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Analysis of underground ventilation networks utilizing the skyline nodal pressure method

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ABSTRACT: A calculation system for the analysis of underground ventilation networks by means of a nodal pressure method using the skyline modified Choleski's decomposition scheme, named 'skyline nodal pressure method', has been studied. The generalized linear equations for solving steady-state ventilation airflow have been developed using a connective function between nodes. The skyline decomposition scheme has an advantage on rapid convergence and the sake of computer memory space as compared with other schemes. Five examples of ventilation networks have been calculated using a popular personal computer. A convergence criterion, that the maximum error of the summed airflow-quantity in continuity at each node is less than 0.1 m^3/min, was used for the calculations. Under the conditions that the fan-pressure is given as a constant value and that the acceleration factor is set at 1.4, the number of the iterations until convergence was not more than 13 for each network. The calculation processing time by the skyline nodal method is roughly in proportion to the total number of nodes to the 3/2 power.

1 INTRODUCTION

In recent years, many reports have been released with respect to analytical systems for underground ventilation networks to predict flow-quantity and air conditions related to secured maintenance of mines using small computers (Wallace and Brunner, 1989). The authors have also developed a calculation code *'MIVENA'* for underground metal mines (Sasaki et al., 1990, 1992). Wang et al.(1985) give an excellent review of the ventilation network theory.

Bhamidipati & Procarione (1985) have compared two well-known methods, the mesh flow-rate and the nodal pressure (head) methods. They have suggested that the nodal pressure method is a superior process due to its simple approach for computer implementation which does not require complex procedures such as the selection of meshes used in the mesh flow method.

The nodal pressure method is based on an approximated linear calculation system incorporating nodal pressures as primary variables. The nodal pressures have to be improved by iterative calculations in order to satisfy Kirchhoff's 1st(current) law.

The nodal methods previously proposed are classified into two types. The first is a conventional method used to improve each of the nodal pressures by a simple equation which incorporates neighboring nodal pressures. Inoue et al.(1989) have recently applied this method to mine ventilation analysis. This method, hereinafter referred to as the 'conventional nodal method', has the merit of requiring extremely small memory space for data and computer programs. However, its convergence process becomes unstable and the processing time increases as the number of nodes increases. This means that attention has to be paid to the convergence process.

The second type is an analytical method which uses a direct matrix operation to solve the simultaneous linear equations. This method, hereinafter referred to as the 'matrix nodal method', has an advantage of achieving solutions with a stable convergent process by handling whole nodal pressures. However, it requires comparatively large working memory space. Bhamidipati & Procarione (1985) have analyzed mine ventilation networks using the matrix nodal method. This method uses the modified Choleski's decomposition scheme(hereinafter referred to as a 'MCD') with a band matrix to solve an admittance matrix as well as the pipe network flows(Isaacs & Mills, 1980).

They have formulated equations related to nodal pressures (not correction factors) as unknown variables. Sasaki et al. (1990) have reported a matrix nodal method used to solve mid-size networks (<200 nodes) by applying the MCD scheme. This method has an exceptional memory usage with a popular personal computer (Melosh et al. 1967, Melosh & Bamford 1969).

The purpose of this study is to introduce a nodal pressure method with rapid calculation, stable convergent process and smaller memory space for the analysis of middle or large size of ventilation networks.

In this study, the fundamental equations of the nodal matrix method are based on linear equations which consider the correction factors for nodal pressure as unknown variables. By introducing a function which expresses the connections between nodes, the linear equations are provided in a more general form and the iterative calculation schemes are explicitly shown.

The calculation schemes used to solve the admittance matrix of the linear equations are based on four kinds of schemes: *a)* original MCD, *b)* band matrix MCD, *c)* skyline MCD and *d)* ICCG scheme. The skyline MCD scheme is an improved MCD method in limiting calculation and memory space of the matrix components by using one dimensional memories. The ICCG scheme refers to incomplete Choleski's decomposition and iterative calculations using the conjugate gradient scheme, which requires relatively small memory space for large scale networks. When the skyline scheme was adapted, the calculation speed was accelerated at least 6 times as compared to that of the original MCD scheme for middle size networks (≈140 nodes).

Two illustrative small ventilation networks and three actual ventilation networks of underground metal mines were developed to evaluate the four schemes. This study permitted a comparative analysis concerning the number of iterations, the calculation speed and the amount of required memory space for the four schemes.

2 ANALYTICAL METHOD

2.1 Definition of symbols and fundamental flow equations

The underground ventilation network dealt with in this study is consisted of *a)* airways, *b)* nodes connecting airways and *c)* junctions connecting some airways to the atmosphere. The junctions of *c)* (intake portals and fan inlets) are named 'boundary nodes'. Ducting or diffusers from exhaust fans are omitted in this study because these have negligible resistances as compared to mine airways.

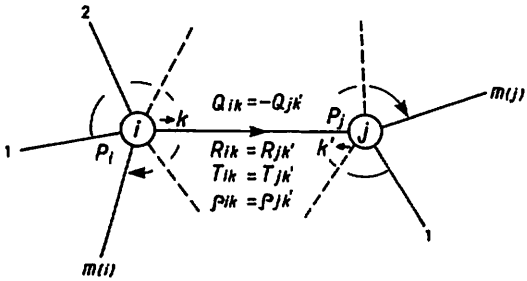


Fig. 1 Schematic diagram indicating connective relations between nodes

Consider a network consisting of a total number of nodes N and let a nodal number be i (=1~N). Figure 1 shows a schematic configuration representing airways connecting two nodes, i and j. The total number of airways connected to node i is represented by $m(i)$. The subscript k(=1~$m(i)$) is used to identify a connected airway or node. The nodal numbers connected to node i are defined using with a function denoted by $J(i,k)$. For example, when the nodal number of the kth node connected to node i is j (see Fig.1), it is defined as,

$$j = J(i,k) \tag{1}$$

Thus all connective relations between nodes are represented by $J(i,k)$. This concept is similar to the connectivity matrix in the finite-element method (Reddy 1984). The use of $J(i,k)$ is also more effective to save computer memory space as compared with the incidence matrix used in the mesh flow method. While only $2N\cdot m(i)_{av}$ of memory space is used in the case of $J(i,k)$, N^2 of memory space is required for the incidence matrix (Wang 1982). For N=100 and $m(i)_{av}$ = 5, the ratio $2N\cdot m(i)_{av}/N^2$ becomes 0.1. Furthermore, the authors have developed a relational data management system to assist the development of input data. This system follows the procedure developed by Ng (1989). This management system has convenient functions used to delete or add nodes and airways by reevaluating $J(i,k)$ and $m(i)$.

