12th U.S./North American Mine Ventilation Symposium 2008 – Wallace (ed)
ISBN 978-0-615-20009-5

Numerical simulation on diffusion phenomena in mine airways by using a method of discrete tracer movements

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ABSTRACT: Diffusion phenomena in mine airways are important to trace gas and dust introduced into mine ventilation flows. Authors have carried out tracer gas measurements to check mine ventilation using concentration-time curves measured in mine airways, however dispersion mechanism to enhance effective diffusion have not been cleared. A numerical model using discrete tracer points dosed into tube flow has been presented to simulate their movements in consideration of turbulent velocity profile and fluctuation. Numerical simulations have been carried out to predict distributions of dosed points in tube flows with turbulent velocity profile, turbulent intensity profile and Reynolds stress. The numerical simulation results show that a combination of turbulent velocity profile and fluctuation creates extremely large dispersion of points in longitudinal direction and concentration curve with long tail was observed. This dispersion by velocity profile and fluctuation contributes large effective diffusion coefficient compared with original diffusion coefficient dominating in uniform velocity fields. Furthermore, the effect of Reynolds stress on the concentration-time curves is discussed.

1 Introduction

In underground mine ventilation, gas and dust diffusions phenomena are very important in order to keep safety operations and quality control. Gasses/dusts generated from mining activities are mainly controlled by the properties and behavior of airflow itself.

Taylor (1953 and 1954) presented the equations to estimate effective diffusion coefficient in straight tubes with turbulent flows by laboratory experiments. Sasaki and Dindiwe (2002) have found that turbulent diffusion coefficient in airways of Kushiro coal mine were larger than the estimation by Taylor equation.

The solution of concentration for advection-diffusion in one dimensional uniform flow is given as follows:

$$C(x,t) = \frac{M}{2A\sqrt{\pi D t}} \exp\left(\frac{-(x-U_m t)^2}{4Dt}\right) \qquad (1)$$

where,
$C(x,t)$ = substance concentration in x-positions and t-time
M = total amount of substance at the origin (x=0, t=0)
t = elapsed time from gas injection (s)
A = cross sectional area of flow (m^2)
D = diffusion coefficient (m/s^2)
x = distance from releasing point (m)
U_m = is average convection velocity (m/s.)

For turbulent tube flow, the effective (virtual) diffusion coefficient, E (m^2/s), was presented by Taylor as,

$$E = 5.0 d u^* \quad ; \quad u^* = U_m\sqrt{\frac{f}{8}} \qquad (2)$$

where,
d = tube diameter (m)
u^* = friction velocity (m/s)
f = friction factor

In this paper, the effective diffusion coefficient, E, is defined as a value which is the optimum value of D to get good agreements with concentration curves derived by numerical simulations or by measurements.

There are some numerical and mathematical models as basic approaches commonly used for dispersion modeling including many variations based upon these equations. Some variations can be treated by statistical functions to represent the randomness of airstream direction, airspeed, and turbulence. Because of the increased computational power available via personal computers, the models have become more complex and varied. With the few measured data on the diffusion coefficient in turbulent flow through mine airways, a simple simulation model is needed to evaluate the turbulent diffusion phenomena in mine-airways. One of expected mathematical approaches is Lagrangian method using probability density function for random walk dispersion (Pope, 1987).

This paper present a numerical diffusion modeling using discrete substance points (tracers) with movements of random-walk based on turbulent velocity intensity, velocity profile and Reynolds stress in tube flows.

2 Numerical Simulation Model and Procedures

A numerical simulation model using discrete points injected into tube flow considers their movements by turbulent velocity profile and intensity of velocity fluctuation in the tube flow. A simple procedure is employed to represent the dispersion by random walk satisfying standard deviations of turbulent velocity fluctuations. The developed method is a kind of Lagradian methods, without any numerical differential calculations of concentration gradient in Gaussian coordinates.

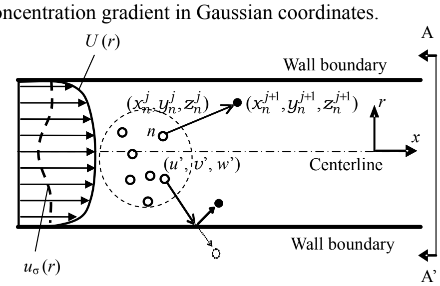

Figure 1 Schematic definition of particle movements in tube flow (*x*-*y* cross section)

