Numerical Simulation on Diffusion Phenomena in Mine Airways by using a Method of Discrete Tracer Movements

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12th US/North America Mine
Ventilation Symp.

Introduction

- Large values of the effective diffusion coefficient were measured in the long mine airways, however its reason and mechanism are not cleared.
- To predict gas diffusion phenomena in mine airways, a simple numerical simulation model is required.

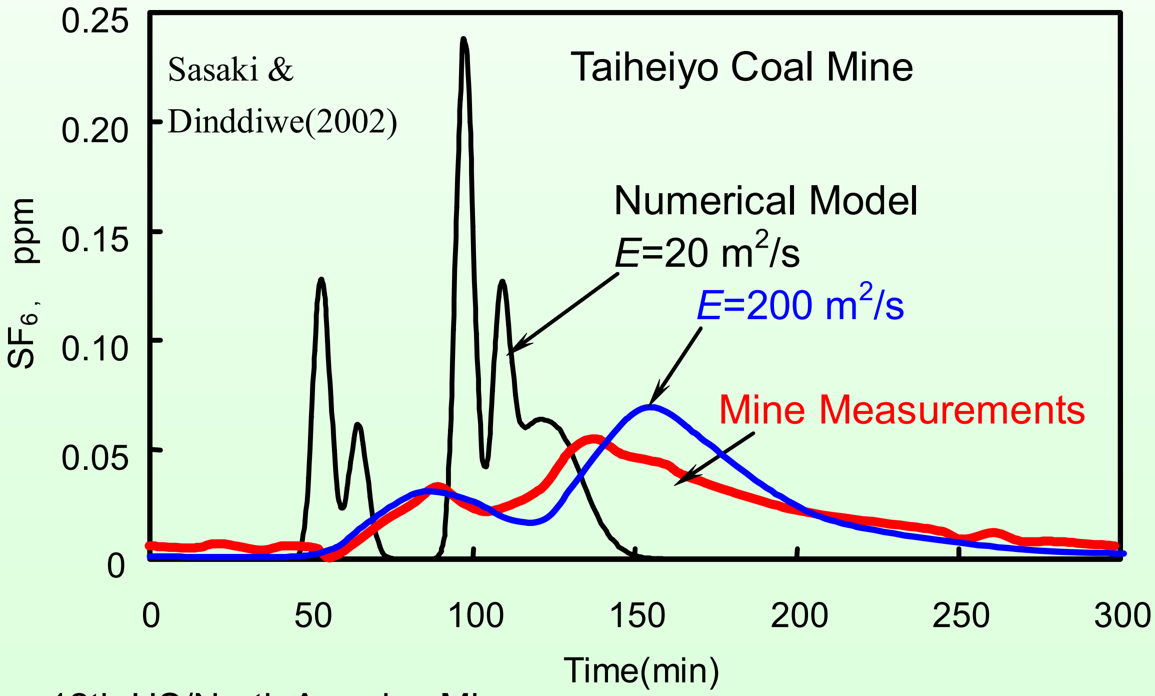
Diffusion in Airways with Long Distance (1 to 4 hours) .

E=200m^2/s ?

Delay?

•Velocity Profile?

•Turbulent Velocity Fluctuation?



12th US/North America Mine Ventilation Symp.

Present Study

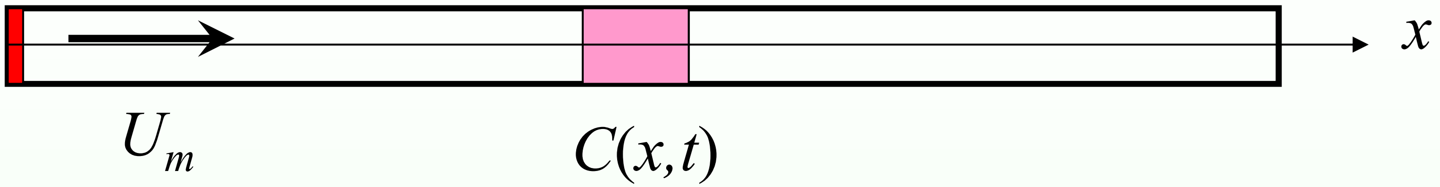
- A numerical diffusion model is presented using discrete points moving with random-walk based on turbulent velocity intensity, velocity profile and Reynolds stress in mine airflows.

- The developed model is a Lagradian method, without any differential calculations of concentration gradient in Gaussian coordinates

Advantages of the Lagradian Method using Discrete Points Movements

- ✓Free of grid generation
- ✓Use a simple algorithm
- ✓Motions of particles (gas) can be treated
- ✓Turbulent mixing effect can be applied
- ✓Numerical visualization is easy

Solution of Diffusion Equation in 1-D Flow



$$C(x,t) = \frac{M}{2A\sqrt{\pi D t}} \exp\left(\frac{-(x - U_m t)^2}{4Dt} \right)$$

$C(x,t)$ = Concentration in *x*-positions and *t*-time

M = Total amount of substance at the origin (x=0, t=0)

t = Elapsed time from gas injection (s)

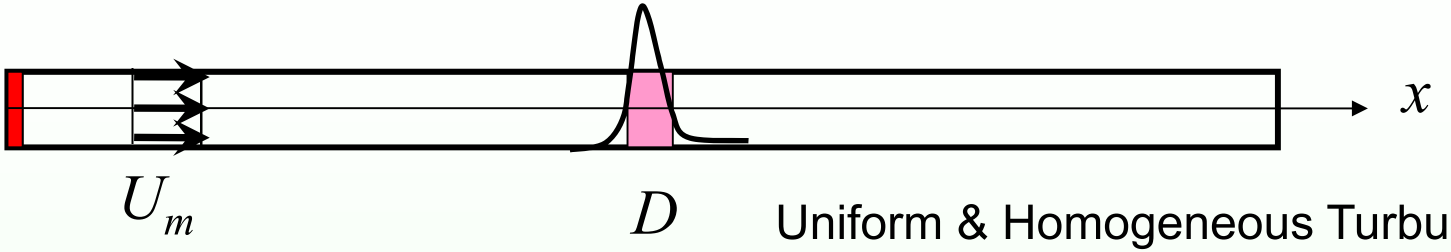
A = Cross sectional area of flow (m^2)