The nodal pressure at node i, p_i(Pa), is defined in this study as the static pressure induced by fans and natural ventilation not including atmospheric static pressure due to elevation difference and airflow dynamic pressure. This is because the elevation difference does not induce airflow and the dynamic pressure is considerably small as compared to the static pressure. Furthermore let the thermal natural ventilation pressure upon an airway from i to j be $\Delta\Phi_{i-j}$(Pa). Then $\Delta\Phi_{i-j}$ is given by an integral value along the airway as,

$$\Delta\Phi_{i-j} = -\int_{h_i}^{h_j} g\cdot[\ \rho(h,\Theta,P_{a0})-\rho_0(h,\Theta_0,P_{a0})]dh \tag{2}$$

where h(m) is the height from the datum level, $g(\mathrm{m/s^2})$ is the gravitational acceleration, $\rho(\mathrm{kg/m^3})$ is the air density at each airway and $\rho_0(\mathrm{kg/m^3})$ is the density of the atmosphere.

The total pressure difference, denoted by ΔH_{i-j} or ΔH_{ik} (Pa), to drive airflow from i to j is given by,

$$\Delta H_{i-j} = \Delta H_{ik} = p_i - p_j + \Delta\Phi_{i-j} \tag{3}$$

If the differences in level and density are given by $\Delta h_{ik}(=h_i-h_j)$ and $\Delta\rho_{ik}$, respectively, then $\Delta\Phi_{ik}(=\Delta\Phi_{i-j})$ can be approximated as,

$$\Delta\Phi_{ik} = g\cdot(\rho_{ik}-\rho_{0ik})(h_i - h_{J(i,k)}) = g\cdot\Delta\rho_{ik}.\Delta h_{ik} \tag{4}$$

where $\rho_{ik}(\mathrm{kg/m^3})$ is the average airflow density upon the airway $(i\to J(i,k))$ and ρ_{0ik} $(\mathrm{kg/m^3})$ is the atmospheric density at h_{ik} $[=(h_i+h_j)/2]$. The airflow quantity, Q_{ik} $(\mathrm{m^3/s})$, is given by

$$\begin{aligned} Q_{ik} &= \delta_{ik}|\Delta H_{ik}/R_{ik}|^{1/n} \\ &= \delta_{ik}|p_i-p_{J(i,k)}+g\cdot\Delta\rho_{ik}.\Delta h_{ik}|^{1/n} \end{aligned} \tag{5}$$

where $R_{ik}(\mathrm{Pa/(m^3/s)^n})$ is the resistance of the airway, and δ_{ik} is the sign expressing the flow direction as follows:

$$\delta_{ik} = \begin{cases} +1 : \Delta H_{ik} \geq 0 \\ -1 : \Delta H_{ik} < 0 \end{cases} \tag{6}$$

Thus Q_{ik} is positive when the airflow is directed from i to $j=J(i,k)$. The index n in Eq.(5) is usually equal to 2 for fully developed turbulent flow (n=2 was used in this study).

In addition, $\Delta\rho_{ik}$ in Eq.(5) is given by,

$$\Delta\rho_{ik} = 1.293\ \frac{P_{a0}\cdot\exp(-h_{ik}/8620)}{101.3} \times\frac{\Theta_{ik}-\Theta_0}{273+(\Theta_{ik}+\Theta_0)/2} \tag{7}$$

where Θ(°C) is the airflow temperature, Θ_0(°C) is the atmospheric temperature and P_{a0}(kPa) is the absolute pressure at the datum elevation (= 101.3 kPa at h_{ik}=0 m).

2.2 Linear equations for correction factors of nodal pressure

The linearized flow admittance, $T_{ik}(\mathrm{m^3/s\cdot Pa})$, between ΔH_i and Q_{ik} is defined as,

$$T_{ik} = (R_{ik}^{1/n}\cdot|\Delta H_{ik}|^{(n-1)/n})^{-1} \tag{8}$$

Equation(5) can thus be rewritten as,

$$Q_{ik} = T_{ik}\cdot\Delta H_{ik} = T_{ik}\cdot(p_i-p_{J(i,k)}+g\cdot\Delta\rho_{ik}.\Delta h_{ik}) \tag{9}$$

This equation provides the approximated linear relationship between Q_{ik} and ΔH_{ik}. When $|\Delta H_{ik}|$ in Eq.(8) is less than 10^{-10}(Pa), the calculation is treated as $|\Delta H_{ik}|=10^{-10}$(Pa) for the sake of the computer implementation.

According to the Kirchhoff's 1st law, the sum of the flows, Q_{ik}, flowed into node i, provided by Eq.(9), is given by,

$$q_i = \sum_{k=1}^{m(i)} Q_{ik} = (\sum_{k=1}^{m(i)} T_{ik})\cdot p_i - \sum_{k=1}^{m(i)}(T_{ik}\cdot p_{J(i,k)}) + \sum_{k=1}^{m(i)}(g\cdot\Delta\rho_{ik}.\Delta h_{ik}) \tag{10}$$

where $q_i(\mathrm{m^3/s})$ is the air quantity supplied into the node i from external air-sources such as a compressed air line which is independent of airflow from intakes. In general, q_i is given as 0 on each node.

