Transmission line model for the near-instantaneous transmission of the ionospheric electric field and currents to the equator

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Abstract The simultaneous onset of the preliminary impulse (PI) of the geomagnetic sudden commencement at high latitude and dayside dip equator is explained by means of the TM0 mode waves propagating at the speed of light in the Earth-ionosphere waveguide (EIW) [Kikuchi et al., 1978]. A couple of issues remain to be addressed in the EIW model: (1) How is the TM0 mode wave excited by the field-aligned currents (FACs) in the polar region? (2) How are the quasi-steady ionospheric currents achieved by the TM0 mode waves? (3) How simultaneous or delayed are the onset and peak of the equatorial PI with respect to the high-latitude PI? To address these issues, we examine the TEM (TM0) mode wave propagation in the finite-length transmission lines replacing the pair of FACs (magnetosphere-ionosphere (MI) transmission line) and the Earth-ionosphere waveguide (ionosphere-ground (IG) transmission line). The issue (1) is addressed by showing that a fraction of the TEM mode wave is transmitted from the MI to IG transmission lines through the polar ionosphere. To address the issues (2) and (3), we examine the properties of the finite-length IG transmission line with finite ionospheric conductivity. It is shown that the ionospheric currents start to grow instantaneously and continue to grow gradually with time constants of 1–10 s depending on the ionospheric conductivity. The MIG transmission line enables us to explain the instantaneous onset and delayed peak time of the equatorial PI and quick electric field response of the low-latitude ionosphere and inner magnetosphere.

1. Introduction
1.1. Simultaneous Onset of the PI

The preliminary impulse (PI) of the geomagnetic sudden commencement (SC) appears as the preliminary reverse impulse (PRI) at the afternoon high latitude and dayside dip equator [Matsushita, 1962]. The equivalent currents of the high-latitude PRI are composed of clockwise/anticlockwise current vortices in the afternoon/morning sector [Nagata and Abe, 1955]. These equivalent current vortices are the Hall currents driven by the dusk-to-dawn electric field transmitted from the magnetosphere by the Alfvén waves with field-aligned currents flowing into/out from the ionosphere in the afternoon/morning sector [Tamao, 1964]. Global MHD simulations reproduce the field-aligned currents and ionospheric electric potentials responsible for the dusk-to-dawn electric field in the high-latitude ionosphere [Slinker et al., 1999; Fujita et al., 2003a].

Araki [1977] found that the PRI occurs simultaneously at the afternoon high latitude and the dayside equator with the temporal resolution of the rapid-run magnetograms (10 s) and suggested that the polar electric field propagates to the equator near-instantaneously. Furthermore, Araki et al. [1985] showed that the preliminary positive impulse (PPI) appears at the nightside equator when the PRI is observed on the dayside. They suggested that the dusk-to-dawn electric field propagates to the day- and nightside equator near-instantaneously during the PI event. Impulsive electric fields associated with the SC are observed by means of the HF Doppler measurements [Davies et al., 1962; Huang et al., 1973], Kikuchi et al. [1985] and Kikuchi [1986] found that the HF Doppler frequency deviations are positive/negative on the dayside/nightside during the PI and vice versa during the main impulse (MI) of SC. They confirmed that the PI electric field is a dusk-to-dawn potential electric field transmitted near-instantaneously (with 10 s resolution) from the polar ionosphere to the low latitude.

In spite of the instantaneous onset of the PI at the equator, Kikuchi et al. [1996] pointed out that the peak of a magnetic impulse could be delayed at the equator by 10 s. Shinohara et al. [1997] also reported that the PI2...
observed at the dayside equator is delayed in phase with respect to the high-latitude Pi2. They explained that the high conductivity of the equatorial ionosphere would result in the induction effect on the currents flowing from the polar ionosphere by calculating the phase delay in a lumped-element circuit.

1.2. Earth-Ionosphere Waveguide

In order to explain the simultaneous onset of the PI at high latitude and equator, Kikuchi and Araki [1979a] examined propagation of the electromagnetic waves in the current-flowing ionospheric $E$ region and found that it takes an hour for the stepwise increase in the polar electric field to make observable effects at the equator. Kikuchi and Araki [1979b] examined propagation modes in the three-layered Earth-ionosphere waveguide (EIW) composed of the fully ionized magnetohydrodynamic (MHD) medium (magnetosphere), conductor (ionospheric $E$ region), and vacuum (neutral atmosphere) terminated by the perfectly conducting ground. They found that the ionospheric electric field and currents can be transmitted at the speed of light by the TM0 mode waves excited by the magnetic field perpendicular to the propagation plane in the north-south meridian. The TM0 mode wave propagation is applicable to the transmission of the PI electric field since the TM0 mode waves have no lower cutoff frequencies. Kikuchi [2005] examined attenuation of the TM0 mode waves due to the finite conductivity of the ionosphere by calculating the Poynting flux transported into the conducting ionosphere and further into the inner magnetosphere. The attenuation of the transmitted electric potential is less than 15% even under the nighttime condition of the low ionospheric conductivity. 

On the other hand, the finite size of the polar electric field results in severe geometrical attenuation [Kikuchi et al., 1978; Kikuchi and Araki, 1979b] and causes low occurrence frequency of the PI at low latitudes [Matsushita, 1962; Araki, 1977; Shinbori et al., 2010]. However, the PI currents are intensified in the equatorial ionosphere by the Cowling effect, being superposed by the Hall currents due to the vertical electric field resulted from the geometry of the horizontal magnetic field [Hirono, 1952; Baker and Martyn, 1953]. The EIW model predicts that the electric field associated with the ionospheric currents is transmitted into the magnetosphere by the Alfvén waves along the magnetic field lines. Thus, the PI electric field is detected by the HF Doppler observations in the low-latitude $F$ region ionosphere [Kikuchi, 1986] and by satellites in the inner magnetosphere [Nishimura et al., 2009]. Furthermore, upward flow of the Poynting flux was observed in the inner magnetosphere as predicted by the EIW model [Nishimura et al., 2010].