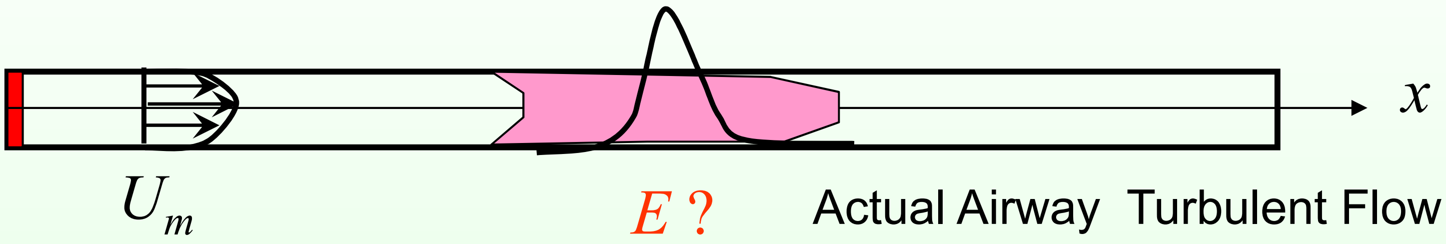
D = Diffusion coefficient (m^2/s)

x = Distance from releasing point (m)

U_m =Velocity (m/s.)

Effective Diffusion Coefficient in Actual Mine Airway?



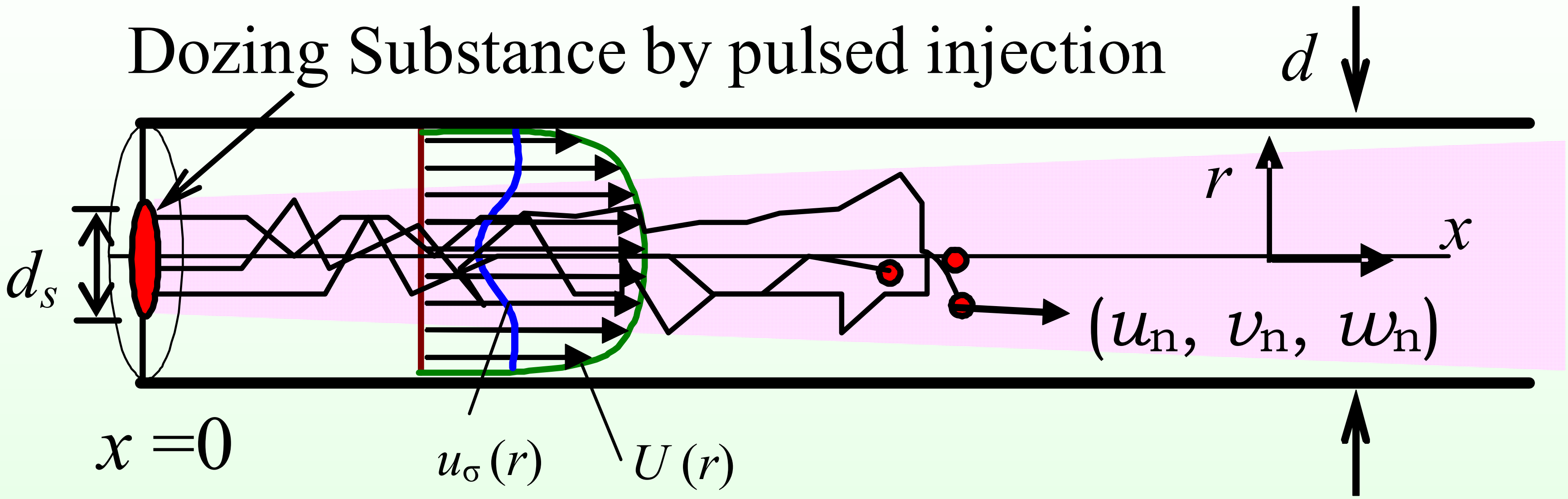


Matching with 1-D Diffusion Solution

$$C(x,t) = \frac{M}{2A\sqrt{\pi E t}} \exp\left(\frac{-(x - U_m t)^2}{4Et} \right)$$

E = Effective Diffusion coefficient (m²/s)

Calculation Scheme to Simulate Particle Movements

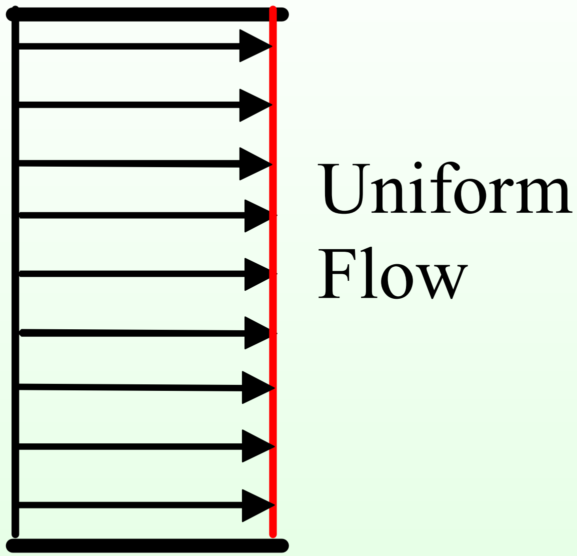


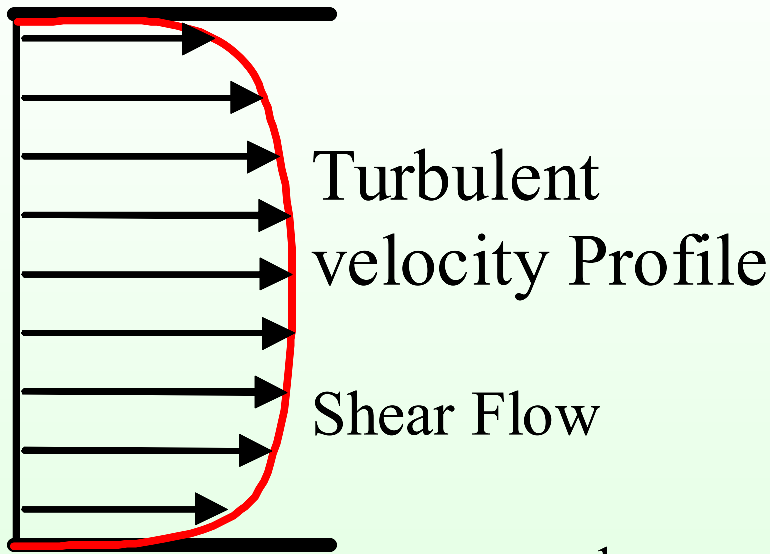
$$\begin{cases} u_n(t) = U_n(x_n, y_n, z_n) + u'_n(u_i, t) \\ v_n(t) = V_n(x_n, y_n, z_n) + v'_n(v_i, t) \\ w_n(t) = W_n(x_n, y_n, z_n) + w'_n(w_i, t) \end{cases}$$

