ABSTRACT

This study suggests a new 3D numerical model for simulating the flow and bed deformation around hydraulic structures, considering the transition process from bed-load motion to suspension. The numerical analysis of the fluid is carried out by solving the Reynolds-averaged Navier-Stokes (RANS) equations coupled with the Volume of Fluid (VOF) method. The temporal change in bed topography is calculated by coupling a stochastic model of sediment pickup, deposition, and transition and a momentum equation of sediment particles to account for the effect of non-equilibrium and transition from bed load to suspension. The numerical model was applied to an experimental scour phenomenon upstream of a slit weir, in which the initial bed elevation was lower than the crest elevation of the slit (i.e., all eroded particles experienced the transition process from bed load to suspension around the slit). A comparison between the numerical results and experimental data indicated that the model could reproduce the scour geometry around the slit weir with sufficient accuracy.

Keywords: Scour, Numerical simulation, Transition, Non-equilibrium sediment transport, OpenFOAM

1. INTRODUCTION

In the vicinity of a cross-river structure with an opening (e.g., check dam or dam with a gate), three-dimensional vortices are formed, causing local scour upstream of the opening. In a check dam, the scour hole influences the trapping and controlling of sediment discharge at the dam. In a hydropower dam, facing a problem of sedimentation around its water intakes, the dam gate can release sediment by causing the scour around the gate to lower the risk of burial of the water intakes. Therefore, for effective sediment management utilizing a cross-river structure, it is very important to evaluate and predict the scour shape upstream of the structure.

In recent years, 3D numerical models for local scour have been developed in many studies. These models can be roughly classified into two types based on the method used; one treats sediment as continuous media and calculates bed deformation using functions of sediment transport (Dixen et al., 2013; Liu & García, 2008; Nagata et al., 2005; Roulund et al., 2005) and the other calculates the particle motion individually using discrete element method (Hajivaliev et al., 2012). Currently, the former is more practical in terms of computational cost. Roulund et al. (2005) successfully reproduced the scour around a circular pile by a 3D model using the bed-load formula. Dixen et al. (2013) improved this model by introducing the effect of externally generated turbulence on the bed load. In order to consider the effect of non-equilibrium on bed-load transport, Nagata et al. (2005) incorporated the momentum equation of bed-load particles into the stochastic model of pickup and deposition and applied this model to the scour around hydraulic structures. Liu & Garcia (2008) used the Volume of Fluid (VOF) method to consider the effect of water surface variation on scour and calculated the bed deformation considering both of the bed load and suspended load.

In the upstream region near a cross-river structure with an opening, bed-load movement in the longitudinal direction is intercepted by the structure. Instead, bed-load particles are entrained into suspension and flow into the opening of the cross-river structure because of the rapid upward flow caused by the contracted flow at the opening (Ota & Sato, 2013). Thus, the entrained particles experience a transition from bed-load motion to suspension. Ashida et al. (1982) showed that, when the bed consists of well-sorted particles, all particles transition as “no motion ⇔ bed load ⇔ suspension” and no particles transition directly from no motion to suspension. Furthermore, Nakagawa et al. (1990) introduced the stochastic model for the transition from bed-load motion to suspension by modeling the orbital motion of saltating particles. In spite of these insights into the transition process, existing 3D simulation models of sediment transport treat sediment entrainment as “no motion ⇔ suspension” for simplification and do not consider the transition from bed load...
motion to suspension. Thus, the applicability of existing models to scour phenomena upstream of cross-river structures becomes questionable, because the scour is mainly caused by the transition from bed load to suspension.

In order to properly simulate the scour upstream of cross-river structures, this study suggests a 3D model of flow and sediment transport that considers the effect of non-equilibrium bed-load transport and the transition process from bed load to suspension. For validation, the model was applied to a laboratory test of scour upstream of a slit weir.

2. NUMERICAL MODEL

2.1 Flow model

The numerical analysis was performed using the interFoam solver, which is one of the standard solvers within the open-source software code OpenFOAM® (OpenCFD Ltd.). The RANS and continuity equations used in the solver are as follows:

\[ \nabla \cdot \mathbf{U} = 0 \] \[ \frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \cdot (\rho \mathbf{U} \mathbf{U}) = -\nabla p^* + \nabla \cdot \mathbf{f} + \rho \mathbf{g} \] where \( \nabla \) is the three-dimensional gradient operator, \( \mathbf{U} \) is the flow velocity vector, \( \rho \) is the fluid density, \( p^* \) is the excess pressure, \( \mathbf{g} \) is the gravitational acceleration vector, and \( \mathbf{f} \) is the body force equivalent to the surface tension. In the solver, the VOF method is used to model the free surface. In order to solve the velocity-pressure coupling, the PISO (pressure implicit with splitting of operator) approach was adopted. The finite volume method (FVM) was used for discretization, the second-order central difference method for spatial discretization and the first-order implicit method for temporal discretization. Further information on the implementation of the model can be found in Jasak (1996).