1.3. Issues to be Addressed

Once the polar electric field is transmitted to the equator, the steady-state electric field and currents can be calculated in the global ionosphere solving the current continuity equation with the ionospheric conductance and the cross polar cap potential associated with a pair of field-aligned currents [Maekawa and Maeda, 1978; Nopper and Carovillano, 1978; Tsunomura and Araki, 1984]. Here one may raise a question how the TM0 mode waves achieve the steady ionospheric currents described by the current continuity equations. This question also requires the waveguide model to satisfy the instantaneous onset and time delay in the peak intensity at the equator. Other than these major issues, we have a couple of issues to be addressed. One is how to excite the TM0 mode wave in the EIW by providing the field-aligned currents on the polar ionosphere. In our original papers, Kikuchi et al. [1978] and Kikuchi and Araki [1979b], we assumed that the source magnetic field for the TM0 mode wave is produced by the Hall currents in the polar ionosphere. However, the Hall currents close with themselves in the ionosphere if the ionosphere is uniform and would not supply the Pedersen currents enhanced by the Cowling effect at the equator. In this paper, we address this issue by showing that a fraction of the magnetic field carried by the field-aligned currents is transmitted into the EIW through the polar ionosphere and acts as the source magnetic field exciting the TM0 mode wave. Another question on our original papers is that the EIW was assumed to be in semi-infinite length and no boundaries are given to the propagating TM0 mode wave. This model applies only to the transient stage when the TM0 mode wave is propagating to the equator at the speed of light. We thus need to clarify the propagation characteristic of the TM0 mode waves in the finite-length waveguide so that the TM0 mode waves can achieve the steady ionospheric currents from high latitude to the equator. Addressing this issue will give an answer to the question on the instantaneous onset and delayed peak time of the equatorial magnetic perturbations. To solve the propagation of the TM0 mode waves in the finite-length EIW with finite ionospheric conductivity, we replace the EIW with a lossy transmission line. The replacement of the EIW with the lossy transmission line will be validated by showing the attenuations of the TM0 and TEM modes being identical.
In the following sections, we first review the TM₀ mode propagation in the three-layered EIW studied in Kikuchi et al. [1978] and Kikuchi and Araki [1979b] (section 2). In this section, we show that the TM₀ mode wave transmits an electric potential of the ionosphere with respect to the ground and that the attenuation of the TM₀ mode wave can be evaluated by calculating the loss of the Poynting flux in the ionosphere. Comparing this result with the transmission line theory, we replace the EIW with a lossy transmission line with the infinitely conducting ionosphere (below referred to as the ionosphere-ground (IG) transmission line). In section 3, we build a transmission line system composed of a pair of field-aligned currents (FACs) and the EIW with the polar ionosphere as a shunt conductor in between (below referred to as the magnetosphere-ionosphere-ground (MIG) transmission line). The pair of FACs and EIW is replaced with the parallel-plane magnetosphere-ionosphere (MI) and ionosphere-ground (IG) transmission lines, respectively. A fraction of the propagating magnetic field is transmitted through the polar ionosphere and excites the TM₀ (TEM) mode wave in the IG transmission line. In the model, the east-west symmetry is assumed so that the electric potential vanishes in the noon-midnight meridian which is far end of the finite-length IG transmission line. In section 4, we obtain the general solution of the telegraphs equations in the parallel-plane transmission line. By applying the Laplace transformation, we calculate the electric potential and currents in the MI transmission line with the polar ionosphere as a load. In section 5, we calculate the electric potential and currents in the MI transmission line with the polar ionosphere as a load. In section 6, we discuss conventional paradigms such as the Fukushima’s theorem [Fukushima, 1976] which is in conflict with the transmission line model and possible applications to the electric field transmission to low-latitude ionosphere and to the inner magnetosphere.

2. Three-Layered Earth-Ionosphere Waveguide

2.1. TM₀ Waveguide Mode

Kikuchi and Araki [1979b] solved equations in the three-layered Earth-ionosphere waveguide (EIW) composed of the vacuum, conducting ionosphere, and fully ionized magnetosphere (MHD) terminated below by the perfectly conducting ground as schematically shown in Figure 1. The propagation is in the x-z plane, and the model is uniform in the y-direction. In the original papers, we assumed that the ionosphere is anisotropic with
dominant Hall conductivity. However, the ionospheric currents carried by the TM0 mode wave are in the north-south direction and connected with the ground surface currents via the displacement current on the wavefront (see Figure 1 in Kikuchi et al. [1978]). This current circuit requires the Pedersen currents in the ionosphere, since the ionospheric Hall currents should be closed in the ionosphere not connecting to the FACs and the equatorial Cowling currents. The Hall conductivity only helps the TM0 mode wave excite the transverse electric (TE) mode wave, which is an evanescent mode in the waveguide [Kikuchi and Araki, 1979b]. Thus, the ionosphere is assumed to be isotropic below.

The electric ($E$) and magnetic ($H$) fields in the vacuum region are governed by the Maxwell equations:

$$\nabla \times E_V = -\mu_0 \frac{\partial H_V}{\partial t},$$

$$\nabla \times H_V = \varepsilon_0 \frac{\partial E_V}{\partial t},$$

where $\mu_0$ and $\varepsilon_0$ denote the magnetic permeability and electric permittivity of the vacuum, respectively. The suffix $V$ refers to the vacuum region.

In the ionosphere, we have conduction currents in place of the displacement currents. Thus, we have

$$\nabla \times E_I = -\mu_0 \frac{\partial H_I}{\partial t},$$

$$\nabla \times H_I = \sigma \cdot E_I,$$

where $\sigma$ denotes the conductivity of the ionosphere, and the suffix $I$ refers to the ionosphere.

In the fully ionized magnetosphere, the electromagnetic fields are transmitted by the magnetohydrodynamic (MHD) waves, which accompany motion of the magnetized plasma. The governing equations are given as

$$\nabla \times E_M = -\mu_0 \frac{\partial H_M}{\partial t},$$

$$\nabla \times H_M = J,$$

$$\rho \frac{du}{dt} = J \times B_0,$$

$$E_M + u \times B_0 = 0,$$

where $\rho$, $u$, $J$, and $B_0$ denote the mass density and velocity of plasma, electric current, and external magnetic field, respectively. The suffix $M$ refers to the magnetosphere.

As was done by Kikuchi and Araki [1979b], the Laplace transformation is applied with respect to time as defined by

$$h_j[x,s] = \int_0^\infty H_j[x,t] e^{-st} dt,$$

$$H_j[x,t] = \frac{1}{2\pi j} \int_{s-j\infty}^{s+j\infty} h_j[x,s] e^{st} ds,$$

where $a$ denotes a real number and $s$ is a complex number defined as $s = a + j\omega$ ($\omega$: angular frequency, $j$: imaginary unit), so the second integral is between minus infinity to plus infinity with respect to $\omega$.