$$\begin{cases} x_n(t+\Delta t) = x_n(t) + u_n(t)\cdot\Delta t \\ y_n(t+\Delta t) = y_n(t) + v_n(t)\cdot\Delta t \\ z_n(t+\Delta t) = z_n(t) + w_n(t)\cdot\Delta t \end{cases}$$

Δt ; Calculation time step

Average Velocity Profile Models





$$U_n = U_m$$

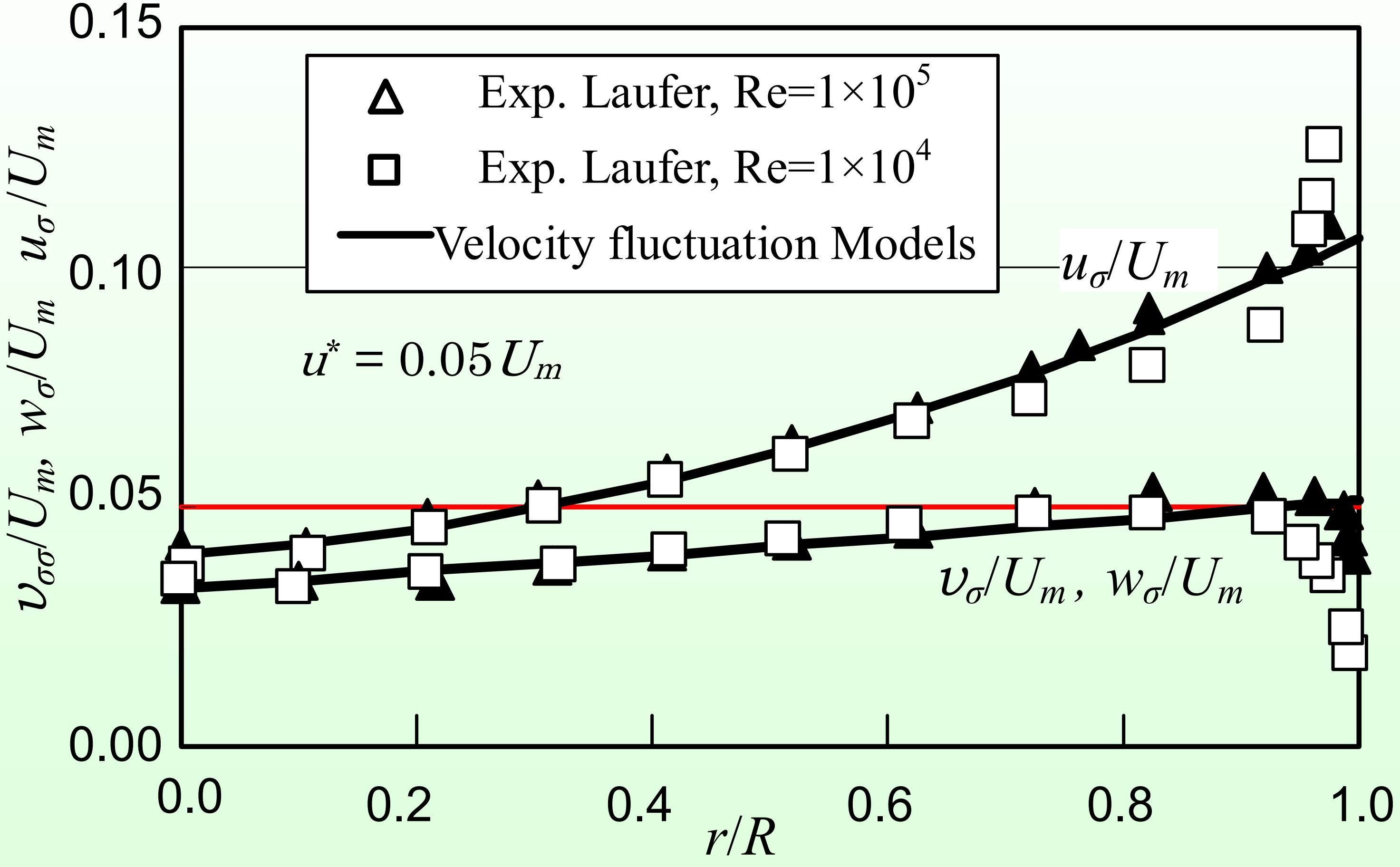
Uniform

$$U_n = \frac{U_{m.}}{0.8174}\left(1 - \frac{r_n^j}{R}\right)^{\frac{1}{7}}$$

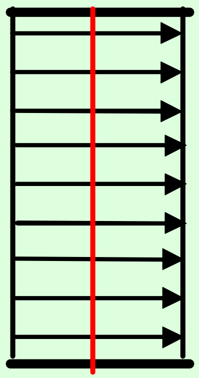
1/7th law

Velocity Fluctuation in Tube Flow

-Turbulent Intensities-

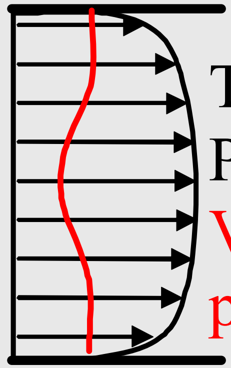


Intensity of Velocity Fluctuation (Standard Deviation of Random Walk)



Uniform Flow;
Homogeneous fluctuation

$$\frac{u_\sigma}{U_m} = \frac{v_\sigma}{U_m} = \frac{w_\sigma}{U_m} = const. = 0.05$$