The \( k-\omega \) SST (shear stress transport) model (Menter et al., 2003), which is often applied to boundary layer flow under inverse pressure gradient, was used to model the closure turbulence. As the local scour phenomenon is mainly caused by the inverse pressure gradient at hydraulic structures, the \( k-\omega \) SST model has often been applied to simulate local scour (e.g., Rouland et al., 2005).

2.2 Bed-deformation model

In the presence of the obstacle and the associated scour hole, the near-bed flow and bed slope, which have significant influence on sediment movement, vary rapidly around the structure; therefore, the assumption of equilibrium sediment transport becomes questionable. In this study, the framework for a non-equilibrium sediment transport model, as suggested by Nagata et al. (2005), has been introduced, with the addition of the transition process from bed-load motion to suspension. The present model calculates: (1) the volume of sediment pickup, (2) the trajectory of sediment motion, (3) the volume of sediment deposition, and (4) the volume of sediment transition from bed-load motion to suspension. The sediment pickup, deposition, and transition are obtained by employing a stochastic model of sediment motion, whereas the sediment movement is calculated by a momentum equation of sediment particles. From sediment pickup, deposition, and transition, the temporal change in bed elevation is obtained.

2.2.1 Calculation of pickup volume

The volume of sediment pickup per unit time from a numerical mesh on the bed surface, \( V_p \), is given by:

\[ V_p = \frac{A_1 d}{A_2} \rho_s S_p \] where \( p_s \) is the pickup rate, \( d \) is the diameter of the sediment particle, \( A_2 \) (\( = \pi/4 \)) and \( A_1 \) (\( = \pi/6 \)) are the two- and three-dimensional shape coefficients of the sand grain, respectively, and \( S_p \) is the area of the bed-surface mesh.

The pickup rate \( p_s \) is obtained from the following equation proposed by Nakagawa et al. (1986), which includes the effect of the local bed slope on sediment motion:

\[ p_s \sqrt{\frac{d}{\sigma^2(\rho - 1)} g} = F_G \sigma L \left( 1 - \frac{k_r \phi \sigma}{\tau_s} \right)^{0.6} \] \[ G = \frac{\cos \psi + k_s \mu_s}{1 + k_s \mu_s} \] \[ \phi = \frac{\mu_s \cos \theta_s - \sin \theta_s \cos \alpha}{\cos \psi + k_s \mu_s} \] where \( \sigma \) is the density of sediment, \( \tau_s \) and \( \tau_c \) are the dimensionless shear stress and dimensionless critical shear stress for sediments, respectively, and \( F_G \) (\( = 0.03 \)), \( k_r \) (\( = 0.7 \)), and \( m_r \) (\( = 3.0 \)) are constants, as suggested by Nakagawa et al. (1986). \( G \) is the coefficient that accounts for the direction deviation between the near-bed velocity and sediment movement, \( \psi \) is the angle between the near-bed velocity and sediment movement, and \( \phi \) is the coefficient that accounts...
for the local bed slope and the direction deviation between the maximum local bed slope and sediment movement. \( \mu_s \) (= 0.7) is the static friction factor, \( k_L \) (= 0.85) is the ratio of lift force to drag force, \( \theta_b \) is the local bed slope, and \( \alpha \) is the angle between the maximum local bed slope and sediment movement.

### 2.2.2 Calculation of deposition and transition volumes

During the transport of sediment particles, some of the particles continue moving, whereas others are deposited on the bed surface at a certain place. The amount of sediment deposition on the bed along the sediment transport trajectory is estimated using the probability density function of the step length:

\[
F_{s}^{(n)} = F_{s}^{(n-1)}[1 - F_{s}^{(n)}]
\]

where the superscript \( (n) \) represents the \( n \)th step after sediment pickup, \( V_{d}^{(n)} \) is the deposition volume of the sediment, \( F_{s}^{(n)} \) is the deposition probability of the bed-load particle, and \( F_{t}^{(n)} \) is the transition probability of the bed-load particle from bed-load motion to suspension. The deposition probability \( F_{s}^{(n)} \) is written as follows:

\[
F_{s}^{(n)} = f_{s}({\Delta s}) \Delta t
\]

where

\[
f_{s}({\Delta s}) = \frac{1}{\lambda} \exp \left( -\frac{{\Delta s}}{\lambda} \right), \quad {\Delta s} = \sum_{\Delta t} h_{sed} \Delta t
\]

where \( f_{s}({\Delta s}) \) is the probability density function of the step length, \( {\Delta s} \) is the distance that the sediment particle moves from the pickup point, and \( \lambda \) is the average step length of the sediment particle, which is estimated using the formula given by Einstein (1942), and \( u_{sed} \) is the bed-load velocity calculated from the momentum equation of the bed load motion, which is mentioned later.