In Kikuchi and Araki [1979b], a transverse magnetic field is given as a boundary condition at $x = 0$ to excite the TM0 mode wave in the waveguide. This assumption will be validated in section 3 by showing that a pair of FACs provides a magnetic field below the ionosphere. If we give a stepwise magnetic field, $H_y U(t)$ ($U$: Heaviside unit step function) at $x = 0$, we can apply the Fourier sine transformation to $h_y[x,s]$ with respect to $x$ as below:

$$h_j[\beta, s] = \sqrt{\frac{2}{\pi}} \int_0^\infty h_j[x,s] \sin[\beta x] dx,$$

$$h_j[\beta, s] = \sqrt{\frac{2}{\pi}} \int_0^\infty h_j[\beta, s] \sin[\beta x] dx,$$

where $\beta$ denotes a real number.
Then we have the Laplace and Fourier transformed governing equations for $H_y$ in the vacuum, ionosphere, and magnetosphere as below:

$$\frac{d^2 \overline{H_y}}{dz^2} - \left( \frac{\beta^2 + \frac{\mu_0 \sigma}{s} \frac{\pi}{c} \right) \overline{H_y} = -H_{y0} \sqrt{\pi \frac{\beta}{c}}$$  \hspace{1cm} (11)  \\
$$\frac{d^2 \overline{H_y}}{dz^2} - (\beta^2 + \mu_0 \sigma \cdot s) \cdot \overline{H_y} = -H_{y0} \sqrt{\pi \frac{\beta}{c}}$$  \hspace{1cm} (12)  \\
$$\frac{d^2 \overline{H_y}}{dz^2} - \frac{s^2}{V_A^2} \overline{H_y} = 0,$$  \hspace{1cm} (13)

where $c$ and $V_A$ denote the light and Alfvén speeds, respectively, and the following relation is used.

$$h_{yV}[0, s] = H_{y0} \frac{1}{s},$$  \hspace{1cm} (14)

We obtain the solutions of (11), (12), and (13) with coefficients $A$'s as

$$\overline{H_y}[\beta, s] = H_{y0} \sqrt{\pi \frac{\beta}{c}} + A_{11} e^{-\frac{\beta}{c} \sqrt{\frac{\beta}{c}}} + A_{12} e^{\frac{\beta}{c} \sqrt{\frac{\beta}{c}}} \frac{1}{s},$$  \hspace{1cm} (15)  \\
$$\overline{E_y}[\beta, s] = \frac{1}{\sqrt{\pi \frac{\beta}{c}}} \left( A_{11} e^{-\frac{\beta}{c} \sqrt{\frac{\beta}{c}}} - A_{12} e^{\frac{\beta}{c} \sqrt{\frac{\beta}{c}}} \right).$$  \hspace{1cm} (16)

in the vacuum region,

$$\overline{H_y}[\beta, s] = \sqrt{\frac{2}{\pi s}} \frac{\beta}{\sqrt{\beta^2 + \mu_0 \sigma s}} + A_{11} e^{-2 \frac{\sqrt{\beta^2 + \mu_0 \sigma s}}{c}} + A_{12} e^{2 \frac{\sqrt{\beta^2 + \mu_0 \sigma s}}{c}},$$  \hspace{1cm} (17)  \\
$$\overline{E_y}[\beta, s] = \frac{1}{s} \left( A_{11} e^{-\frac{\beta}{c} \sqrt{\frac{\beta}{c}}} - A_{12} e^{\frac{\beta}{c} \sqrt{\frac{\beta}{c}}} \right).$$  \hspace{1cm} (18)

in the conducting ionosphere, and

$$\overline{H_y}[\beta, s] = A_{11} e^{-\frac{\beta}{c} \sqrt{\frac{\beta}{c}}} + A_{12} e^{\frac{\beta}{c} \sqrt{\frac{\beta}{c}}} \frac{1}{s},$$  \hspace{1cm} (19)  \\
$$\overline{E_y}[\beta, s] = h_{yV}[0, s] \frac{1}{s} \left( A_{11} e^{-\frac{\beta}{c} \sqrt{\frac{\beta}{c}}} - A_{12} e^{\frac{\beta}{c} \sqrt{\frac{\beta}{c}}} \right),$$  \hspace{1cm} (20)

in the magnetosphere. The coefficients $A$'s should be determined by the boundary conditions at the interfaces between the different regions; the horizontal electric and magnetic fields are continuous, the horizontal electric field vanishes on the ground, and the electric and magnetic fields vanish at the infinite $z$.

The inverse Fourier and Laplace transforms of the first term in (15) is obtained as

$$H_{yV}[x, t] = H_{y0} U \left[ t - \frac{x}{c} \right],$$  \hspace{1cm} (21)

which represents the zeroth-order transverse magnetic (TM$_0$) mode wave propagating at the speed of light in the $x$-direction with uniform intensity in the $z$-direction [Kikuchi and Araki, 1979b]. The second and third terms of (15) represent attenuation of the TM$_0$ mode wave due to dissipation in the conducting ionosphere and leakage to the magnetosphere. Later we estimate the attenuation of the TM$_0$ mode wave by evaluating the transport of the Poynting flux from the vacuum region into the ionosphere instead of solving all the equations.

### 2.2. Horizontal Transmission of the Ionospheric Electric Potential

From (2) and (21), we obtain the vertical electric field of the TM$_0$ mode wave as,

$$E_{zV}[x, t] = -Z_0 H_{y0} U \left[ t - \frac{x}{c} \right],$$  \hspace{1cm} (22)

where $Z_0$ is the wave impedance in the vacuum. Since $E_{zV}$ is uniform in the $z$-direction, a height integration of $E_{zV}$ provides the electric potential of the ionosphere with respect to the ground, $V[x,t]$. Thus, the
electric potential given by one of the field-aligned currents pair (FAC (+) in Figure 1), \( V_0 \), is transmitted in the \( x \)-direction at the speed of light as expressed by

\[
V[x, t] = V_0 U \left[ t - \frac{x}{c} \right]. \tag{23}
\]

Likewise, the negative electric potential carried by the FAC (−) propagates in the waveguide. These two potentials with opposite polarities will achieve steady-state ionospheric currents as shown in section 5.

2.3. Upward Transmission of the Poynting Flux

The horizontal propagation in the conducting ionosphere described by the first term of (17) never plays a role in transmitting the electric field from high latitude to the equator because the propagation is diffusion over long distance with a time lag of the order of an hour [Kikuchi and Araki, 1979a]. The rest of the terms depending on \( z \) in (17) and (18) represent a vertical diffusion mode excited by the TM_0 mode wave at the lower edge of the ionosphere. This means that a fraction of the Poynting flux carried by of the TM_0 mode wave transmits into the ionosphere. If we let the conductivity and thickness of the ionosphere be \( 2 \times 10^{-4} \, \text{mho/m} \) and 30 km for the daytime condition, we obtain 1 s for the electromagnetic fields to achieve the quasi-steady distribution in the \( z \)-direction [Kikuchi and Araki, 1979a]. Compared with the time scale of our concern (\( > 1 \) min), we can assume that the electric and magnetic fields distribute in the ionosphere instantaneously and those at the upper edge of the ionosphere can be a source for the Alfvén wave propagating upward in the magnetosphere.

