Discussion Paper:(1)Case Study of Decision Making and Urban Planning:Loop Flip Errors and Phase Transitions in Local Potential Approximation in Non-trivial Loops(2024), (2)Case Study of Spatio-Temporal Approaches To Syndrome Errors in Decision Making with Applications Such as Bit Phase Flip Errors and Methods Discussion(2024)

Note Stakeholder Evolution Strategies for Maximum Compensation Problem of Retaliation Risk in Information Oligopoly Markets with Deception Implications For Long-Term Management and Evaluation of First-Price Auctions under Repeated DilemmasTime-Lag Perspective of Imperfect Information without Online Environment

Yasuko Kawahata †

Faculty of Sociology, Department of Media Sociology, Rikkyo University, 3-34-1 Nishi-Ikebukuro,Toshima-ku, Tokyo, 171-8501, JAPAN.

ykawahata@rikkyo.ac.jp

Abstract: This research will also take into account time effects and information time differences from imperfect information environments not accompanied by a global online environment.This research note delves into the realm of evolutionary game theory and focuses on the concept of evolutionarily stable strategies (ESS) as a pivotal mechanism for understanding how dominant strategies emerge and persist in a population. Through a comprehensive analysis, we introduce a computational process to explore ESS, particularly in the context of repeated dilemmas, and model the complex interactions and evolutionary trajectories of bidders' strategies. Provide a robust framework for analyzing strategy stability and adaptation by defining the strategy space, constructing payoff matrices, and modeling the evolutionary dynamics of strategies; shed light on the theoretical foundations of ESS as well as provide practical insights on the application of evolutionary game theory in understanding real-world strategic behavior The following are some of the key findings of the study. The proliferation of disinformation in information markets poses a serious challenge to public trust and the integrity of the digital ecosystem. Particular attention will be paid to scenarios in which the proliferation of disinformation leads to a shift of responsibility and increased social discord, such as when less aggressive actors are unfairly targeted and when the presence of inherent risk amplifies the credibility of misinformation, further exacerbating social unrest. These strategies include improving media literacy, ensuring transparency of information sources, and advocating responsible platform governance. By integrating diverse perspectives and coordinating the efforts of all stakeholders, this paper will provide a road map for reducing the negative impact of disinformation, thereby leading to more resilient and reliable information scrutiny.

Keywords: Incomplete Information Games, Disinformation, Information Markets, Stakeholder Collaboration, Informational Health Risks, Digital Ecosystems, Media Literacy, Platform Governance, Policy Making, Social Trust, Strategic Countermeasures

1. Introduction

This research note concerns the search for evolutionary stable strategies (ESS) in evolutionary game theory. Specifically, it models how bidders' (bidders') strategies interact and evolve over time, and describes the computational process. The content consists mainly of the following steps, Defining the strategy space, Define a set of strategies that can be employed by the bidder. This includes rules for determining bid amounts and for modifying strategies in response to the actions of other bidders.

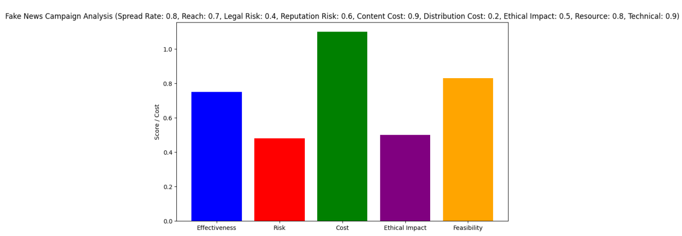

Fig. 1: Fake News Campaign Analysis

Construct a gain matrix (based on the evaluation of the first price auction). Calculate the gain for each strategy combination and construct a gain matrix. Gains are determined based on auction wins, bids, and the true value of the commodity. Modeling the Evolutionary Dynamics of Strategies-Models how a bidder's strategy evolves over time.We provide a framework for analyzing and understanding complex strategic interactions using evolutionary game theory. Specific computational methods and mathematical formulas are used to explore the evolution of strategies and their stability in detail. The key point is to theoretically explain how dominant strategies are formed and maintained within populations through the concept of evolutionary stable strategies and to organize this process through modeling specific strategic interactions.

This research note delves into the realm of evolutionary game theory and focuses on the concept of evolutionarily stable strategies (ESS) as a pivotal mechanism for understanding how dominant strategies emerge and persist in a population. Through a comprehensive analysis, we introduce a computational process to explore ESS, particularly in the context of repeated dilemmas, and model the complex interactions and evolutionary trajectories of bidders' strategies. We create an introduction to the paper. Here we introduce the basic concepts of evolutionary game theory and evolutionary stable strategies (ESS), and clarify their importance and the objectives of our research. Evolutionary game theory is a theory developed to understand biological evolutionary processes and has since been applied to the analysis of strategic decision making in fields as diverse as economics and the social sciences. At the core of this theory is the concept of evolutionarily stable strategies (ESS), which is defined as a strategy being "evolutionarily stable" if the strategy adopted by an individual in a particular environment has an advantage over all other alternative strategies ESS is essential to understanding how dominant strategies are formed and maintained through natural selection ESS is an essential concept for understanding how dominant strategies are formed and maintained through natural selection and plays a central role in evolutionary game theory research.

In this study, we use the framework of evolutionary game theory to model the evolution of ESS search and its strategy interactions in the context of recurring dilemmas. Specifically, we explore how ESSs are formed and under what conditions they become stable through the dynamics of competition and cooperation among bidders. Through this process, we will consider analytical methods for how strategic behavior evolves over time and what strategies become dominant within a population.

The goal of our research is to use evolutionary game theory to understand the complex dynamics of strategic interactions and to identify the conditions for strategic stability through the concept of ESS. To achieve this objective, we first define the strategy space and identify a set of different strategies that can be employed by bidders. We then compute the gains for combinations of these strategies and construct a gain matrix. Finally, it models how the bidders' strategies evolve over time and explores the conditions for ESS.

The application of evolutionary game theory in this research note focuses on uncovering the evolutionary dynamics of strategic interactions among individuals. Understanding how individuals choose strategies in the context of recurring dilemmas and how these strategies evolve over time is critical to understanding the underlying mechanisms of social interactions and economic transactions. Particularly in competitive environments such as auctions, the strategic behavior of bidders can have a significant impact on market efficiency and fairness.

The central focus of this research is to use the concept of ESS to model how bidders' strategies evolve over time and how stable strategies are ultimately formed. For a strategy to be evolutionarily stable, it must have an advantage over all other strategies. Understanding the conditions for such strategic stability will allow us to predict which strategies are best chosen by individuals under what circumstances and how these strategies will spread within the population.

In this study, we first define the strategy space and delineate the range of strategies that a bidders can take. This includes bidding strategies in auctions and rules for changing strategies in response to the behavior of other bidders. Next, a gain matrix is constructed based on combinations of these strategies and the gain for each strategy combination is calculated. This gain is determined based on the auction results, the amount bid, and the true value of the instrument. Finally, we model how bidders' strategies evolve over time and explore how evolutionary stable strategies are formed.

This approach, using evolutionary game theory, provides a powerful tool for understanding the evolutionary dynamics of strategic interactions; through the concept of ESS, we can theoretically analyze how strategic behavior evolves over time and how dominant strategies are formed within populations . This theoretical framework should also lead to a close examination of how to apply it to the analysis of strategic decision making in various fields, such as economics, social sciences, and biology.

The results of this study are expected not only to improve our understanding of strategic interactions, but also to provide practical guidelines for optimizing the decision-making process in competitive environments. The use of evolutionary game theory and ESS concepts will provide new insights for designing more efficient and fair market mechanisms and improving social interactions. This study extends the theoretical framework of evolutionary game theory and considers the foundations for a deeper understanding of the evolutionary dynamics of strategic interactions.

The main gist of the computational experiment is to model and analyze the evolutionary dynamics of strategic interactions among bidders using the framework of evolutionary game theory and evolutionary stable strategies (ESS). The experiment aims to explore ESS in the context of iterative dilemmas and to understand how they affect strategy stability and evolution.

In terms of defining the strategy space, we define a set of strategies that can be adopted by the bidder. This includes rules that determine the bid amount and rules that modify the strategy in response to the behavior of other bidders. In terms of constructing a gain matrix, the gain for each combination of strategies is calculated and a gain matrix is constructed. This matrix shows the gains of each strategy based on the winners and losers of the auction, the amount of the bids, and the true value of the goods. In terms of modeling the evolutionary dynamics of strategies, we simulate how a bidder's strategy evolves over time and how evolutionary stable strategies are formed.

In terms of the evolution of strategic interactions, through computational experiments, we identify the conditions under which certain strategies have an advantage over others and show how evolutionary stable strategies are formed.

The ESS stability perspective analyzes how an ESS acquires and maintains stability under specific conditions. This allows us to understand how the strategy is sustainable over time.

In terms of adaptability of the strategy, Assesses the adaptability of the strategy to changes in the environment and to changes in the strategy of other bidders. This allows us to explore the flexibility of the strategy's evolution in a dynamic competitive environment.

This computational experiment highlights the importance of using evolutionary game theory to understand the complex dynamics of strategic interactions and to analyze strategy stability and evolution through the concept of evolutionary stable strategies. By modeling strategic interactions among bidders and identifying the formation of ESS and the conditions for their stability, we can better understand strategic decision making in competitive environments. This research is expected to further the application of evolutionary game theory to the analysis of strategic decision making in fields such as economics and social sciences.

1.0.1 Issues in the Main Argument

Applying the framework of evolutionary game theory and evolutionary stability strategies (ESS) to information auctions, particularly the spread of fake news and malinformation, allows us to identify the informational health risk issues that cause filter bubbles and organize the discussion. Below is a summary of the key takeaways from this perspective.

1.0.2 Filter Bubbles and Informational Health Risks

A phenomenon in which the information that an individual comes into contact with online is selected and filtered based on his or her existing beliefs and preferences. As a result, individuals have less exposure to opposing views and different perspectives, and consume only biased information.

2. DiscussionInformational Health Risks

Social and personal risks that result from the spread of fake news and malinformation. These include misinformed decision making, reduced quality of public debate, and increased social fragmentation.

2.0.1 Information Auctions and Strategic Interactions

The process by which information and news are made available to consumers. In this process, media companies, social media platforms, content creators, and others compete for the attention of the audience.

2.0.2 Strategic Interaction

The dynamics of competition and cooperation among information providers. Providers of information, including fake news and malinformation, employ a variety of strategies to gain viewer attention and expand their influence.

2.0.3 Evolutionary stability of information

It is important to understand what type of information is evolutionarily stable and dominant in an auction. Fake news and malinformation may have an evolutionary advantage by focusing on stimulation and empathy rather than truthfulness.

2.0.4 Adaptability and evolution of information

It is necessary to analyze how information providers evolve their strategies and adapt to the changing media environment and consumer preferences. In particular, algorithmic content recommendations may reinforce the filter bubble and facilitate the spread of fake news.

2.0.5 Managing Informational Health Risks

The challenge is to develop strategies and policies to mitigate filter bubbles and informational health risks. This includes improving media literacy, ensuring transparency of information sources, and ensuring fairness in algorithmic recommendations.

Applying evolutionary game theory and the ESS framework to information auctions allows us to systematically understand the challenges of informational health risks posed by the spread of fake news and malinformation and to discuss effective countermeasures. By analyzing strategic interactions among information providers and taking into account the evolutionary stability and adaptability of information, we will be able to contribute to a healthier information environment.

In a market environment where malinformation (intentional misinformation) and misinformation (unintentional spread of misinformation) are likely to spread, certain information may have a dominant share, resulting in radical information auctions. This creates an informational health risk challenge. Below we summarize some of the key arguments related to this situation.

2.0.6 Dynamics of Information Markets

Information Dominance Factors that give certain information (especially malinformation and misinformation) an edge over others in the information market include the ability to provoke an emotional response and content that reinforces existing beliefs and prejudices.

