

## Optimization of pit limit designs by the newly developed BPITC approach

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**ABSTRACT:** The new concept for final pit shape in open pit design namely Best Positive Inverted Truncated Cone (BPITC) algorithm is presented for feasibility study and scheduling.

Five mineral deposit models were used for eventual surface mine design optimization. Initially, geological reserve models were simulated by the utilization of GSLIB using random and regular drill hole data. Effects of varying anisotropy on the obtained metal quantities were also investigated. The BPITC was successfully applied to 100 times simulated deposit models. The optimization results by BPITC were compared with those obtained by Positive Moving Cone (PMC) and Dynamic Cone (DC) based algorithms for the same mineral deposit models. The obtained results indicate that BPITC is superior to all the other optimizers. Conclusively, the frequency distribution curves of profits and pit incremental feature by BPITC give effective information on mineral project decision-making and mining sequencing respectively.

### 1 INTRODUCTION

For many years now, the Mining Industry has been challenged by the problems involved in the way to determine the final open pit configuration coupled with mining sequencing. This hardship is experienced owing to considerably many variables or parameters (i.e. grade, mineral quantities, geometrical size, geological characteristics, etc.) required to decide the pit limit.

To help solve the above stated hardships, the scope of this study is to first utilize the twenty geologic diamond drill hole (DDH) data sets, categorized into five models namely model 1, model 2, model 3, model 4 and model 5 to develop regular three dimensional (3-D) block models of reserves by the utilization of relevant GSLIB tools since great variety of geo-statistical methods are now available (Deutsch & Journel 1992). Initially, geological reserve models of the respective data sets were generated by the utilization of a geo-statistical modeling package known as GSLIB. To accomplish the purpose of this whole study, additionally, other geo-statistical tools such as kriging, variogram calculations and simulation tools were used to help in making decisions whether or not to develop the mineral deposits. Sensitivity analyses on the effects of varying anisotropic factors on the results of modeling mineral deposit models, were performed

during simulation of the data sets. It was justified that for any type of deposit, the usage of correct anisotropic factors lead to correct evaluation of reserves.

The above developed geological reserve models were then transformed into economical block models using objective mathematical function formulations and relevant mining and metallurgical cost items. The mathematical models used allowed for flexibility in basic mining, metallurgical and economic parameters used. The ultimate open pit limits were then optimized by algorithms based on the heuristic and mathematical categories. The heuristic category algorithms considered in this study include conventional Positive Moving Cone, hereinafter referred to as Positive Moving Cone (PMC), while the mathematical algorithm category optimizers include the Dynamic Cone (DC) based algorithms.

The calculated results of profits were compared for all the models. This revealed that all the profits or benefits computed for all the five model deposits show that a newly developed scheme named as BPITC can always indicate optimal solutions for final pit designs.

In conclusion, the frequency distribution curves of profits and pit incremental feature by BPITC give effective information on mineral project decision-making and mining sequencing respectively.