The improved nodal pressures of the $(M+1)$th iteration, $\{p_i\}^{M+1}$, are obtained from the Mth nodal pressures $\{p_i\}^M$, and the Mth correction factors $\{\Delta p_i\}^M$(Pa):

$$\{p_i\}^{M+1} = \{p_i\}^M + \{\Delta p_i\}^M \tag{11}$$

By substituting Eq.(11) into $\{p_i\}$ in Eq.(10), the following equation is obtained (Sasaki et al., 1990),

$$[A_{ij}]\{\Delta p_i\}^M = \{B_i\}^M \tag{12}$$

where the admittance matrix $[A_{ij}]$, of order $N\times N$, and the vector $\{B_i\}$, of order N, are expressed as,

$$\left.\begin{aligned} A_{ii} &= \sum_{k=1}^{m(i)} T_{ik} \quad (i=1\sim N) \\ A_{ij} &= -T_{ik} \quad (j=J(i,k)) \\ A_{ij} &= 0 \quad (j\neq J(i,k)) \\ B_i &= q_i - \sum_{k=1}^{m(i)} Q_{ik} \end{aligned}\right\} \tag{13}$$

Equation(12) expresses the simultaneous linear equations relating to $\{\Delta p_i\}^M$ as unknown values. If $\{p_i\}^M$ are treated as unknown values as suggested by Bhamidipati et al.(1985), a large amount of double precision variables are needed for the computer implementation (refer to Sasaki et al., 1990).

Assuming that $j=J(i,k)$, defined by Eq.(1), and concurrently that the k'th node connected to node j in opposite side is defined as $i = J(j,k')$ (see Fig. 1), then the relation between A_{ij} and A_{ji} is given as,

$$A_{ij} = A_{iJ(i,k)} = -T_{ik} = -T_{jk'} = A_{jJ(j,k')} = A_{ji} \quad (14)$$

Above Eq.(14) shows that $[A_{ij}]$ is a symmetric matrix (Rao 1987). The vector $\{B_i\}$ in Eqs.(13) is composed of the errors associated with the continuity of flow quantity at each node i after the Mth iterative calculation. Equation (12) takes the structure that $\{p_i\}^{M+1}$ converges, i.e. $\{\Delta p_i\}^M \approx \{0\}$, as the errors of flow quantity at whole nodes decrease.

Assume that the total number of the boundary nodes is ΔN and that the nodal number at one of the boundary nodes is τ. Since ΔN pieces of inflow and exhaust air-quantities, $q_\tau(m^3/s)$, at the boundary nodes are unknown, the ranking of Eq.(12) is N-ΔN. Thus $\{\Delta p_i\}^M$ cannot be obtained by directly solving Eq.(12). At these boundary nodes, however, the nodal pressures, $\{P_\tau\}^M$, are given and $\{\Delta p_\tau\}^M$ are set as 0, i.e.

$$\left.\begin{aligned} A_{\tau\tau} &= 1 \\ B_\tau &= 0 \\ A_{\tau j} &= 0 \quad (j \neq \tau) \end{aligned}\right\} \quad (15)$$

For example, the pressure at an intake-portal node is given as 0, and the pressure at a fan-inlet node is determined on the fan characteristic curve. The ranking of Eq.(12) increases by one, however, $[A_{ij}]$ in Eq.(12) is no longer a symmetric matrix. In order to convert it to a symmetric matrix, conversions of $[A_{ij}]$, given by,

$$A_{i\tau} = 0 \quad (i \neq \tau) \quad (16)$$

are used, and the result is expressed as follows,

$$\begin{array}{c} \qquad\qquad\quad \tau \\ \begin{bmatrix} A_{11} & \cdots & 0 & \cdots & A_{1N} \\ \cdot & \cdots & \cdots & \cdots & \cdot \\ \cdot & \cdots & 0 & \cdots & \cdot \\ 0 & \cdots 0 & 1 & 0 \cdots & 0 \\ \cdot & \cdots & 0 & \cdots & \cdot \\ \cdot & \cdots & \cdots & \cdots & \cdot \\ A_{N1} & \cdots & 0 & \cdots & A_{NN} \end{bmatrix} \end{array} \begin{Bmatrix} \Delta p_1 \\ \cdot \\ \Delta p_\tau \\ \cdot \\ \Delta p_N \end{Bmatrix} = \begin{Bmatrix} B_1 \\ \cdot \\ 0 \\ \cdot \\ B_N \end{Bmatrix} \quad (17)$$

If the all pressures of the boundary nodes are settled, the ranking of Eq.(17) should be N.

2.3 Skyline modified Choleski's decomposition scheme

In solving large-scale simultaneous linear equations using a personal computer, attention should be paid to calculation speed and effective use of memory space as com-pared with the conventional nodal method, because the direct matrix operation needs a comparably large memory space for the case of large-scale networks.

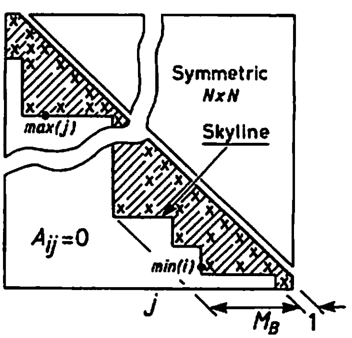


Fig. 2 Schematic diagram of the coefficient matrix [A] (x-mark in the figure representing the matrix elements of $A_{ij} \neq 0$)

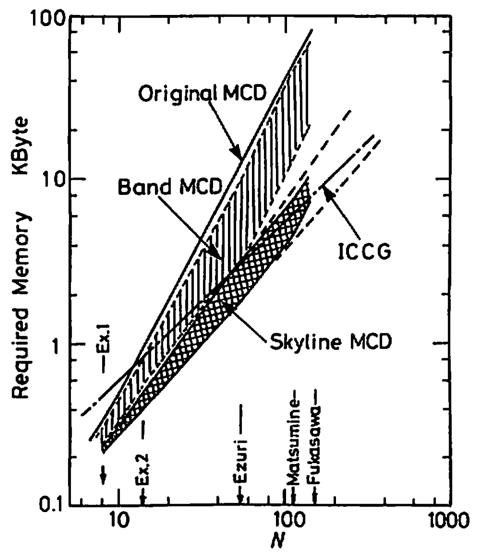


Fig. 3 Comparison of the required memory for decomposition (the lower boundary lines of skyline and band MCD shows the case of the optimal nodal numbers)

In general, since it is relatively easy with the finite-element method to make a half band width be small, only a band matrix with elements that are nonzero in the neighborhood of diagonal elements is considered to save computer memory (Collins, 1973). However in the case of ventilation networks, there is a possibility of an extreme case in which the number difference between connected nodes is almost N (see Table 1). When the continuous arrayed memory is restricted within an extent of memory space, the termination and the numbering of nodes are considered not to exceed the limited width.

