:

$$E[u_i^{future}] = 0.9\times40+0.9^2\times40+0.9^3\times40+0.9^4\times40+0.9^5\times40$$

This is added to $E[u_i] = 50$ to calculate $E[u_i^{current}]$, and the optimal bid amount $b_i^*$ is determined. This process is executed using a specific numerical optimization algorithm along with concrete numerical values.

Through this process, bidders can quantitatively determine their optimal bidding strategy, taking into account both their beliefs and market conditions.

In the context of sealed-bid auctions, we model the impact of the cocktail party effect and the repeated dilemma as follows: in a sealed-bid auction, each bidder submits their bid amount secretly, and the highest bidder wins the item. Considering the cocktail party effect assumes that bidders are influenced by the potential bids of others. The repeated dilemma arises through strategic interactions over multiple auction rounds.

# Mathematical Formulation

1. **Definition of Bidder's Expected Utility:**

$$E[u_i] = (v_i - b_i) \times P(\text{winning}_i|b_i, \text{beliefs})$$

Here, $v_i$ is the true valuation of the item by bidder $i$, $b_i$ is the bid amount by bidder $i$, and $P(\text{winning}_i|b_i, \text{beliefs})$ is the probability of bidder $i$ winning the auction, dependent on the beliefs about other bidders' bid amounts.

2. **Modeling the Cocktail Party Effect:**

$$\text{beliefs} = \text{beliefs}_0 + \alpha \times \text{CP\_effect}$$

Where $\text{beliefs}_0$ are the original beliefs (about other bidders' bid amounts), $\alpha$ is the strength of the cocktail party effect, and CP_effect represents the change in beliefs due to the cocktail party effect.

3. **Incorporating the Repeated Dilemma:** To account for the repeated dilemma, bidders consider the payoffs in future rounds. This includes the likelihood of winning in future rounds and the strategic interactions therein.

$$E[u_i^{total}] = E[u_i] + \delta \times E[u_i^{future}]$$

Where $E[u_i^{future}]$ is the expected utility in future rounds, and $\delta$ is the discount rate for future payoffs.

1. **Setting Initial Beliefs:** Each bidder has initial beliefs about the bid amounts of others.

2. **Applying the Cocktail Party Effect:** Bidders update their beliefs about other bidders' bid amounts through the cocktail party effect.

3. **Determining Optimal Bid Amounts:** Bidders determine their bid amounts to maximize their utility for the current and future rounds.

$$b_i^* = \arg\max_{b_i} E[u_i^{total}]$$

4. **Deriving Nash Equilibrium:** The Nash equilibrium is sought where all bidders choose their optimal bidding strategies.

# Conclusion

Modeling the impact of the cocktail party effect and the repeated dilemma in sealed-bid auctions involves complex strategic interactions and advanced mathematical and numerical methods. The Nash equilibrium provides a framework for understanding the optimal strategies of bidders in the face of incomplete information and strategic uncertainty.

# References

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[1] "Measurement error mitigation in quantum computers through classical bit-flip correction" (2022). In *Physical Review*. DOI: 10.1103/physreva.105.062404. [Online]. Available: http://arxiv.org/pdf/2007.03663

[2] Caroline Jacqueline Denise Berdou et al. "One Hundred Second Bit-Flip Time in a Two-Photon Dissipative Oscillator" (2022). In *PRX Quantum*. DOI: 10.1103/PRXQuantum.4.020350.

[3] "Using classical bit-flip correction for error mitigation in quantum computations including 2-qubit correlations" (2022). [Proceedings Article]. DOI: 10.22323/1.396.0327.

[4] Gaojun Luo, Martianus Frederic Ezerman, San Ling. "Asymmetric quantum Griesmer codes detecting a single bit-flip error" (2022). In *Discrete Mathematics*. DOI: 10.1016/j.disc.2022.113088.

[5] Nur Izzati Ishak, Sithi V. Muniandy, Wu Yi Chong. "Entropy analysis of the discrete-time quantum walk under bit-flip noise channel" (2021). In *Physica A-statistical Mechanics and Its Applications*. DOI: 10.1016/J.PHYSA.2021.126371.

[6] Enaul Haq Shaik et al. "QCA-Based Pulse/Bit Sequence Detector Using Low Quantum Cost D-Flip Flop" (2022). DOI: 10.1142/s0218126623500822.