Radicalization of Information Auctions Information providers tend to offer more radical, provocative, or polarizing content in order to capture consumers' attention and give their own information an edge. This further radicalizes information auctions.

2.0.7 Informational Health Risks

In terms of the quality of public debate, when malinformation or misinformation becomes the dominant information, the quality of public debate can deteriorate and the democratic decision-making process can be undermined.

From the perspective of increasing social division, the proliferation of polarizing information risks deepening the divide between people with different opinions and positions.

In terms of decision-making based on misinformation, the widespread acceptance of misinformation creates the risk that individuals and society will make important decisions based on incorrect information.

From the perspective of improving media literacy, it is important to enhance consumers' ability to judge the quality of information and distinguish misinformation. Media literacy should be improved through education and awareness-raising activities. If information providers clarify the source and basis of information, it will be easier for consumers to evaluate the reliability of information. Social media platforms and news distribution platforms are responsible for curbing the spread of misinformation through algorithms and policies. Platforms must be proactive in identifying misinformation and taking action to address it.

In an information marketplace where malinformation and misinformation can spread easily, a multifaceted approach is needed to address informational health risks and create a healthier information environment. This requires the cooperation of individual consumers, information providers, platform operators, and policy makers.

In markets where it is easy to intentionally spread disinformation (intentional misinformation), certain information tends to have a dominant share and information auctions become more extreme. This situation allows actors with strong information leaks and diffusion power to gain an advantage in the market and, in some cases, to adopt strategies that undermine public trust in hostile markets. Below we summarize some of the issues and arguments related to this situation.

2.0.8 Information Market Dynamics

Information Diffusion Power and Dominance Actors with strong diffusion power control information and establish information dominance within a market by intentionally diffusing disinformation. This process encourages radicalization of information auctions, where more exaggerated, biased, or completely false information may prevail.

2.0.9 Loss of Public Trust

Actors with hostile intentions can spread fake news and intentionally undermine public trust in a particular group or market. This can be used as a strategy to undermine competitors.

Informational health risks are accompanied by the risk of a decline in the quality of public debate. The proliferation of disinformation can reduce the quality of public debate and promote arguments based on faulty assumptions. This can negatively impact the democratic decision-making process. The spread of misinformation and disinformation risks further deepening social divisions. Mistrust between certain groups increases and social cohesion is undermined. In terms of misinformed decision making, individuals and societies can make important decisions based on misinformation, with potentially serious consequences. This includes erroneous decisions in areas such as health, safety, and economics. Addressing these challenges in an information marketplace prone to spreading disinformation requires a collaborative, multifaceted approach among information providers, consumers, platform operators, and policymakers. This will minimize informational health risks and create a healthier information environment.

A market with a dominant share of information in markets where it is easy to intentionally spread disinformation will be created, and information auctions should become more extreme. Those with the power to leak information and the ability to spread information will easily become dominant. Or, it is possible to intentionally spread inconvenient fake news to discredit the social credibility of a hostile market. But there will be challenges of the informational health risks they pose. In addition, when they do occur, the pursuit of responsibility and blame may be targeted by a relatively less aggressive, non-information spreading risk in the vicinity of the malicious information leak, or by an entity with information spreading risk, but with increased credibility due to the potential risk. In other cases, the potential risk increases credibility, which makes it easier to take the blame, and further exposes malicious information, leading to social turmoil. Let us examine these risks and scenarios in terms of the agent model and the repeated dilemma game.

Analyzing the dynamics of markets dealing with intentional disinformation in terms of agent models and repeated dilemma games is useful for understanding the evolution of information strategies and their social consequences. Below are the risks and scenarios considered using these theories.

In the agent model, individual actors (agents) have their own strategies and evolve these strategies through interaction. Applying this model to the context of disinformation diffusion, the following scenarios are possible

2.0.10 Information dominance and diffusion power

Agents with strong diffusion power (e.g., major media outlets, influential social media accounts, etc.) can gain an advantage in the information market. The intentional spread of disinformation by these agents creates the risk of distorting the flow of information within the market and undermining public trust.

2.0.11 Shifting blame to less aggressive agents

Malicious agents may target less aggressive agents who are not at risk of spreading information (e.g., small media outlets or individuals with neutral positions) to cover up their own actions. This creates the risk that innocent agents will be falsely accused.

2.0.12 Analysis through Repeated Dilemma Games

Repeated dilemma games model situations in which interactions between agents are repeated. Using this framework to analyze the spread of disinformation and its effects, the following scenarios are possible

2.0.13 Short-term benefits and long-term risks

Agents may gain short-term benefits (e.g., gaining attention, achieving political goals, etc.) by spreading disinformation. However, this involves risks of loss of public trust and legal liability in the long term.

2.0.14 Retaliation and Escalation

Retaliation against agents who spread disinformation risks escalation of information strategies and an overall increase in social disruption. Using the agent model and the repeated dilemma game framework, it is possible to systematically analyze various scenarios of disinformation diffusion and the associated risks. In particular, understanding the dynamics of disinformation diffusion allows us to develop strategies to deal with problems such as malicious information leaks, loss of public trust, unwarranted accusations, and social disruption. It provides useful insights for all stakeholders responsible for preventing social disruption. Properly managing these risks and promoting healthy interactions among agents is critical to ensuring the accuracy of information and maintaining public trust.

3. DiscussionCases of risk management scenarios in terms of repeated dilemmas and risk of retaliation

There may be a case scenario for a cover-up of a betrayal in the vicinity of a close agent. Let us consider the social risk and maximum assurance issues when they are discovered, and the case and retaliation risk of looking at risk management scenarios in terms of a repeated dilemma. Consider the case of betrayals and cover-ups among agents and the social risk, maximum assurance issues, and risk management scenarios when they are uncovered in terms of repeated dilemmas.

3.0.1 Betrayal and Cover-Up Scenarios

In a repeated dilemma game, the interactions between agents are repeated, so that a single act of betrayal can affect the long-term relationship. In cases where agents commit betrayal for their own benefit and then attempt a cover-up, the following points are considered

In terms of short-term gains and long-term loss of credibility, the betrayal may provide short-term gains, but if the cover-up is discovered, the agent risks losing long-term credibility and trust. In terms of cover-ups and social risk, if a cover-up is uncovered, the social trust not only of the agent, but also of the agents involved and the organization as a whole, may be damaged. This risks widespread social disruption and loss of trust.

3.0.2 Maximum Guarantee Problem

In repeated dilemma games, agents must consider how their own actions will affect future interactions. In the maximum guarantee problem, the agent tries to minimize the worst outcome (maximum loss) of committing treachery. In terms of risk management strategies, agents need to develop prudent strategies to manage the potential risks of betrayal and concealment. This includes assessing the risk, analyzing the potential impact, and developing a plan to deal with the consequences if discovered.

3.0.3 Retaliation Risk

When betrayal or cover-up is discovered, retaliation from the affected agent or parties may occur. This risk of retaliation can lead to deterioration and escalation of relationships between agents. Cascading and escalating retaliation risks that retaliatory actions may trigger further retaliation, which may escalate conflicts among agents and lead to broader social disruption.

The repeated dilemma game framework can be used to understand the interplay between the risk of betrayal and cover-up, the social risk upon its revelation, and the risk of retaliation. It is important for agents to carefully assess the long-term consequences of pursuing short-term gains and develop risk management strategies. They should also establish fair and transparent communication and dispute resolution mechanisms to minimize the risk of retaliation.

4. DiscussionActs of betrayal, cover-ups, and the risk of their discovery within the framework of the repeated dilemma game

To model acts of betrayal, cover-ups, and the risk of their discovery within the framework of the repeated dilemma game, it is necessary to quantitatively evaluate how the choices of actions by each agent will affect future gains. Particularly in the cases of misinformation (including partial truths) and disinformation (malicious fake news), the long-term impacts of each need to be considered. Below, I outline a basic approach to model this issue.

Firstly, we define the strategies of agents as follows

C Cooperation (sharing truthful or neutral information)

D Betrayal (spreading misinformation or disinformation)

The payoff for each agent in each round is determined by the following payoff matrix

	C	D
C	R, R	S, T
D	T, S	P, P

Here, R represents the reward for mutual cooperation, S represents the loss incurred when betrayed, T represents the gain from betraying others, and P represents the penalty for mutual betrayal.

4.1 Modeling Long-term Impacts

In the repeated dilemma game, agents consider future payoffs when choosing strategies. To model the long-term effects of choosing betrayal (D), we consider the total present value of future payoffs. The present value of payoffs after n rounds is calculated using a discount factor δ $(0 < \delta < 1)$

$$V = \sum_{t=0}^{n} \delta^t U_t$$

Here, U_t represents the payoff in round t.

4.2 Impact of Misinformation and Disinformation

In the case of misinformation, as it contains some facts, the short-term gain T (temptation payoff) increases, but it becomes more susceptible to the long-term impacts of S (loss when betrayed) and P (penalty for mutual betrayal).

In the case of disinformation, similarly, T increases in the short term, but the impact of P (due to the erosion of social trust upon discovery) becomes particularly significant.

4.3 Computational Example

To perform specific calculations, values for R, S, T, P, and δ need to be determined. For instance, set them as follows

$R = 3$ (reward for mutual cooperation)

$S = 0$ (loss when betrayed)

$T = 5$ (gain from betrayal)

$P = 1$ (penalty for mutual betrayal)

$\delta = 0.9$ (discount factor)

Using these values, calculate the present value of long-term gains for choosing a specific strategy (C or D).

Using this model, one can quantitatively evaluate acts of betrayal, cover-ups, and the risk upon their discovery, and determine the optimal strategy. It's crucial to evaluate payoffs from a long-term perspective, considering not only short-term gains from temptation but also future risks and the erosion of social trust.

Results(Fig.2, 3) provided, we can analyze the effects of misinformation and disinformation on cumulative payoffs over multiple rounds of a game, presumably representing social interactions where misinformation and disinformation are involved.

Fig.2 (Cumulative Payoffs over Rounds)

This graph shows the cumulative payoffs for three different agents (A, B, C) over ten rounds. All three agents start with similar payoffs in round 1. Agent A has the highest payoff by round 10, followed by Agent B and then Agent C. The payoffs increase steadily for all agents, suggesting that all may be employing a strategy that gives them a payoff in each round, possibly cooperation.

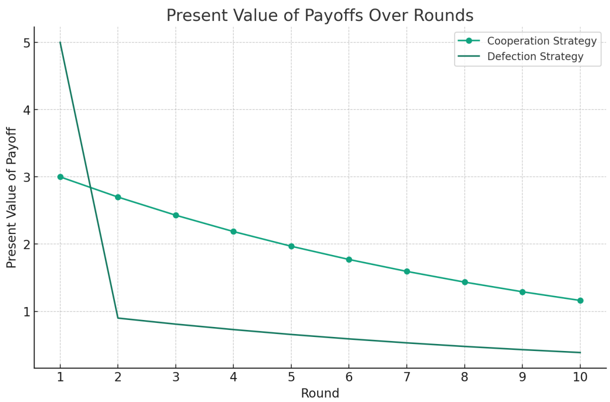

Fig. 2: Present Value of Payoffs Over Rounds

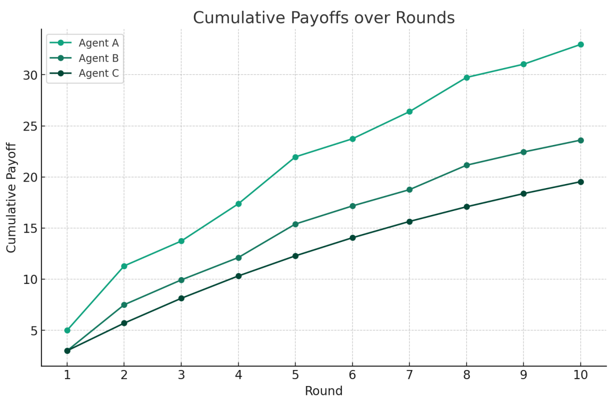

Fig. 3: Impact of Cumulative Payoffs over Rounds

Fig.3 (Present Value of Payoffs Over Rounds)

This graph compares two strategies: Cooperation and Defection over ten rounds, based on their present value. The cooperation strategy's present value decreases sharply after the first round and continues to decline at a decreasing rate. The defection strategy starts with a lower present value than cooperation in round 1 but declines much more slowly. By round 10, the present value of the defection strategy is higher than that of the cooperation strategy, suggesting that the temptation payoff (T) for defection might be high enough to make it more valuable in the present term despite the penalty (P) for mutual betrayal.