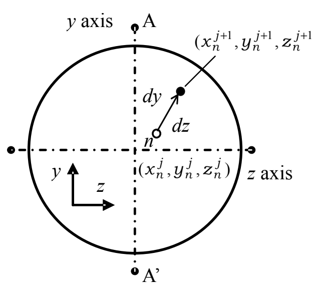

Figure 2 Schematic definition of radial particle movements (*y*-*z* cross section)0

2.1 Definitions of Discrete Particles and Tube Flows

The simulations of diffusion phenomena have been investigated in two types of flow fields that are uniform (constant), laminar, and turbulent flows including molecular random walks and velocity fluctuation. For cases of laminar and uniform flows, velocity fluctuation intensities are assumed a constant value over a cross section based on diffusion coefficient, while profiles of velocity fluctuating intensities of the turbulent flow are given as interpolated functions of radius based on the measured values (Laufer, 1954). Mean velocity and corresponding velocity fluctuation in *x*, *y*, and *z* directions are denoted as (U, V, W) and (u', v', w'), respectively.

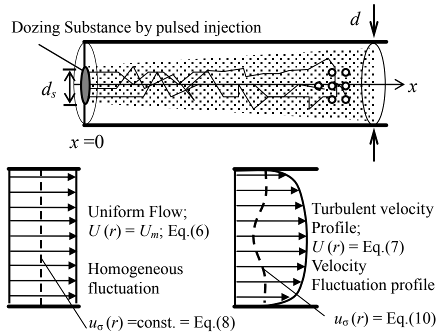

Figure 3 Schematic definition of flow velocity profiles in a tube (d_s = initial distributed diameter of dosed substance)

Velocity components at (x, y, z) are expressed as

$$(u, v, w) = (U+u', V+v', W+w') \tag{3}$$

Suppose a position and velocity of one of the discrete points dosed into channel flow from the origin (x_0, y_0, z_0) are denoted as (x_n, y_n, z_n) and (u_n, v_n, w_n),

$$\begin{aligned} x_n(t+\Delta t) &= x_n(t) + u_n(t)\cdot\Delta t \\ y_n(t+\Delta t) &= y_n(t) + v_n(t)\cdot\Delta t \\ z_n(t+\Delta t) &= z_n(t) + w_n(t)\cdot\Delta t \end{aligned} \tag{4}$$

where t and Δt are elapsed time from dozing and time step of movement, respectively.

Suppose velocity fluctuations, (u', v', w'), satisfy the normal (Gaussian) function, corresponding intensities (root mean square values) of velocity fluctuations are equal to standard deviations (u_i, v_i, w_i). Thus, in present simulations for diffusion of discrete points in the flow field, the velocity components (u_n, v_n, w_n) are expressed by

$$\begin{aligned} u_n(t) &= U_n(x_n, y_n, z_n) + u_n'(u_i, t) \\ v_n(t) &= V_n(x_n, y_n, z_n) + v_n'(v_i, t) \\ w_n(t) &= W_n(x_n, y_n, z_n) + w_n'(w_i, t) \end{aligned} \tag{5}$$

where (u'_n, v'_n, w'_n) are given by generating random numbers which probability densities satisfy the normal functions of $(u_\sigma, v_\sigma, w_\sigma)$ as their standard deviations.

2.2 Velocity Profiles of Tube Flow

Two types of velocity profile such as uniform and turbulence, have been investigated.

Superscripts *n* and *i* represent the number identifying the dozed points and number of random movements by time step Δt, respectively. Thus, radial distance of the particle after *j*-th random movement, r_n^j , is defined as follows;.

$$x_n^j = x_n(j\Delta t),\quad y_n^j = y_n(j\Delta t)\quad ,\quad z_n^j = z_n(j\Delta t)$$
$$r_n^j = \sqrt{y_n^{j\,2} + z_n^{j\,2}} \tag{6}$$

Two velocity profiles in longitudinal x-direction were supposed by following equations

1) Uniform flow;

$$U_n = U_m \tag{7}$$

2) Turbulent velocity profile flow;

$$U_n = \frac{U_m}{0.8174}\left(1-\frac{r_n^j}{R}\right)^{\frac{1}{7}} \tag{8}$$

where U_m is longitudinal average velocity and R is radius of the airway. The velocity values, $V(r)$ and $W(r)$, in y and z direction perpendicular to the longitudinal flow direction (x) are set as zero for straight airways.

