

A Probabilistic Method to Determine Whether the Speed of Light Is Constant

Motohisa Osaka

Department of Basic Science, Nippon Veterinary and Life Science University, Tokyo, Japan

Email: osaka@nms.ac.jp

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Abstract

Although the formula of mass-energy equivalence was derived from the hypothesis that the speed of light in free space is constant, conversely, the purpose of this research is to show that a method of probabilistically determining whether the speed of light is constant is derived from this formula. By considering the formula of mass-energy equivalence to be a function of the energy of an object moving at speed V , the probability density function (PDF) of the energy can be obtained using the inverse function of this formula, if the speed of light obeys a probability distribution. The main result is that the PDF of the energy diverges to infinity at a certain energy value regardless of the PDF of the speed of light. Thus, when the speed calculated from this value enters a certain range of the speed of light as V increases stepwise from below 299,792,458 m/s, the PDF of the energy should increase abruptly. If not, then the speed of light is constant. This is the method of probabilistically determining whether the speed of light is constant. An experimental method is proposed to confirm this.

Keywords

Special Relativity, Light Speed, Mass-Energy Equivalence

1. Introduction

Albert Einstein published the theory of special relativity in 1905 [1]. The theory is on the relationship between space and time. One of its results is mass-energy equivalence: $E = mc^2$, where E is the energy of an object when it is moving, m is its mass while moving and c is the speed of light. This is derived from two hypotheses. One is that the speed of light in free space is constant for all observers, regardless of their relative motion or of the motion of the light source. This hypothesis is generally considered verified by the Michelson-Morley experiment,

which shows the differences between the speed of light in the direction of motion of the earth and that in different directions are within experimental errors [2]. This was supported by similar experiments with higher resolutions [3] [4]. However, these experiments do not show that the speed of light in free space is constant in all inertial systems. A team at University of Glasgow reported that photon group velocity was reduced using time-correlated photon pairs and that the delay was several micrometers over a propagation distance of the order of 1 m [5]. Although they showed that adding spatial structure to an optical beam of single photons reduced the speed of light, the significance of their study was considered to be limited. The findings do not affect the formula of energy-mass equivalence because c in the formula is still regarded as the maximum speed of all objects in free space. However, the question is whether the maximum speed is constant. This study presents a probabilistic method derived from $E = mc^2$ to determine whether the speed of light is constant.

2. Mathematical Steps

It assumes that the speed of light obeys a probability distribution.

The formula $E = mc^2$ is also expressed as

$$E = \frac{m_0}{\sqrt{1 - \left(\frac{V}{c}\right)^2}} c^2, \quad (1)$$

where m_0 is the rest mass of the object, and V is its speed; $V < c$.

The assumptions are:

- 1) m_0 and V are constant.
- 2) c obeys a probability distribution between c_a and c_b ; $c_a < c_b$. Two probability distributions are adopted: a uniform distribution and a triangular distribution.

Then, the probability density function (PDF) of E is calculated by obtaining the inverse function of E as follows.

Step 1: Determining the inverse function of E

As $0 < V/c < 1$, V/c is defined by $\sin\theta$ ($0 < \theta < \pi/2$).

From (1),

$$E = \frac{m_0 V^2}{\cos\theta - \cos^3\theta}. \quad (2)$$

Then, $\cos\theta$ is represented as y :

$$c = \frac{V}{\sqrt{1 - y^2}}. \quad (3)$$

Representing $m_0 V^2$ as P_0 , the following third-order equation for y is obtained from (2):

$$f(y) = y^3 - y + \frac{P_0}{E} = 0. \quad (4)$$

A positive value of E results from any value of θ on the interval $0 < \theta < \pi/2$. In

other words, at least one of the three roots of $f(y)$ is between 0 and 1. As the roots of $f(y)$ are intersections of $g(y) = y^3 - y = y(1-y)(1+y)$ and $h(y) = -P_0/E$, all three roots are real. Two of them are between 0 and 1 and the remaining root is negative. The two roots between 0 and 1 are denoted as y_1 and y_2 ; $y_1 \leq y_2$. As $0 < y < 1$, the negative root is neglected. Since y_1 and y_2 are determined by E , it is necessary to find out whether the inverse function of E , $c \equiv h(E)$, is a two-valued or single-valued function.

Step 2: Determining whether E is a two-valued or single-valued function

From (1), E has the minimum $\frac{3\sqrt{3}}{2}P_0$ ($P_0 = m_0 V^2$) at $c_m \equiv \frac{\sqrt{6}}{2}V$. Three cases are examined on the basis of whether c_m is between c_a and c_b .