Turbulent velocity Profile;
Velocity Fluctuation profile by Laufer

$$\begin{cases} \dfrac{u_\sigma}{U_m} = 0.0484.\left(\dfrac{r_n^i}{R}\right)^2 + 0.0179..\left(\dfrac{r_n^i}{R}\right) + 0.0397 \\ \dfrac{v_\sigma}{U_m} = \dfrac{w_\sigma}{U_m} = 0.00013.\left(\dfrac{r_n^i}{R}\right)^2 + 0.0171.\left(\dfrac{r_n^i}{R}\right) + 0.0329 \end{cases}$$

Assumption:

Standard Deviation of Random Walk velocity of Particles
= Velocity Fluctuation Intensity (root mean square value)

How to Move Particles?

$$u' = R(u_\sigma, t) \cdot U_m$$

Random Number following Gaussian Distribution

Generate a Random Number of Velocity Fluctuation following Gaussian Probability Density Function with Standard Deviation (Velocity intensity)

⇩

$$\Delta x_n = (U(r_n^i) + u').\Delta t$$

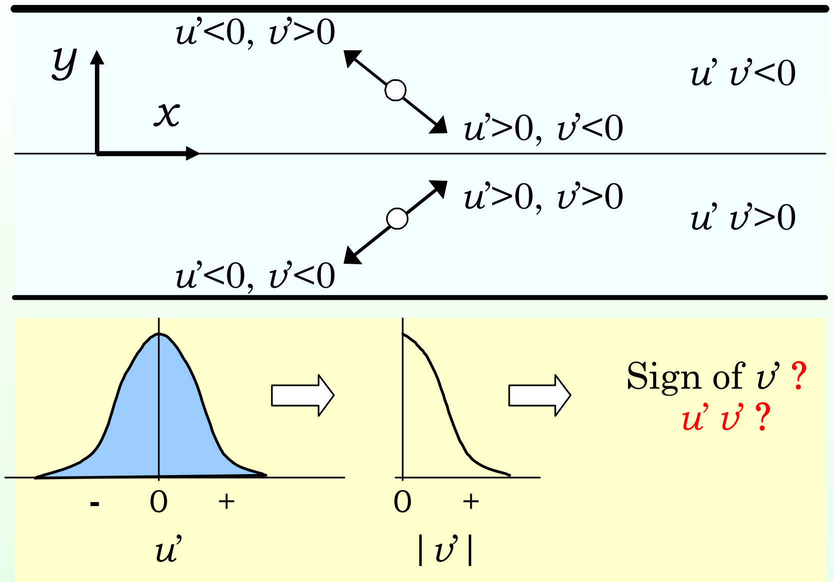
Calculate Movement of Random Walk during calculation time step

⇩

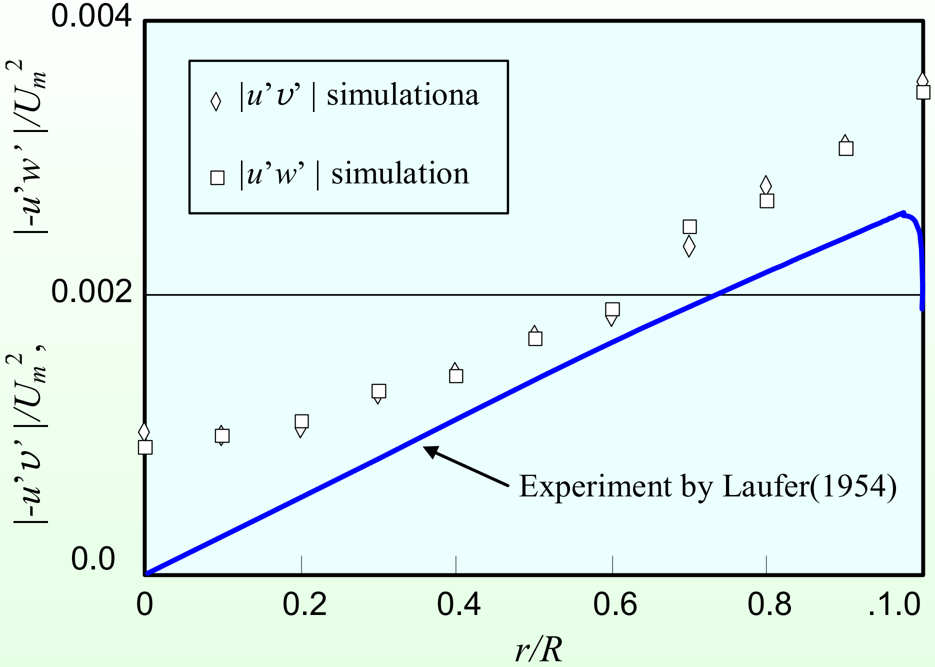
$$x_n^{i+1} = x_n^i + \Delta x_n$$

Make particle point Move to new position

Reynolds Stress Model



Calculation Scheme



Calculation Results

Diffusion Coefficient of Turbulent Flow

Diffusion coefficient, *D*, is related to turbulent velocity intensity, u_σ and turbulent mixing length, L_σ (Rouse, 1959).

Liepmann, H. & Laufer, J.(1947)