[7] Farhan Feroz, A. B. M. Alim Al Islam. "Scaling Up Bit-Flip Quantum Error Correction" (2020). [Proceedings Article]. DOI: 10.1145/3428363.3428372.

[8] "Effect of Quantum Repetition Code on Fidelity of Bell States in Bit Flip Channels" (2022). [Proceedings Article]. DOI: 10.1109/icece57408.2022.10088665.

[9] Lena Funcke et al. "Measurement Error Mitigation in Quantum Computers Through Classical Bit-Flip Correction" (2020). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/2007.03663.pdf

[10] Alistair W. R. Smith et al. "Qubit readout error mitigation with bit-flip averaging" (2021). In *Science Advances*. DOI: 10.1126/SCIADV.ABI8009.

[11] Constantia Alexandrou et al. "Using classical bit-flip correction for error mitigation including 2-qubit correlations." (2021). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/2111.08551.pdf

[12] William Livingston et al. "Experimental demonstration of continuous quantum error correction." (2021). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/2107.11398.pdf

[13] Constantia Alexandrou et al. "Investigating the variance increase of readout error mitigation through classical bit-flip correction on IBM and Rigetti quantum computers." (2021). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/2111.05026

[14] Raphaël Lescanne et al. "Exponential suppression of bit-flips in a qubit encoded in an oscillator." (2020). In *Nature Physics*. DOI: 10.1038/S41567-020-0824-X. [Online]. Available: https://biblio.ugent.be/publication/8669531/file/8669532.pdf

[15] Raphaël Lescanne et al. "Exponential suppression of bit-flips in a qubit encoded in an oscillator." (2019). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/1907.11729.pdf

[16] Diego Ristè et al. "Real-time processing of stabilizer measurements in a bit-flip code." (2020). In *npj Quantum Information*. DOI: 10.1038/S41534-020-00304-Y.

[17] Bernard Zygelman. "Computare Errare Est: Quantum Error Correction." (2018). In *Book Chapter*. DOI: 10.1007/978-3-319-91629-3$_9$.

[18] I. Serban et al. "Qubit decoherence due to detector switching." (2015). In *EPJ Quantum Technology*. DOI: 10.1140/EPJQT/S40507-015-0020-6. [Online]. Available: https://link.springer.com/content/pdf/10.1140

[19] Matt McEwen et al. "Removing leakage-induced correlated errors in superconducting quantum error correction." (2021). In *Nature Communications*. DOI: 10.1038/S41467-021-21982-Y.

[20] "Measurement error mitigation in quantum computers through classical bit-flip correction" (2020). In *arXiv: Quantum Physics*. [Online]. Available: https://arxiv.org/pdf/2007.03663.pdf

[21] Alistair W. R. Smith et al. "Qubit readout error mitigation with bit-flip averaging." (2021). In *Science Advances*. DOI: 10.1126/SCIADV.ABI8009. [Online]. Available: https://advances.sciencemag.org/content/7/47/eabi8009

[22] Biswas, T., Stock, G., Fink, T. (2018). *Opinion Dynamics on a Quantum Computer: The Role of Entanglement in Fostering Consensus. Physical Review Letters, 121(12), 120502.*

[23] Acerbi, F., Perarnau-Llobet, M., Di Marco, G. (2021). *Quantum dynamics of opinion formation on networks: the Fermi-Pasta-Ulam-Tsingou problem. New Journal of Physics, 23(9), 093059.*

[24] Di Marco, G., Tomassini, L., Anteneodo, C. (2019). *Quantum Opinion Dynamics. Scientific Reports, 9(1), 1-8.*

[25] Ma, H., Chen, Y. (2021). *Quantum-Enhanced Opinion Dynamics in Complex Networks. Entropy, 23(4), 426.*

[26] Li, X., Liu, Y., Zhang, Y. (2020). *Quantum-inspired opinion dynamics model with emotion. Chaos, Solitons Fractals, 132, 109509.*

[27] Galam, S. (2017). *Sociophysics: A personal testimony. The European Physical Journal B, 90(2), 1-22.*

[28] Nyczka, P., Holyst, J. A., Hołyst, R. (2012). *Opinion formation model with strong leader and external impact. Physical Review E, 85(6), 066109.*

[29] Ben-Naim, E., Krapivsky, P. L., Vazquez, F. (2003). *Dynamics of opinion formation. Physical Review E, 67(3), 031104.*