The vertical distributions of the electric and magnetic fields in the ionosphere are expressed as below [Kikuchi and Araki, 1979b].

\[
H_{\mu}[x, z, t] = \left( 1 - \frac{\sigma z}{\Sigma_A + \Sigma_I} \right) H_{\nu}[x, t], \tag{24}
\]

\[
E_{\alpha}[x, z, t] = \frac{1}{\Sigma_A + \Sigma_I} H_{\nu}[x, t], \tag{25}
\]

where \( \Sigma_A \) and \( \Sigma_I \) denote the Alfvén conductance of the magnetosphere and height-integrated ionospheric conductivity, respectively, as defined by

\[
\Sigma_A = \frac{1}{\mu_0 V_A}, \quad \Sigma_I = \sigma \cdot d \ (d = \text{thickness of the ionosphere}). \tag{26}
\]

The magnetic and electric fields propagating in the magnetosphere are derived from the first terms of (19) and (20) as given by

\[
H_{\mu M}[x, z, t] = \frac{\Sigma_A}{\Sigma_A + \Sigma_I} H_{\nu}[x, t] \cdot U \left( t - \frac{z}{V_A} \right), \tag{27}
\]

\[
E_{\alpha M}[x, z, t] = \frac{1}{\Sigma_A + \Sigma_I} H_{\nu}[x, t] \cdot U \left( t - \frac{z}{V_A} \right). \tag{28}
\]

From (25) and (28), we deduce that the electric field in the ionosphere is mapped upward into the magnetosphere at the Alfvén speed. The magnetic field, on the other hand, decreases with height in the ionosphere (24), and therefore the magnetic field above the ionosphere would be considerably reduced for the daytime ionospheric conditions. Indeed, an intense electric field of the PI of SC is observed in the magnetosphere together with weak magnetic field [Nishimura et al., 2010] and no Pi2 is observed above the ionosphere as the Pi2 is observed on the ground [Sutcliffe and Lühr, 2010]. From the current circuit point of view, the electric current carried by the TM_0 mode wave is divided into the ionospheric current \( \Sigma_A E_{\alpha I} \) and the wavefront current \( \Sigma_I E_{\alpha M} \) of the upward propagating Alfvén wave. Thus, under a condition of large ionospheric conductance to Alfvén conductance ratio (\( \Sigma_I > \Sigma_A \)), most currents flow in the ionosphere and therefore the three-layered EIW can be replaced with a parallel-plane transmission line. Then the TM_0 mode is identical with the TEM mode in the transmission line.
It should be noted that the electric current carried by a single TM_0 mode wave is deduced from the wave parameters in (22) and (25) as below:

\[ J = (\Sigma_I + \Sigma_A)E_W = \frac{E_W}{Z_0}. \] (29)

The expression (29) never contains the potential difference and ionospheric conductivity needed to calculate the steady ionospheric currents. We will show in section 5 that continuous propagation of the TM_0 mode waves with multiple reflections in a finite-length transmission line achieves the steady-state currents with some time constants.

### 2.4. Attenuation of the TM_0 Mode Wave

Using (21) and (22), we obtain the horizontal Poynting flux \( S_{xW} \) in the EIW, while from (24), (25), (27), and (28), we obtain the upward Poynting fluxes \( S_{uW} \) and \( S_{zM} \) in the ionosphere and magnetosphere, respectively, as given below:

\[ S_{xW} = E_W \times H_W = Z_0 h_{sh}^2 \delta(x, t), \] (30)
\[ S_{uW} = \frac{\Sigma_I + \Sigma_A - \sigma(z - h)}{(\Sigma_A + \Sigma_I)^2} h_{sh}^2 \delta(x, t), \] (31)
\[ S_{zM} = \frac{\Sigma_A}{(\Sigma_A + \Sigma_I)^2} h_{sh}^2 \delta(x, t) \cdot t - \frac{Z}{V_A}. \] (32)

We here evaluate an upward leakage of the Poynting flux, which would cause attenuation of the TM_0 mode wave. We let the Poynting flux across the section at \( x \) be \( S_{xW}[x] \) and at \( x + \Delta x \) be \( S_{xW}[x + \Delta x] \). Then, the energy transmitting into the ionosphere is a divergence of the Poynting flux from the space bounded by \( x \) and \( x + \Delta x \) (Figure 1) as expressed below:

\[ h(S_{xW}[x] - S_{xW}[x + \Delta x]) = \Delta x S_{xW}[x, z = h]. \] (33)

Using (30), we obtain the following differential equation.

\[ \frac{\partial S_{xW}[x]}{\partial x} = -\frac{1}{h Z_0 (\Sigma_A + \Sigma_I)} S_{xW}[x]. \] (34)

We thus obtain the attenuation factor of the Poynting flux and electric field as follows:

\[ \frac{S_{xW}[x]}{S_{xW}[0]} = e^{-\frac{Z_0 h}{h (\Sigma_A + \Sigma_I)}}, \] (35)
\[ \frac{E_{xW}[x]}{E_{xW}[0]} = e^{-\frac{Z_0 h}{h (\Sigma_A + \Sigma_I)}}. \] (36)

Applying the parameters, \( V_A = 1000 \text{ km/s}, \Sigma_I = 8 \text{ mho}, x = 8000 \text{ km} \), we obtain the attenuation factor of the electric field as 0.988. The result indicates no appreciable attenuation for the daytime condition. For the nighttime condition, the ionospheric conductance is comparable to or smaller than the Alfvén conductance. In an extreme case of poorly conducting ionosphere, null-conductance, the ratio is 0.87. In this extreme case, the current associated with the TM_0 mode wave flows on the wavefront of the Alfvén wave in the magnetosphere. At any case, the attenuation of the TM_0 mode wave is not significant, but it suffers severe geometrical attenuation as studied by Kikuchi et al. [1978] and Kikuchi and Araki [1979b]. The curvature of the Earth would make some modification on the propagation of the TM_0 mode. However, we should recall that the TM_0 mode wave is characterized by the electric field perpendicular to the conductors of the waveguide. This condition should be valid in a curved waveguide. More discussion will be given in the discussion section.

It is to be noted that the waveguide is in semi-infinite length in Figure 1, but in the real situation, the waveguide has a finite length from the polar region to the equator, say, 8000 km. We will examine the propagation of the TM_0 mode wave in a finite-length waveguide by replacing the waveguide with a parallel-plane transmission line in section 5.
3. Magnetosphere-Ionosphere Transmission System

3.1. FAC-EIW Coupled System

The electric potentials and FACs should be supplied by dynamos in the plasma region at the magnetopause, cusp, and mantle in the outer magnetosphere [Slínskey et al., 1999; Fujita et al., 2003a, 2003b; Tanaka, 1995; Siscoe et al., 2000] (more explanation in the discussion section). The FACs can be replaced with a transmission line [Sato and Iijima, 1979; Watanabe et al., 1986], connecting with the Earth-ionosphere waveguide (EIW) [Kikuchi et al., 1978; Kikuchi and Araki, 1979b] as schematically shown in Figure 2. The FACs and the EIW form a magnetosphere-ionosphere-ground (MIG) transmission line with the polar ionosphere as a shunt conductor in-between. The MIG transmission line has a three-dimensional structure, but here we show a dawn-dusk section for the sake of simplicity (see discussion section). Hereafter, the MI transmission line and the IG transmission line are used to refer the pair of FACs and the EIW, respectively.