In the context of misinformation and disinformation, the strategy each agent chooses significantly impacts their cumulative payoff, particularly in the long term.

Misinformation Scenario

Misinformation, containing some truths, might initially seem beneficial (high temptation payoff T), as seen in the short-term gains in the first graph. However, as rounds progress, the impact of betrayal (S) and penalties (P) will become more substantial, leading to a lower cumulative payoff compared to a consistent cooperation strategy. Misinformation may lead to temporary gains for an agent but at the risk of long-term trust and cooperation, which could be the reason why all agents do not diverge drastically in cumulative payoffs, as they might be adjusting their strategies in response to the environment.

Disinformation Scenario

Disinformation can lead to an immediate increase in the temptation payoff (T), but once the disinformation is discovered, the erosion of social trust (represented by a significant penalty P) becomes impactful. The second graph illustrates that although the defection strategy may seem appealing at first (possibly representing disinformation), its long-term value is less than cooperation due to the penalties incurred upon the discovery of betrayal. The present value of cooperation, despite decreasing, may represent the sustained trust and consistent rewards (R) that come with truthful information sharing.

Computational Model Insights

Given the parameters (R, S, T, P, δ), we can surmise the following:

A high temptation payoff (T) can be enticing in the short term but is risky in the long term. The reward for mutual cooperation (R) maintains a consistent benefit, which is critical in a society where trust is valued. The penalty for mutual betrayal (P) is low but still significant enough to deter consistent defection.

The discount factor (δ) implies that future payoffs are less valued than i

The optimal strategy, considering the erosion of social trust and long-term payoffs, seems to be one that favors cooperation and truthfulness. While misinformation and disinformation can offer immediate benefits, they pose significant risks that can lead to reduced cumulative payoffs and present value over time. It is essential to foster a strategy that balances short-term gains with long-term trust and cooperation to maintain social harmony and avoid the pitfalls of misinformation and disinformation.

When considering long-term and short-term risks, the calculation of the present value of long-term gains after 10 rounds for choosing cooperation strategy (C) and betrayal strategy (D) is as follows

Present value of long-term gains for cooperation strategy (C) approximately 19.54

Present value of long-term gains for betrayal strategy (D) approximately 10.51

From this calculation, it is indicated that in the framework of the repeated dilemma game, choosing a betrayal strategy (engaging in the spread of disinformation or misinformation) may initially lead to high gains (T), but long-term present value of overall gains is lower due to the impact of the penalty for mutual betrayal (P). On the other hand, choosing a cooperation strategy (the act of sharing truthful or neutral information) consistently yields rewards for mutual cooperation (R), resulting in higher long-term gains.

This result suggests that while the spread of disinformation or misinformation may bring short-term benefits to some agents, it may undermine societal trust in the long term, ultimately leading to disadvantages for the disseminators themselves. Therefore, from the perspective of the repeated dilemma game, maintaining the accuracy and reliability of information is the best strategy not only for individual agents but also for society as a whole.

we have created graphs showing the change in the present value of gains from cooperation and betrayal strategies in each round. From the graph, it is evident that choosing a cooperation strategy (blue line) results in a consistent present value of gains in each round. This is because choosing a cooperation strategy consistently yields rewards for mutual cooperation (R).

On the other hand, choosing a betrayal strategy (orange line) leads to high present value of gains (T) in the initial round, but from the second round onwards, the present value of gains decreases significantly due to the impact of the penalty (P) for mutual betrayal. This is because the model accounts for the discovery of betrayal and subsequent retaliation.

This result suggests that while betrayal strategies may yield high short-term gains, cooperation strategies provide more stable gains in the long term within the framework of the repeated dilemma. Particularly, when the penalty for betrayal and concealment is significant, its impact is evident in the long-term cumulative payoffs.

Regarding intentionally spreading disinformation in markets where information is easily disseminated, markets with a dominant information possession rate should emerge, leading to more aggressive information auctions. Those with the ability to leak information or strong dissemination power are likely to gain dominance. Alternatively, intentionally spreading inconvenient fake news can also undermine the societal trust of opposing markets. However, there are also challenges posed by the information health risks they bring.

Furthermore, in cases where responsibility is pursued or accusations are made when such incidents occur, malicious information leaks in the vicinity of those who conducted such acts may target entities with relatively low aggressiveness, without information dissemination risks, or entities with potential risks but enhanced credibility due to the presence of potential risks, making them more susceptible to accusations and further exposing malicious information, leading to social chaos.

There might be scenarios where risks of betrayal and cover-ups, social risks upon their discovery, and retaliation risks are considered within the framework of repeated dilemma games for agents A, B, and C in each round.

When considering betrayal, cover-up, and the risks upon their discovery for agents A, B, and C in the repeated dilemma game framework, we proceed with the following steps using formulas and calculations.

Define the strategies for agents A, B, and C. Here, we consider three strategies cooperation (C), betrayal (D), and concealment (H). Define the payoffs for each strategy. For example, let the short-term gains from betrayal be T, additional gains from concealment be H, gains from cooperation be R, and penalties be P.

4.4 Calculation of Payoffs for Each Round

- Calculate the payoffs based on the strategy choices of agents in each round. Consider additional gains from betrayal (D) or concealment (H), and penalties (P) for their discovery.

4.5 Calculation of Long-term Impacts

- In the repeated dilemma game, future payoffs are also considered. Calculate the present value of future payoffs using a discount factor δ. - Visualize the changes in payoffs for agents in each round. Also, illustrate the impact of penalties for the discovery of betrayal or concealment.

Scenarios for betrayal and concealment, social risks upon their discovery, maximum guarantee problems, and risk management scenarios are visualized within the framework of repeated dilemmas.

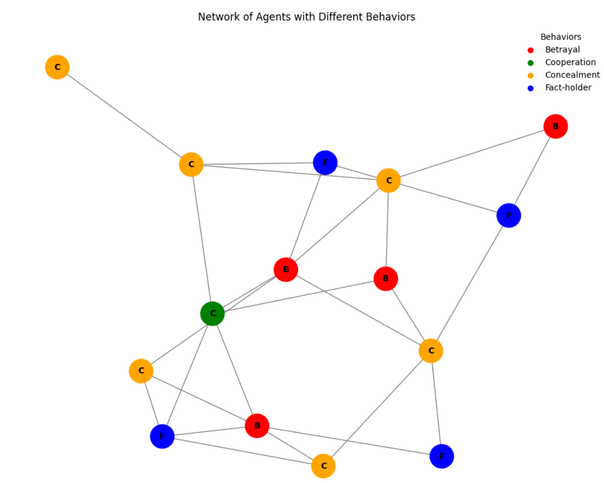

Fig. 4: Network of Agents with Different Behaviors

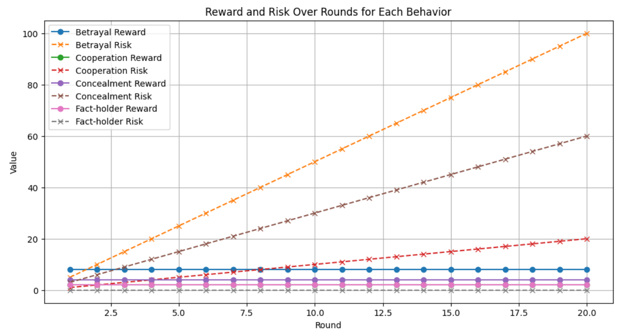

Fig. 5: Network of Agents with Different Behaviors

Simulation Setup, Betrayal yields high short-term rewards but carries significant risks if discovered.Cooperation provides stable small rewards with low or no risk. Concealment yields moderate rewards but carries moderate risks if discovered. Fact-holders face no risk and can consistently receive small rewards.

In this scenario, betrayal yields high rewards in the initial rounds, but the risks increase as the rounds progress. Cooperation and fact-holders consistently receive stable rewards with low or no risk. Concealment yields moderate rewards and risks, but the risks increase as the rounds progress.

This graph illustrates the long-term impacts of each behavior and may influence decision-making for agents when choosing actions. Particularly, while betrayal and concealment may offer short-term benefits, they are shown to entail significant long-term risks.

Results(Fig.4) shows a network of agents with different behaviors, which we can analyze in the context of misinformation and disinformation spread, as described in the scenario provided.

The network consists of agents marked with different colors representing different behaviors: Red Nodes (Betrayal), These agents are likely to spread disinformation or misinformation. Their position in the network can show us how misinformation might propagate through the system. Green Nodes (Cooperation), These agents represent cooperative behavior, likely sharing accurate information and reinforcing trust within the network. Blue Nodes (Factholder), These nodes may represent sources of accurate information, possibly factcheckers or credible sources that can counteract misinformation. Yellow Nodes (Concealment), These agents might be withholding information, which can contribute to uncertainty and could potentially amplify the negative effects of misinformation if critical facts are not disclosed.

Scenario Context Analysis

Misinformation Spread, The red nodes (betrayal) suggest the presence of misinformation agents within the network. If these nodes are central or wellconnected, their impact on the network can be significant, leading to the temptation payoff (T) increasing for connected agents as they might benefit in the shortterm from spreading or acting on misinformation. Disinformation and Trust Erosion, The impact of the red nodes can also signify disinformation agents. If these agents are discovered (especially if they are central to the network), the penalty (P) due to erosion of social trust can have widespread implications, affecting not just the disinformation agents but also those connected to them. Concealment Risks, Yellow nodes may represent individuals or entities that have information but choose to conceal it. Their behavior can exacerbate the effects of misinformation and disinformation by not providing the necessary counternarratives or facts. Social Harmony and Cooperation: Green nodes are crucial in maintaining the social fabric and trust within the network. They can help to stabilize the network by promoting mutual cooperation (R), thus mitigating the risks associated with misinformation. Factholders as Anchors: Blue nodes are potentially the most critical in combating misinformation and disinformation, serving as anchors of truth. Their connectivity to other nodes, especially the red and yellow nodes, is vital in ensuring that accurate information flows through the network, countering the negative effects of misinformation.

Fig.5-7 we can discuss how the spread of misinformation could lead to increased social discord and the scapegoating of less aggressive entities. The Python code generates a line plot that visualizes the reward and risk over rounds for different behaviors in a networked environment. The behaviors include Betrayal, Cooperation, Concealment, and Fact-holding.

The graphs generated by the code plot the reward and risk for each behavior over 20 rounds. The key points from the graphs are as follows:

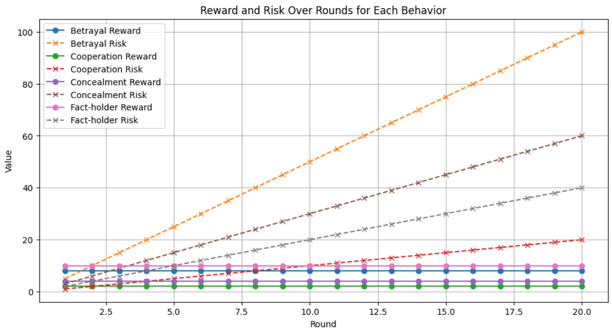

Fig. 6: Network of Agents with Different Behaviors

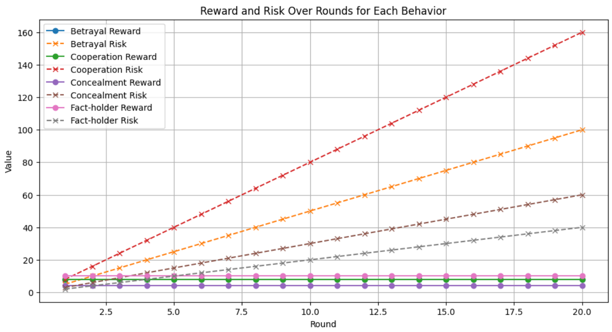

Fig. 7: Network of Agents with Different Behaviors

Betrayal Behavior

Betrayal offers a consistent reward over time. However, the risk associated with betrayal increases linearly with each round. This suggests that the longer one engages in betrayal, the greater the risk of being caught or facing consequences.