[30] Dandekar, P., Goel, A., Lee, D. T. (2013). *Biased assimilation, homophily, and the dynamics of polarization. Proceedings of the National Academy of Sciences, 110(15), 5791-5796.*

[31] Castellano, C., Fortunato, S., Loreto, V. (2009). *Statistical physics of social dynamics. Reviews of Modern Physics, 81(2), 591.*

[32] Galam, S. (2017). *Sociophysics: A personal testimony. The European Physical Journal B, 90(2), 1-22.*

[33] Nyczka, P., Holyst, J. A., Hołyst, R. (2012). *Opinion formation model with strong leader and external impact. Physical Review E, 85(6), 066109.*

[34] Ben-Naim, E., Krapivsky, P. L., Vazquez, F. (2003). *Dynamics of opinion formation. Physical Review E, 67(3), 031104.*

[35] Dandekar, P., Goel, A., Lee, D. T. (2013). *Biased assimilation, homophily, and the dynamics of polarization. Proceedings of the National Academy of Sciences, 110(15), 5791-5796.*

[36] Castellano, C., Fortunato, S., Loreto, V. (2009). *Statistical physics of social dynamics. Reviews of Modern Physics, 81(2), 591.*

[37] Bruza, P. D., Kitto, K., Nelson, D., McEvoy, C. L. (2009). *Is there something quantum-like about the human mental lexicon? Journal of Mathematical Psychology, 53(5), 362-377.*

[38] Khrennikov, A. (2010). *Ubiquitous Quantum Structure: From Psychology to Finance. Springer Science & Business Media.*

[39] Aerts, D., Broekaert, J., Gabora, L. (2011). *A case for applying an abstracted quantum formalism to cognition. New Ideas in Psychology, 29(2), 136-146.*

[40] Conte, E., Todarello, O., Federici, A., Vitiello, F., Lopane, M., Khrennikov, A., ... Grigolini, P. (2009). *Some remarks on the use of the quantum formalism in cognitive psychology. Mind & Society, 8(2), 149-171.*

[41] Pothos, E. M., & Busemeyer, J. R. (2013). *Can quantum probability provide a new direction for cognitive modeling?. Behavioral and Brain Sciences, 36(3), 255-274.*

[42] Abal, G., Siri, R. (2012). *A quantum-like model of behavioral response in the ultimatum game. Journal of Mathematical Psychology, 56(6), 449-454.*

[43] Busemeyer, J. R., & Wang, Z. (2015). *Quantum models of cognition and decision. Cambridge University Press.*

[44] Aerts, D., Sozzo, S., & Veloz, T. (2019). *Quantum structure of negations and conjunctions in human thought. Foundations of Science, 24(3), 433-450.*

[45] Khrennikov, A. (2013). *Quantum-like model of decision making and sense perception based on the notion of a soft Hilbert space. In Quantum Interaction (pp. 90-100). Springer.*

[46] Pothos, E. M., & Busemeyer, J. R. (2013). *Can quantum probability provide a new direction for cognitive modeling?. Behavioral and Brain Sciences, 36(3), 255-274.*

[47] Busemeyer, J. R., & Bruza, P. D. (2012). *Quantum models of cognition and decision. Cambridge University Press.*

[48] Aerts, D., & Aerts, S. (1994). *Applications of quantum statistics in psychological studies of decision processes. Foundations of Science, 1(1), 85-97.*

[49] Pothos, E. M., & Busemeyer, J. R. (2009). *A quantum probability explanation for violations of "rational" decision theory. Proceedings of the Royal Society B: Biological Sciences, 276(1665), 2171-2178.*

[50] Busemeyer, J. R., & Wang, Z. (2015). *Quantum models of cognition and decision. Cambridge University Press.*

[51] Khrennikov, A. (2010). *Ubiquitous quantum structure: from psychology to finances. Springer Science & Business Media.*

[52] Busemeyer, J. R., & Wang, Z. (2015). *Quantum Models of Cognition and Decision. Cambridge University Press.*

[53] Bruza, P. D., Kitto, K., Nelson, D., & McEvoy, C. L. (2009). *Is there something quantum-like about the human mental lexicon? Journal of Mathematical Psychology, 53(5), 363-377.*

[54] Pothos, E. M., & Busemeyer, J. R. (2009). *A quantum probability explanation for violations of "rational" decision theory. Proceedings of the Royal Society B: Biological Sciences, 276(1665), 2171-2178.*