$$D \cong u_\sigma L_\sigma$$

$$L_\sigma \cong u_\sigma \frac{\Delta t}{2}$$

$$D \cong (u_\sigma)^2 \frac{\Delta t}{2}$$

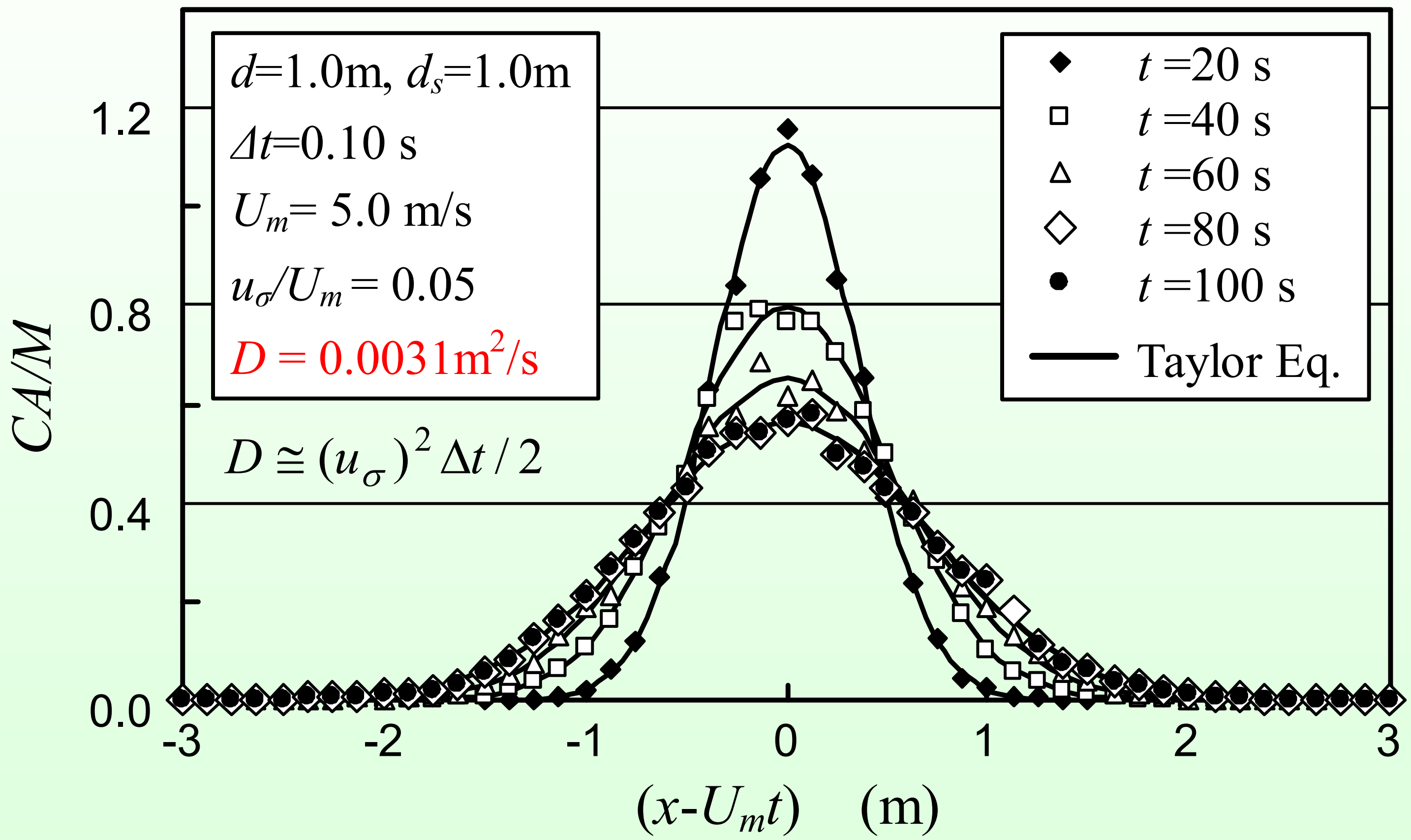
Δt : Time step of Movement

ΔL_σ: Turbulent mixing length

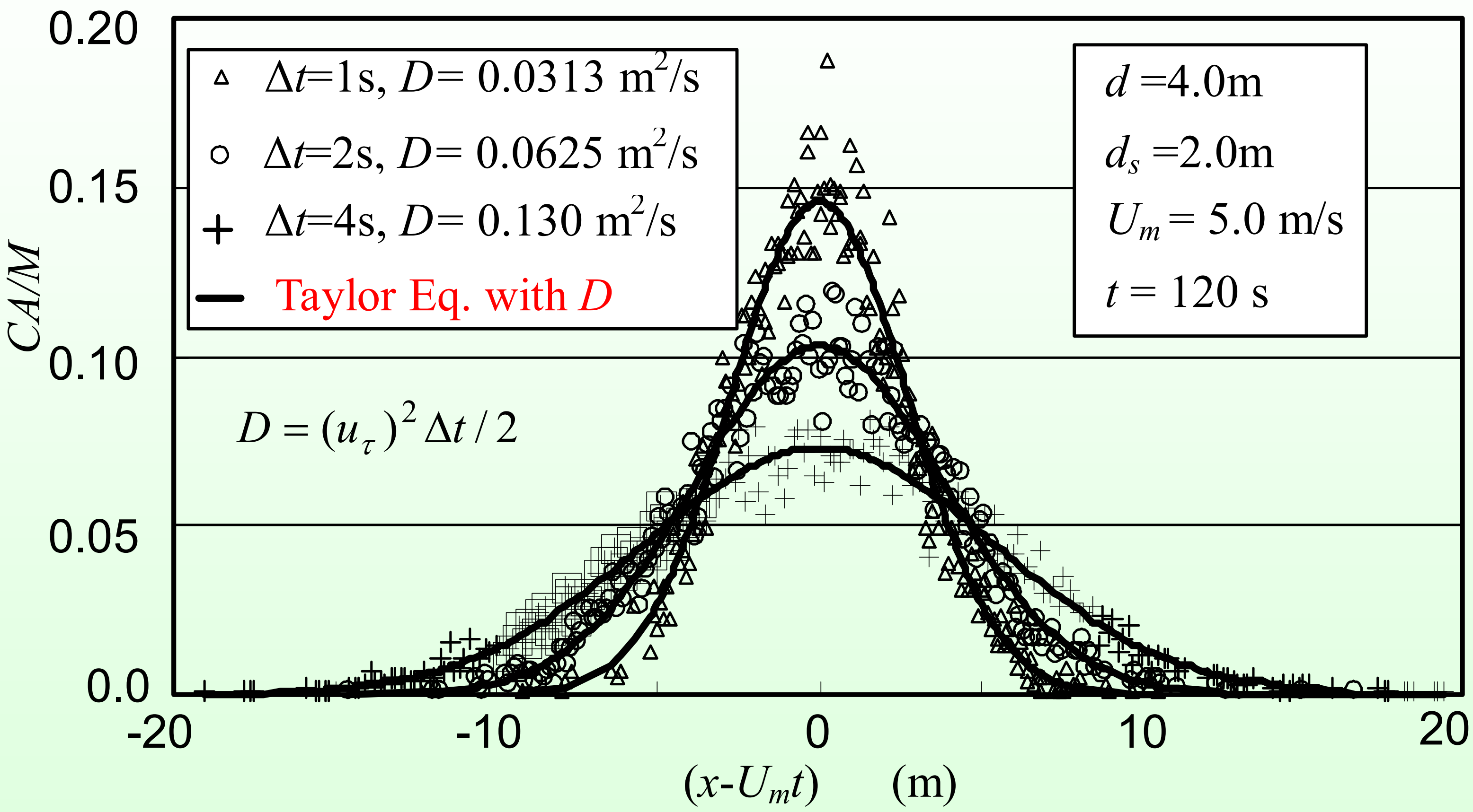
u_σ: turbulent velocity intensity

>> Molecular Diffusion Coefficient

Uniform Flow with Homogeneous Turbulence (=0.05U_m)