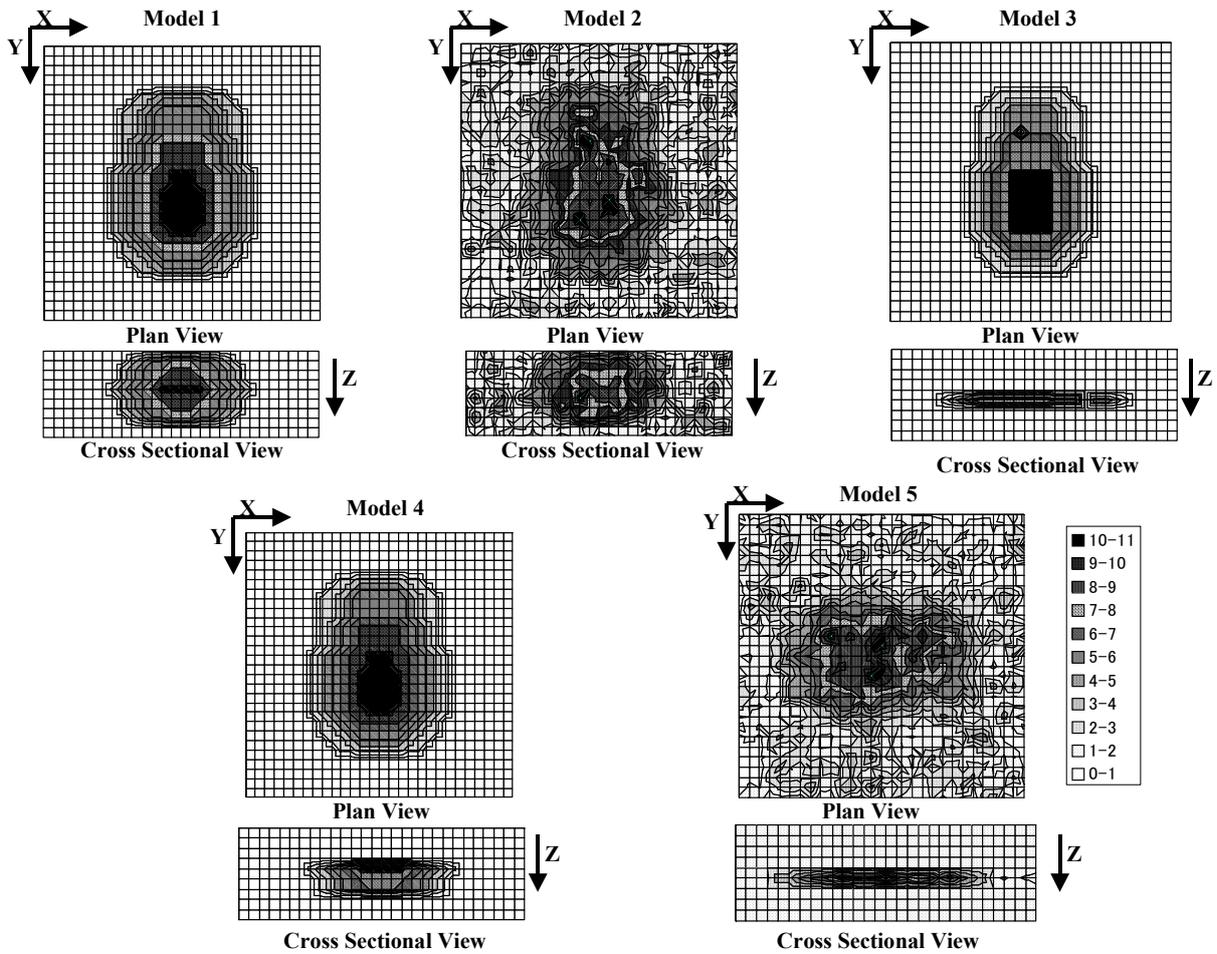


Fig. 1. Mineral contour maps for models

## 2 GEO-STATISTICAL MODELING OF MINERAL DEPOSITS

For the available data sets, GSLIB was used for modeling of geologic reserve models to generate 3-D visible models to eventually determine the optimum ultimate pit design configurations. The used deposit models were considered to be of different characteristics. Models 1 and 2 represent thick type mineral deposit models while models 3, 4 and 5 represent thin type ones. This was also a very good test for optimizers and the trend of their results. Figure 1 above, shows pictorial representation of the mineral deposit models.

The types of data used in the 5 models mentioned above were either regularly (gridded) spaced 3-D data or irregularly (random) spaced 3-D data patterns (Fig. 2). For both kinds of data patterns, the contained data sets could further be categorized as 225 drill hole (DH) and 100 DH data types (Fig. 2). Semi-variograms were studied in the X, Y and Z directions in addition to the general direction. After the experimental (expt) points of the obtained graphs

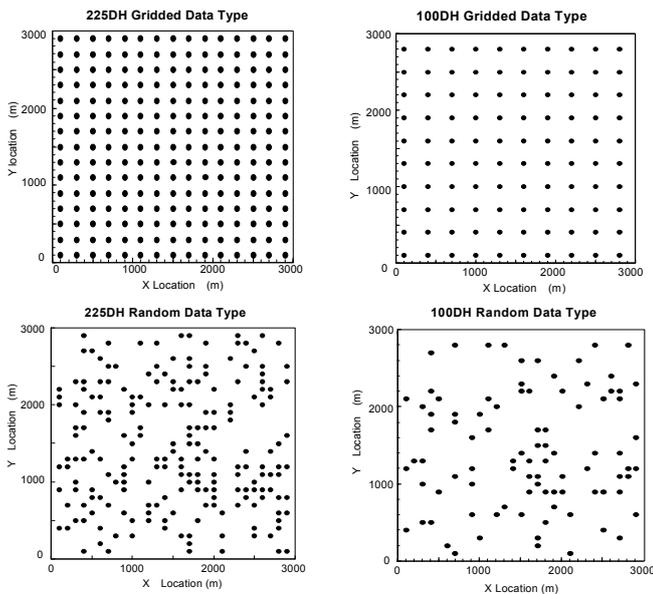


Fig. 2. Drill hole data patterns for models

for variance were developed for all possible sample pairs, they were fitted by theoretical or standard (std) gaussian model functions. Various variations of nugget effect, the sill and the radius of influence were tested, for fitting theoretical models to the experimental semi-variogram values. Examples of the resulting experimental semi-variogram models and fitted standard or theoretical gaussian models for the models 2 and 4 are as depicted in Figures 3 and 4. The generated experimental variograms do show that the mineralization is not continuous in all the studied directions since it is evident that the ranges of influence (ROI) in the studied directions varied distinctively. These are anisotropies in the continuity of mineralization and are called geometrical anisotropies.