Case 1: $c_m \leq c_a$

Then $V \leq \frac{\sqrt{6}}{3}C_a$. As E increases monotonically between c_a and c_b ,

$$\frac{V}{\sqrt{1-y_1^2}} < c_a \quad \text{and} \quad c_a \leq \frac{V}{\sqrt{1-y_2^2}} \leq c_b. \quad (5)$$

Therefore, only y_2 is accepted. In this case $c = h(E)$ is a single-valued function (Figure 1(a)).

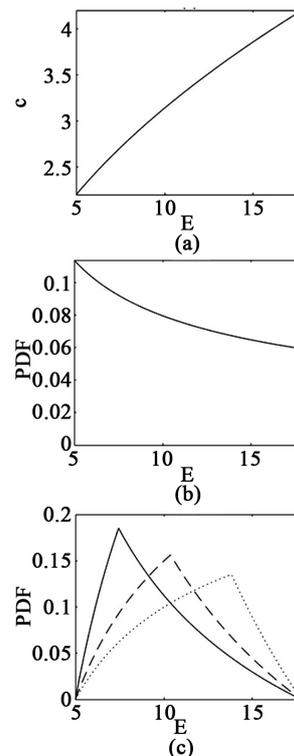


Figure 1. Case of $c_m \leq c_a$. The object speed and limits of c are set to $V = 0.5$, $c_a = 2.2$ and $c_b = 4.2$. Then, $c_m \approx 0.6124$. (a) Inverse function of E for c ; (b) Probability distribution of E when c obeys the uniform distribution; (c) Probability distribution of E for three cases: the c -value of the vertex of the triangular probability distribution of c is 2.7 (solid line), 3.2 (dashed line) or 3.7 (dotted line). The probability distribution of E is triangle-like with the vertex moving rightward with the probability distribution of c .

Case 2: $c_a < c_m < c_b$

Then $\frac{\sqrt{6}}{3}C_a < V < \frac{\sqrt{6}}{3}C_b$. As E is parabolic and downward convex between c_a and c_b , the following relationship occurs:

$$c_a < \frac{V}{\sqrt{1-y_1^2}} < \frac{V}{\sqrt{1-y_2^2}} < c_b. \tag{6}$$

Both y_1 and y_2 are accepted and $c = h(E)$ is a two-valued function (**Figure 2(a)**). Otherwise, the following occurs:

$$\frac{V}{\sqrt{1-y_1^2}} < c_a \text{ or } c_b < \frac{V}{\sqrt{1-y_2^2}}. \tag{7}$$

Then $c = h(E)$ is single-valued function (only y_2 is accepted in **Figure 2(a)**).

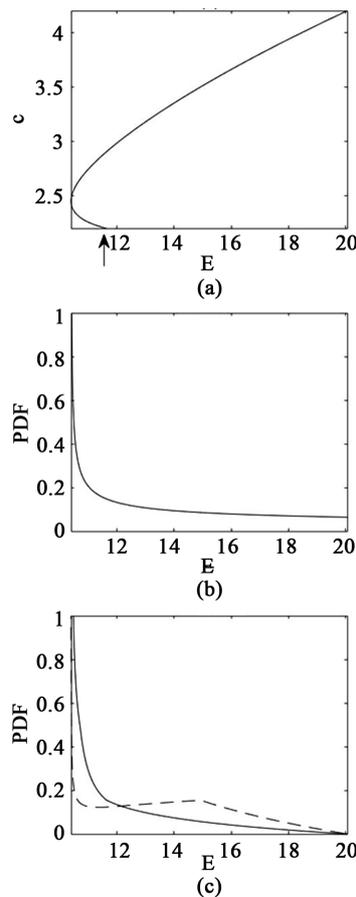


Figure 2. Case of $c_a < c_m < c_b$. The object speed and limits of c are set to $V = 2$, $c_a = 2.2$ and $c_b = 4.2$. Then, $c_m \approx 2.4995$. (a) The inverse function of E is a two-valued function between minimum E and the value marked with an arrow and a one-valued function between the marked value and maximum E ; (b) Probability distribution of E when c obeys the uniform distribution; (c) Probability distribution of E for two cases: the c -value of the vertex of the triangular probability distribution of c is less than c_m ($2.3 < c_m$) (solid line), and larger than c_m ($3.7 > c_m$), (dashed line). In the former, the PDF of E decreases monotonically. In the latter, it has a vertex. If $c_a < c_m < c_b$, the probability density of E is maximum at minimum E in either distribution of c .

Case 3: $c_m \geq c_b$

Then $V \geq \frac{\sqrt{6}}{3}C_b$. As E decreases monotonically between c_a and c_b ,

$$c_a \leq \frac{V}{\sqrt{1-y_1^2}} \leq c_b \quad \text{and} \quad c_b < \frac{V}{\sqrt{1-y_2^2}}. \quad (8)$$

Therefore, only y_1 is accepted. In this case $c = h(E)$ is also a single-valued function (**Figure 3(a)**).