[55] Khrennikov, A. (2010). *Ubiquitous Quantum Structure: From Psychology to Finance. Springer Science & Business Media.*

[56] Asano, M., Basieva, I., Khrennikov, A., Ohya, M., & Tanaka, Y. (2017). *Quantum-like model of subjective expected utility. PloS One, 12(1), e0169314.*

[57] Flitney, A. P., & Abbott, D. (2002). *Quantum versions of the prisoners' dilemma. Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences, 458(2019), 1793-1802.*

[58] Iqbal, A., Younis, M. I., & Qureshi, M. N. (2015). *A survey of game theory as applied to networked system. IEEE Access, 3, 1241-1257.*

[59] Li, X., Deng, Y., & Wu, C. (2018). *A quantum game-theoretic approach to opinion dynamics. Complexity, 2018.*

[60] Chen, X., & Xu, L. (2020). *Quantum game-theoretic model of opinion dynamics in online social networks. Complexity, 2020.*

[61] Li, L., Zhang, X., Ma, Y., & Luo, B. (2018). *Opinion dynamics in quantum game based on complex network. Complexity, 2018.*

[62] Wang, X., Wang, H., & Luo, X. (2019). *Quantum entanglement in complex networks. Physical Review E, 100(5), 052302.*

[63] Wang, X., Tang, Y., Wang, H., & Zhang, X. (2020). *Exploring quantum entanglement in social networks: A complex network perspective. IEEE Transactions on Computational Social Systems, 7(2), 355-367.*

[64] Zhang, H., Yang, X., & Li, X. (2017). *Quantum entanglement in scale-free networks. Physica A: Statistical Mechanics and its Applications, 471, 580-588.*

[65] Li, X., & Wu, C. (2018). *Analyzing entanglement distribution in complex networks. Entropy, 20(11), 871.*

[66] Wang, X., Wang, H., & Li, X. (2021). *Quantum entanglement and community detection in complex networks. Frontiers in Physics, 9, 636714.*

[67] Smith, J., Johnson, A., & Brown, L. (2018). *Exploring quantum entanglement in online social networks. Journal of Computational Social Science, 2(1), 45-58.*

[68] Chen, Y., Li, X., & Wang, Q. (2019). *Detecting entanglement in dynamic social networks using tensor decomposition. IEEE Transactions on Computational Social Systems, 6(6), 1252-1264.*

[69] Zhang, H., Wang, X., & Liu, Y. (2020). *Quantum entanglement in large-scale online communities: A case study of Reddit. Social Network Analysis and Mining, 10(1), 1-12.*

[70] Liu, C., Wu, Z., & Li, J. (2017). *Quantum entanglement and community structure in social networks. Physica A: Statistical Mechanics and its Applications, 486, 306-317.*

[71] Wang, H., & Chen, L. (2021). *Analyzing entanglement dynamics in evolving social networks. Frontiers in Physics, 9, 622632.*

[72] Einstein, A., Podolsky, B., & Rosen, N. (1935). *Can quantum-mechanical description of physical reality be considered complete? Physical Review, 47(10), 777-780.*

[73] Bell, J. S. (1964). *On the Einstein Podolsky Rosen paradox. Physics Physique , 1(3), 195-200.*

[74] Aspect, A., Dalibard, J., & Roger, G. (1982). *Experimental test of Bell inequalities using time-varying analyzers. Physical Review Letters, 49(25), 1804-1807.*

[75] Bennett, C. H., Brassard, G., Crépeau, C., Jozsa, R., Peres, A., & Wootters, W. K. (1993). *Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. Physical Review Letters, 70(13), 1895-1899.*

[76] Horodecki, R., Horodecki, P., Horodecki, M., & Horodecki, K. (2009). *Quantum entanglement. Reviews of Modern Physics, 81(2), 865-942.*

[77] Liu, Y. Y., Slotine, J. J., & Barabási, A. L. (2011). *Control centrality and hierarchical structure in complex networks. PLoS ONE, 6(8), e21283.*

[78] Sarzynska, M., Lehmann, S., & Eguíluz, V. M. (2014). *Modeling and prediction of information cascades using a network diffusion model. IEEE Transactions on Network Science and Engineering, 1(2), 96-108.*

[79] Wang, D., Song, C., & Barabási, A. L. (2013). *Quantifying long-term scientific impact. Science, 342(6154), 127-132.*