The ROI for the variograms generated for model 2 are longer than those obtained for model 4 because model 2 is a thick deposit model and model 4 is thin type (Figs 3 & 4).

Anisotropies were quantified by calculating the ratios of the ranges for two different directions for all the models studied (Table 1).

Anisotropies were varied in the Z directions from 0.15 to 1.0, of which 0.30 approximately was the optimal one for models 2, 4 and 5 whereas, about 0.40 was the optimal one for models 1 and 3. It was evident that once anisotropy deviated from the correct one, a different contour map could be obtained (Fig. 5).

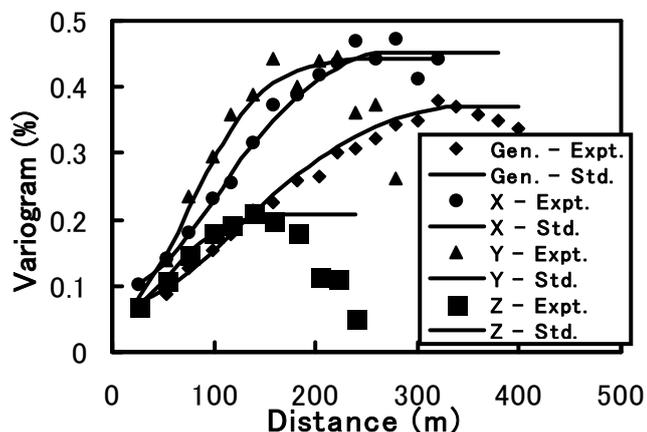


Fig. 3. Semi-variograms for model 2

The total metal amount (TMA) contained also varied with anisotropy (Figs 6 & 7). This showed justification of the effect of varying anisotropy on the predictable shape and quantity of mineral deposits. Usage of wrong anisotropy leads to wrong definition of deposit.

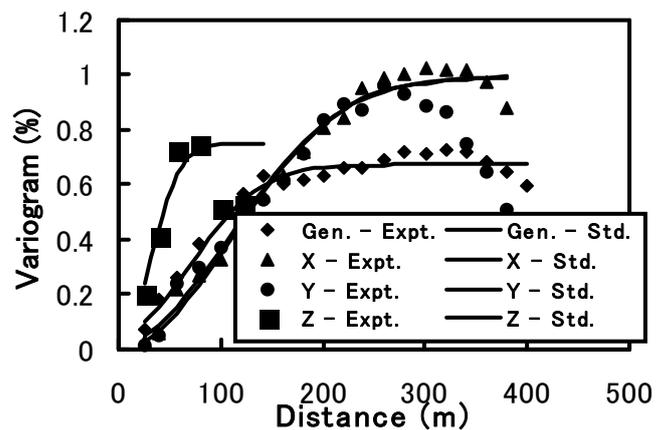


Fig. 4. Semi-variograms for model 4

Table 1. Anisotropic factors for mineral models

Model	X	Y	Z	ANIS(1)	ANIS(2)
1	167.70	188.50	71.70	0.89	0.38
2	158.97	173.30	47.38	0.92	0.27
3	134.09	135.70	54.10	0.99	0.40
4	151.70	152.40	41.90	1.00	0.27
5	139.13	159.13	47.60	0.87	0.30

The gaussian standard model chosen for kriging calculations and simulation incorporated the geometric anisotropies, which were discovered in the experimental variograms.

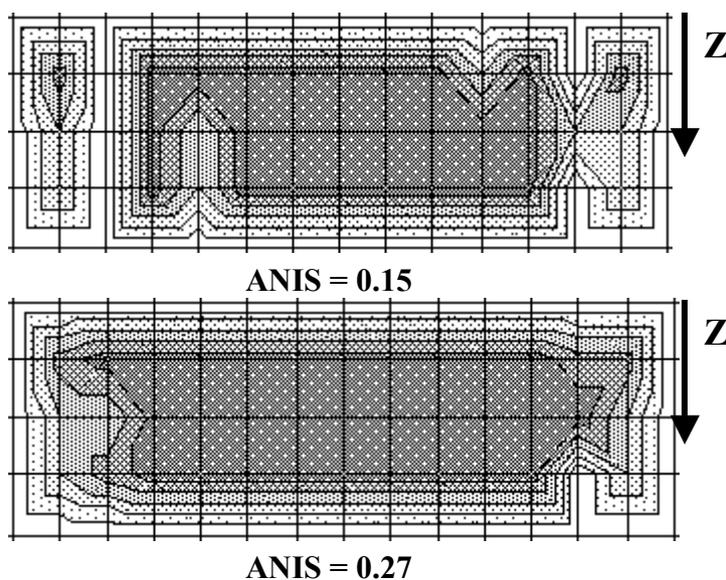


Fig. 5. Effects of varying anisotropy (ANIS) on pre-defined shapes of mineral deposit models