Four decomposition schemes of the symmetric admittance matrix were compared in this study. Furthermore the calculations of the decomposition are restricted to the regime containing nonzero elements in $[A_{ij}]$, as illustrated in Fig. 2. The minimum column number of each row, $j_{min}(i)$, and the maximum row number of each column, $i_{max}(j)$, are defined to express the boundary elements of the nonzero regime. For the case of Fukasawa Mine (N=141, Fig. 4(d)), the occupied area ratio of the nonzero regime is approximately 17% in the whole $[A_{ij}]$, and it should be smaller for a case of larger N. Thus, calculation speed is particularly enhanced by restricting numerical calculations to the regime in the under triangular matrix represented by,

$$A_{ij} \; ; \quad i = j \sim i_{max}(j), \; j = j_{min}(i) \sim i \tag{18}$$

This method is usually called as the skyline method or the wave front method (Melosh & Bamford 1967). The one dimensional arrayed memory for the nonzero regime, expressed by Eq.(18), is used due to the economy of memory space. Figure 3 shows a comparison of the memory quantity required for the matrix operation as function of N in each scheme. The skyline MCD scheme is favorable for N < 500.

2.4 Fan characteristics

When a main fan characteristic exhibiting the relation between the air quantity Q_f (m^3/s) and the fan-inlet static pressure P_f(>0, Pa) at a standard rotational speed n_0(rpm) is approximated with a Γth degree polynomial, obtained by the least squares method, the relation is provided as,

$$P_f(Q_f, n_0) = \sum_{\gamma=0}^{\Gamma} C_\gamma \cdot Q_f^\gamma \tag{19}$$

where C_γ($Pa \cdot s/m^3$) is the γth order coefficient. Since a fan can be controlled by changing its rotational speed, P_f on a driving rotational speed n_d(rpm) is given as

$$P_f(Q_f, n_d) = \beta^2 \cdot \sum_{\gamma=0}^{\Gamma} C_\gamma \cdot (Q_f/\beta)^\gamma \tag{20}$$

where, $\beta = n_d/n_0$ (Γ=3 was used in this study).

2.5 Initial values of nodal pressures

The initial values of the whole nodal pressures, defined as $\{p_i\}^0$, were given as a same constant pressure,

$$p_i = -K_0 \cdot P_f(0, n_d) \qquad (i \neq \tau) \tag{21}$$

where K_0 is an initial value factor and $P_f(0, n_d)$ is the fan inlet pressure at Q_f =0, obtained from Eq.(20). On the other hand, initial values upon boundary nodes are given by,

$$\left. \begin{array}{ll} p_{\tau 0} = -P_f(0, n_d) & \text{at fan-inlet nodes} \\ p_{\tau 0} = 0 & \text{at intake-portal nodes} \end{array} \right\} \tag{22}$$

2.6 Improvements of fan pressure

The flow quantity Q_f and the static pressure P_f at the fan inlet are improved in accordance with the following method.

With the flow quantity Q_f^M calculated from the Mth iterative nodal pressures $\{p_i\}^M$ and fan inlet pressure P_f^M, the equivalent admittance of flow T_f^M between Q_f^M and P_f^M is defined as,

$$T_f^M = P_f^M / Q_f^M \tag{23}$$

When the improved Q_f is expressed as Q_f^*, the $P_f(Q_f^*, n_d)$ determined from the Eq.(20) is equal to the total pressure loss of the network $T_f^M \cdot Q_f^*$. Thus an algebraic equation concerning Q_f^* is obtained as follows:

$$T_f^M \cdot Q_f^* - \beta^2 \cdot \sum_{\gamma=0}^{\Gamma} C_\gamma \cdot (Q_f^*/\beta)^\gamma = 0 \tag{24}$$

The Q_f^* is determined by Newton-Raphson's method. Finally, the $(M+1)$th improved P_f^{M+1} can be obtained by substituting Q_f^* to Eq.(20). The convergence of P_f synchronizes with the convergence of $\{p_i\}$. The nodal pressure p_τ^{M+1} at the fan-inlet node τ for the $(M+1)$th iteration is given as,

$$p_\tau^{M+1} = -P_f^{M+1} \tag{25}$$

In the case that a principal fan is

installed in the network, the whole nodal pressures, p_i^M, are corrected by,

$$p_i^{(M+1/2)} = p_i^M \cdot P_f^{M+1} / P_f^M \quad (i \neq \tau) \qquad (26)$$

where $p_i^{(M+1/2)}$ is the adjusted nodal pressure and becomes the approximated value for the sake of the $(M+1)$th iteration. Equation (26) is a simple method to adjust the nodal pressures due to the influence of the fan pressure with an assumption that the flow between an individual nodal pressure and the fan node pressure is approximately constant.