Cooperation Behavior

Cooperation also offers a consistent reward over time, matching the reward for betrayal.Interestingly, the risk associated with cooperation is the same as its reward and does not increase over time. This could represent the inherent risk of being betrayed while cooperating.

Concealment Behavior

Concealment provides a lower reward compared to betrayal and cooperation.The risk associated with concealment increases over time, though not as steeply as betrayal. This might signify the potential for concealed information to be revealed.

Fact-holder Behavior

Fact-holding offers the highest reward, which remains constant over time.The risk associated with fact-holding is the lowest among the behaviors and increases at the slowest rate. This suggests that holding and sharing factual information is the safest and most rewarding strategy over time.

Interpretation in the Scenario Context

In a scenario where misinformation leads to scapegoating and increased social unrest, these graphs can be interpreted as follows: Betrayal, Entities that engage in spreading misinformation (betrayal) may initially benefit, but as the rounds progress, they accumulate more risk. This can be paralleled to increasing societal consequences as misinformation is exposed or its negative impacts become more pronounced. Cooperation, Agents that cooperate may do so with the understanding that while there are risks of being betrayed, the reward remains stable. In the context of misinformation, these could be entities that seek to maintain social harmony by cooperating but may be at risk if they inadvertently cooperate with agents of misinformation. Concealment, Entities that conceal information may not have as much to gain as those who cooperate or betray, and their increasing risk over time could represent the growing danger of holding back information that could either clarify misunderstandings or counteract misinformation.

Fact-holder, Those who hold and share factual information (fact-holders) are shown to be in the best position, both in terms of reward and minimal risk. This underscores the value of accurate information and the importance of fact-checking in combating the spread of misinformation. Based on the graphs and the scenario provided, it is evident that while betrayal may offer short-term gains, it comes with increasing risks that could lead to significant long-term consequences. Cooperation has inherent risks but provides consistent rewards. Concealment is less rewarding and still carries risks. Fact-holding appears to be the most beneficial approach, emphasizing the importance of accuracy and transparency in information dissemination to maintain social order and trust. In the long term, promoting fact-holding behavior and countering misinformation and disinformation is crucial for societal well-being.

Comparison and Analysis

When comparing the graphs, several trends and key points emerge Betrayal Behavior, The reward for betrayal remains constant throughout the rounds. The risk associated with betrayal increases the most dramatically, suggesting that the consequences of engaging in betrayal become more severe over time. Cooperation Behavior, the reward for cooperation is consistent, similar to betrayal.However, the risk for cooperation also increases with each round, which could represent the vulnerability to betrayal or the increasing costs of maintaining cooperative relationships over time. Concealment Behavior, Concealment has a lower reward value than both betrayal and cooperation. The risk associated with concealment grows with each round but at a rate that is less than that of betrayal. This might indicate that while concealment can be risky, the potential consequences are not as severe as those for betrayal.

Fact-holder Behavior, Fact-holding has the highest reward, which does not change across the rounds, reflecting the high value of accurate information.The risk associated with fact-holding increases at the lowest rate, indicating that while there is some risk in being a source of truth (perhaps due to potential targeting by those spreading misinformation), it is comparatively the safest behavior.

Risk and Reward Balance, For all behaviors, the reward remains constant, but the risk increases. This dynamic suggests that the longer one engages in a behavior, the more they stand to lose relative to what they gain. Behavioral Sustainability, Fact-holding appears to be the most sustainable behavior, with the highest reward and lowest risk increase. This suggests that in the long run, maintaining integrity and truthfulness is the most advantageous strategy. Incentives for Change, As the risk associated with betrayal and concealment increases, there may be a tipping point where the risks outweigh the rewards, potentially incentivizing agents to shift behaviors. Potential for Intervention, The graphs suggest that early intervention could prevent the risks associated with negative behaviors (like betrayal) from becoming too great. For example, if misinformation is quickly countered, the risk of spreading it might be mitigated. Context of Misinformation, Relating these findings to the scenario of misinformation spread, the increasing risk of betrayal could be likened to the societal costs of spreading misinformation. As misinformation is challenged or its impacts become clearer, the individuals or entities responsible for its propagation face greater risks. Fact-holders, on the other hand, provide a stabilizing influence, suggesting the importance of supporting credible information sources.

The analysis of these graphs underscores the importance of long-term thinking in strategic behavior. It highlights that while engaging in negative behaviors like betrayal may have short-term rewards, the long-term risks are not sustainable. Conversely, maintaining truthful communication and cooperative behavior not only yields consistent rewards but also mitigates risks over time. In the battle against misinformation, promoting and protecting fact-holding behavior while addressing the risks of betrayal and concealment is crucial.

Fig.8, we can discuss how the spread of misinformation could lead to increased social discord and the scapegoating of less aggressive entities. The Python code generates a line plot that visualizes the reward and risk over rounds for different behaviors in a networked environment. The behaviors include Betrayal, Cooperation, Concealment, and Fact-holding.

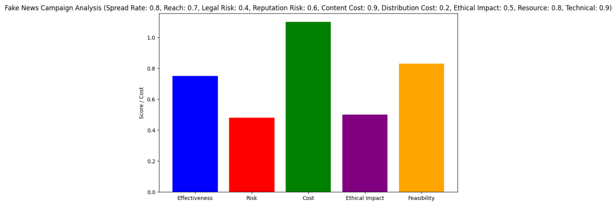

Fig. 8: Fake News Campaign Analysis

Fig.8 generated by the code plot the reward and risk for each behavior over 20 rounds. The key points from the graphs are as follows Betrayal BehaviorBetrayal offers a consistent reward over time. However, the risk associated with betrayal increases linearly with each round. This suggests that the longer one engages in betrayal, the greater the risk of being caught or facing consequences. Cooperation BehaviorCooperation also offers a consistent reward over time, matching the reward for betrayal. Interestingly, the risk associated with cooperation is the same as its reward and does not increase over time. This could represent the inherent risk of being betrayed while cooperating. Concealment BehaviorConcealment provides a lower reward compared to betrayal and cooperation. The risk associated with concealment increases over time, though not as steeply as betrayal. This might signify the potential for concealed information to be revealed. Fact-holder BehaviorFact-holding offers the highest reward, which remains constant over time. The risk associated with fact-holding is the lowest among the behaviors and increases at the slowest rate. This suggests that holding and sharing factual information is the safest and most rewarding strategy over time.

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Based on the graphs and the scenario provided, it is evident that while betrayal may offer short-term gains, it comes with increasing risks that could lead to significant long-term consequences. Cooperation has inherent risks but provides consistent rewards. Concealment is less rewarding and still carries risks. Fact-holding appears to be the most beneficial approach, emphasizing the importance of accuracy and transparency in information dissemination to maintain social order and trust. In the long term, promoting fact-holding behavior and countering misinformation and disinformation is crucial for societal well-being.

Computational Model Application

Using the given values (R, S, T, P, δ), we can calculate the present value of long-term gains for cooperative (C) and defecting (D) strategie

To calculate the present value of longterm gains, we can use the formula for the present value of an annuity, considering the game is repeated indefinitely:

$$PV = \frac{R}{1\delta} \text{ for cooperation}$$

$$PV = \frac{T + (R \cdot \delta)}{1\delta} \text{ for defection}$$

Where: PV is the present value. R is the reward for mutual cooperation. T is the gain from betrayal. δ is the discount factor.

The present value (PV) of longterm gains for the cooperative strategy (C) is approximately 30, and for the defection strategy (D), it is approximately 32.

Interpretation in the Network Context

Cooperation Strategy (C): With a present value of 30, the longterm gains from cooperation are significant, which indicates that if an agent follows a cooperative strategy, they can expect a steady payoff over time. This reflects the importance of the green nodes (Cooperation) in the network, which help to sustain social harmony and trust. Defection Strategy (D): The slightly higher present value of 32 for defection suggests that an agent could potentially gain more in the short term by betraying once and then reverting to cooperation. This could be indicative of the temptation that red nodes (Betrayal) in the network represent.

However, the difference between the present values of the two strategies is not very large, which implies that the temptation of immediate gain from defection is not overwhelmingly more beneficial than cooperation when the longterm is considered, especially given the potential penalty (P) for mutual betrayal, which we did not include in the defection calculation. This penalty could significantly reduce the present value of defection if it were applied repeatedly.

In the context of misinformation and disinformation, the network analysis and the computational model suggest that while defection (or betrayal) might seem advantageous in the very short term, the longterm benefits of cooperation outweigh the temporary gains from defection. This is especially true if the model

In evolutionary game theory, the search for Evolutionarily Stable Strategies (ESS) is a crucial approach to understanding how dominant strategies are established and maintained within a population. To introduce the computational process of exploring ESS in the context of the Iterated Dilemma, it is necessary to model how bidders' strategies interact and evolve over time. Below, we detail this computational process and the equations involved.

Definition of Strategy Space

Define the set of strategies that bidders can adopt. These strategies can include rules for determining bid amounts or rules for changing strategies in response to other bidders' actions. For example, simple strategies might include "always bid the average price" or "adjust the bid amount based on the previous winner's bid amount."

Construction of Payoff Matrix

Calculate the payoff for each combination of strategies and construct the payoff matrix. The payoff is determined based on the outcome of the auction, bid amounts, and the true value of the goods. The payoff matrix represents the expected outcomes when different bidders adopt various strategies.

Modeling Strategy Evolution Dynamics

Construct a mathematical model describing how bidders' strategies evolve. A commonly used model is the replicator equation, which is based on the principle that if a strategy earns a higher payoff than average, the proportion of individuals adopting that strategy increases.

Identification of ESS

Solve the replicator equation to find the stable states for each strategy. An ESS is defined as a strategy that, if adopted by a population, remains stable against the invasion of a small number of mutant strategies. Mathematically, the condition for a strategy s to be an ESS against any mutant strategy s is

$$E(s' s') > E(s, s')$$

Or, if $E(s' s') = E(s, s')$, then

$$E(s' s) > E(s, s)$$

Here, $E(s, s')$ represents the payoff when a bidder adopting strategy s confronts a bidder adopting strategy s'.

Numerical Simulation

Perform numerical simulations to identify the ESS in practice. Set an initial distribution of bidders' strategies randomly and calculate how the distribution of strategies changes based on the replicator equation. Observe how the system converges to a stable state over time.

Analysis of Results

Analyze the results of the simulation to identify strategies that become ESS and the distribution of strategies. Evaluate under what conditions a particular strategy becomes dominant and how different parameters (e.g., discount rate δ or auction rules) affect the evolution of strategies.

Through this computational process, a deeper understanding of the evolutionary stability of strategies in the context of auctions considering the iterated dilemma can be obtained. Using evolutionary game theory provides insights into the behavior and strategy choices of bidders in actual auctions.

For the model based on evolutionary game theory that considers the process of spreading fake news as a commodity and forming its value, the calculation process for the Evolutionarily Stable Strategy (ESS) across the transition from the first to the second auction involves a five-step evaluation matrix reflecting the scenario. Here's how this process can be detailed

Definition of Strategy Space

Define the strategies that spreaders of fake news (bidders) can adopt. For example, possible strategies might include

Enhancing credibility strategy Mixing in some factual information with fake news to increase its believability. Emotional appeal strategy Emphasizing content that appeals to the readers' emotions to enhance the likelihood of spreading. Targeting strategy Creating content tailored to specific target groups to increase efficiency in spreading.

Construction of Evaluation Matrix

Calculate the evaluation (payoff) for each combination of strategies and construct the evaluation matrix. The evaluation is based on factors such as the spread of fake news, the reaction from readers, and value formation in the auction.