In order to evaluate the transition probability \( F_{t}^{(n)} \), a probability density function with Poisson distribution is introduced, taking into account that the transition event occurs only once per time \( \Delta t \). The transition probability \( F_{t}^{(n)} \) is written as follows:

\[
F_{t}^{(n)} = \sum f_{t}({\Delta t}) \Delta t
\]

where

\[
f_{t}({\Delta t}) = p_{t} \exp(-p_{t}{\Delta t})
\]

where \( f_{t}({\Delta t}) \) is the probability density function of transition from bed-load motion to suspension at the \( n \)th step after sediment pickup and \( p_{t} \) is the transition rate, which represents the expected value of the frequency of transition. The transition rate \( p_{t} \) is calculated using the formula introduced by Nakagawa et al. (1990) through theoretical and experimental investigation of the transition phenomenon:

\[
p_{t} = \frac{d}{(\sigma/\rho - 1)g} = F_{t} \left[ \frac{u_{*}}{w_{*}} \right]^{-3/2}
\]

where \( F_{t} (= 0.0175) \), \( n (= 0.4) \), and \( m (= 1.10) \) are constants, \( (u_{*}/w_{*}) \) is the dimensionless shear stress for suspension, and \( (u_{*}/w_{*}) \) is the dimensionless critical shear stress for suspension, estimated using the suggestion of van Rijn (1984):

\[
\left( \frac{u_{*}}{w_{*}} \right)_{0} = \begin{cases} 4/3 & \text{for } 1 < d_{s} \leq 10 \\ 0.4 & \text{for } d_{s} > 10 \end{cases}
\]

where \( u_{*} \) is the frictional shear velocity, \( w_{*} \) is the settling velocity of the particle, and \( d_{s} = [(\sigma/\rho - 1)g d^{3}/\gamma]^{1/3} \) is the dimensionless particle diameter.

### 2.2.3 Calculation of sediment trajectory

By disregarding inter-particle collisions, the movement velocity vector \( u_{med} \) of a sediment particle is calculated using its motion equation. Two unit vectors parallel to the local bed surface are defined as \( p_{x} \) on the \( x-z \) plane and \( p_{y} \) on the \( y-z \) plane. Then, the motion equation of the sediment particle in the \( p_{y} \) \( (j = 1, 2) \) direction is written as follows:

\[
m_{med} \frac{\partial \hat{u}_{med}}{\partial t} = D_{j} + W_{j} - F_{j}
\]

where

\[
m_{med} = \rho \left( \frac{\sigma}{\rho} + C_{w} \right) A_{d} d^{3}
\]
The temporal variation in bed elevation is expressed as follows:

\[ V_{\text{sed}} = \sum_{t=1}^{n} V_{\text{sed},t} \]

where \( V_{\text{sed}} \) is the volume of the deposited sediments in each computational cell at each time step, and \( V_{\text{sed},t} \) represents the total volume of deposited sediments in each computational cell at each time step, and \( S_j \) is the projected area of the computational cell in which the sediment is deposited.

In the present scheme, the position at which the deposition volumes were obtained did not necessarily coincide with the numerical grid points. Therefore, the mesh region with the location of the moving sediment particle was found at each time step, and the deposition volume was distributed proportionally to each grid point based on the relative location of the particle.

In this study, the numerical model was applied only to the early stages of scouring, during which the suspended particles do not settle on the scour hole again because of the very high shear stresses on the scour hole. Thus, it should be noted that, for simplification, it was assumed that the suspended load does not transition back to bed load.

3. APPLICATION TO SCOUR EXPERIMENT

The present numerical model was applied to a scour experiment around a slit weir. In the experiment, bed load could not pass through the weir because the initial bed elevation was lower than the crest elevation of the slit. Therefore, all eroded sediments experienced the transition process from bed load to suspension. Thus, this experiment was used to validate the proposed numerical model that included the transition process.

3.1 Experimental setup

A scour experiment was conducted at the Central Research Institute of Electric Power Industry (CRIEPI) in a circulating flume that was 10.5 m long, 0.50 m wide, and 0.35 m deep. As shown in Figure 1, a slit weir made of Plexiglas was placed in the flume, and a sediment bed was positioned upstream of the slit weir. The bed elevation was leveled at 10 mm below the crest elevation of the slit. Laser displacement sensors (IL-600, Keyence Co., Ltd.) were used to scan the sediment surface at a resolution of 1 mm and 4 mm in the longitudinal and lateral directions, respectively. Further details of the experimental apparatus can be found in Ota & Sato (2013).