Booster fans installed in series to provide the required air quantity are treated by the conventional method used by Sasaki et al. (1990). Consider the approximate resistance based on pressure loss near the booster fan, predicted from its pressure ΔP_f(Pa) and flow quantity ΔQ_f (m^3/sec), defined as,

$$\Delta R_f = \Delta P_f / (\Delta Q_f)^2 \qquad (27)$$

Then ΔR_f and $\pm\Delta P_f$ are added to the terms of R_{ik} and ΔH_{ik} respectively. The sign of ΔP_f is decided taking into consideration of the flow direction. The airway flow rate with the booster fan whose flow direction is set as $i \to j$, is given by,

$$\begin{aligned} Q_{ik} &= \delta_{ik} \left| \frac{\Delta H_{ik} + \Delta P_f}{R_{ik} + \Delta R_f} \right|^{1/n} \quad ; \ j = J(i,k) \\ Q_{jk'} &= \delta_{ik} \left| \frac{\Delta H_{jk'} - \Delta P_f}{R_{jk'} + \Delta R_f} \right|^{1/n} \quad ; \ i = J(j,k') \end{aligned} \qquad (28)$$

2.7 Convergence acceleration factor

In order to accelerate the convergence, an acceleration factor ω is effective(Inoue et al., 1989). The Mth solution $\{p_i\}^M$ is given using ω as follows,

$$\{p_i\}^M = \{p_i\}^{M-1} + \omega\{\Delta p_i\}^{M-1} \qquad (29)$$

However if ω is too large, the convergent process becomes unstable. Taking into account such a situation, the optimum value of ω was decided by preliminary calculations. The fastest convergence has been obtained with most of the networks dealt with in this study, when ω was set at 1.4 (refer to §3.1).

2.8 Convergence criterion

The calculation processing time until convergence is varied depending on the convergence criterion used. In this study the error quantity in continuity upon individual nodes, denoted as ΔQ_i(m^3/s), was calculated as (Bhamidipati(1985) and Sasaki et al.(1990)),

$$\Delta Q_i = q_i - \sum_{k=1}^{n(i)} Q_{ik} \qquad (30)$$

The maximum absolute value of ΔQ_i denoted by $|\Delta Q_i|_{max}$ could be regarded as a reasonable convergence criterion. The minimum limit of $|\Delta Q_i|_{max}$ used to verify convergence should be set in accordance with the amount of total inflow quantity in each network. However, in this study a fixed convergence criterion

$$60 \times |\Delta Q_i|_{max} < 0.1 \text{ m}^3/\text{min} \qquad (31)$$

was applied to all networks, and the number of iterations until satisfying Eq.(31), denoted by M_c, was obtained. The criterion of Eq.(31) is believed to be a relatively strict one compared with 6 m^3/min used by Bhamidipati et al.(1985).

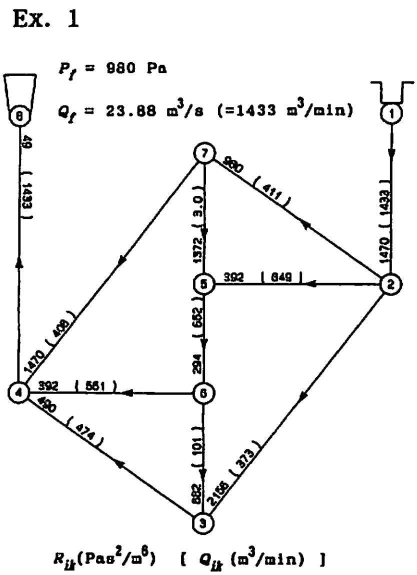
2.9 Procedures of iterative calculations

The solution can be obtained by the following iterative calculation procedure:

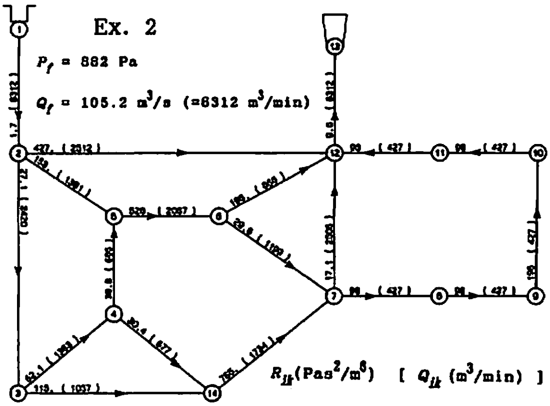
1) Read the network data and fan characteristics, and calculate natural ventilation pressures, $\Delta\Phi_{ik}$.
2) Set the initial pressures, $\{p_i\}^0$, using Eqs.(21) and (22).
3) Find the fan operating point Q_f and P_f by Eqs.(24). (in case of one main fan, adjust p_i using Eq.(26))
4) Calculate T_{ik} and derive $[A]$ and $\{B\}$ using Eqs. (8) & (13).
5) Set the boundary nodal pressures and adjust $[A]$ and $\{B\}$ using Eqs.(15) & (16) as Eq.(17).
6) Compute the solution $\{\Delta p_i\}$ using a decomposition scheme and correct $\{p_i\}$ by Eq.(29), Q_{ik} using Eq.(9) and ΔQ_i using Eq.(30).
7) Iterate calculations in 3)~ 6) until the solution satisfies the convergence criterion in Eq.(31).
8) Save or print out the solutions (p_i and Q_{ik}).

2.10 Examples of ventilation networks

Two illustrative networks and three actual underground ventilation networks, which are trackless copper mines (the Ezuri Mine, the Matsumine Mine and the Fukasawa Mine) run by the Hanaoka Mining Co. LTD., were developed to evaluate the developed method. Fig. 4(a)-(e) represent the ventilation networks



(a)



(b)

analyzed. Placement of nodes in the actual networks was determined in consideration of airway junctions and types of the airways. The nodal numbers were manually given. Table 1 gives the total number of nodes N, the total number of airways $L(=\Sigma m(i)/2)$, the half band width M_B (see Fig. 2), the average number difference between connected nodes (denoted by ΔI), the fan characteristics and air quantities for the developed examples. The ΔI is defined as follows:

$$\Delta I = \sum_{i=1}^{N} \left\{ \sum_{k=1}^{m(i)} |i-J(i,k)| \right\} \Big/ \sum_{i=1}^{N} m(i) \qquad (32)$$

In the case of the skyline MCD and ICCG schemes, the calculation speed is affected by ΔI. The networks indicated by '*' in Table 1 represent the cases with optimal distribution of nodal numbers to reduce ΔI and the half band width by means of the optimization method (Sasaki et al.,1990).