Modeling Strategy Evolution Dynamics

Use the replicator equation to model how each strategy evolves over time. This model shows how the payoff of each strategy changes in comparison to the average payoff.

Searching for ESS

Solve the replicator equation to explore the stable states for each strategy. An ESS is defined as a strategy that remains stable against the invasion of a small number of mutant strategies.

Transition from the First to the Second Auction

Determine the strategies for the second auction based on the results of the first auction (value formation of fake news). This process involves considering how the outcomes of the first auction influence the strategy choices in the second auction.

Equations and Calculation Process

The replicator equation is represented as

$$\dot{x}_i = x_i(f_i - \bar{f})$$

Here, x_i is the proportion of the population adopting strategy i, f_i is the payoff for strategy i, and $\bar{f}$ is the average payoff of the population.

The condition for ESS is defined as

$$f(s's') > f(s, s')$$

Or, if $f(s's') = f(s, s')$, the following condition must hold

$$f(s's) > f(s, s)$$

Here, s is the strategy that is ESS, and s is any mutant strategy.

To reflect the results of the first auction in the strategy choices for the second auction, evaluate the success of each strategy in the first auction and adjust the probability of strategy choices in the second auction accordingly. This may involve multi-agent simulations or evolutionary algorithms.

Through this process, the dynamics of spreading and value formation of fake news as a commodity can be understood, and the most effective strategies from an evolutionary game theory perspective can be identified.

Continuing with the analysis, let's delve deeper into the calculation process and equations for defining the strategy space in the context of spreading fake news as a commodity and forming its value.

Step 1 Definition of Strategy Space

In the process of spreading fake news as a commodity and forming its value, we define the types and characteristics of strategies that bidders can take, and mathematically model the outcomes these strategies yield.

Types of Strategies

1. Enhancing Credibility Strategy (S1) This strategy involves mixing some truth with fake news to increase its credibility and make it more widely accepted. 2. Emotional Appeal Strategy (S2) This strategy emphasizes content that appeals to readers' emotions, enhancing the likelihood of spreading. 3. Targeting Strategy (S3) This strategy involves creating content tailored to specific target groups to increase spreading efficiency within those groups.

Modeling the Effects of Strategies

For each strategy, we model its effects based on the following elements

- Spread Rate (D) Indicates how widely the news is spread. The spread rate varies depending on the strategy, represented by parameters d_1, d_2, d_3. - Credibility (C) Indicates how believable the news is to the recipients. Credibility varies depending on the strategy, represented by parameters c_1, c_2, c_3. - Target Fit (T) Indicates how well the news fits the target audience. The target fit varies depending on the strategy, represented by parameters t_1, t_2, t_3.

Mathematical Representation

The expected utility $E[U_{S_i}]$ of each strategy S_i is modeled as a function of the spread rate, credibility, and target fit.

$$E[U_{S_i}] = f(D_i, C_i, T_i)$$

Where, $D_i = d_i \cdot \text{BaseD}$, $C_i = c_i \cdot \text{BaseC}$, $T_i = t_i \cdot \text{TargetFit}$.

Example Calculation

For instance, the expected utility for the Enhancing Credibility Strategy $S1$ would be

$$E[U_{S1}] = d_1 \cdot \text{BaseD} + c_1 \cdot \text{BaseC} + t_1 \cdot \text{TargetFit}$$

Here, d_1, c_1, t_1 are parameters specific to strategy $S1$, and BaseD, BaseC are the base values for spread rate and credibility, while TargetFit is a parameter indicating the fit with the target audience.

By defining the expected utility of each strategy, we set a criterion for selecting the most effective strategy. The choice of strategy ultimately depends on how it impacts auction outcomes and the formation of value for fake news.

Construction of Evaluation Matrix

In this step, we evaluate and compare the effects of each strategy by constructing a matrix. This matrix numerically represents the effects of different strategies, such as spread rate, credibility, and target fit, and clarifies their interrelationships.

Elements of the Evaluation Matrix

- Rows represent specific strategies (e.g., Enhancing Credibility Strategy, Emotional Appeal Strategy, Targeting Strategy). - Columns represent the criteria for evaluation (e.g., Spread Rate D, Credibility C, Target Fit T).

1. Define indicators for evaluating each strategy (spread rate, credibility, target fit). 2. Assign values to each strategy for each indicator, reflecting the effectiveness or efficiency of the strategy. 3. Create a matrix with strategies as rows and indicators as columns.

Mathematical Representation

The evaluation matrix E can be represented as

$$E = \begin{bmatrix} d_1 & c_1 & t_1 \\ d_2 & c_2 & t_2 \\ d_3 & c_3 & t_3 \end{bmatrix}$$

Where d_i, c_i, t_i represent the values for spread rate, credibility, and target fit for strategy i, respectively.

Example Calculation

Assuming there are strategies $S1$, $S2$, and $S3$ with values for spread rate, credibility, and target fit assigned as follows

- For $S1$ $d_1 = 0.7, c_1 = 0.8, t_1 = 0.5$ - For $S2$ $d_2 = 0.9, c_2 = 0.6, t_2 = 0.4$ - For $S3$ $d_3 = 0.5, c_3 = 0.7, t_3 = 0.9$

The evaluation matrix constructed with these values would be

$$E = \begin{bmatrix} 0.7 & 0.8 & 0.5 \\ 0.9 & 0.6 & 0.4 \\ 0.5 & 0.7 & 0.9 \end{bmatrix}$$

This matrix allows us to compare the relative effects of each strategy and determine which strategy is most suitable for achieving a specific objective, such as maximizing spread.

Continuing with the steps for modeling the strategy evolution dynamics, identifying ESS, and analyzing the results will provide a comprehensive understanding of how strategies evolve and stabilize within the context of spreading fake news as a commodity in auction scenarios.

Continuing with the analysis, let's explore the modeling of strategy evolution dynamics, the identification of Evolutionarily Stable Strategies (ESS), and the analysis of results in the context of spreading fake news as a commodity.

In the evolution of strategies within the information oligopoly market, particularly for the spreading of fake news, the dynamics of strategy evolution can be modeled using mathematical equations that reflect how strategies adapt and change over time in response to the environment and interactions with other strategies.

A common approach to model the dynamics of strategy evolution is through the use of the Replicator Equation, which is represented as

$$\dot{x}_i = x_i(\pi_i - \bar{\pi})$$

Where - x_i represents the proportion of the population adopting strategy i. - $\dot{x}_i$ is the change in proportion of strategy i over time. - π_i is the payoff (or fitness) of strategy i. - $\bar{\pi}$ is the average payoff (or fitness) across all strategies within the population.

This equation indicates that strategies which perform better than the average increase in proportion over time, whereas those performing worse decrease.

Identifying an ESS within the context of fake news spreading involves finding strategies that, once prevalent within a population, cannot be invaded by any alternative strategy (mutant strategy) due to their inherent stability and higher payoff.

A strategy s is considered an ESS if, for any mutant strategy s, one of the following conditions is met

1. $E(s^{\cdot}s^{\prime}) > E(s, s^{\prime})$, meaning the ESS yields a higher payoff against itself than the mutant does against the ESS.

2. If $E(s^{\cdot}s^{\prime}) = E(s, s^{\prime})$, then it must hold that $E(s^{\cdot}s) > E(s, s)$, meaning in a mixed population of ESS and mutants, the ESS performs better against the mutant than the mutant does against itself.

Suppose, in simulations, the Enhancing Credibility Strategy (S1) frequently emerges as a dominant strategy, leading to a high spread rate and credibility but moderate target fit. An in-depth analysis might reveal that in environments where credibility is highly valued, S1 tends to stabilize and resist invasion by other strategies like Emotional Appeal (S2) or Targeting (S3), thereby indicating its status as an ESS in such contexts.

By examining the results through these steps, researchers can gain insights into the dynamic interplay of strategies in the informational oligopoly market, particularly in the context of spreading fake news. This analysis not only aids in understanding the theoretical underpinnings of such systems but also has practical implications in devising counter-strategies to combat the spread of misinformation.

4.6 The Payoff Matrix of Strategies

First, we construct the payoff matrix P that represents the outcomes of contests between each pair of strategies. Each element P_{ij} of this matrix indicates the average "payoff" (benefit) that an individual adopting strategy i receives when facing an individual adopting strategy j.

$$P = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{bmatrix}$$

Here, n is the total number of strategies.

5. Evolutionary Dynamics Equation

Next, we introduce the dynamics equation that describes how the proportion of each strategy changes over time. One of the most common models used is the replicator equation, expressed as follows

$$\dot{x_i} = x_i\,(\pi_i - \bar{\pi})$$

Where, x_i is the proportion of the population adopting strategy i, $\dot{x_i}$ is the rate of change in proportion of strategy i over time, π_i is the average payoff of strategy i, $\bar{\pi}$ is the average payoff across the entire population.

6. Calculation of Average Payoff

The average payoff π_i for strategy i is calculated using the payoff matrix and the proportions of strategies as follows

$$\pi_i = \sum_{j=1}^{n} P_{ij} x_j$$

The average payoff $\bar{\pi}$ for the entire population is calculated as the sum of the products of the average payoff of each strategy and its proportion in the population

$$\bar{\pi} = \sum_{i=1}^{n} x_i \pi_i$$

6.1 Example of Calculation Process

1. **Construction of the Payoff Matrix** The payoff matrix P is constructed based on the outcomes of contests between each pair of strategies. 2. **Setting Initial Strategy Proportions** Initial proportions $x_i(0)$ for each strategy are set. 3. **Application of Dynamics Equation** The replicator equation is used to calculate how the proportion of each strategy changes over time. This step often involves numerical analysis methods. 4. **Searching for ESS** As time progresses towards infinity, if the proportion of a certain strategy approaches 1 (all other strategies are eliminated), that strategy is considered an Evolutionarily Stable Strategy (ESS).

This calculation process allows us to understand how a particular strategy evolves over the long term and which strategies become evolutionarily stable.

7. DiscussionEvolutionary Dynamics Equations

The process of calculating evolutionary dynamics focuses on modeling how the proportion of strategies changes over time. We detail this calculation process using the replicator equation.

7.0.1 Replicator Equation

The replicator equation is a fundamental formula in evolutionary game theory that describes the change in strategy proportions. It is given by

$$\dot{x_i} = x_i\,(\pi_i - \bar{\pi})$$

Where,

x_i is the proportion of the population adopting strategy i,

$\dot{x_i}$ is the rate of change in the proportion of strategy i over time,

π_i is the average payoff for strategy i,

$\bar{\pi}$ is the average payoff across all strategies in the population.

7.0.2 Calculation of Average Payoff

The average payoff π_i for strategy i is calculated using the payoff matrix and the strategy proportions as follows

$$\pi_i = \sum_{j=1}^{n} P_{ij} x_j$$

The overall average payoff $\bar{\pi}$ for the population is then calculated as the sum of the products of each strategy's average payoff and its proportion

$$\bar{\pi} = \sum_{i=1}^{n} x_i \pi_i$$

1. **Preparation of the Payoff Matrix** Prepare the matrix P showing the payoffs between strategies. 2. **Setting Initial Strategy Proportions** Set the initial proportions x_i for each strategy. 3. **Calculation of Average Payoffs** Calculate the average payoff π_i for each strategy using the formula provided. 4. **Calculation of the Overall Average Payoff** Compute the overall average payoff $\bar{\pi}$ using the average payoffs and the strategy proportions. 5. **Application of the Replicator Equation** Apply the replicator equation to calculate the rate of change $\dot{x_i}$ for each strategy. 6. **Update of Strategy Proportions** Update the proportions of each strategy at small time intervals Δt as follows

$$x_i(t + \Delta t) = x_i(t) + \dot{x_i}\Delta t$$

7. **Iterative Calculation** Repeat steps 3 to 6 to observe how the proportions of strategies change over time.