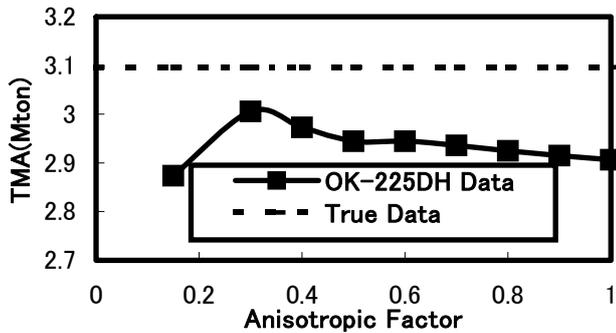


Fig. 6. Effects of varying anisotropy on total metal amount (TMA) for model 2

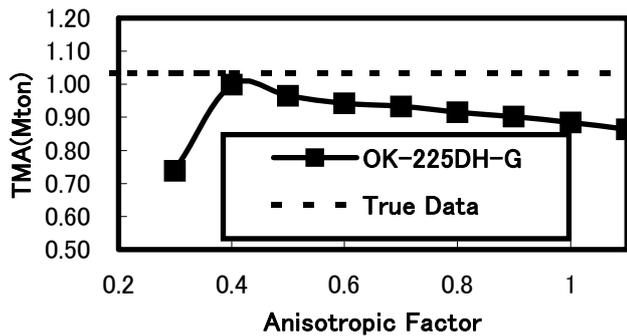


Fig. 7. Effects of varying anisotropy on total metal amount (TMA) for model 4

Despite the endless list of types of kriging, ordinary kriging (OK) was employed to pre-determine the grade of required blocks from the available DDH assay data. The output from this process consists of a single grade and a single kriging variance.

A gaussian related algorithm called sequential gaussian simulation (SIM) was applied to the data during this study because it is the most straightforward algorithm for generating realizations of a multivariate field (Deutsch & Journel 1992). The principle behind this algorithm is that each variable is simulated sequentially according to its normal cumulative condition distribution function fully characterized through a kriging system type. The conditioning (hard) data and all previously simulated (soft) values found within a neighborhood of the location being simulated were used to produce results. This technique generates one or more simulated deposits having the same mean, variance, histogram, and variogram function as the real deposit being simulated.

The available grade data for the 5 models were simulated 100 times and Figures 8 and 9 show the calculated total metal contained per geo-statistical simulation for model 4.

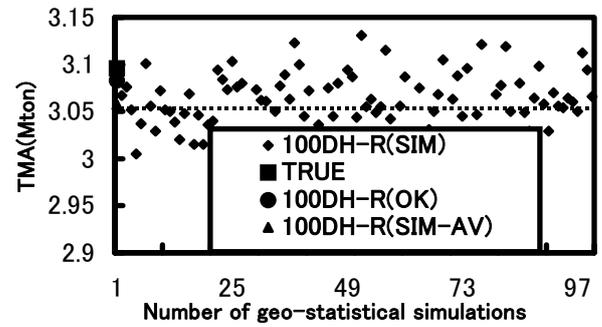


Fig. 8. Comparison of total metal amount (TMA) for simulated, kriged & true data for model 2

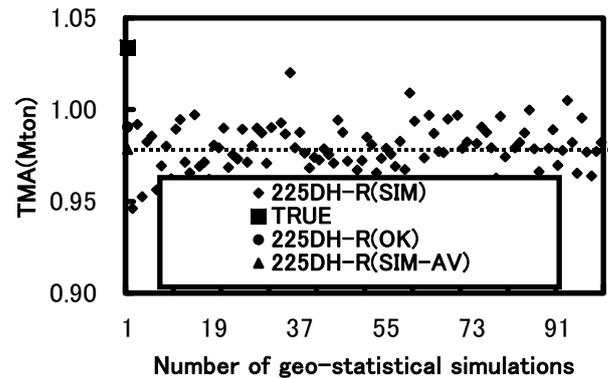


Fig. 9. Comparison of total metal amount (TMA) for simulated, kriged & true data for model 4

### 3 PIT LIMIT DESIGN HISTORICAL EVOLUTION

#### 3.1 Positive Moving Cone (PMC) Based Algorithms

Many authors have researched and published papers on the subject of ultimate open pit design algorithms both in the heuristic category, such as the Positive Moving Cone (PMC) algorithm, in the mathematical category, such as the graph theory algorithm, linear programming and network flow algorithm.