The characteristic curves of the surface main fans are illustrated in Fig. 5. With respect to the analyses of the three actual networks, the airflow temperatures upon individual airways are incorporated in the calculation as input data and the term $\Delta\Phi_{ik}$ was considered in the numerical calculations. The airflow temperatures(θ_{ik}) were determined using a forecast analytical system for temperature and humidity developed by authors (see Sasaki et al. 1992).

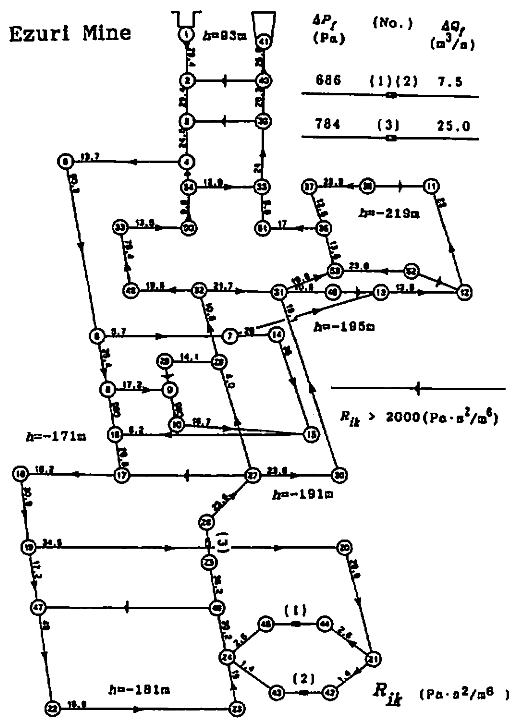


Fig. 4(c)

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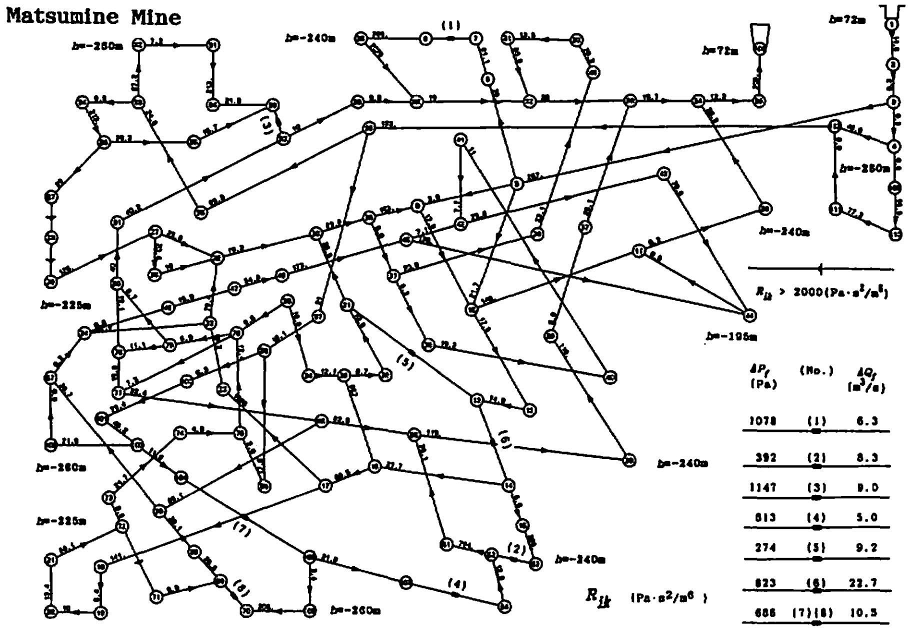
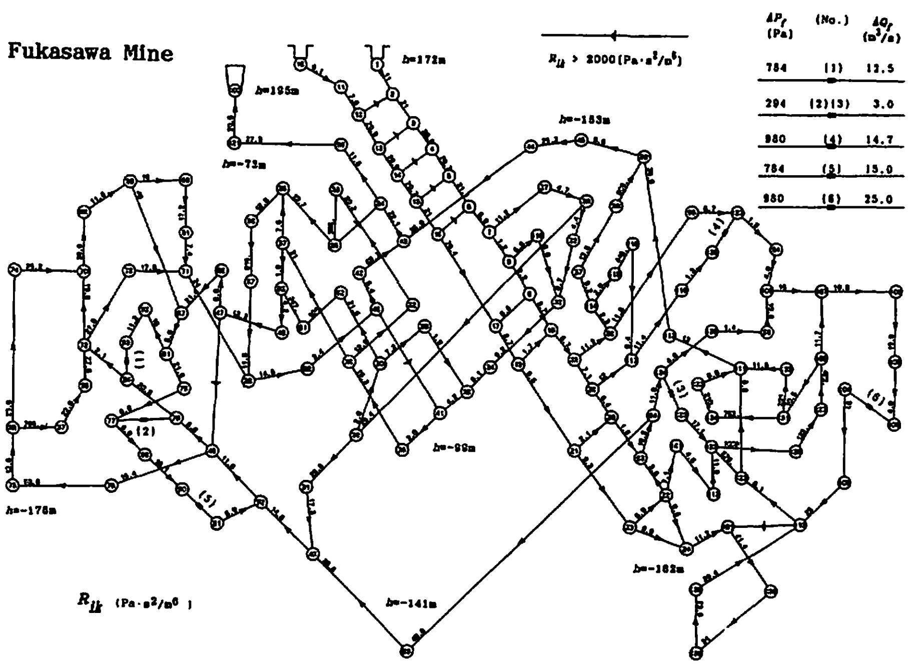


(d)



(e)

Fig. 4 Developed ventilation networks, a)Example 1 (N=8), b)Example 2 (N=14), c)The Ezuri Mine (N=51), d)The Matsumine Mine (N=112), e)The Fukasawa Mine(N=141)
(The symbol of an arrow in a box shows booster fans)