The east-west symmetry is assumed in the MIG transmission line system. The electric potential vanishes on the noon-midnight meridian plane which can be replaced with a perfectly conducting sheet (dashed line in Figure 2). We thus have two transmission lines designated to the positive (right half) and negative (left half) potentials separated by the perfectly conducting sheet. From the charge neutrality in the whole system, the ground should be of zero potential and is also replaced by a perfect conductor. Then, the transmission line is simplified to be a combination of parallel-plane MI and IG transmission lines as shown for the positive potential in Figure 3. Likewise, a mirror-imaged transmission line is applied to the negative electric potential (not shown). The positive and negative electric potentials will be transmitted to the equator and achieve the quasi-steady ionospheric currents in the global ionosphere as is studied in section 5.

In Figure 3, we give the size of the MI and IG transmission lines with the suffixes 1 and 2, respectively, as

\[
\begin{align*}
&d_1 = 80000 \text{ km (length of the MI transmission line)} \\
&d_2 = 8000 \text{ km (length of the IG transmission line)} \\
&w = 2000 \text{ km (width of the MI and IG transmission lines)} \\
&l = 2000 \text{ km (spacing of the MI transmission line)} \\
&h = 100 \text{ km (spacing of the IG transmission line)}.
\end{align*}
\]
The parameters of the magnetosphere and ionosphere are given below:

\[ V_A = 1000 \text{ km/s (Alfven speed)} \]
\[ \Sigma_1 = 8 \text{ mho (height-integrated conductivity of polar ionosphere)} \]
\[ \Sigma_2 = 0.3-100 \text{ mho (height-integrated conductivity of mid-equatorial ionosphere)} \]  

Thus, we have the wave impedance in the MI and IG transmission lines as below:

\[ Z_{01} = \mu_0 V_A = 1.26 \text{ ohm} \]
\[ Z_{02} = \mu_0 c = 376.7 \text{ ohm}. \]  

The characteristic impedance of the MI transmission line, \( Z_1 \), is equal to \( Z_{01} \) since the width and spacing are the same and that of the IG transmission line, \( Z_2 \), is \( h/w \) times \( Z_{02} \). Then we have

\[ Z_1 = \mu_0 V_A = 1.26 \text{ ohm} \]
\[ Z_2 = 376.7 \frac{h}{w} = 18.8 \text{ ohm}. \]

### 3.2. Excitation of the TM\(_0\) Mode Wave

One of the issues of the present paper — How is the TM\(_0\) mode wave excited? — is addressed by showing that a fraction of the electric current flows from the MI transmission line into the IG transmission line. Letting the resistivity of the polar ionosphere be \( R_1 = 1/\Sigma_1 \), we obtain the reflection and transmission coefficients at the polar ionosphere for the electric potential as below:

\[ C_r = \frac{R_1Z_2 - Z_1(R_1 + Z_2)}{R_1Z_2 + Z_1(R_1 + Z_2)} = -0.821 \]
\[ C_t = \frac{2R_1Z_2}{R_1Z_2 + Z_1(R_1 + Z_2)} = 0.179. \]
At the arrival of the source electric potential, $V_0$, the negative reflection coefficient suppresses the electric potential at the foot of FAC to be 0.179 $V_0$, which is an electric potential relative to the ground ($=E_z/C_1$) in the IG transmission line. Since the vertical electric field is associated with the transverse magnetic field $H_y$ through the relation, $H_y = E_z/C_1$, the transverse magnetic field can be a source for the TM$_0$ mode wave as postulated in Kikuchi and Araki (1979b). The electric currents in the ionosphere and on the ground surface are closed by the displacement current on the wavefront of the TM$_0$ mode wave (Figure 2). Therefore, in the transient stage when a single TM$_0$ mode wave is propagating, the load conductance of the MI transmission line is a sum of the conductance of the polar ionosphere, $\Sigma_1$, and the conductance of the IG transmission line, $1/Z_2 = 0.053$ mho. It should be noted that the ionospheric current carried by a single TM$_0$ mode wave is much less than the steady ionospheric currents proportional to the ionospheric conductance (8 mho). However, as is shown in section 5, the ionospheric current grows as the TM$_0$ mode waves propagate back and forth in the finite-length IG transmission line.

### 3.3. Transmission Line Equations

We formulate transmission line equations for the electric potential, $V$, and current, $I$, in the MI and IG transmission lines. The governing equations in the transmission line are given as follows:

\[
RL + L \frac{\partial I}{\partial t} = -\frac{\partial V}{\partial x},
\]

(42)

\[
C \frac{\partial V}{\partial t} = -\frac{\partial I}{\partial x},
\]

(43)

where $R$, $L$, and $C$ denote the resistance, inductance, and capacitance per unit length, respectively. Then we have equations for $V$ and $I$ as

\[
\frac{\partial^2 V}{\partial x^2} = \frac{1}{V_{\text{ph}}^2} \left( \frac{\partial^2 V}{\partial t^2} + \alpha \frac{\partial V}{\partial t} \right),
\]

(44)

\[
\frac{\partial^2 I}{\partial x^2} = \frac{1}{V_{\text{ph}}^2} \left( \frac{\partial^2 I}{\partial t^2} + \alpha \frac{\partial I}{\partial t} \right),
\]

(45)

where $V_{\text{ph}} = \sqrt{\frac{1}{LC}}$ (phase velocity), $\alpha = \frac{R}{L}$ (attenuation rate).

Letting the Laplace transforms of $V$ and $I$ be $v$ and $i$, respectively, and assuming that $V=I=0$ at $t=0$, we have,

\[
v(x,s) = Ae^{-\gamma x} + Be^{\gamma x},
\]

(47)

\[
i(x,s) = \frac{1}{Z} (Ae^{-\gamma x} - Be^{\gamma x}),
\]

(48)

where $A$ and $B$ are complex numbers and $\gamma$ is given below:

\[
\gamma = \frac{1}{V_{\text{ph}}} \sqrt{s(s + \alpha)}.
\]

(49)

Letting the internal resistivity of the dynamo and the load resistance be $r$ and $R_L$, respectively, we have the boundary conditions at $x=0$ and $d$ as follows:

\[
V[x = 0, t] = V_0 U[t] - r \cdot i[x = 0, t]
\]

\[
V[x = d, t] = R_L \cdot i[x = d, t].
\]

(50)

The Laplace transforms of (50) are given by

\[
v(0,s) = \frac{V_0}{s} - r \cdot i(0,s)
\]

\[
v(d,s) = R_L \cdot i(d,s).
\]

(51)
From (47), (48), and (51), we obtain the voltage and currents in the transmission line as follows:

\[ v_x(s) = \frac{V_0}{s} \left( \frac{1 + \Gamma e^{-2(d-x)}}{1 - \Gamma'^* e^{-2d}} \right) \]

\[ i_x(s) = \frac{V_0}{s} \frac{1}{r + Z} \frac{e^{-\gamma x}}{1 - \Gamma e^{-2d}} \]

where \( \Gamma \) and \( \Gamma'^* \) are reflection coefficients at the load and dynamo, respectively, which are given by

\[ \Gamma = \frac{R_L - Z}{r + Z} \]

\[ \Gamma'^* = \frac{r - Z}{r + Z} \]