Step 3: Calculation of PDF of E

From the PDF of c , $f_c(c)$, and the inverse function of E , $c = h(E)$, the PDF of the random variable E , $f_E(E)$, can be obtained as

$$f_E(E) = f_c[h(E)] \left| \frac{d}{dE} h(E) \right| \quad (9)$$

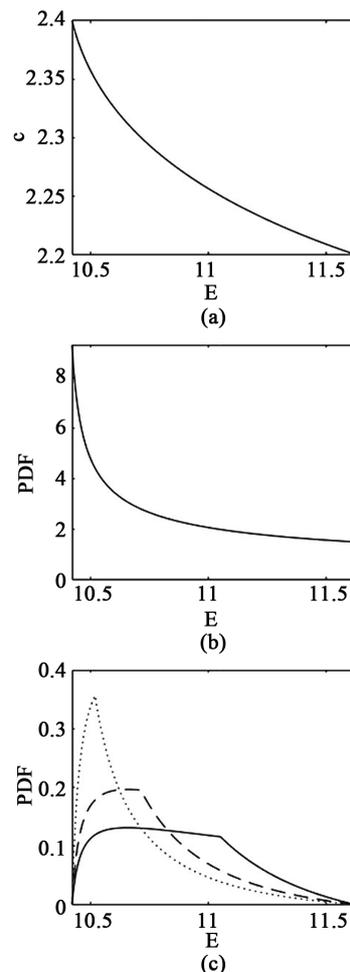


Figure 3. Case of $c_m \geq c_b$. The object speed and limits of c are set to $V = 2$, $c_a = 2.2$ and $c_b = 2.4$. Then, $c_m \approx 2.4995$. (a) Inverse function of E for c ; (b) Probability distribution of E when c obeys the uniform distribution; (c) Probability distribution of E for three cases: the c -value of the vertex of the triangular probability distribution of c is 2.25 (solid line), 2.3 (dashed line) or 2.35 (dotted line). The probability distribution of E changes from trapezoid-like shape to triangle-like shape as the vertex of the probability distribution of c moves rightward.

where $\left| \frac{d}{dE} h(E) \right|$ is the Jacobian of the transformation. The absolute value should be taken since the PDF must be positive:

$$\left| \frac{dc}{dE} \right| = \left| \frac{d}{dE} h(E) \right| = \left| \frac{dc}{d\theta} \right| \left| \frac{d\theta}{dE} \right| \tag{10}$$

From $c = V/\sin\theta$ and $y = \cos\theta$,

$$\left| \frac{dc}{d\theta} \right| = \sqrt{\frac{P_0}{m_0}} \left| -\frac{\cos\theta}{1 - \cos^2\theta} \right| = \sqrt{\frac{P_0}{m_0}} \frac{y}{1 - y^2}. \tag{11}$$

From (2) and $y = \cos\theta$,

$$\left| \frac{d\theta}{dE} \right| = \frac{P_0}{E^2} \frac{1}{\sin\theta |1 - 3\cos^2\theta|} = \frac{P_0}{E^2} \frac{1}{\sqrt{1 - y^2} |1 - 3y^2|}. \tag{12}$$

From (10), (11) and (12),

$$\left| \frac{dc}{dE} \right| = \left| \frac{d}{dE} h(E) \right| = \frac{P_0 \sqrt{P_0}}{\sqrt{m_0}} \frac{y}{(1 - y^2) \sqrt{1 - y^2} |1 - 3y^2|} \frac{1}{E^2}. \tag{13}$$

From (9) and (13), the PDF of E can be obtained:

$$\begin{aligned} f_E(E) &= f_c[h(E)] \left| \frac{d}{dE} h(E) \right| \\ &= f_c[h(E)] \frac{P_0 \sqrt{P_0}}{\sqrt{m_0}} \frac{y}{(1 - y^2) \sqrt{1 - y^2} |1 - 3y^2|} \frac{1}{E^2} \end{aligned} \tag{14}$$

As $0 < y < 1$, $f_E(E)$ diverges to infinity at $y = \frac{1}{\sqrt{3}}$. From (3), $c = \frac{\sqrt{6}}{2} V$. This value of c is equal to c_m . Then $E = \sqrt{3} m_0 c_m^2$.

Step 4: Setting of parameters

As the value of m_0 has no qualitative effect on the relationship between E and c , m_0 is set equal to 1. The values of V , c_a and c_b are set arbitrarily, depending on whether c_m is within the range of c : $c_a \leq c \leq c_b$.