The scour experiment was carried out under the clear scour condition, as this study was focused on the bed deformation caused by the local flow around the slit weir. The flow rate was \( 4.0 \times 10^{-3} \) m³/s, and at 0.5 m upstream of the slit weir, the approaching flow depth was 72.7 mm, the approaching flow velocity was 0.11 m/s, and the Froude and Reynolds numbers were 0.13 and 8.2 \times 10^{3}, respectively. It should be noted that these hydraulic parameters did not vary with time while the bed was scoured in the experiment.
First, the slit was closed with a plate that had the same shape as the slit (50 mm height, 100 mm width, and 15 mm thickness), and the flume was filled gradually until the water overflowed the wing. The slit was then opened, and the experiment was initiated. After a given time had passed, the slit was closed, and the bed topography was measured. After the bed topography measurement, the experiment was resumed. The bed topography measurements were carried out at 40 s, 80 s, 160 s, 320 s, and 640 s after the start of the experiment.

3.2 Numerical condition

The numerical conditions used in the simulation were similar to the experimental conditions. The cross section at 0.5 m upstream of the slit weir was taken as the inlet boundary. The computational cells consisted of cubes, and their total number was 478,520. At the initial condition, the edge length was set to 5.0 mm, but finer edge length was applied in the near-bed region in geometric progression. The mesh points moved in the vertical direction when the bed deformed, and
the non-slip condition was imposed at the bed and the wall boundary. In the present calculation method for bed deformation, it was necessary to keep track of the exact location and speed of every sediment particle that leaves each numerical grid point on the bed surface. This made it impractical to calculate the sediment pickup volume at all time steps because of computational memory limitations. To overcome this difficulty, the calculation of bed deformation was carried out only every 0.01 s (20 times the interval of flow calculation). Furthermore, parallel computation with domain decomposition was used to accelerate the simulation. As OpenFOAM comes with parallel-computing support, the present model easily used these modules to implement both the flow-field solver and the sediment-transport solver simultaneously. The computations were done on the supercomputer SGI ICE X at the CRIEPI.

3.3 Numerical result

Figure 2 shows a snapshot of the flow and the bed at 160 s after the start of the simulation. The streamlines were obtained by using tracers from the inlet boundary, which was 10 mm higher than the bed. The bed colors show the variation in the transition rate \( p_t \) (i.e., average frequency of transition from bed load to suspension). Vortex flow occurs in the vicinity of the weir, where bed load is entrained into suspension at the maximum rate of 0.3 Hz.

Figure 3 compares the contour lines of scour depth obtained from simulations and experiment. Both the simulation and experimental results show a crater-shaped scour hole around the slit and an increase in the size of the scour hole with time. In order to validate the simulation quantitatively, the temporal variation in the scour volume was compared with that of the experiment, as shown in Figure 4. It was found that the simulation underestimated the experimental scour volume by up to 30%. A possible reason for this underestimation could be that the initial coefficients of step length and transition rate had been based on the turbulence of uniform flow in an open channel. Therefore, the time scale of the turbulence in the local flow around the slit could have influenced the frequency of transition from bed load to suspension.

Figure 5 shows the horizontal locations of maximum scour depth in the simulation and experiment. Both results indicate that the bed was most-deeply scoured around the boundary between the slit and the wing. This can be attributed to the fact that the bed particles around this boundary are most likely to be entrained because of the rotational flow from wing to slit (Ota & Sato, 2013). Thus, the simulation reproduced this feature of the scour quite well. Figure 6 shows the maximum scour depth in simulation and experiment, and it can be seen that the simulation underestimates the maximum scour depth in the experiment, as in the case of scour volume. Furthermore, it was observed that the difference in maximum scour depth between simulation and experiment was larger than the difference in scour volume (see Figure 4). A possible reason for this could be that strong vortex flow appears at the location of the maximum scour depth, thus requiring further investigation of the influence of this vortex flow on the transition rate.

The evaluation of the scour shape upstream of a cross-river structure is important for practical use, and the proposed numerical model was capable of reproducing the scour shape observed in the experiment, although it tended to underestimate the size of the scour hole.
4. CONCLUSIONS

This study suggested a new 3D simulation method of modeling flow and sediment transport, considering the transition process from bed-load motion to suspension. This numerical analysis coupled the RANS equations of the fluid with the VOF method and the $k$-$\omega$ SST turbulence model using the OpenFOAM library. The temporal change in bed topography was calculated by coupling a stochastic model of sediment pickup, deposition, and transition and a momentum equation of sediment particles to account for the effect of non-equilibrium and transition on sediment transport.
For validation of the numerical model, it was applied to the scour experiment around a slit weir, in which the initial bed elevation was lower than the crest elevation of the slit (i.e., all eroded particles experienced the transition process into suspension around the slit). The simulation showed good agreement with the experiment in terms of the scour shape but underestimated the maximum scour depth. Further research is needed to investigate the effect of transition rate, which is an important parameter in the simulation of the transition process.

REFERENCES