Since \( |\Gamma|, |\Gamma'^*| < 1 \), the solutions (52) and (53) can be expressed in terms of a series of propagating waves in the positive and negative directions as given below:

\[ v_x(s) = \frac{V_0}{s} \left\{ \sum_{n=1}^{\infty} (\Gamma')^{n-1} e^{-2(n-1)d-x} + \Gamma \sum_{n=1}^{\infty} (\Gamma')^{n-1} e^{-2(n-1)d+x} \right\} \]

\[ i_x(s) = \frac{V_0}{s} \left\{ \frac{1}{r + Z} \sum_{n=1}^{\infty} (\Gamma')^{n-1} e^{-2(n-1)d-x} - \Gamma \frac{1}{r + Z} \sum_{n=1}^{\infty} (\Gamma')^{n-1} e^{-2(n-1)d+x} \right\} \]

In the following sections, we examine the propagation in the MI and IG transmission lines, separately.

Figure 4. Electric potential, current, and power calculated at the (a) dynamo in the outer magnetosphere \((z = 0)\), (b) midmagnetosphere \((z = 0.5d_1)\), and (c) polar ionosphere \((z = d_1)\) of the MI transmission line \((d_1 = 80,000 \text{ km})\) for the daytime ionospheric conductance \((8 \text{ mho})\). The electric potential and internal resistivity of the dynamo are assumed to be 50 kV and 0 ohm, respectively, and the Alfvén speed is 1000 km/s.
4. Magnetosphere-Ionosphere Transmission Line

In the magnetosphere, the conductivity along the magnetic field lines can be assumed to be infinite. Then we have \( \alpha = 0 \) and obtain the inverse Laplace transforms of (55) and (56) as shown below [McLachlan, 1962]. In this section, we use \( z \) instead of \( x \) as the distance from the magnetospheric dynamo to tell the vertical transmission from the horizontal transmission.

\[
V_1(z, t) = V_0 \frac{Z_1}{r + Z_1} \left[ \sum_{n=1}^{\infty} \left( \frac{\Gamma}{\Gamma + 1} \right)^{n-1} U \left( t - \frac{2(n-1)d_1 + z}{V_A} \right) + \Gamma \sum_{n=1}^{\infty} \left( \frac{\Gamma}{\Gamma + 1} \right)^{n-1} U \left( t - \frac{2nd_1 - z}{V_A} \right) \right] \tag{57}
\]

\[
l_1(z, t) = V_0 \frac{1}{r + Z_1} \left[ \sum_{n=1}^{\infty} \left( \frac{\Gamma}{\Gamma + 1} \right)^{n-1} U \left( t - \frac{2(n-1)d_1 + z}{V_A} \right) - \Gamma \sum_{n=1}^{\infty} \left( \frac{\Gamma}{\Gamma + 1} \right)^{n-1} U \left( t - \frac{2nd_1 - z}{V_A} \right) \right] \tag{58}
\]

The \( V_1 \) and \( l_1 \) are composed of the waves propagating between the magnetospheric dynamo and polar ionosphere. If the dynamo supplies a constant voltage irrespective of the magnetospheric and ionospheric parameters, the internal resistivity of the dynamo should be zero.

4.1. MI Transmission Line With Daytime Ionospheric Conditions

Figure 4 shows the electric potential and currents calculated at the (a) dynamo, \( z = 0 \), (b) midmagnetosphere, \( z = 0.5d_1 \), and (c) polar ionosphere, \( z = d_1 \) of the MI transmission line with the daytime ionospheric conductance (8 mho) which is one order of magnitude greater than the wave conductance (0.79 mho) in the magnetosphere. The electric potential and internal resistivity of the dynamo are assumed to be 50 kV and 0, respectively, and the Alfven speed is 1000 km/s. The electromagnetic energy is plotted in the bottom panels. The electric potential at the ionosphere increases gradually and approaches to the steady-state value, 50 kV, respectively, and the Alfven speed is 1000 km/s. The electromagnetic energy is plotted in the bottom panels.

The gradual increases in the electric potential and currents are a result of consecutive reflections at the dynamo and ionosphere, where the reflection coefficients are \(-1.0\) and \(-0.819\), respectively. When the first down coming wave arrives at the ionosphere, the reflection reduces the electric potential to be 0.181 \( V_0 \). The reflected
wave returns to the dynamo and the electric potential recovers $V_0$. After consecutive reflections, the electric potential increases step-by-step as shown in Figure 4c. On the other hand, the electric potential oscillates at midmagnetosphere (Figure 4b), which is equivalent to the field line resonance damping with the time constant. The energy transmitted from the dynamo to the ionosphere increases similarly at the three locations.

Figure 5 shows the electric potential and currents at the polar ionosphere for the ionospheric conductance of (a) 8 mho and (b) 16 mho with the same transmission line parameters as employed above. The time constant is doubled for the doubled ionospheric conductance. Likewise, the time constant is proportional to the length of the transmission line and inversely proportional to the propagation velocity (plots not shown). Figure 5c shows the electric potential and current in the lumped-element LR circuit which is applied to the MI transmission line with a high ionospheric conductance to wave conductance ratio to calculate the exact time constant (see below).

### 4.2. Lumped-Element LR Circuit Analogy

The lumped-element circuit is composed of the total inductance of the FAC, $L_T = \mu_0 d_1$, and the resistivity of the ionosphere, $R_i = 1/\Sigma_1$. In this LR circuit, the voltage across the resistivity and current in the circuit are obtained with the source $V_0$ as below:

$$V(t) = V_0 \left(1 - e^{-t/\tau_1}\right),$$

(59)

$$I(t) = \frac{V_0}{R_i} \left(1 - e^{-t/\tau_2}\right).$$

(60)
Temporal variations of \( V(t) \) and \( I(t) \) with the parameters used in Figure 5b are plotted in Figure 5c. It is apparently seen that both results are identical except small amplitude fluctuations in Figure 5b. We thus obtain the time constant \( T \) as

\[
T = L_1/R_1 = \mu_0\Sigma_1 d_1.
\]

Using (61), we obtain 804 s and 1608 s for the conductivities 8 and 16 mho, respectively. The time constant leads to that the ionospheric currents grow gradually even when the interplanetary magnetic field (IMF) turns southward abruptly as discussed later.