2.3 Velocity fluctuation model

Two profiles of velocity fluctuations in longitudinal x-direction of uniform, laminar and turbulent flows in airways were assumed as follows;

1) Uniform homogeneous turbulent flow;
The intensities of velocity fluctuations in three components are set as the constant value ($0.05U_m$) which is derived from average intensities of velocity fluctuations in turbulent velocity profile (see Figure 4);

$$\frac{u_\sigma}{U_m} = \frac{\upsilon_\sigma}{U_m} = \frac{w_\sigma}{U_m} = const. = 0.05 \tag{9}$$

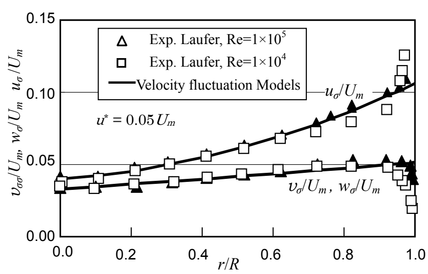

Figure 4 Value of u_σ, υ_σ and w_σ over normalized radial position (r/R) based on assumption that shear velocity is $u^* \approx 0.05U_m$.

2) Turbulent velocity profile flow;
Laufer (1954) presented measurement results of turbulent intensities in tube flows against shear velocity, u^* for two Reynolds numbers (Re=5×10^4 and 5×10^5). Authors interpolated his measurement results and presented the equations (2nd order polynomial equation) against r/R as shown in Figure 4.

Assume the friction factor; $f = 0.02$, the shear velocity is evaluated by,

$$u^* = 0.05U_m \tag{10}$$

u_σ, υ_σ and w_σ can be expressed by following Eq. (11).

$$\left.\begin{aligned} \frac{u_\sigma}{U_m} &= 0.0484\left(\frac{r_n^i}{R}\right)^2 + 0.0179\left(\frac{r_n^i}{R}\right) + 0.0397 \\ \frac{\upsilon_\sigma}{U_m} &= w_\sigma = 0.00013\left(\frac{r_n^i}{R}\right)^2 + 0.0171\left(\frac{r_n^i}{R}\right) + 0.0329 \end{aligned}\right\} \tag{11}$$

2.4 Inherent Diffusion Coefficient

1) Laminar flow
In the laminar flow, the diffusion coefficient can be given by the molecular diffusion coefficient

2) Turbulent flow
In the case of turbulent flow, the diffusion coefficient is related to turbulent velocity intensity, u_σ and turbulent mixing length, L_σ (see Rouse, 1959). Liepmann, H. and Laufer, J.(1947) presented the following relationship (see Rouse(1959))

$$D \cong u_\sigma L_\sigma \tag{12}$$

Authors have assumed that L_σ is evaluated from turbulent velocity fluctuation and time step for movements of points, as expressed by,

$$L_\sigma \cong u_\sigma \Delta t/2\,;\qquad D \cong (u_\sigma)^2 \Delta t/2 \tag{13}$$

where Δt is time step to move the points by velocity fluctuation intensity, thus the time step should be given based on the diffusion coefficient for the turbulent flow.

$$\Delta t \cong \frac{2D}{(u_\sigma)^2} \tag{14}$$

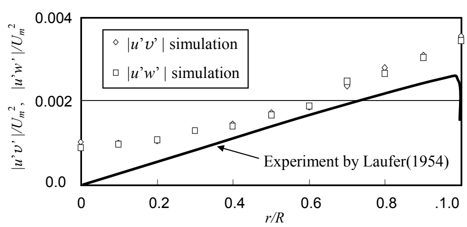

Figure 5 Simulated radial distribution of normalized Reynolds stresses for turbulent velocity profile and velocity fluctuations by present numerical model.

2.5 Reynolds Stress Correlations

In fluid flow, quantitatively, mass of particles translated from slower region to a faster layer in shear flows. Reynolds stress showing a relationship of longitudinal and vertical velocity fluctuations in the flow including shear layers such as the tube flow. In present study, effects of

Reynolds stress correlations have been investigated by considering that fluctuations of point movements in z and y have a relation with fluctuation in x direction, i.e, there is a correlation between x axis velocity fluctuation, u', with y and r z axis velocity fluctuations, v' and w', expressed as

$$-u'v'>0 \quad \text{and} \quad -u'w'>0 \tag{15}$$

In the simulations, the value of u' including its sign was firstly given as a random number following Eq.(11), then absolute values of $|v'|$ and $|w'|$ were generated randomly, but their signs were decided to satisfy Eq.(15).

Figure 5 shows distributions of $|-u'v'|$ and $|-u'w'|$ calculated by present calculation schemes comparing with Laufer's measurements. The simulated values of $|-u'v'|$ and $|-u'w'|$ are relatively larger than that of the measurements, since the signs of v' and w' are always decided based on the sign of u'. It means that the correlations between the fluctuations become slightly higher values and contribute accumulations of Reynolds stresses which are larger than that of actual flows.