Table 1 Characteristics of the ventilation networks

Network	N	L	M_s+1	ΔI	P_f	$60 \cdot Q_f$
	-	-	-	-	Pa	m^3/min
Example 1	8	11	6	3.3	980	1433
*			4	1.7		
Example 2	14	19	12	3.4	882	6312
*			7	2.2		
Ezuri Mine	51	67	41	9.6	≈880	≈3900
*			11	2.9		
Matsumine Mine	112	137	109	8.4	≈1580	≈4900
*			26	6.5		
Fukasawa Mine	141	179	120	13.0	≈910	≈5200
*			35	5.2		

(* ; the cases with the optimized nodal numbers)

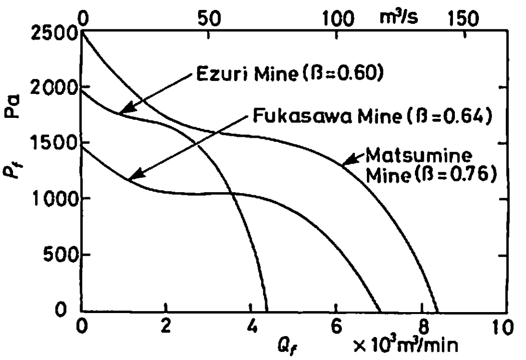


Fig. 5 Characteristics of the main surface fans in the three actual mine networks

3 RESULTS AND DISCUSSIONS

The solutions of Q_{lk} for Ex. 1 and Ex. 2 are showed in Figs. 4(a) and 4(b). The calculated airflow rate agree well with measured values in the actual networks within ±18% of the total intake airflow-quantities. It should be noted that the uncertainty of the measurements was estimated to be more than 20% because the mines were running at the time of measurements.

In this study, calculations were performed using the matrix nodal method and a personal computer. The program was written in the Microsoft BASIC Ver.7.1™.

3.1 Convergence acceleration and initial-value factors

Figure 6 shows the value of $|\Delta Q_i|_{max}$ and the maximum absolute value of Δp_i, $|\Delta p_i|_{max}$, plotted against respective iterations of Mth degree for the Fukasawa Mine. It is noticed that both $|\Delta p_i|_{max}$ and $|\Delta Q_i|_{max}$ are reduced exponentially with M. This means that the calculation system is reasonable and its convergence process is very stable. In addition when the convergence criterion expressed by Eq.(31) is satisfied, $|\Delta p_i|_{max}$, is in the range of 10^{-3}~10^{-2}(Pa).

The effects of ω and K_0 are shown in Figs. 7 and 8 respectively. M_c decreases as ω becomes larger as long as the value of ω remains less than 1.4. It is also noticed that the solutions converge with almost the same M_c for any K_0 given in the range of 0.1~0.9 at ω=1.4. Thus the convergence process is almost not influenced by K_0.

When ω=1.4, K_0=0.3 and the fan pressure is given as a constant value, M_c is no more than 13 for each of the networks analyzed.

3.2 Effect of the fan characteristics on the number of iterations

Table 2 shows the variation of M_c for the following conditions

i) the fan pressure P_f is kept constant

ii) the fan characteristic with the operation point unsettled is given as $P_f(Q_f)$

iii) the individual nodal pressure is adjusted using Eq.(26) simultaneously with the $P_f(Q_f)$ in case of *ii)*.

In case *i)*, the convergence has been completed with M_c≤13. However in case *ii)*,

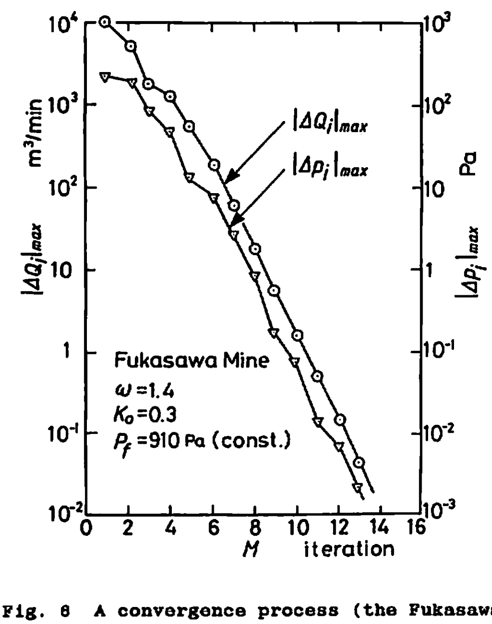


Fig. 6 A convergence process (the Fukasawa Mine)

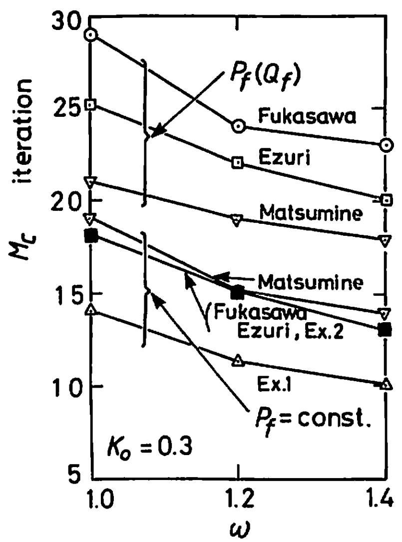


Fig. 7 Effects of the convergence acceleration factor ω on the number of iterative calculations until convergence (M_c)

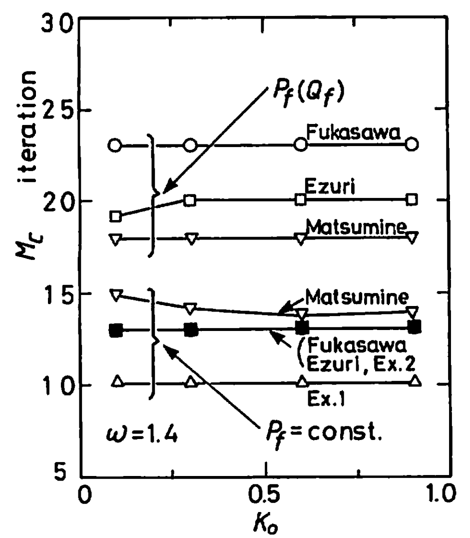


Fig. 8 Effects of the initial set factor K_0 on the number of iterative calculations until convergence (M_c)