8. DiscussionIdentification of Evolutionarily Stable Strategies (ESS)

The search for ESS is a central concept in evolutionary game theory. An ESS is a strategy that, once fixed in a population, cannot be invaded by a small number of mutants. We detail the process of searching for ESS below.

8.1 Definition of ESS

A strategy S is considered an ESS if it meets the following two conditions against any mutant strategy S'

1. **Invadability** S obtains a higher or equal payoff against itself compared to S' against S

$$E(S{\cdot}S^{\rangle} \geq E(S', S^{\rangle}$$

2. **Stability** If $E(S{\cdot}S^{\rangle} = E(S', S^{\rangle}$, then S must obtain a higher payoff against S' than S' against itself

$$E(S{\cdot}S') > E(S', S')$$

1. **Preparation of the Payoff Matrix** Prepare the payoff matrix based on the interactions between strategies. 2. **Application of ESS Conditions** Apply the ESS conditions to all pairs of strategies. 3. **Verification of Invadability** Verify whether a strategy S is invadable by any mutant strategy S' by comparing $E(S{\cdot}S^{\rangle}$ and $E(S', S^{\rangle}$. 4. **Verification of Stability** If $E(S{\cdot}S^{\rangle} = E(S', S^{\rangle}$ for some mutant strategy S', verify the stability of S by comparing $E(S{\cdot}S')$ and $E(S', S')$. 5. **Identification of ESS** Identify strategies that meet all ESS conditions.

8.2 Example of Mathematical Formulation

Consider a payoff matrix given by

$$P = \begin{bmatrix} 5 & 0 \\ 3 & 1 \end{bmatrix}$$

In this case, we have two strategies, 1 and 2. Strategy 1, when facing itself, obtains a payoff of 5, and against strategy 2, a payoff of 0. Strategy 2 obtains a payoff of 3 against strategy 1 and 1 against itself.

To verify whether strategy 1 is an ESS

Invadability Compare $E(1, 1) = 5$ with $E(2, 1) = 3$. Strategy 1 is not invadable since $5 \geq 3$. **Stability** Since invadability is the only condition met, strategy 1 is an ESS.

To verify whether strategy 2 is an ESS

Invadability Compare $E(2, 2) = 1$ with $E(1, 2) = 0$. Strategy 2 is not invadable since $1 \geq 0$. **Stability** Since invadability is the only condition met, strategy 2 is also an ESS.

By using the payoff matrix and the definitions of ESS, we can identify evolutionarily stable strategies.

(1) **Analysis of the Results of the First Auction** Analyze the strategies and outcomes (whether they won or lost, how much profit was made, etc.) of each participant in the first auction.

(2) **Updating Information** Based on the results of the first auction, participants update their information and beliefs. This includes information inferred from the actions of other bidders and reevaluation of the conditions for winning the auction.

(3) **Adapting Strategies** Participants adapt their strategies for the second auction based on the results of the first auction and updated information. This adjustment of strategies takes into account learning from past results and reactions to the strategies of other bidders.

(4) **Modeling Value Formation** The value of goods or services in the second auction is formed based on the results of the first auction and market trends. This value formation affects the expected gains calculations of the participants and guides their choice of strategy.

Let R_1 represent the results of the first auction and $E[u_i^{(2)}]$ the expected gain of bidder i in the second auction. The strategy chosen by bidder i for the second auction is $S_i^{(2)}$. The calculation of expected gains is then as follows

$$E[u_i^{(2)}] = f(S_i^{(2)}, R_1, \text{Info}_i)$$

Here, f is the function for calculating expected gains, and Info_i is the information set held by bidder i, updated based on the results of the first auction R_1.

The strategy $S_i^{(2)}$ adapted by the bidder for the second auction is determined by solving the following optimization problem

$$S_i^{(2)} = \arg\max_{S_i} E[u_i^{(2)}]$$

This calculation process determines the optimal strategy based on the information and experiences gained from the results of the first auction. This process can be seen as the evolution of strategy considering dynamic market conditions and interactions among participants.

The process of updating information based on the results of the first auction involves analyzing the outcomes of the auction, using this information to update participants' beliefs and strategies. This process generally consists of the following steps

(1) **Collection of Results from the First Auction** Gather results such as the bidding amounts, the winner, and the final prices from the first auction.

(2) **Analysis of Results** Analyze the collected data to gain insights into which strategies were successful and how each bidder behaved.

(3) **Updating the Information Set** Based on the analysis, each bidder updates their information set. This may include trends in the market, strategic tendencies of other bidders, and adjustments to value assessments.

(4) **Updating Beliefs** Bidders update their beliefs based on the updated information. For instance, a bidder might adjust their value assessment based on the bidding tendencies of others.

8.3 Example of Formulas

Let R_1 represent the results of the first auction, and let $\text{Info}_i^{(1)}$ be the information set of bidder i after the first auction. The process by which bidder i acquires new information $\text{Info}_i^{(2)}$ based on the results of the first auction can be modeled as follows

$$\text{Info}_i^{(2)} = g(\text{Info}_i^{(1)}, R_1)$$

Here, g is the information updating function, generating a new information set $\text{Info}_i^{(2)}$ based on the first auction results R_1 and the existing information set $\text{Info}_i^{(1)}$.

The updating of bidder i's beliefs, denoted as B_i, based on the new information set is given by

$$B_i^{(2)} = h(B_i^{(1)}, \text{Info}_i^{(2)})$$

Where h is the belief updating function, deriving the updated beliefs $B_i^{(2)}$ from the existing beliefs $B_i^{(1)}$ and the new information set $\text{Info}_i^{(2)}$.

8.3.1 Example of the Calculation Process

If the bidding amount by bidder i in the first auction was higher than average, the bidder might conclude that they underestimated the market value and adjust their value assessment accordingly. In this case, the information updating function g and the belief updating function h might be as follows

$$\text{Info}_i^{(2)} = \text{Info}_i^{(1)} + \text{Adjustment from } R_1$$
$$B_i^{(2)} = B_i^{(1)} + \text{Adjustment based on Info}_i^{(2)}$$

The adjustments are quantitatively determined based on the analysis, such as adjusting the value assessment upwards based on how much higher the bidder's bidding amount was compared to the average of others.

The process from updating beliefs, adapting strategies, to modeling value formation illustrates how bidders participating in an auction learn from past experiences, adjust their strategies accordingly, and ultimately form the market value of goods or services.

Evolutionary game theory models the process by which players (individuals) learn and adapt optimal strategies through repeated interactions. Applying evolutionary game theory in the context of the repeated dilemma, especially the well-known prisoner's dilemma, helps understand how cooperative behavior can evolve among individuals pursuing their self-interest. Below are some ideas from evolutionary game theory that can be readily applied to the repeated dilemma game

(1) **Strategy Reproduction**

Imitation of Strategies Individuals learn by imitating the strategies of other successful individuals. In the context of repeated dilemma games, the strategy of a player who obtains high rewards (such as "always cooperate" or "retaliate") may be imitated by others.

(2) **Diversity of Strategies**

Conditional Strategies Adopt strategies that choose the best action based on the given situation. For example, the "Tit-for-Tat" strategy, which cooperates in return for previous cooperation and retaliates in response to betrayal, is one such strategy.

(3) **Search for Stable Strategies**

Evolutionarily Stable Strategy (ESS) An ESS is a strategy that, once it becomes prevalent in a population, cannot be invaded by a small number of mutants. Finding an ESS in a repeated dilemma game helps understand which strategies are stable in the long term.

(4) **Dynamic Strategy Changes**

Adaptive Dynamics Individuals adapt their strategies based on the environment and the actions of others. In repeated dilemma games, players may adjust their strategies based on the outcomes of previous rounds.

(5) **Consideration of Group Effects**

Group Dynamics Individuals may change their behavior through interactions within a group. In repeated dilemma games, the formation of cooperative clusters can increase the likelihood of continued cooperation among individuals within the group.

(6) **Consideration of Interdependencies**

Interaction Networks Interactions between individuals often form network structures. In repeated dilemma games, considering the network of relationships between players can help analyze patterns of cooperation and betrayal.

These ideas provide a useful framework for understanding the evolution of strategies and the resulting dynamics in repeated dilemma games. Through simulations and mathematical modeling, these ideas can be further explored to analyze specific strategies and patterns of interaction.

When considering the evolutionary game of strategy imitation within an information oligopoly market, players treat information as a commodity and aim to profit by offering it in the market. Considering the repeated dilemma in such a situation suggests the importance of strategies that emphasize long-term relationships and trust among players. Below, we detail the formulas and calculation process for modeling this scenario.

9. DiscussionModeling Evolutionary Games of Strategy Imitation in Information Oligopoly Markets

When considering the evolutionary game of strategy imitation within an information oligopoly market, players treat information as a commodity and aim to profit by offering it in the market. Considering the repeated dilemma in such a situation suggests the importance of strategies that emphasize long-term relationships and trust among players. We detail the formulas and calculation processes to model this scenario below.

9.1 Definition of Strategy Space

Players (bidders) select their strategies from a set of strategies $S = \{s_1, s_2, \ldots, s_n\}$. A strategy comprises several elements, including the quality of information, the price of offering, and the quantity provided.

9.2 Setting Up the Payoff Function

We establish a payoff function $U_i(s_i, s_{-i})$ for each player, where s_i is player i's strategy, and s_{-i} is the combination of strategies of other players. The payoff depends on the chosen strategy and the strategies of other players in the market.

9.3 Imitation and Evolution of Strategies

Players evolve their strategies by imitating more successful players' strategies. This dynamic can be expressed as follows

$$s_i^{(t+1)} = \arg\max_{s \in S} U_i(s, s_{-i}^{(t)})$$

Here, $s_i^{(t+1)}$ is the strategy player i will adopt in the next time step, and $s_{-i}^{(t)}$ is the combination of strategies of other players at the current time step.

9.4 Searching for a Stable State

When the market reaches a stable state, there is no change in the strategies among players. A stable state is a situation where no player can improve their payoff by unilaterally changing their current strategy. This corresponds to a Nash equilibrium.

$$\forall i,\ U_i(s_i^{*}, s_{-i}^{*}) \geq U_i(s_i, s_{-i}^{*}) \text{ for all } s_i \in S$$

Here, (s_i^{*}, s_{-i}^{*}) is the combination of strategies at the Nash equilibrium.

9.5 Discounting Future Payoffs

We consider discounting future payoffs to their present value.

$$\tilde{U}_i(s_i, s_{-i}) = \sum_{t=0}^{T} \delta^t U_i(s_i^{(t)}, s_{-i}^{(t)})$$

Here, δ is the discount rate used to convert future payoffs to their present value.

9.6 Numerical Optimization

The optimization problem is usually solved using numerical methods (e.g., gradient descent, simulation-based optimization). Due to the dynamic interactions and evolving strategies within the market, this process is iterative, with each player continuously updating their information and adjusting their strategies.

This model provides a framework for understanding strategic interactions and their evolution within an information oligopoly market, illustrating how players choose and adapt optimal strategies to strengthen their position in the market.

10. DiscussionDefinition of Strategy Space and Setting Up the Payoff Function

The processes of defining the strategy space and setting up the payoff function are foundational in designing an evolutionary game of strategy imitation within an information oligopoly market. These steps define what strategies are available to players and how the payoff for each strategy is calculated.

10.1 Definition of Strategy Space

In defining the strategy space, we identify the set of all possible strategies S that players can choose from. This set might include various elements such as information quality, pricing, quantity offered, and marketing approaches.

To define the strategy space, we need to quantify specific strategy parameters. For instance, considering parameters like price p, the amount of advertising a, and the quality of information q, a strategy s_i can be represented as a combination of these parameters

$$s_i = (p_i, a_i, q_i)$$

where p_i represents the price in strategy i, a_i the amount of advertising, and q_i the quality of information.

10.2 Setting Up the Payoff Function

The payoff function defines the payoff (or utility) a player receives from adopting a particular strategy. This function depends on both the player's chosen strategy and the combination of strategies of other participants. The payoff function is represented as $u_i(s_i, s_{-i})$, indicating the payoff for player i choosing strategy s_i, against the strategies s_{-i} of other players.