2.6 Movements of Discrete Tracer points

Discrete tracer points representing gas or dusts are pulsed injected at upstream origin (x=0). Number of points used in present simulations was 10,000. The larger number of points can provide the smoother stochastic curves for the concentration (density) of the points.

It is assumed that those points are placed randomly at initial time (t=0) in a circular area with diameter, d_S ($\leq d$), at the origin. Random movements of each point can be treated as tracking tracers under the influence of the tube flow characteristics.

Typically, the movement of a point, Δx, Δy and Δz are described by a stochastic probability function. Random number generated in a computer was used for the velocity fluctuations to calculate random movements of all points at every calculation step. Equations (16) to (18) show the calculation sequence of the point's movement in the flow which is close to random walk (see Figures 1 and 2).

$$u' = R(t)Um \tag{16}$$

$$\Delta x_n = (U(r_n^i) + u').\Delta t \tag{17}$$

$$x_n^{i+1} = x_n^i + \Delta x_n \tag{18}$$

For y and z components, the same calculation sequences were applied. When a point move out from wall boundary (r>R), those were replaced in the tube flow region by making rebound or reflection to the opposite direction in y and z directions (see Figure 1).

3 Simulation Results and Discussion

Simulations have been conducted for several conditions considering parameters that give influences on turbulent diffusion expressed by point movements.

3.1 Diffusion in Uniform Flow by Homogeneous Isotropic Turbulence

Firstly, simulations have been carried out on diffusion in ideal flow with uniform velocity and homogeneous isotropic turbulence (similar to grid generated turbulence flow) in order to compare with Taylor equation (1) which is the solution to predict one dimensional diffusion flow.

Figure 6 shows simulation results of stochastic concentration-distance curves at different elapsed times for tube flow of d = 1 m, U_m = 5.0 m/s. The simulated concentration-distance curves for different elapsed time, t, have good agreements with that calculated by Taylor equation with D=0.0031m^2/s which is evaluated by Eq.(13), as shown in Figure 6. This means that diffusion coefficient of present simulation model by random walk affected velocity fluctuation in the uniform flow can be evaluated by equation (13).

The longitudinal diffusion width spreading the dozed points is around 2 m at t =100s, and it is increasing with root value of the elapsed time.

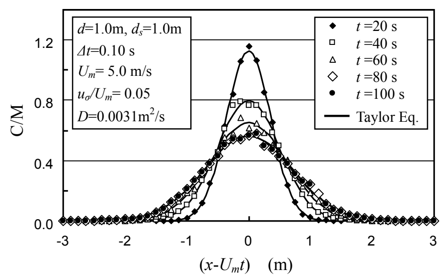

Figure 6 One of simulation results of concentration distance curves compared with Taylor equation (1) for uniform flow.

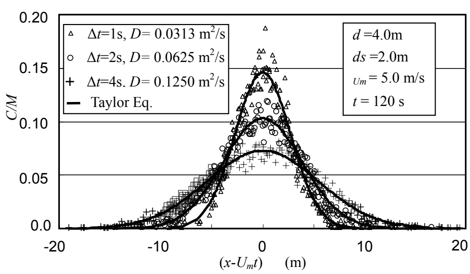

Figure 7 Comparison of concentration-distance curves by different calculation time steps for uniform flow.

3.2 Effect of Simulation Time Step on Diffusion Coefficient

Figure 7 shows an effect of time step, Δt, on concentration-distance curves. At the same elapsed time, t=120s, the diffusion width in longitudinal flow direction is increasing with the time step. The simulated curves have good agreements with that of Taylor equation using the diffusion coefficient calculated by Eq. (13).

3.3 Dispersion in Tube Flow with Turbulent Velocity Profile

Effects of the velocity profile have been investigated by the numerical simulations. Figure 8 shows one of simulated results on the distribution of 10,000 points at t= 120s after dozing from the origin of the tube with d=4.0m, U_m=5.0m/s. It is observed that the points are extremely spread over regime of around 300m (x = 400 to 700 m) which is two order larger than that of uniform flow. Bear (1972) also described a same kind of dispersion effect in porous media.

The line of Taylor equation shows a history matched result by changing diffusion coefficient in Eq. (1). The optimum value was 20m^2/s which becomes the effective diffusion coefficient, E, for the simulated curve. The value of E is 160 times of the original diffusion coefficient, D, calculated by Eq.(13). On the other hand, the value of E calculated by Eq.(2) presented by Taylor is E = 5.0du*=5 m^2/s for u^*/U_m= 0.05.