Table 2 Effects exerted by the characteristics of fans and Eq.(26) on the number of the iterations M_c (original MCD, band MCD and skyline MCD schemes)

Network	P_f =const.	$P_f(Q_f)$ (Fig. 5)	
		-	use Eq.(26)
Example 1	10	-	-
Example 2	13	-	-
Ezuri Mine	13	56	18
Matsumine Mine	13	45	15
Fukasawa Mine	13	40	17

(unit; iteration)

Table 3 Comparisons of computer processing time an iteration, Δt_{CPU}(sec/iteration), among four decomposition schemes

Network	Original MCD	Band MCD	Skyline MCD	ICCG
Ezuri Mine	2.21	2.39	2.03	(3.6)
*	2.21	0.68	0.38	(2.7)
Matsumine Mine	16.9	21.4	2.03	(12)
*	16.9	4.79	1.47	(9.5)
Fukasawa Mine	32.7	27.6	3.50	(13)
*	32.7	8.10	1.83	(11)

[* ; the cases with optimized nodal numbers, ( ); the average time in the ICCG scheme.]

applied to the actual networks, M_c increased to 40~56. This was due to the fact that extra iterations were necessary to make the effect of the adjusted P_f distribute throughout the network nodes. In case *iii)*, M_c is reduced to 17~18, less than the half of ii), by adjusting the effect of P_f upon the individual nodes utilizing Eq.(26). The adjusting method using Eq.(26) is a simplified one, but it is effective in reducing the iterations for a network with a main fan.

3.3 Calculation processing time

The reducing process of $|\Delta Q_i|_{max}$ against M is almost independent from the type of hardware and programming language used. However, in order to distinguish the merits of the actual system, the calculation processing time is practically important.

Table 3 shows a comparison of the calculation processing time required for an iteration Δt_{CPU} (sec) by four decomposition schemes. The results of Δt_{CPU} for cases with optimized nodal numbers are also shown for comparison. The calculation by the skyline MCD is the fastest among the four schemes for every network. When the skyline MCD was used, the solutions could be gained with Δt_{CPU} = 1.8~3.5(sec/iteration) even in the case of Fukasawa Mine(N=141). In the case of the fan characteristics given as $P_f(Q_f)$, as shown in Fig. 6, the whole calculation time until convergence is 31~61(sec), which is obtained by multiplying M_c(=17) by Δt_{CPU}.

The difference of assigning optimizing nodal numbers is expressed as the difference of the calculation time. In the case of Fukasawa mine, the calculation time for optimized nodal numbers is almost 1/2 of that with manually numbering (see Table 3). Accordingly, with a large scale network of almost N=500, it is expected that the overall calculation time becomes smaller by incorporating a routine of automatically reassigning nodal numbers as in the case of finite-element methods.

Figure 9 shows Δt_{CPU} plotted against the total number of nodes N. The difference in Δt_{CPU} between the skyline MCD and other schemes becomes larger as N increases. In the case of the original MCD or the band MCD, Δt_{CPU} is almost multiplied in proportion to N^2. In the case of the skyline MCD, Δt_{CPU} is almost multiplied in proportion to $N^{3/2}$. For the skyline MCD, Δt_{CPU} is believed to be proportional to N as long as ΔI does not change. However, ΔI unavoidably becomes larger as the connections between nodes become more complicated or with the increase of N (see Table 1). Thus, Δt_{CPU} increases with increasing values of N.

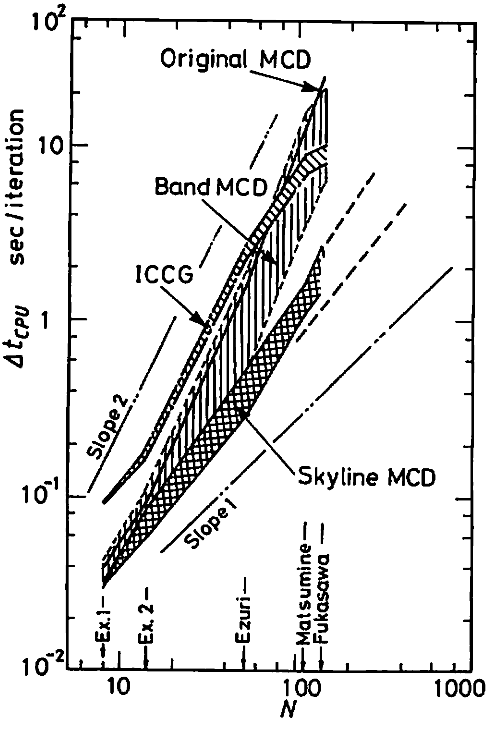


Fig. 9 Relationship between the total number of nodes N and calculation time an iteration, Δt_{CPU}.

4 CONCLUSIONS

The nodal pressure method has been utilized with some success to predict ventilation network flow distribution. The calculation system of the linear equations for the correction of nodal pressures as unknown variables was described. By introducing the connective function between nodes (Eq.(1)), the fundamental equations for the networks were expressed in a more general form, and the calculation schemes were explained. The use of the connective function is effective to save computer memory as compared with the case of incident matrix used in the mesh flow method.

The influence exerted by the different decomposition schemes, the total number of nodes, fan characteristics and optimization of nodal numbers upon the number of iterations until convergence, M_c, has been investigated. Calculations were made with two simple illustrative networks and three actual mine networks as examples of analysis. From the comparative investigations, the skyline modified Choleski's decomposition scheme has advantages on calculation speed and memory space as compared with the other schemes. A condition that the maximum absolute error in continuity of airflow quantity upon nodes is less than 0.1m^3/min was employed as the convergence criterion.

Convergence was achieved with $M_c \le 13$ for the condition of constant fan pressure with a convergence acceleration factor of 1.4. In case of the skyline decomposition scheme, the relationship between the calculation time an iteration Δt_{CPU} and the total number of nodes N is $\Delta t_{CPU} \propto N^{3/2}$ for the examples dealt with in this study.

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