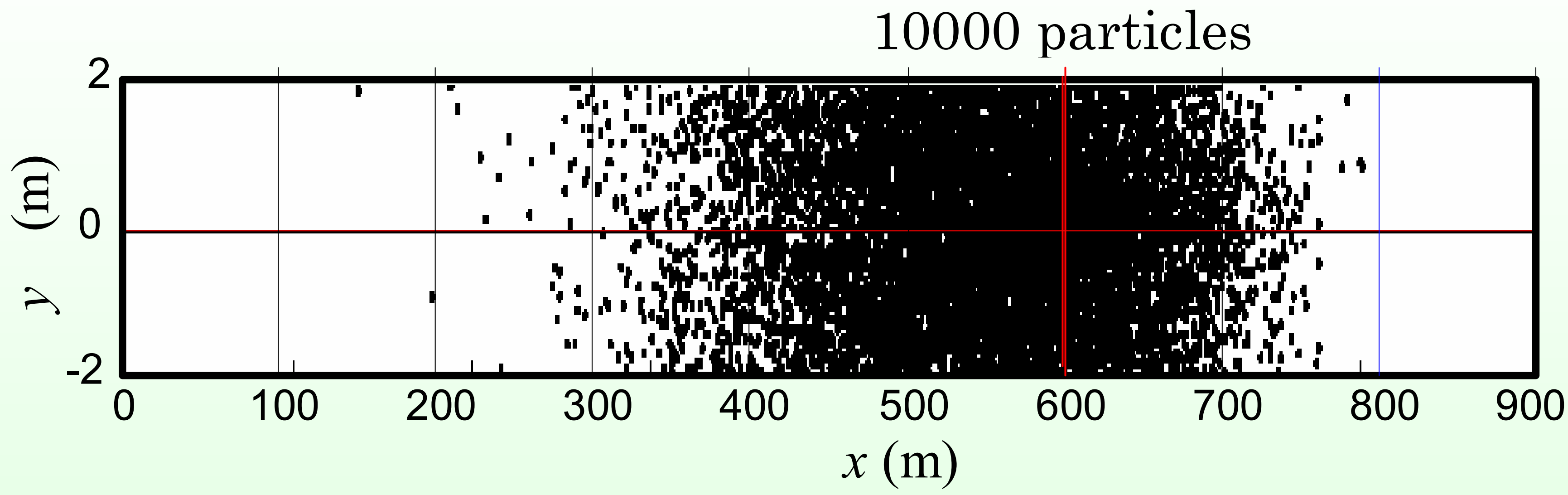


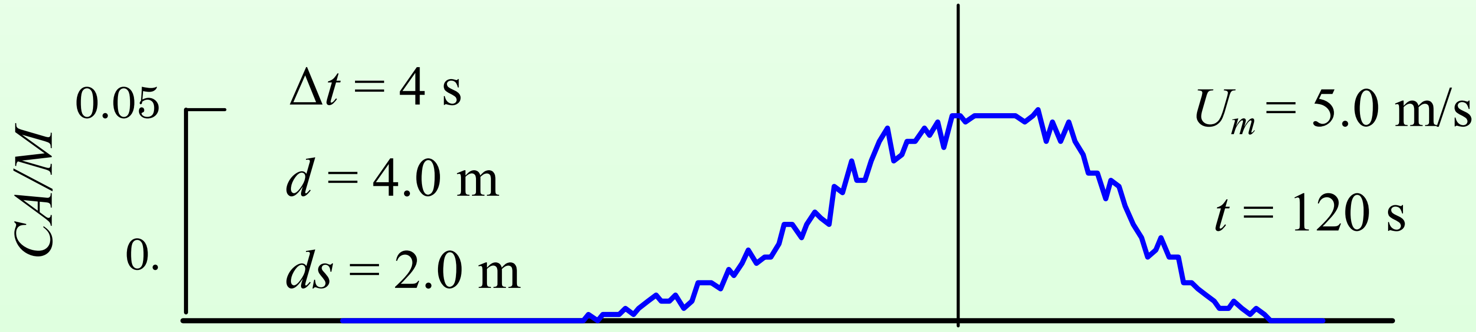
Effect of Calculation Time Step on Diffusion for Uniform & Homogeneous Flow