These results show a possibility that the effective diffusion coefficient can be evaluated using the present simulation model.

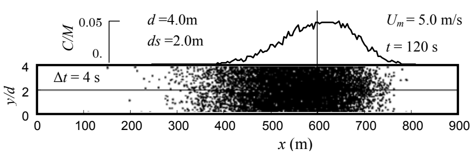

Figure 8 One of simulation results showing longitudinal distribution of points in tube flow with turbulent velocity profile and Reynolds stresses (number of points is 10000)

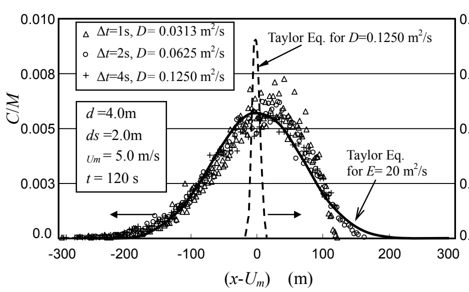

Figure 9 Simulation results of longitudinal concentration-distance curves in tube flow with turbulent velocity profile (Reynolds stress model).

3.4 Comparison with Tracer Gas Measurement Results in A Mine Ventilation

Figure 10 shows the simulated concentration-time curve compared with the measurement result against a pulsed injection of tracer gas (SF6) into a mine airway (Widodo, 2007). The condition of the mine airway, which is relatively straight, has average cross-sectional area of 11.0 m^2, average equivalent diameter of 3.17m and average velocity of 2.3 m/s. The gas monitor was placed at 1119 m from the dozing point where a SF_6 balloon was broken in order to measure the tracer gas concentration with time interval of about 40s. The simulation was done using the average condition of the mine airway, thus it did not reflect the detail of the mine airway. However, the simulated result of Reynolds stress model shows almost good agreement of peak concentration and diffusion width shown in Figure 10.

In the Figure, the curve of Reynolds stress = 0 is also plotted. It is clear that traveling time from the origin to the monitor position becomes shorter by Reynolds stress. The reason of the early arriving is a stochastic function of Reynolds stress that moves points from slower region to faster region in the tube flow.

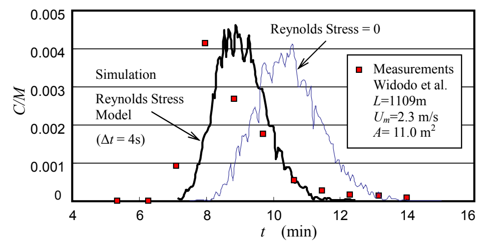

Figure 10 Simulated concentration-time curve at monitor position at x=1119 m, compared with that of tracer gas measurements carried out in a mine (Widodo, 2007).

4 Summary and Conclusion

The large effective diffusion coefficient was measured in mine airways; however its flow mechanism has not been cleared. In this study, numerical simulations on diffusion phenomena in mine airways have been done to predict tracer gas concentration-distance or time curves to investigate mine ventilation airflow. A numerical diffusion model has been presented using discrete substance points (tracers) with movements of random-walk based on turbulent velocity intensity, velocity profile and Reynolds stress in tube flows.

A numerical simulation method using discrete points dosed into tube flow has been proposed to simulate those movements by turbulent velocity profile and intensity of velocity fluctuation in the flow. A simple procedure is employed to represent the effect of dispersion by random walk satisfying standard deviations of turbulent velocity fluctuation. The developed method is a kind of Lagradian methods, without any differential calculations of concentration gradient in Gaussian coordinates.

Numerical simulations have been carried out on distribution of tracers in tube flows with ordinary turbulent velocity profile following the 1/7th law, turbulent intensity profile interpolating Laufer's experiments and Reynolds stress correlation.

The numerical simulation results show that a combination of turbulent velocity profile and fluctuation creates extremely large dispersion in longitudinal direction and long tail of tracer concentration is observed. This dispersion leads to extremely large effective diffusion coefficient compared with original diffusion coefficient dominating in uniform velocity fields. Numerical modeling for discrete tracer movements based on random-walk movements has been done.

The substance concentration-distance curves or concentration-time curves were simulated and analyzed by comparison with Taylor equation and measurements carried out at a mine airway. From those simulation results, it is concluded that the substance dispersions in mine airways are strongly affected by turbulent velocity profile and intensity.

Reynolds stress has a function to make smaller travelling time of tracer points, because it moves them from slower region to faster region by correlations between velocity fluctuations in tube flow.

It has been shown a possibility that the effective diffusion coefficient can be evaluated from concentration-distance or concentration-time curves simulated by the numerical model presented.

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