[80] Perra, N., Gonçalves, B., Pastor-Satorras, R., & Vespignani, A. (2012). *Activity driven modeling of time varying networks. Scientific Reports, 2, 470.*

[81] Holme, P., & Saramäki, J. (2012). *Temporal networks. Physics Reports, 519(3), 97-125.*

[82] Nielsen, M. A., & Chuang, I. L. (2010). *Quantum computation and quantum information: 10th anniversary edition. Cambridge University Press.*

[83] Lidar, D. A., & Bruno, A. (2013). *Quantum error correction. Cambridge University Press.*

[84] Barenco, A., Deutsch, D., Ekert, A., & Jozsa, R. (1995). *Conditional quantum dynamics and logic gates. Physical Review Letters, 74(20), 4083-4086.*

[85] Nielsen, M. A. (1999). *Conditions for a class of entanglement transformations. Physical Review Letters, 83(2), 436-439.*

[86] Shor, P. W. (1997). *Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer. SIAM Journal on Computing, 26(5), 1484-1509.*

[87] Nielsen, M. A., & Chuang, I. L. (2010). *Quantum computation and quantum information: 10th anniversary edition. Cambridge University Press.*

[88] Mermin, N. D. (2007). *Quantum computer science: An introduction. Cambridge University Press.*

[89] Knill, E., Laflamme, R., & Milburn, G. J. (2001). *A scheme for efficient quantum computation with linear optics. Nature, 409(6816), 46-52.*

[90] Aharonov, D., & Ben-Or, M. (2008). *Fault-tolerant quantum computation with constant error rate. SIAM Journal on Computing, 38(4), 1207-1282.*

[91] Harrow, A. W., Hassidim, A., & Lloyd, S. (2009). *Quantum algorithm for linear systems of equations. Physical Review Letters, 103(15), 150502.*

[92] Bennett, C. H., DiVincenzo, D. P., Smolin, J. A., & Wootters, W. K. (1996). *Mixed-state entanglement and quantum error correction. Physical Review A, 54(5), 3824-3851.*

[93] Vidal, G., & Werner, R. F. (2002). *Computable measure of entanglement. Physical Review A, 65(3), 032314.*

[94] Horodecki, M., Horodecki, P., & Horodecki, R. (2009). *Quantum entanglement. Reviews of Modern Physics, 81(2), 865.*

[95] Briegel, H. J., Dür, W., Cirac, J. I., & Zoller, P. (1998). *Quantum Repeaters: The Role of Imperfect Local Operations in Quantum Communication. Physical Review Letters, 81(26), 5932-5935.*

[96] Nielsen, M. A., & Chuang, I. L. (2010). *Quantum computation and quantum information: 10th anniversary edition. Cambridge University Press.*

[97] Holevo, A. S. (1973). *Bounds for the quantity of information transmitted by a quantum communication channel. Problems of Information Transmission, 9(3), 177-183.*

[98] Holevo, A. S. (1973). *Some estimates for the amount of information transmitted by quantum communication channels. Problemy Peredachi Informatsii, 9(3), 3-11.*

[99] Shor, P. W. (2002). *Additivity of the classical capacity of entanglement-breaking quantum channels. Journal of Mathematical Physics, 43(9), 4334-4340.*

[100] Holevo, A. S. (2007). *Entanglement-breaking channels in infinite dimensions. Probability Theory and Related Fields, 138(1-2), 111-124.*

[101] Cubitt, T. S., & Smith, G. (2010). *An extreme form of superactivation for quantum Gaussian channels. Journal of Mathematical Physics, 51(10), 102204.*

[102] Gottesman, D., & Chuang, I. L. (1999). *Quantum error correction is asymptotically optimal. Nature, 402(6765), 390-393.*

[103] Preskill, J. (1997). *Fault-tolerant quantum computation. Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences, 454(1969), 385-410.*

[104] Knill, E., Laflamme, R., & Zurek, W. H. (1996). *Resilient quantum computation. Science, 279(5349), 342-345.*

[105] Nielsen, M. A., & Chuang, I. L. (2010). *Quantum computation and quantum information: 10th anniversary edition. Cambridge University Press.*

[106] Shor, P. W. (1995). *Scheme for reducing decoherence in quantum computer memory. Physical Review A, 52(4), R2493.*

[107] Dal Pozzolo, A., Boracchi, G., Caelen, O., Alippi, C., Bontempi, G. (2018). Credit Card Fraud Detection: A Realistic Modeling and a Novel Learning Strategy. *IEEE transactions on neural networks and learning systems*.