Gershon 1983 reported that the PMC technique, in spite of being fast, does not guarantee that the best solution is obtained.

Wright 1990 editorialized that PMC technique should employ the "re-testing" procedure as suggested by Korobov 1974.

Yamatomi et al. 1994 presented their findings by application of computational algorithm for 3-D open pit designs in order to obtain the most profit-able pit configuration. The traditional PMC algorithm was

modified by including “refilling” of negative blocks while complying with slope angles constraints.

### 3.2 *Dynamic Cone (DC) Based Algorithms*

The first optimization technique applied to the problem of UPL was dynamic programming (DP) based on Lerchs and Grossmann (LG) 1965 algorithm as cited by Wright 1987; Yamatomi et al. 1994.

Sevim and Lei 1994 was in consent with the modifications done by Dowd and Onur 1993 to the Korobov 1974 algorithm, which was based on the PMC technique. It was claimed that this algorithm guaranteed the optimality by correctly allocating the values of the blocks in the common sections of the cones to the appropriate cones.

Wright 1987 proclaimed that they managed to successfully employ DP applications directly to solve 3-D open pit design problems by using a 3-D algorithm. The above-mentioned algorithm was employed with a 3-D incremental minimum removal cone over each block considered.

Yamatomi et al. 1994 recommended that CPVs be employed instead of BCVs in the decision-making RF used when determining the pit values. Therefore, Yamatomi’s algorithm identified that the block under assessment had twelve neighboring blocks among which the best possible one must be selected from. Notwithstanding the above, the authors conceded that their improved algorithm could not always obtain the optimal solution due to existence of multi-stage paths, among which only one was really optimal.

### 3.3 *Other Algorithms*

Knowles 1999 introduced a Mixed Integer Linear Programming (MILP) approach to determine and optimize the sequence of mining. This optimization processing involved minimization of the NPV of annual costs while achieving a minimum annual production goal. The author pointed out that MILP was difficult to solve to proven optimality and that proving optimality took an extremely long time.

Other optimization techniques namely network analysis have been applied to UPL problem solving but these have not been widely used (Sevim & Lei 1994). The network analysis and linear programming concepts of pit limits determination are far beyond the scope of the current study.

## 4 OPEN PIT LIMIT OPTIMIZATION

Prior to applying optimizers, the geologic ore grade reserve models were initially partitioned into 8410 cubic blocks. There were 29 x 29 x 10 unit blocks in the X, Y, and Z directions, respectively. Each unit of block was considered to be a cube with each side measuring 25 meters and of density equal to 4 267.2 kg/m<sup>3</sup>. Bounding techniques were applied to discard the un-necessary blocks.

The generated 3-D “mineral inventories” of the deposit models were then transformed into “economic block models” in which each block was assigned a block net value (BNV) basing on its volumetric reserves and quality and on the economics of open pit mining. Further the mineral concentration was assumed to be that of copper at a market price of \$1 800 per ton. Additionally a the metals contained in a mining area were mined whilst taking into account a cutoff grade of 0.25.

### 4.1 *Optimization by DC Based Algorithms*

After determining block column values (BCVs) for the block models, DC-3N algorithm uses a 3-neighbor RF formula to calculate the pit values. DC-12N uses a 12-neighbor RF formula to determine the optimal path while the BCVs are calculated in exactly the same way as DC-3N.

### 4.2 *Optimization by PMC Based Algorithms*

PMC involves the examination of the block model starts in the top most left corner and proceeds from left to right on every level, searching for all positive valued blocks on the first level, prior to those on the second level and then moving onto the third level. PMC mines only positive pits.

BPMC algorithm starts excavating from the lowest level towards the top. The algorithm initially mines maximum positive pit in a mining area and then continues to survey within the mine area. It decides the size and position of the cone with maximum benefit and then iterates the same procedure.

BPITC decides size, position and depth of the cone with maximum benefit and then iterates the same procedure. This algorithm can be simply explained as outlined in the phases below;

#### **Phase 1:**

Assess value for cone of the lowest block,  $b_{ijk}$  in the biggest possible pit. If the value of the lowest block for the cone is negative, then leave the negative

block un-mined. Otherwise, if this block is positive, then mine it and the sub or intermediate pit and the respective pit topography outcome are noted.

**Phase 2:**

Evaluate other blocks on the lowest level and repeat Phase 1. Otherwise, move to Phase 3.

**Phase 3:**

Proceed to subsequent levels upwards and each time repeating Phases 1 and 2 until the top level of the block model has been reached.