Larger Time Step produces larger diffusion coefficient in uniform flow

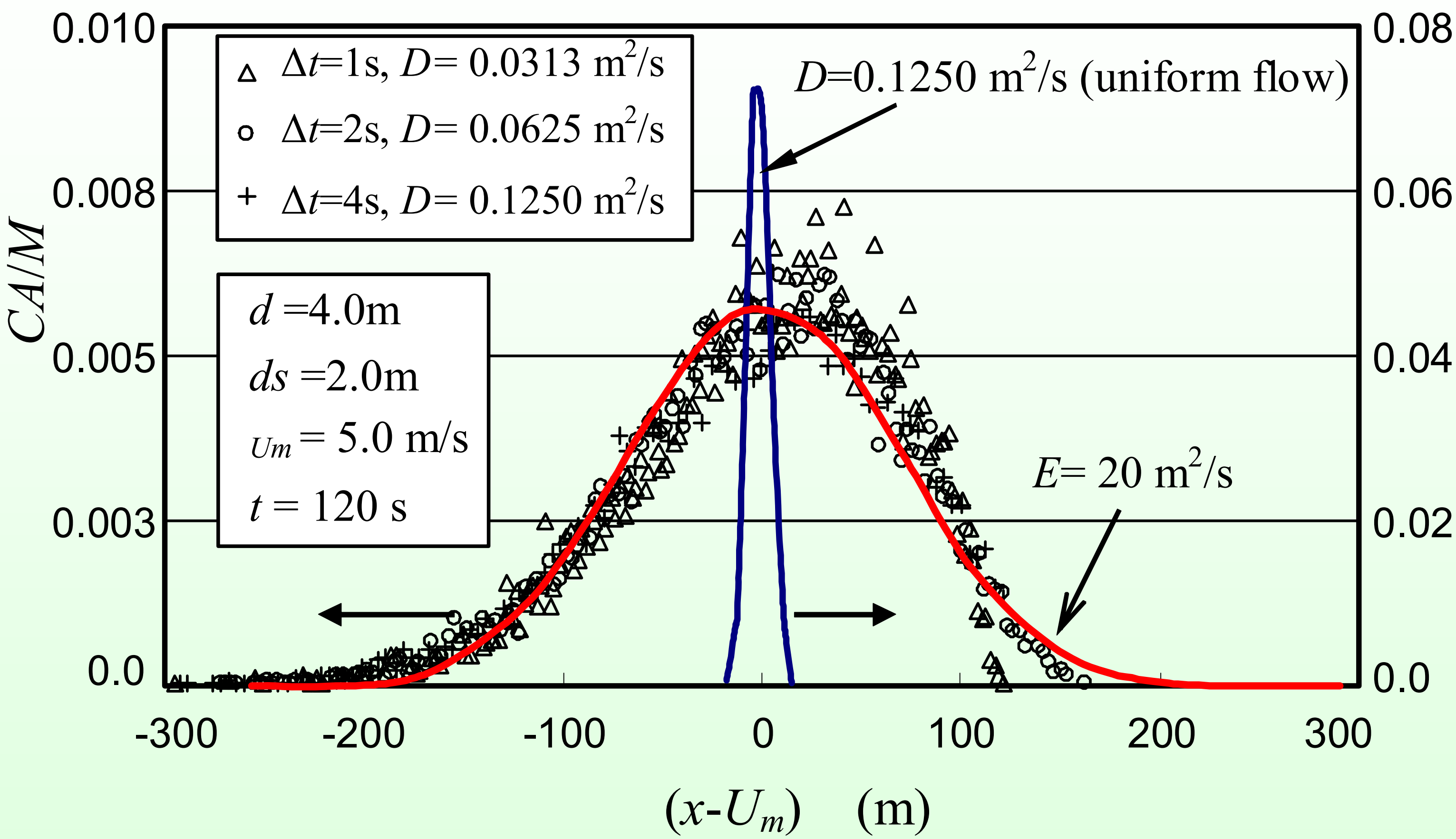
Calculation Result for Turbulent Flow with Velocity Profile





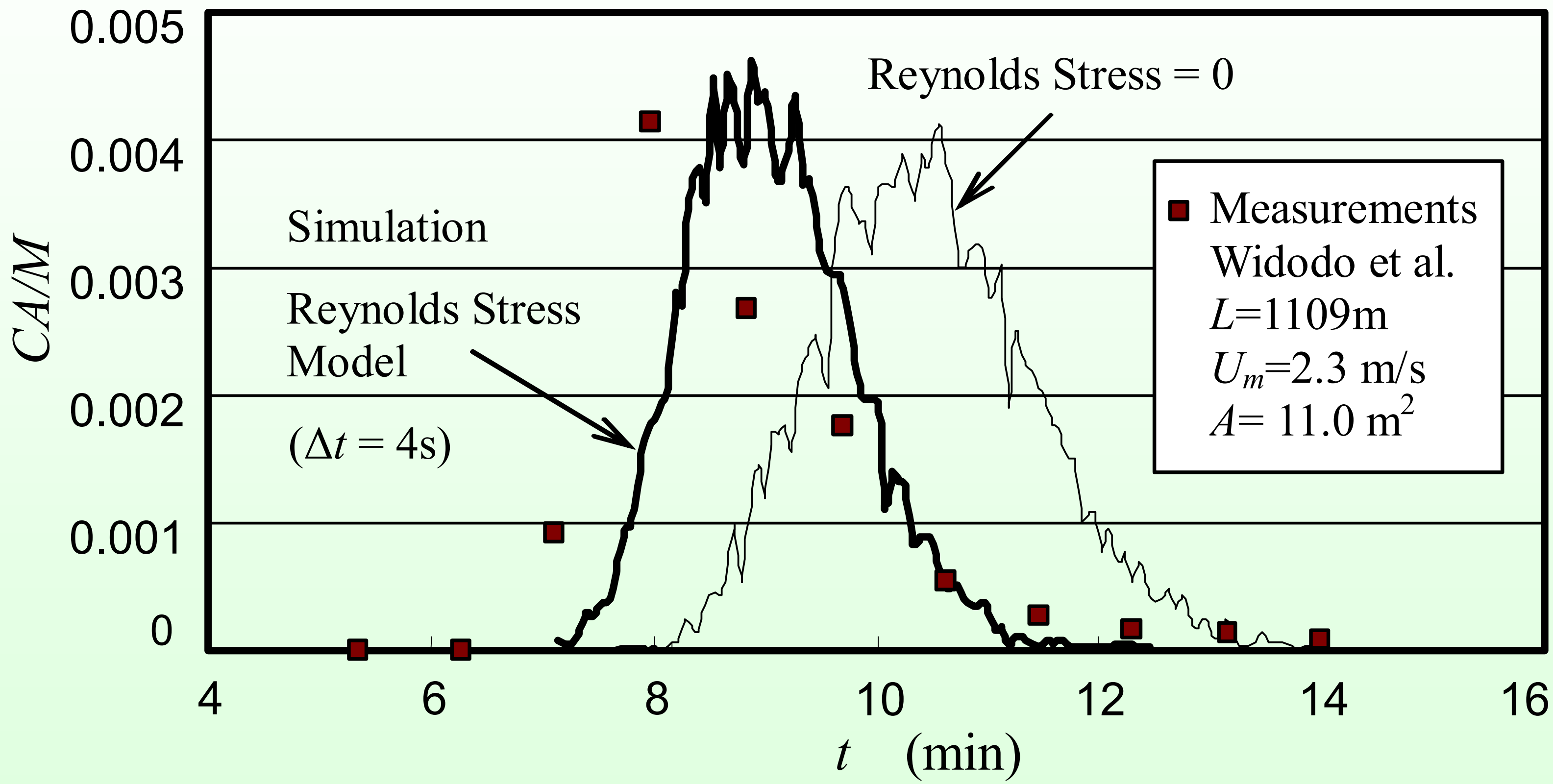
Diffusion length is over 400 m!