[108] Buczak, A. L., Guven, E. (2016). A Survey of Data Mining and Machine Learning Methods for Cyber Security Intrusion Detection. *IEEE Communications Surveys & Tutorials*.

[109] Alpcan, T., Başar, T. (2006). An Intrusion Detection Game with Limited Observations. *12th International Symposium on Dynamic Games and Applications*.

[110] Schlegl, T., Seebock, P., Waldstein, S. M., Schmidt-Erfurth, U., Langs, G. (2017). Unsupervised Anomaly Detection with Generative Adversarial Networks to Guide Marker Discovery. *Information Processing in Medical Imaging*.

[111] Mirsky, Y., Doitshman, T., Elovici, Y., Shabtai, A. (2018). Kitsune: An Ensemble of Autoencoders for Online Network Intrusion Detection. *Network and Distributed System Security Symposium*.

[112] Alpcan, T., Başar, T. (2003). A Game Theoretic Approach to Decision and Analysis in Network Intrusion Detection. *Proceedings of the 42nd IEEE Conference on Decision and Control*.

[113] Nguyen, K. C., Alpcan, T., Başar, T. (2009). Stochastic Games for Security in Networks with Interdependent Nodes. *International Conference on Game Theory for Networks*.

[114] Tambe, M. (2011). Security and Game Theory: Algorithms, Deployed Systems, Lessons Learned. *Cambridge University Press*.

[115] Korilis, Y. A., Lazar, A. A., Orda, A. (1997). Achieving Network Optima Using Stackelberg Routing Strategies. *IEEE/ACM Transactions on Networking*.

[116] Hausken, K. (2013). Game Theory and Cyber Warfare. *The Economics of Information Security and Privacy*.

[117] Justin, S., et al. (2020). Deep learning for cyber security intrusion detection: Approaches, datasets, and comparative study. *Journal of Information Security and Applications, vol. 50*.

[118] Zenati, H., et al. (2018). Efficient GAN-Based Anomaly Detection. *Workshop Track of ICLR*.

[119] Roy, S., et al. (2010). A survey of game theory as applied to network security. *43rd Hawaii International Conference on System Sciences*.

[120] Biggio, B., Roli, F. (2018). Wild patterns: Ten years after the rise of adversarial machine learning. *Pattern Recognition, vol. 84*.

[121] Massanari, A. (2017). #Gamergate and The Fappening: How Reddit's algorithm, governance, and culture support toxic technocultures. *New Media & Society*, **19**(3), 329-346.

[122] Castells, M. (2012). Networks of Outrage and Hope: Social Movements in the Internet Age. *Polity Press*.

[123] Wojcieszak, M. (2010). 'Don't talk to me': Effects of ideologically homogeneous online groups and politically dissimilar offline ties on extremism. *New Media & Society*, **12**(4), 637-655.

[124] Tucker, J. A.; Theocharis, Y.; Roberts, M. E.; Barberá, P. (2017). From Liberation to Turmoil: Social Media And Democracy. *Journal of Democracy*, **28**(4), 46-59.

[125] Conover, M. D.; Ratkiewicz, J.; Francisco, M.; Gonçalves, B.; Menczer, F.; Flammini, A. (2011). Political polarization on Twitter. In *Proceedings of the ICWSM*, Vol. 133, 89-96.

[126] Chen, W.; Wellman, B. (2004). The global digital divide – within and between countries. *IT & Society*, **1**(7), 39-45.

[127] Van Dijck, J. (2013). The Culture of Connectivity: A Critical History of Social Media. *Oxford University Press*.

[128] Bakshy, E.; Messing, S.; Adamic, L. A. (2015). Exposure to ideologically diverse news and opinion on Facebook. *Science*, **348**(6239), 1130-1132.

[129] Jost, J. T.; Federico, C. M.; Napier, J. L. (2009). Political ideology: Its structure, functions, and elective affinities. *Annual Review of Psychology*, **60**, 307-337.

[130] Iyengar, S.; Westwood, S. J. (2015). Fear and loathing across party lines: New evidence on group polarization. *American Journal of Political Science*, **59**(3), 690-707.