4.3. MI Transmission Line With Nighttime Ionospheric Conditions

Figure 6 shows the electric potential and currents calculated at the (a) dynamo, \( z = 0 \), (b) midmagnetosphere, \( z = 0.5d_1 \), and (c) polar ionosphere, \( z = d_1 \), of the MI transmission line with the nighttime ionospheric conductivity (0.2 mho) which is smaller than the wave conductance (0.79 mho) in the magnetosphere. In the midmagnetosphere and ionosphere, the electric potential and current oscillate and approach the steady-state values. It is to be noted that the temporal variations of the current are different at different locations, but the current continuity is certified by the polarization currents on the wavefront of the Alfvén waves.

5. Ionosphere-Ground Transmission Line

5.1. Semi-Infinite IG Transmission Line

We first examine the characteristics of the TM\( 0 \) (TEM) mode wave in a semi-infinite length transmission line with finite ionospheric conductivity to validate the use of the lossy parallel-plane transmission line in place of the three-layered EIW. In the IG transmission line model, the dynamo is located at one end \((x = 0)\) with voltage \( V_0 \) and internal resistivity \( r \sim 0 \), since the reflection coefficient at \( x = 0 \) is \(-0.99\) as derived from the conductivity of the polar ionosphere and the characteristic impedance of the IG transmission line (54). If we assume that the electric potential and currents vanish at \( z = 0 \), (b) midmagnetosphere, and (c) polar ionosphere, \( z = d_1 \), of the MI transmission line with the nighttime ionospheric conductivity (0.2 mho) which is smaller than the wave conductance (0.79 mho) in the magnetosphere. In the midmagnetosphere and ionosphere, the electric potential and current oscillate and approach the steady-state values. It is to be noted that the temporal variations of the current are different at different locations, but the current continuity is certified by the polarization currents on the wavefront of the Alfvén waves.

Thus, the lossy transmission line can replace the EIW, while the EIW model helps us evaluate the electric field transmitting upward into the magnetosphere along the field lines (25, 28) as was done by Nishimura et al. (2010).
Figure 7 shows the electric potential and currents at a distance of 5000 km from the polar ionosphere calculated from (64) and (65) for three cases of ionospheric height-integrated conductivities, (a) 30, (b) 5, and (c) 0.2 mho. For the stepwise increase of 50 kV at $x = 0$, the electric potential at 5000 km increases stepwise in all cases (upper panels), but the electric currents (lower panels) decrease rather rapidly for the low ionospheric conductivity. Less energy is transported in the less conductive transmission line and the effective characteristic impedance of the transmission line becomes higher.

5.2. Finite-Length IG Transmission Line

Figure 8 shows a schematic diagram of the magnetosphere-ionosphere-ground transmission line system which connects the magnetospheric dynamo with the equatorial ionosphere via the polar ionosphere. The transmission system is assumed to be symmetrical with respect to the equator as well as to the noon-midnight meridian without losing the generality. The ionospheric currents are shown along the dawn and dusk terminators and equator (solid arrows). The ground surface currents are in opposite direction to the ionospheric currents (dotted arrows). The east-west symmetry makes the electric potential be zero on the noon-midnight meridian and the north-south symmetry makes the equatorial current flow along the equator. From the MIG transmission line system, we pick out the IG transmission line with finite length from the polar ionosphere to the equator (Figure 3). The conductivity is not uniform in the real ionosphere, particularly at the equator where the Pedersen current is substantially enhanced by the Cowling effect [Hirano, 1952]. However, the uniform ionosphere is employed below to examine how the steady ionospheric currents are achieved by the TM$_0$ mode waves and how the ionospheric conductivity affects their propagation.

We now give the dynamo electric potential at $x = 0$ and null potential at $x = d_2$, which provides the following relations,

$$
V_1(x = 0,t) = V_0 \cdot U(t) - t \cdot l_2(x = 0,t),
$$

$$
V_2(x = d_2,t) = 0.
$$

(68)

The internal resistivity of the dynamo should be the resistance of the polar ionosphere (1/8 ohm), which is much smaller than the characteristic impedance of the IG transmission line (18.8 ohm). Under these conditions, we have

$$
\Gamma = \Gamma' = -1
$$

(69)
The electric potential and current in the IG transmission line are obtained as,

\[ v_2(x,s) = \frac{v_0}{s} \sum_{n=1}^{\infty} e^{-\gamma(n-1)d_x} - \frac{v_0}{s} e^{-\gamma 2n d_x} \],

(70)

\[ i_2(x,s) = \frac{v_0}{Z} \sum_{n=1}^{\infty} e^{-\gamma(n-1)d_x} + \frac{v_0}{s} \sum_{n=1}^{\infty} e^{-\gamma 2n d_x} \],

(71)

The inverse transforms of (70) and (71) are obtained as

\[ V_2(x,t) = V_0 \left\{ \frac{\pi}{Z} \sum_{n=1}^{\infty} \left[ e^{-\frac{\pi^2}{4} U(t-t_{n1})} + \frac{\alpha}{2} \int_{t_{n1}}^{t} e^{-\frac{\pi^2}{4} U(t-t)} dt \right] \right\},

(72)

\[ I_2(x,t) = V_0 \frac{1}{Z} \sum_{n=1}^{\infty} e^{-\frac{\pi^2}{4} U(t-t_{n2})} \frac{\alpha}{2} \int_{t_{n2}}^{t} e^{-\frac{\pi^2}{4} U(t-t)} dt \left\{ \frac{t_{n1}}{t_{n2}} \right\} + \frac{\pi}{Z} \sum_{n=1}^{\infty} e^{-\frac{\pi^2}{4} U(t-t_{n2})} \frac{\alpha}{2} \int_{t_{n2}}^{t} e^{-\frac{\pi^2}{4} U(t-t)} dt \left\{ \frac{t_{n1}}{t_{n2}} \right\},

(73)

where \( t_{n1} = \frac{2(n-1)d_x + x}{c} \) and \( t_{n2} = \frac{2nd_x - x}{c} \).

Figure 8. Magnetosphere-ionosphere-ground transmission line system in east-west and north-south symmetries. The ionospheric currents are shown with solid arrows and the ground surface currents with dotted arrows. The equatorial electric field and currents are in dawn-dusk direction due to the north-south symmetry and the electric potential vanishes in the noon/midnight meridian due to the east-west symmetry.
waves propagating from $x=0$ and the second terms represent waves reflected back from $x=d_2$. Each traveling wave is composed of the attenuated TM$_0$ mode wave and gradually increasing diffusion mode same as those in the semi-infinite transmission line. In spite of the diffusion, the transmitted electric potential can be approximated by a step function with attenuated amplitude within the time scale of our concern.