Effect of Velocity Profile on Diffusion

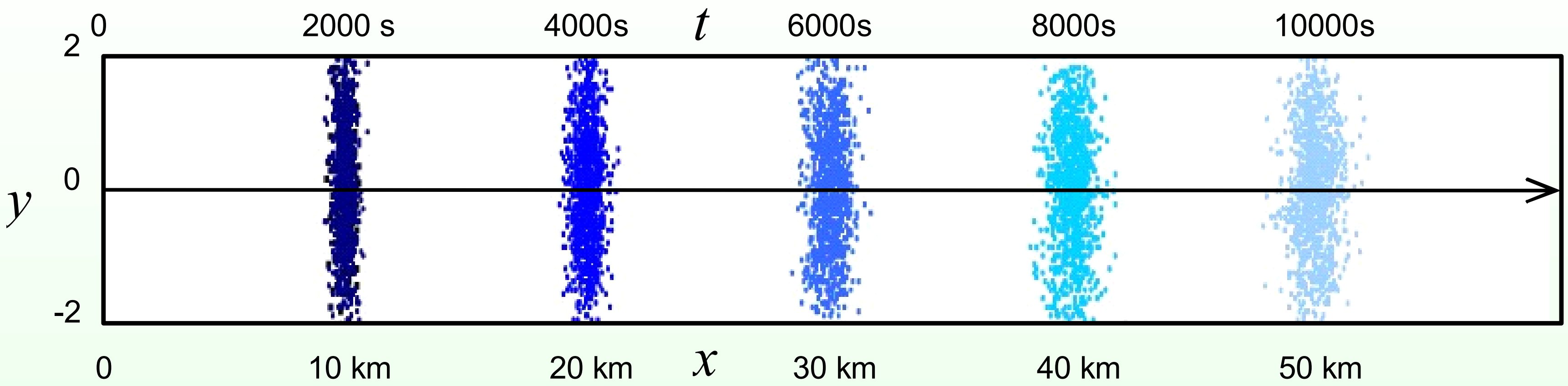


Effective diffusion coefficient for turbulent flow with velocity profile is not sensitive to calculation time step.

Effect of Reynolds Stress on Diffusion



Diffusion in Long Airways ~ 50 km



E=20 m^2/s

E=200 m^2/s ; Measured Value in the Mine

Further modification of present model is required by considering dead spaces along mine airways in order to express large diffusion coefficient.

Conclusion#1 (Numerical Model)

- ✓ A numerical simulation method, a Lagradian method, using discrete points dosed into tube flow has been proposed to simulate those movements by turbulent velocity profile and intensity of velocity fluctuation.

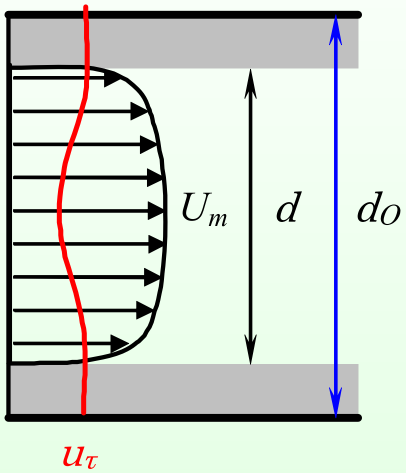
- ✓ A simple procedure is employed to represent the effect of dispersion by random walk satisfying Gaussian probability density function for standard deviation as turbulent intensity of velocity fluctuation.

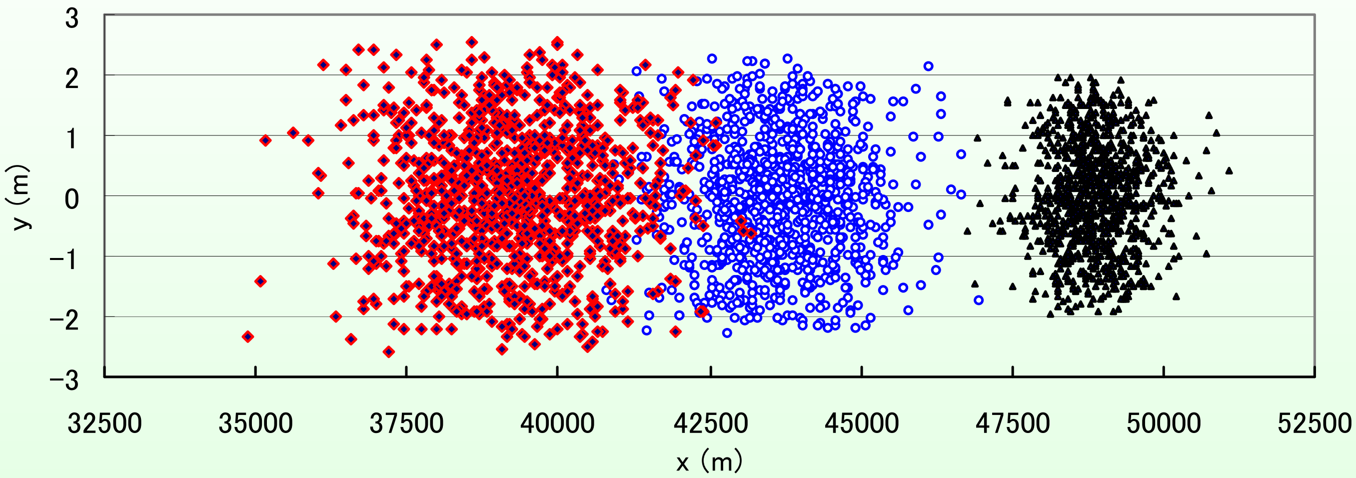
Conclusion#2 (Results)

- ✓ The Combination of turbulent velocity profile and fluctuation creates large dispersion of gas in longitudinal direction and long tail of gas concentration.
- ✓ Dispersions leads to large effective diffusion coefficient (20m^2/s) compared with original diffusion coefficient dominating in uniform velocity fields(≈0.1 m^2/s).
- ✓ Reynolds stress has a function to make slower travelling time of tracer points.


Thanks for your attention!

Modification of Model in considering Dead Spaces and Leaks





do/d=1.288	*do/d*=1.15	do/d=1.00
E=120 m²/s	*E*=60 m²/s	*E*=20 m²/s