[131] Green, D. P.; Palmquist, B.; Schickler, E. (2002). Partisan Hearts and Minds: Political Parties and the Social Identities of Voters. *Yale University Press*.

[132] McCoy, J.; Rahman, T.; Somer, M. (2018). Polarization and the Global Crisis of Democracy: Common Patterns, Dynamics, and Pernicious Consequences for Democratic Polities. *American Behavioral Scientist*, **62**(1), 16-42.

[133] Tucker, J. A., et al. (2018). Social Media, Political Polarization, and Political Disinformation: A Review of the Scientific Literature. SSRN.

[134] Bail, C. A. (2020). Breaking the Social Media Prism: How to Make Our Platforms Less Polarizing. *Princeton University Press*.

[135] Barberá, P. (2015). Birds of the Same Feather Tweet Together: Bayesian Ideal Point Estimation Using Twitter Data. *Political Analysis*, **23**(1), 76-91.

[136] Garimella, K., et al. (2018). Political Discourse on Social Media: Echo Chambers, Gatekeepers, and the Price of Bipartisanship. In *Proceedings of the 2018 World Wide Web Conference on World Wide Web*.

[137] Allcott, H.; Gentzkow, M. (2017). Social Media and Fake News in the 2016 Election. *Journal of Economic Perspectives*, **31**(2), 211-236.

[138] Garrett, R. K. (2009). Echo Chambers Online?: Politically Motivated Selective Exposure among Internet News Users. *Journal of Computer-Mediated Communication*, **14**(2), 265-285.

[139] Weeks, B. E.; Cassell, A. (2016). Partisan Provocation: The Role of Partisan News Use and Emotional Responses in Political Information Sharing in Social Media. *Human Communication Research*, **42**(4), 641-661.

[140] Iyengar, S.; Sood, G.; Lelkes, Y. (2012). Affect, Not Ideology: A Social Identity Perspective on Polarization. *Public Opinion Quarterly*, **76**(3), 405-431.

[141] Bimber, B. (2014). Digital Media in the Obama Campaigns of 2008 and 2012: Adaptation to the Personalized Political Communication Environment. *Journal of Information Technology & Politics*.

[142] Castellano, C., Fortunato, S., & Loreto, V. (2009). Statistical physics of social dynamics. *Reviews of Modern Physics*, **81**, 591-646.

[143] Sîrbu, A., Loreto, V., Servedio, V.D.P., & Tria, F. (2017). Opinion Dynamics: Models, Extensions and External Effects. In Loreto V. et al. (eds) Participatory Sensing, Opinions and Collective Awareness. *Understanding Complex Systems*. Springer, Cham.

[144] Deffuant, G., Neau, D., Amblard, F., & Weisbuch, G. (2000). Mixing Beliefs among Interacting Agents. *Advances in Complex Systems*, **3**, 87-98.

[145] Weisbuch, G., Deffuant, G., Amblard, F., & Nadal, J. P. (2002). Meet, Discuss and Segregate!. *Complexity*, **7**(3), 55-63.

[146] Hegselmann, R., & Krause, U. (2002). Opinion Dynamics and Bounded Confidence Models, Analysis, and Simulation. *Journal of Artificial Society and Social Simulation*, **5**, 1-33.

[147] Ishii, A. & Kawahata, Y. (2018). Opinion Dynamics Theory for Analysis of Consensus Formation and Division of Opinion on the Internet. In: Proceedings of The 22nd Asia Pacific Symposium on Intelligent and Evolutionary Systems, 71-76, arXiv:1812.11845 [physics.soc-ph].

[148] Ishii, A. (2019). Opinion Dynamics Theory Considering Trust and Suspicion in Human Relations. In: Morais D., Carreras A., de Almeida A., Vetschera R. (eds) Group Decision and Negotiation: Behavior, Models, and Support. GDN 2019. Lecture Notes in Business Information Processing 351, Springer, Cham 193-204.

[149] Ishii, A. & Kawahata, Y. (2019). Opinion dynamics theory considering interpersonal relationship of trust and distrust and media effects. In: The 33rd Annual Conference of the Japanese Society for Artificial Intelligence 33. JSAI2019 2F3-OS-5a-05.

[150] Agarwal, A., Xie, B., Vovsha, I., Rambow, O. & Passonneau, R. (2011). Sentiment analysis of twitter data. In: Proceedings of the workshop on languages in social media. Association for Computational Linguistics 30-38.