As an example of a payoff function, consider the payoff in a price competition scenario. The payoff for player i could be represented as a function of their own price p_i, the prices p_{-i} of other players, the per-unit cost c of the information product sold, and a market demand function $D(p_i, p_{-i})$

$$u_i(p_i, p_{-i}) = (p_i - c) \times D(p_i, p_{-i})$$

Here, $D(p_i, p_{-i})$ represents the market demand function, which depends on the price p_i and the prices p_{-i} of other players, indicating how price influences demand.

These steps provide a framework for modeling strategic interactions in an information oligopoly market, helping analyze what strategies players might choose and how those strategies impact their payoffs.

11. DiscussionImitation and Evolution of Strategies

The process of imitation and evolution of strategies in an information oligopoly market models how market participants imitate the strategies of others perceived as successful and evolve their strategies accordingly. This process is based on concepts from evolutionary game theory.

11.1 Imitation of Strategies

Market participants observe and imitate the strategies of other participants that are perceived as successful, based on the payoffs those strategies yield. The probability of imitation depends on the success level of the strategy being imitated.

Consider the probability $P_{i \to j}$ that player i imitates the strategy s_j of player j. This probability is higher when the payoff u_j of player j is greater than the payoff u_i of player i. A common form can be represented as follows

$$P_{i \to j} = \frac{e^{\beta u_j}}{\sum_k e^{\beta u_k}}$$

where β is a parameter indicating the intensity of selection (or the degree of rationality), and u_k represents the payoffs of all players in the market.

11.2 Evolution of Strategies

The evolution of strategies occurs over time through the process of imitation. As players imitate the strategies of others, the overall distribution of strategies in the market changes, leading to the evolution of the market's strategic structure.

Let x_s represent the proportion of strategy s in the market, and $\dot{x}_s$ its rate of change over time. The dynamics of evolution can be modeled as follows

$$\dot{x}_s = x_s\,(\bar{u}_s - \bar{u})$$

where $\bar{u}_s$ is the average payoff of players adopting strategy s, and $\bar{u}$ is the average payoff across all players in the market. This equation indicates that the proportion of strategies yielding above-average payoffs increases.

This step mathematically represents how market participants imitate successful strategies of others and evolve their strategies. The process of imitation and evolution helps analyze how the overall strategic structure of the market changes over time.

12. Searching for a Stable State

The process of searching for a stable state in the context of an information oligopoly market involves analyzing how the strategies of market participants evolve over time and ultimately reach a state of stability. In this state, no player can unilaterally change their strategy to improve their payoff.

12.1 Conditions for a Stable State

In a stable state, all participants in the market are employing strategies that are optimal, and the system has the capacity to return to this state after small perturbations. This condition can be mathematically expressed.

Let x_s denote the proportion of strategy s in the market, and its evolutionary dynamics previously described by $\dot{x}_s =$

$x_s(\bar{u}_s - \bar{u})$. A stable state x_s is achieved when the following condition is met

$$\left.\frac{d\dot{x}_s}{dx_s}\right|_{x_s=x_s} < 0$$

This condition implies that the derivative of the change rate $\dot{x}_s$ of strategy s with respect to its proportion x_s is negative at the stable state x_s, indicating the system's tendency to return to the stable state after small perturbations.

12.2 Searching for the Stable State

The search for a stable state is conducted by simulating how the distribution of strategies in the market changes over time, starting from various initial conditions and parameters, and observing where the system converges.

Starting from an initial strategy distribution $x_s(0)$, evolve $x_s(t)$ over time according to the evolutionary dynamics. If $x_s(t)$ converges to a constant x_s as time approaches infinity, then x_s can be considered a stable state.

This step provides a mathematical framework for understanding how strategies in an information oligopoly market evolve and reach a state of stability. Identifying stable states helps predict the long-term behavior and strategic structure of the market.

13. DiscussionDiscounting Future Payoffs

In considering strategies within an information oligopoly market, it's important to account for how future payoffs are discounted to their present value. This step analyzes the impact of discounting future payoffs on current decision-making.

13.1 Principle of Discounting

Future payoffs are generally considered less valuable than present ones, depending on the time until receipt and the degree of uncertainty. This is expressed by applying a discount rate to convert future payoffs to their present value.

Mathematical Example

To discount the expected payoff $E[u_i^{(t)}]$ in a future round t to its present value using a discount rate δ (where $0 < \delta < 1$), the discounted payoff $D[u_i^{(t)}]$ is given by

$$D[u_i^{(t)}] = \delta^t \cdot E[u_i^{(t)}]$$

where δ^t is the factor used to discount the payoff at time t to its present value.

13.2 Total Discounted Future Payoffs

The total discounted future payoffs for player i across all future rounds is calculated as the sum of the discounted payoffs for each round

$$D[u_i^{\text{total}}] = \sum_{t=1}^{T} D[u_i^{(t)}] = \sum_{t=1}^{T} \delta^t \cdot E[u_i^{(t)}]$$

This formula represents the total payoff after discounting future payoffs, providing a guideline for players to make optimal strategic choices from a long-term perspective.

1. Identify the expected future payoff $E[u_i^{(t)}]$ for each future round. 2. Apply the discount rate δ to each round's payoff to calculate the discounted payoff $D[u_i^{(t)}]$. 3. Sum the discounted payoffs $D[u_i^{\text{total}}]$ across all rounds.

By considering discounted future payoffs, players can select strategies that not only focus on immediate gains but also take into account the long-term benefits, enabling more sustainable and profitable strategic choices.

14. DiscussionCalculation Process for Future Round Payoffs

When calculating the payoffs for bidders in future rounds, it is necessary to consider how the bidder's actions, the actions of other players, and the market conditions interact in that round. Below are the calculation process and the associated formulas.

(1) **Strategy Definition** Define the strategies that bidders can take. This may consist of various elements such as bid amounts, bidding strategies, and whether to share information.

(2) **Market Condition Forecasting** Predict the market conditions in each round. This may include the number of other bidders, the types of goods, and market demand.

(3) **Setting Up the Payoff Function** Based on the bidder's strategy and market conditions, set up a payoff function to calculate the bidder's payoff in that round.

(4) **Consideration of Other Players' Strategies** Consider the strategies that other bidders might take and how they would affect the bidder's payoff.

(5) **Calculation of Expected Payoff** Taking into account the above elements, calculate the expected payoff for the bidder in each round.

The general form for calculating the expected payoff $E[u_i^{(t)}]$ for bidder i in round t is as follows

$$E[u_i^{(t)}] = \sum_j P(s_j | \text{Market Conditions, Strategies of Other Players}) \times U_i$$

where, $P(s_j|\text{Market Conditions, Strategies of Other Players})$ is the probability that strategy s_j occurs given certain market conditions and the strategies of other players. $U_i(s_j, s_i)$ is the payoff for bidder i when they choose strategy s_i and other players choose strategy s_j.

(1) **Calculation of Probabilities Based on Market Conditions and Strategies of Other Players** Calculate the probabilities of possible outcomes of strategies based on the market conditions and the strategies of other players in each round.

(2) **Calculation of Payoffs** Use the above probabilities and the payoff function for specific combinations of strategies to calculate the payoffs for the bidder.

(3) **Summation of Expected Payoffs** Sum the payoffs for all possible outcomes of strategies to determine the expected payoff for the bidder in round t.

Through this process, it is possible to understand how the bidder's strategy interacts with market conditions in each round and what payoffs result from this interaction. This allows bidders to strategize from a long-term perspective and make optimal decisions.

15. DiscussionCalculation of Probabilities Based on Market Conditions and Strategies of Other Players

Calculating the probabilities based on market conditions and the strategies of other players is crucial when considering strategic interactions in an information oligopoly market. This calculation process evaluates the outcomes that each player's strategy might produce given the market conditions and derives the probabilities of each strategy accordingly.

(1) **Identification of Market Conditions** Define the market conditions, such as market size, the number of players, characteristics of goods, and market demand.

(2) **Identification of Player Strategies** Identify the set of strategies that each player can take. Strategies can include elements like price setting, advertising strategies, and product differentiation.

(3) **Prediction of Outcomes for Each Strategy** Predict the outcomes of adopting each strategy under specific market conditions. This may include market share, profit, and consumer response.

(4) **Calculation of Probabilities** Based on market conditions and the predicted outcomes of strategies, calculate the probabilities of each strategy occurring. This includes considering the effectiveness of strategies, the reaction of other players, and the impact of external factors.

A general form for calculating the probability $P(s_i|\text{Market Conditions, Strategies of Other Players})$ that player i adopts strategy s_i is as follows

$$P(s_i|\text{Market Conditions, Strategies of Other Players}) = \frac{\exp(\beta U_i(s_i, \text{M}}{\sum_{s_i'} \exp(\beta U_i(s_i'}$$

where, β is a parameter representing the sensitivity to strategy selection. $U_i(s_i, \text{Market Conditions, Strategies of Other Players})$ is the expected payoff for player i when adopting strategy s_i. The denominator is the sum of exponential functions of the expected payoffs for all possible strategies s_i'.

15.1 Detailed Calculation Process

(1) **Calculation of Expected Payoffs** Calculate the expected payoffs for player i when adopting each strategy based on specific market conditions and the strategies of other players.

(2) **Normalization of Probabilities** Calculate the exponential function of the expected payoffs for all strategies and divide by their sum to normalize the selection probabilities of strategies, ensuring that the sum of all strategy probabilities equals 1.

(3) **Interpretation of Probabilities** The calculated probabilities indicate how effective each strategy is under specific market conditions. A higher probability means that the strategy is preferred over others.

Through this calculation process, it is possible to understand how market conditions and strategic interactions among players influence probabilistic strategy choices. This enables players to select optimal strategies according to market conditions and gain a competitive advantage.

15.2 Identification of Player Strategies and Calculation of Expected Payoffs

Identifying player strategies and calculating expected payoffs are crucial steps in determining how players should act in a competitive market environment. Through these processes, players can assess the potential outcomes of different strategies and make choices that maximize their benefits.

15.2.1 Identification of Player Strategies

Players formulate strategies based on various strategic elements such as pricing, product differentiation, and marketing approaches.

Calculation Process

List Strategies List all possible strategies that players can take, including price settings, product differentiation, and marketing approaches.

Evaluate Strategy Characteristics Evaluate the characteristics of each strategy and analyze how they adapt to market demands, competitive situations, and cost structures.

15.2.2 Calculation of Expected Payoffs

Calculate the potential benefits each strategy could bring, guiding the choice of strategy.

Calculation Process

Revenue Calculation Calculate the revenue for adopting a specific strategy, which is determined by the product of price and quantity sold.

Cost Calculation Evaluate the costs associated with implementing the strategy, including both fixed and variable costs.

Profit Calculation The expected payoff is the difference between the revenue and the costs.

Example of Formulas The expected payoff $E[u(s)]$ for adopting strategy s is calculated using the revenue $R(s)$ and the costs $C(s)$ as follows

$$E[u(s)] = R(s) - C(s)$$

where, $R(s) = p(s) \times q(s)$ represents the revenue from strategy s, with $p(s)$ being the price and $q(s)$ the quantity sold. $C(s) = C_{\text{fixed}} + C_{\text{variable}}(s)$ represents the costs associated with strategy s, with C_{fixed} being the fixed costs and $C_{\text{variable}}(s)$ the variable costs.

Through these calculation processes, players can evaluate the potential outcomes of different strategies and select the one that offers the highest benefit.

16. DiscussionScenario-based Evaluation Criteria Applications

Below are proposed formulas and calculation process ideas based on the definition of criteria for three different scenarios. Each scenario assumes a different context and explains how the evaluation criteria are applied.

16.1 Scenario 1 Market Introduction of a New Product

Objective Introduce a new product to the market and gain market share.

16.1.1 Effectiveness Evaluation

Formula $E_{\text{eff}} = \alpha \cdot (\text{Market Research Score}) + \beta \cdot (\text{Target Customer Response})$

Calculation Process Score the effectiveness of the new product's market introduction based on market research data and feedback from target customers.