BPMC finds and mines the maximum pit using a cone whilst BPITC finds and mines a truncated cone (Fig. 10) with highest pit value (PV). Two distinctive features for BPITC are that; (i) BPITC finds the maximum PV and (ii) optimizes the truncated variable distance.

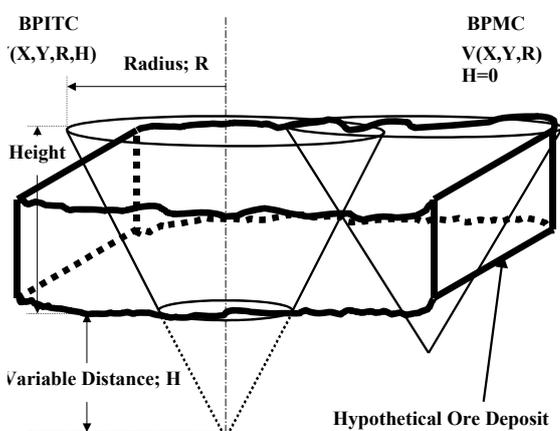


Fig. 10. Distinction of BPITC from BPMC

The optimization results by BPITC have been compared with those obtained by other PMC and DC based algorithms mentioned above for the same mineral deposit models. Figures 11 and 12 depict the mode of scheme employed by each algorithm during assessment of blocks for models 2 and 4. A group of iterations can therefore be considered as one stage during scheduling.

For each of the applied optimizing algorithms, once the whole economic block model was fully examined, the optimum pit values were then collected to give the optimum path for the 3-D open pit. This gave rise to the pit topographies whose data was then used to develop visible 3-D open pit models (Fig. 13). Figure 14 reflects the stages of mining incorporated in the BPITC program for model 4. Furthermore, Figures 15 and 16 show that at the first iterations, the number of blocks are the lowest against the highest benefits for models 2 and 4. This is also a meritorious feature for BPITC in

that it indicates that the investment of a mineral project could easily be recovered at these initial stages of mining mineral deposits.

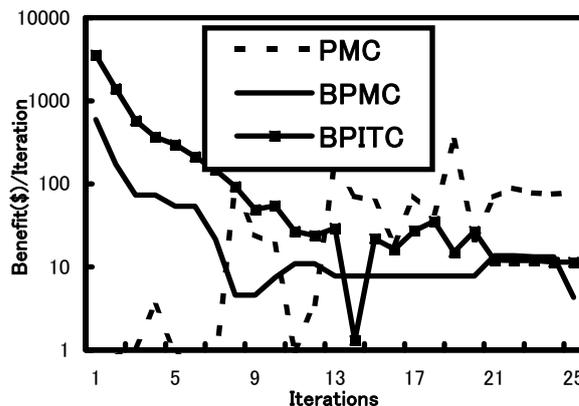


Fig. 11. Comparison of algorithms' mode of benefit assessment for model 2

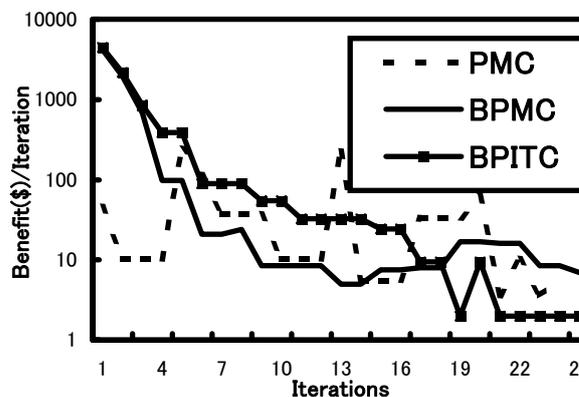


Fig. 12. Comparison of algorithms' mode of benefit assessment for model 4

Comparison of the benefit results reveal that PMC based algorithms give better results for thick type mineral deposit models while DC based algorithms give better results for thin type mineral deposit models. The obtained results for all the models used indicate that BPITC is superior to all the other optimizers (Figs 17 & 18). This makes it the best moving cone method for maximization of profits computed during pit designs.

Figures 19 and 20 depict the obtained total metal amount (TMA) frequency distribution curves of the varied anisotropies for models 2 and 4. It is vivid that the TMA obtained for the optimal anisotropies for all the models are the highest and are with the highest frequencies. This indicates the importance of using the correct anisotropic factors during pre-definition of ore grade reserves.