3. Results

Case 1: $c_m \leq c_a$

The object speed and limits of c are set to $V = 0.5$, $c_a = 2.2$ and $c_b = 4.2$. Then $c_m \approx 0.6124$. **Figure 1(a)** shows the relationship between E and c . The inverse function of E is a single-valued monotonically increasing function. **Figure 1(b)** shows the PDF of E , which decreases monotonically when c obeys the uniform distribution. **Figure 1(c)** shows three cases: the c -value of the vertex of the triangular probability distribution of c is 2.7, 3.2 or 3.7.

Case 2: $c_a < c_m < c_b$

The object speed and limits of c are set to $V = 2$, $c_a = 2.2$ and $c_b = 4.2$. Then $c_m \approx 2.4495$. **Figure 2(a)** shows the relationship between E and c . The inverse function of E is a two-valued function between minimum E and a certain value marked with an arrow and a single-valued function between the marked value

and maximum E . In **Figure 2(b)**, the PDF of E decreases monotonically when c has the uniform probability distribution. **Figure 2(c)** shows two cases: the c -value of the vertex of the triangular probability distribution of c is less than c_m , ($2.3 < c_m$), and higher than c_m , ($3.7 > c_m$).

Case 3: $c_m \geq c_b$

The object speed and limits of c are set to $V = 2$, $c_a = 2.2$ and $c_b = 2.4$. Then $c_m \approx 2.4495$. In **Figure 3(a)**, c is a single-valued monotonically decreasing function. In **Figure 3(b)** the PDF of E again decreases monotonically when c has the uniform distribution. **Figure 3(c)** shows three cases: the c -value of the vertex of the triangular probability distribution of c is 2.25, 2.3 or 2.35.

4. Discussion

When the speed of light is assumed to be variable, its probability distribution is unknown. Although this study only examines two probability distributions for it, the probability distribution of E has certain characteristics. If the distribution of c is uniform, the probability density of E is always maximum at minimum E and decreases monotonically regardless of whether c_m is within the range of c or not. If the distribution of c is triangular and $c_m \leq c_a$, the distribution of E is also triangle-like with the vertex moving rightward with c . If the distribution of c is triangular and $c_m \geq c_b$, the probability distribution of E changes from trapezoid-like shape to triangle-like shape. In both cases the vertex of the probability distribution of E moves together with the probability distribution of c . In contrast with these cases, if $c_a < c_m < c_b$, the PDF of E is always maximum at minimum E in either distribution of c . This is because the PDF of E diverges to infinity at minimum E in either distribution of c from Equation (14). In practice, the PDF of E at minimum E increases even faster as the calculation step becomes smaller. This equation shows that if $c_a < c_m < c_b$, the PDF of E is always maximum at minimum E : $E = \sqrt{3}m_0c_m^2$ regardless of the distribution of c (that is, diverges to infinity). This suggests that even if the distribution of c is unknown, E will rapidly increase as soon as c_m enters a certain range of c as the speed V of an object increases.

On this basis, the following method is proposed to detect any range of c . As the speed of light is defined as $299,792,458 \text{ m/s} \equiv c_L$, $c_a \leq c_L \leq c_b$.

If

$$c_m = \frac{\sqrt{6}}{2}V < c_a \leq c_L \leq c_b \quad (15)$$

then

$$V < \frac{\sqrt{6}}{3}c_L. \quad (16)$$

For example, the speed of one thousand electrons or protons is increased stepwise by an accelerator from below $\frac{\sqrt{6}}{3}c_L$ to c_L . For each step, a frequency

distribution of E will be obtained. As V is increased from below $\frac{\sqrt{6}}{3}c_L$, c_m will enter the range of c at the critical value of V . Then the probability density of minimum E will increase sharply. Since the speed of light has been measured with very fine precision [4], the range of c would be very narrow. Then the speed will need to be more finely increased bit by bit (in steps of 100 m/s if possible). As V increases after c_m exceeds c_b , the probability density of minimum E will decrease abruptly. If these phenomena are observed, then c is variable. If not, then c is constant.

5. Conclusion

If it is possible that the speed of light in free space is variable, then a probabilistic method to detect the variability is applicable. This assumes that c obeys a probability distribution. From mass-energy equivalence, the PDF of E can be obtained using the inverse function of E . The energy is minimum at $c_m = \frac{\sqrt{6}}{2}V$, and the PDF of E diverges to infinity at $E = \sqrt{3}m_0c_m^2$. Thus, when c_m enters the range of c as V is increased stepwise from below $\frac{\sqrt{6}}{3}c_L$, the PDF of E increases abruptly regardless of the PDF of c . If this is observed by accelerating a beam of electrons or photons, it will show that c is variable; otherwise, c is constant.

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Conflicts of Interest

The author declares that there is no conflict of interests regarding the publication of this paper.

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