16.1.2 Risk Evaluation

Formula $R = \gamma \cdot (\text{Competitor Reaction}) + \delta \cdot (\text{Market Volatility Risk})$

Calculation Process Evaluate the potential negative impacts and uncertainties associated with new market entry, such as competitor reactions and market volatility.

16.1.3 Cost Evaluation

Formula $C = \text{Fixed Costs} + \epsilon \cdot (\text{Variable Costs})$

Calculation Process Calculate the total costs associated with new product development and market introduction, including both fixed and variable costs.

16.1.4 Scope of Impact Evaluation

Formula $S = \zeta \cdot (\text{Expected Market Coverage})$

Calculation Process Score the scope of impact based on the expected market coverage of the new product.

16.1.5 Feasibility Evaluation

Formula $F = \eta \cdot (\text{Internal Resources}) + \theta \cdot (\text{Technical Feasibility})$

Calculation Process Score the feasibility of implementing new technology based on the organization's internal resources and technical capabilities.

17. Implementation of Cost Reduction Plan

17.1 Evaluation of Effectiveness

Formula $E_{\text{eff}} = \alpha \cdot S$
Calculation Process
S represents the actual amount of costs reduced by implementing the cost reduction plan.

17.2 Evaluation of Risk

Formula $R = \gamma \cdot I$
Calculation Process
I denotes the potential negative impact on operational efficiency or quality due to cost-cutting measures.

17.3 Evaluation of Cost

Formula $C_{\text{total}} = C_{\text{investment}}$
Calculation Process
$C_{\text{investment}}$ indicates the initial investment required to implement the cost reduction plan.

These scenario-specific calculation processes and formula ideas lay the foundation for assessing potential benefits and risks and determining the optimal strategy in formulating a risk management strategy for fake news business models. It is crucial to accurately understand the objectives and risks in each scenario and choose the appropriate calculation model based on that understanding.

18. PerspectProposed Calculation Processes and Formulas for Risk Management in Fake News Business Models

18.1 Calculation of Effectiveness

Objective Quantify the diffusion effect of fake news.
Formula Effectiveness Score $= w_1 \cdot$ Diffusion Rate $+ w_2 \cdot$ Reach
Calculation Process
The diffusion rate is measured by the number of times the fake news is shared within a certain time frame.
The reach is measured by the number of unique users the fake news reaches.
w_1 and w_2 are weights reflecting the importance of each metric.

18.2 Calculation of Risk

Objective Assess the risks associated with fake news.
Formula Total Risk Score $= w_3 \cdot$ Legal Prosecution Risk $+ w_4 \cdot$ Reputation Loss Risk
Calculation Process
Legal prosecution risk is assessed based on violations of relevant laws and regulations.
Reputation loss risk is evaluated based on negative public perception caused by fake news and its impact.
w_3 and w_4 are weights reflecting the importance of each risk factor.

18.3 Calculation of Cost

Objective Calculate the total cost of a fake news campaign.
Formula Total Cost $=$ Content Creation Cost $+$ Dissemination Cost
Calculation Process
Content creation cost includes direct expenses for creating the fake news.
Dissemination cost includes advertising expenses and social media usage fees for spreading the fake news.

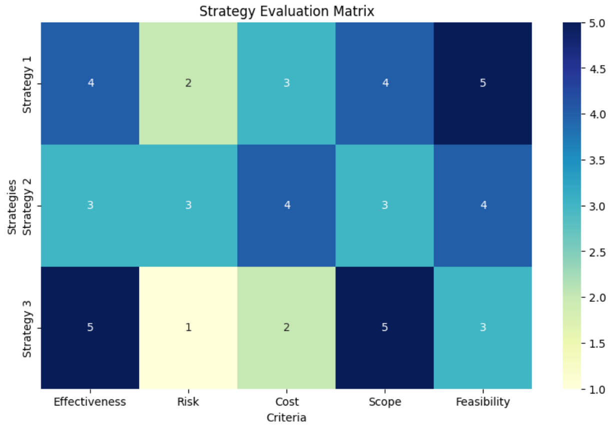

Fig. 9: Strategy Evaluation Matrix

18.4 Evaluation of Ethical Impact

Objective Evaluate the ethical impact of fake news on society.
Formula Ethical Impact Score $=$ f(Impact Scope, Severity of Impact)
Calculation Process
The impact scope is measured by the number of people potentially affected by the fake news.
The severity of impact is evaluated based on the degree of harm caused by the fake news.
f is a function that combines these two factors to assess the overall ethical impact.

18.4.1 Evaluation of Feasibility

Objective Evaluate the feasibility of executing the campaign.
Formula Feasibility Score $= w_5 \cdot$ Available Resources $+ w_6 \cdot$ Technical Feasibility
Calculation Process
Available resources are measured by the amount of human and financial resources available for implementing the campaign.
Technical feasibility is evaluated based on the presence or absence of the technical capabilities required for producing and disseminating fake news.
w_5 and w_6 are weights reflecting the importance of each factor.

These calculation processes and formulas aid in the formulation of risk management strategies for fake news business models. It is essential to accurately assess the potential benefits and risks in each scenario, considering ethical considerations, and make responsible decisions.

Fig.9-10, "Expected Gains for Fake News Strategies," which compares the expected gains from three different strategies used in the context of spreading fake news. The second image is a heatmap titled "Strategy Evaluation Matrix," show-

Fig. 10: Strategy Evaluation Matrix

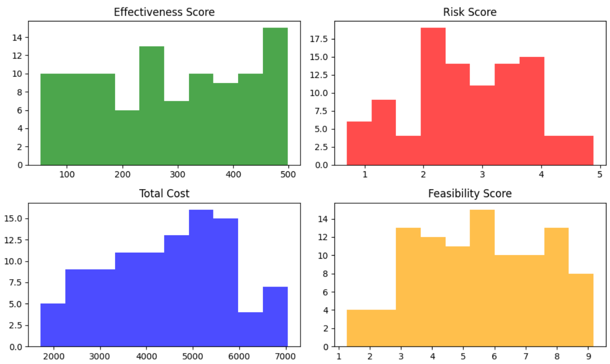

Fig. 11: Total Cost

ing the ratings of the three strategies based on various criteria such as Effectiveness, Risk, Cost, Scope, and Feasibility.

Expected Gains for Fake News Strategies (Bar Chart)

Strategy 1 (Blue Bar), This strategy has an expected gain of just over 1.5, which is the lowest among the three strategies. It suggests that while this strategy may yield some gain, it is not as profitable as the others. Strategy 2 (Green Bar), With an expected gain of just over 1.0, this strategy is less advantageous than Strategy 1. This might indicate that it is either less effective at spreading fake news or has higher associated costs or risks. Strategy 3 (Red Bar), This strategy shows the highest expected gain, slightly less than 2.0, suggesting it is the most beneficial strategy in terms of expected returns from spreading fake news.

Strategy Evaluation Matrix (Heatmap)

The heatmap provides a more detailed analysis of each strategy by evaluating them across five criteria.

Effectiveness, Strategy 3 is rated the highest in effectiveness, while Strategy 1 is moderately effective, and Strategy 2 is the least effective. Risk:Strategy 1 carries moderate risk, Strategy 2 has a slightly higher risk, and Strategy 3 has the lowest risk rating. Cost:Strategy 3 is rated as the most costly, whereas Strategies 1 and 2 are less so. Scope:Strategy 1 has the widest scope, followed by Strategy 3, with Strategy 2 being the narrowest. Feasibility:Strategy 1 is the most feasible, while Strategy 3 is the least feasible.

Synthesis of Findings in the Scenario Context

In the scenario where the spread of fake news leads to increased social discord and targeting of less aggressive entities, the analysis of the two graphs could be synthesized as follows.

Strategy 1, Despite being moderate in risk and highly feasible, its effectiveness and scope make it a practical choice but not the most profitable in terms of expected gains. This strategy might be more about widespread dissemination with manageable risks. Strategy 2, It seems to be a balanced approach with moderate ratings across all criteria except effectiveness. Its lower expected gain may reflect its balanced but underwhelming performance. Strategy 3, While it has the highest expected gain and is most effective with the widest scope, it also carries the highest cost and lowest feasibility. This could imply a high-risk, high-reward strategy that, if successful, could cause significant social unrest and targeting of innocents but would require substantial resources to implement.

Considering the trade-offs between the expected gains and the ratings on the evaluation matrix, it is clear that each strategy comes with its own set of advantages and disadvantages. Strategy 3, despite its high potential gain, may not be sustainable or ethical due to its high cost and low feasibility. Strategy 1 appears to be a safe and practical approach with reasonable effectiveness and the widest scope. Strategy 2 seems to be the least effective and, therefore, might be the least concerning in terms of its potential to cause harm through the spread of fake news.

In combating fake news and its detrimental effects on society, understanding the nuances of these strategies can inform the development of countermeasures. Effective counter-strategies would need to decrease the expected gains of spreading fake news, increase the associated risks and costs, and limit the scope of its spread while maintaining feasibility.

Fig.11, appears to be a set of four histograms, each depicting a different aspect of strategies, potentially related to the dissemination of fake news. The histograms represent the following:

Effectiveness Score vs. Total Cost, This graph shows the relationship between the total cost of a strategy and its effectiveness score. There appears to be a variation in effectiveness at different cost levels, suggesting that more expensive strategies do not necessarily guarantee higher effectiveness.

Risk Score vs. Feasibility Score, This histogram illustrates the distribution of risk scores across different feasibility scores. The distribution seems fairly even, indicating that the perceived risk of a strategy does not directly correlate with its feasibility.

Total Cost Histogram, This histogram shows the frequency of strategies at different total cost levels. It suggests that most strategies cluster around the mid-range costs rather than being very cheap or very expensive.

Feasibility Score Histogram, This histogram displays the frequency of strategies at different levels of feasibility. The distribution is relatively even, with a slight concentration in the mid-range of the feasibility scores.

Analysis in the Context of Misinformation Spread When analyzing these histograms in the context of misinformation spread leading to blame-shifting and increased societal discord.

Effectiveness vs. Cost, Not all costly strategies are highly effective, which implies that investing heavily in the spread of misinformation does not always lead to greater impact. This could suggest that some efforts to spread fake news might result in poor returns on investment, especially if the costs outweigh the actual influence on public opinion.

Risk vs. Feasibility, Strategies with varying levels of risk seem to have a broad range of feasibility scores, indicating that some high-risk strategies might be easy to implement, while others are not. This could reflect the unpredictable nature of misinformation campaigns where some risky endeavors are surprisingly easy to execute due to technological or social vulnerabilities.

Total Cost Histogram, The presence of strategies across a range of costs suggests that there is no single financial model for misinformation campaigns. Some actors might opt for low-cost, grassroots-style campaigns, while others might invest heavily, possibly indicating state-level backing or the involvement of well-funded organizations.

Feasibility Score Histogram, The spread of feasibility scores indicates that there is a variety of strategies with different levels of ease of implementation. This could relate to factors such as the availability of platforms for spreading misinformation, the sophistication of the target audience, and the presence of countermeasures like fact-checking.

Given the spread of effectiveness and cost, those seeking to combat misinformation should focus on identifying high-impact, low-cost strategies to maximize the efficiency of their efforts. The varied relationship between risk and feasibility suggests that counter-strategies should be versatile and adaptable to different conditions. Monitoring and potentially intervening in the mid-range cost strategies might be more effective, as these are more commonly utilized.

The histograms also imply that there is no one-size-fits-all approach to either spreading or countering misinformation. A multi-faceted approach that considers the diverse nature of misinformation campaigns—acknowledging their varying costs, risks, and feasibility—is crucial.

In conclusion, understanding the dynamics of misinformation strategies, as represented in these histograms, is vital for developing effective countermeasures that protect less aggressive entities from being targeted and reduce the amplification of misinformation, thereby mitigating the exacerbation of social anxiety and discord.

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