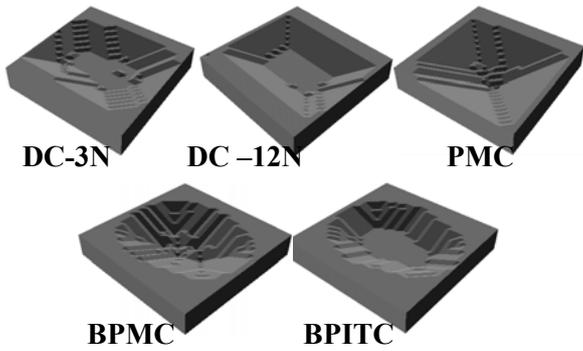


Fig. 13. Pictorial representation for designed 3-D open pit models for model 4

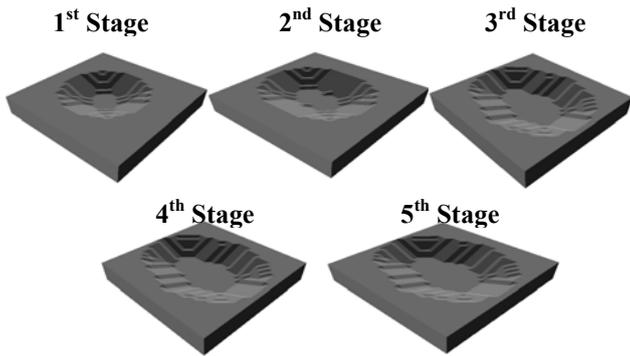


Fig. 14. Stages of mining sequencing by BPITC for model 4

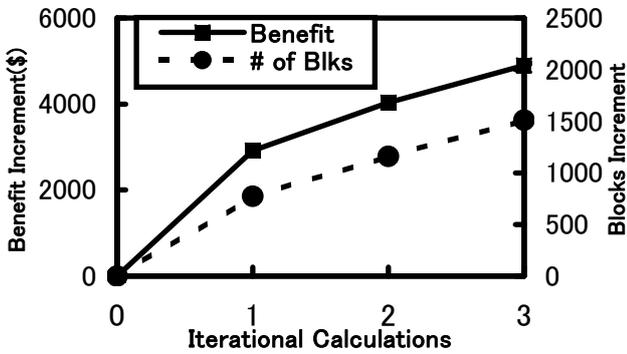


Fig. 15. Benefit versus number of blocks per iterational calculation for model 2

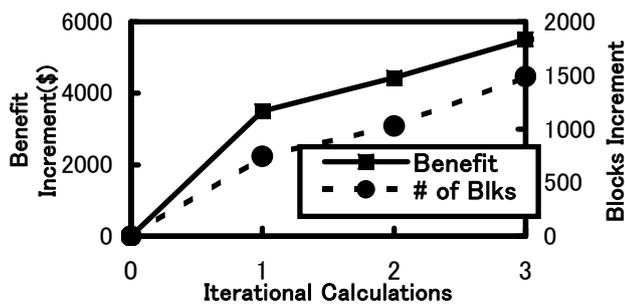


Fig. 16. Benefit versus number of blocks per iterational calculation for model 4

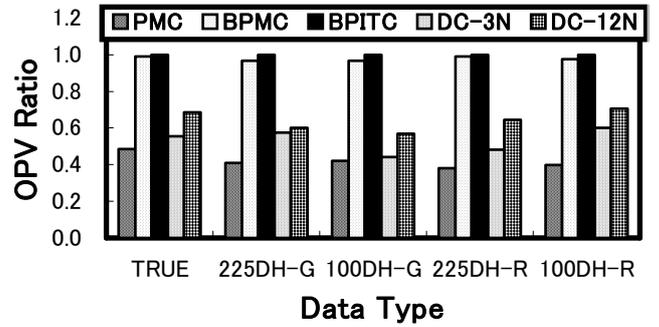


Fig. 17. Histogram for optimum pit values (OPV) for model 2 (100 & 225 show the number of drill holes)

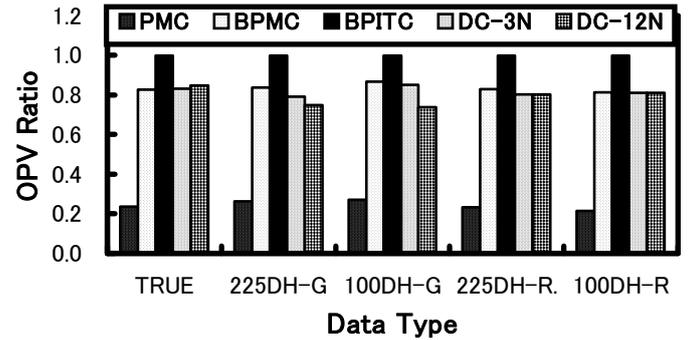


Fig. 18. Histogram for optimum pit values (OPV) for model 4 (100 & 225 show the number of drill holes)