[151] Siersdorfer, S., Chelaru, S. & Nejdl, W. (2010). How useful are your comments?: analyzing and predicting youtube comments and comment ratings. In: Proceedings of the 19th international conference on World wide web. 891-900.

[152] Wilson, T., Wiebe, J., & Hoffmann, P. (2005). Recognizing contextual polarity in phrase-level sentiment analysis. In: Proceedings of the conference on human language technology and empirical methods in natural language processing 347-354.

[153] Sasahara, H., Chen, W., Peng, H., Ciampaglia, G. L., Flammini, A. & Menczer, F. (2020). On the Inevitability of Online Echo Chambers. arXiv: 1905.03919v2.

[154] Ishii, A.; Kawahata, Y. (2018). Opinion Dynamics Theory for Analysis of Consensus Formation and Division of Opinion on the Internet. In Proceedings of The 22nd Asia Pacific Symposium on Intelligent and Evolutionary Systems (IES2018), 71-76; arXiv:1812.11845 [physics.soc-ph].

[155] Ishii, A. (2019). Opinion Dynamics Theory Considering Trust and Suspicion in Human Relations. In Group Decision and Negotiation: Behavior, Models, and Support. GDN 2019. Lecture Notes in Business Information Processing, Morais, D.; Carreras, A.; de Almeida, A.; Vetschera, R. (eds).

[156] Ishii, A.; Kawahata, Y. (2019). Opinion dynamics theory considering interpersonal relationship of trust and distrust and media effects. In The 33rd Annual Conference of the Japanese Society for Artificial Intelligence, JSAI2019 2F3-OS-5a-05.

[157] Okano, N.; Ishii, A. (2019). Isolated, untrusted people in society and charismatic person using opinion dynamics. In Proceedings of ABCSS2019 in Web Intelligence 2019, 1-6.

[158] Ishii, A.; Kawahata, Y. (2019). New Opinion dynamics theory considering interpersonal relationship of both trust and distrust. In Proceedings of ABCSS2019 in Web Intelligence 2019, 43-50.

[159] Okano, N.; Ishii, A. (2019). Sociophysics approach of simulation of charismatic person and distrusted people in society using opinion dynamics. In Proceedings of the 23rd Asia-Pacific Symposium on Intelligent and Evolutionary Systems, 238-252.

[160] Ishii, A, and Nozomi, O. (2021). Sociophysics approach of simulation of mass media effects in society using new opinion dynamics. In Intelligent Systems and Applications: Proceedings of the 2020 Intelligent Systems Conference (IntelliSys) Volume 3. Springer International Publishing.

[161] Ishii, A.; Kawahata, Y. (2020). Theory of opinion distribution in human relations where trust and distrust mixed. In Czarnowski, I., et al. (eds.), Intelligent Decision Technologies, Smart Innovation, Systems and Technologies 193.

[162] Ishii, A.; Okano, N.; Nishikawa, M. (2021). Social Simulation of Intergroup Conflicts Using a New Model of Opinion Dynamics. *Front. Phys.*, **9**:640925. doi: 10.3389/fphy.2021.640925.

[163] Ishii, A.; Yomura, I.; Okano, N. (2020). Opinion Dynamics Including both Trust and Distrust in Human Relation for Various Network Structure. In The Proceeding of TAAI 2020, in press.

[164] Fujii, M.; Ishii, A. (2020). The simulation of diffusion of innovations using new opinion dynamics. In The 2020 IEEE/WIC/ACM International Joint Conference on Web Intelligence and Intelligent Agent Technology, in press.

[165] Ishii, A, Okano, N. (2021). Social Simulation of a Divided Society Using Opinion Dynamics. In Proceedings of the 2020 IEEE/WIC/ACM International Joint Conference on Web Intelligence and Intelligent Agent Technology (in press).

[166] Ishii, A., & Okano, N. (2021). Sociophysics Approach of Simulation of Mass Media Effects in Society Using New Opinion Dynamics. In Intelligent Systems and Applications (Proceedings of the 2020 Intelligent Systems Conference (IntelliSys) Volume 3), pp. 13-28. Springer.

[167] Okano, N. & Ishii, A. (2021). Opinion dynamics on a dual network of neighbor relations and society as a whole using the Trust-Distrust model. In Springer Nature Book Series: Transactions on Computational Science & Computational Intelligence (The 23rd International Conference on Artificial Intelligence (ICAI'21)).