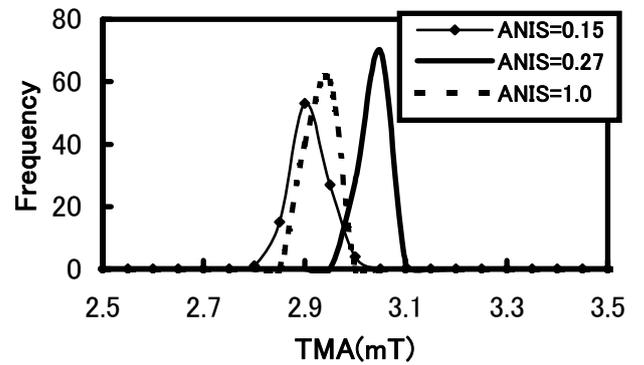


Fig. 19. Total metal amount (TMA) frequency distribution for various anisotropies for model 2

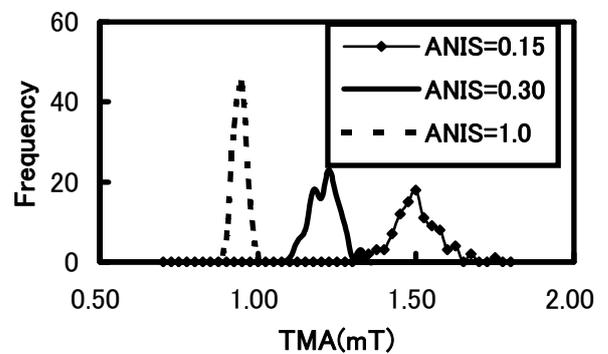


Fig. 20. Total metal amount (TMA) frequency distribution for various anisotropies for model 4

Figures 21 & 22 show the frequency distribution curves of optimum pit values (OPV) for models 2 and 4. It is clear that the curves for BPITC have the highest frequencies and occur towards the highest profits' or benefits' area of the graphs.

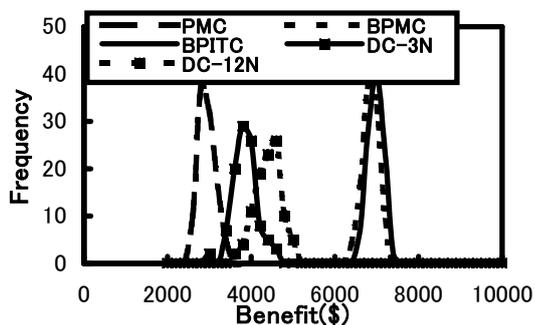


Fig. 21. Pit value frequency distribution curves for model 2

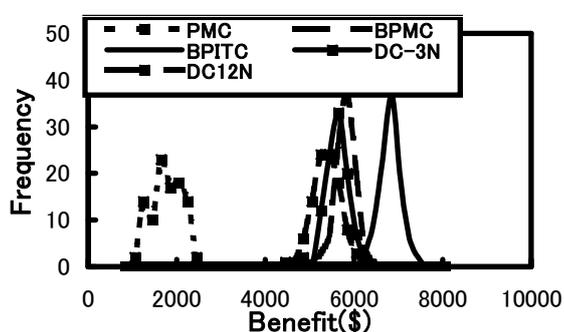


Fig. 22. Pit value frequency distribution curves for model 4

## 5 CONCLUSIONS

Geo-statistical analyses results show that it is possible to accurately predict the ore grade reserves for eventual visual reference for pit limits design. Subsequently, it was justified that for any type of deposit, the usage of correct anisotropic factors lead to correct evaluation of reserves. The optimization results by BPITC have been compared with those obtained by other PMC and DC based algorithms for the same mineral deposit models. Comparisons reveal that PMC based algorithms give better results for thick type mineral deposit models while DC based algorithms give better results for thin type mineral deposit models. The obtained results for all the models used indicate that BPITC is superior to all the other optimizers. This makes it the best moving cone method for maximization of profits computed during pit designs.

Even if the BPITC optimizer gives the best and true optimal solutions, it has a limitation of not realizing the time effect or present value of money. This factor makes it not to be reliable over time.

Perhaps, this concept needs consideration in the near future. Notwithstanding this, the conclusions from this study can be vital guidelines for Mine Planners or Engineers and the Mining Industry as a whole. Ultimately, the frequency distribution curves of profits and the pit incremental feature by BPITC give effective information on mineral project decision-making and mining sequencing respectively.

1. It was justified that for any type of deposit, the usage of correct anisotropic factors lead to correct evaluation of reserves.

2. All the profits/benefits computed for all the five model deposits show that a newly developed scheme named as BPITC can always indicate optimal solutions for final pit designs.

3. Conclusively, the frequency distribution curves of profits and pit incremental feature by BPITC give effective information on mineral project decision-making and mining sequencing respectively.

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