The ionospheric conductivity is substantially dependent on the latitude, i.e., solar zenith angle, but we employ the uniform ionosphere of 8000 km length to calculate temporal variations of $V_2$ and $I_2$. The simplified model calculation will clarify how the quasi-steady ionospheric currents are achieved by the TM$_0$ mode waves. Figure 9 shows $V_2$ and $I_2$ at $x=5000$ km with $V_0=50$ kV as the source electric potential using the typical ionospheric conductivity at the daytime midlatitude, 30 mho [Tsunomura, 1999]. The plots are displayed as a function of time from 0 to 10 s (upper panels) and from 0 to 0.2 s (lower panels). As seen in Figure 9a, the electric potential oscillates at high frequencies because of the propagation at the speed of light between the two boundaries. The source electric potential is first observed at 0.017 s and shortly made zero by the reflected wave from the end of the IG transmission line (lower panel of Figure 9a). The electric potential then recovers the source value at the dynamo and the back and forth propagation results in oscillations at the frequency 18.8 Hz. The TM$_0$ mode waves suffer attenuation due to the finite conductivity of the ionosphere and therefore the amplitude of oscillations decreases as shown in the upper panel of Figure 9a. The electric potential approaches a steady-state value (18.75 kV) determined by the relation,

$$V_2(x) = V_0 \left[ \frac{d_2-x}{d_2} \right]. \quad (75)$$

On the other hand, the ionospheric currents (Figure 9b) do not oscillate, but increase gradually (upper panel of Figure 9b). The electric current carried by the single TM$_0$ mode wave is obtained from $E_z$ and $Z_2$, which is a tiny fraction of the steady-state current intensity. The perfect reflections at the two boundaries double the
current and therefore the current increases step-by-step at every reflection as shown in the lower panel of Figure 9b. The steady-state current intensity (375 kA) is calculated by the relation:

\[ I_2(t, x) = \frac{w}{d_2} \Sigma \cdot V_0. \]  

(76)

5.3. Quasi-Steady Ionospheric Currents Under Day and Night Conditions

Here we calculate \( I_2 \) with other sets of height-integrated conductivity: 100, 30, 10, 3, 1, and 0.3 mho for daytime equatorial-midlatitudes and 3, 1, and 0.3 mho for nighttime low-midlatitudes [Tsunomura, 1999]. Figure 10a shows temporal variations of \( I_2 \) in the same format as in Figure 9. It is observed that the current starts to increase instantaneously but increases gradually with the time constant larger for larger ionospheric conductivity. The steady-state current intensity is larger for larger ionospheric conductivity, although the steady-state electric potential (18.75 kV) is common in all cases.

We apply the lumped-element LR circuit analogy to the IG transmission line similarly to the MI transmission line except that \( R \) is the resistance of the transmission line itself instead of the load resistor. Thus, \( L \) and \( R \) are proportional to the length of the transmission line as given below:

\[ L_T = \mu_0 d_2 \frac{h}{w}, \]

\[ R_T = \frac{d_2}{wL}. \]  

(77)

Figure 10. (a) The electric currents at \( x = 5000 \) km in the finite-length IG transmission line with the conductivity of 100, 30, 10, 3, 1, and 0.3 mho, and (b) currents in the lumped-element circuit with 100, 10, and 1 mho. The time constant is longer as the conductance increases. The lumped-element circuit reproduces the current in the IG transmission line.
The electric current in the circuit is readily obtained as below:

\[ I_2(t) = \frac{V_0}{R_T} \left( 1 - e^{-\frac{R_T}{C_1}} \right). \tag{78} \]

The temporal variations for 100, 30, and 1 mho are shown in Figure 10b, which are exactly identical with those of the IG transmission line. The exact time constant in which the current reaches 63% of the steady current is obtained from (78) as

\[ T = \frac{L_T}{R_T} = \mu_0 h \Sigma. \tag{79} \]

It is to be noted that the time constant of the IG transmission line is not proportional to the length of the transmission line, but proportional to the height of the ionosphere as well as to the height-integrated ionospheric conductivity.

We now have the exact time constant for the six cases in Figure 10a as below:

\begin{align*}
T &= 12.6 \text{ s for } \Sigma = 100 \text{ mho} \\
T &= 3.78 \text{ s for } \Sigma = 30 \text{ mho} \\
T &= 1.26 \text{ s for } \Sigma = 10 \text{ mho} \\
T &= 0.38 \text{ s for } \Sigma = 3 \text{ mho} \\
T &= 0.13 \text{ s for } \Sigma = 1 \text{ mho}. \tag{80}
\end{align*}

For the propagation from the polar to equatorial latitudes, we may apply 100 mho while 10 mho to the low latitude. Thus, the difference in the time constant between the low latitude and equator of about 11 s well accounts for the observed delay in the peak time of the magnetic impulse at the equator (10 s) reported by Kikuchi et al. [1996]. For the storm/substorm geomagnetic disturbances, the time delay is almost nothing so that the transmission from the polar ionosphere to the equator can be considered to be instantaneous. The short time constant enables us to use the steady-state current continuity equations in calculating the distribution of transmitted electric field and currents in the global ionosphere [Maekawa and Maeda, 1978; Nopper and Carovillano, 1978; Tsunomura and Araki, 1984]. In the magnetosphere-ionosphere-ground transmission line system (Figure 2), a substantial amount of electromagnetic energy dissipates in the low-latitude ionosphere, which can be comparable to that dissipated in the polar ionosphere. In other words, the mid-equatorial latitude ionosphere is a load for the FACs during geomagnetic disturbances of our concern.

6. Discussion

6.1. Excitation of the TM_0 Mode Wave in Conflict With the Fukushima’s Theorem

According to the Fukushima’s theorem [Fukushima, 1969, 1976], no magnetic fields are produced by the field-aligned currents below the ionosphere. This is because the magnetic fields due to the FACs are canceled by those due to the Pedersen currents in the uniform ionosphere. The Fukushima’s theorem is useful to understand that the ground magnetic disturbances at high latitudes are mostly caused by the ionospheric Hall currents. The TM_0 mode wave scenario seems to conflict with the Fukushima’s theorem, but it should be noted that Fukushima assumed a priori no currents in the space below the ionosphere [Fukushima, 1976].

This assumption may be due to the single FAC model, which never closes with the ground surface currents (Figure 11a). Applying the single FAC results, Fukushima concluded that the pair of FACs does not produce magnetic fields below the ionosphere [Fukushima, 1969].

In the present paper, however, we started with a pair of FACs and showed that the current circuit is completed between the ionosphere and ground via the wavefront of the propagating TM_0 mode wave in the transient stage (Figure 11b) and via the equivalent conducting sheet in the noon-midnight meridian in the steady state as discussed below. The displacement currents at the wavefront (thick vertical dotted lines in the Figure 11b) propagate at the speed of light horizontally to low latitudes carrying the positive/negative potential to the right/left in the figure. It should be emphasized that the current closure is achieved by the pair of FACs and the two wavefront currents with opposite directions. One may say that the single FAC assumed by Fukushima [1976] is an extreme case of the
A pair of FACs attained by increasing the spacing between the two FACs to infinity. However, the characteristic impedance of two line conductors, \( Z \), increases to the infinity as the spacing increases to infinity as below:

\[
Z = \frac{Z_0}{\pi} \log_2 \frac{D}{a}
\]

where \( a \) and \( D \) denote the radius of conductors and the spacing, respectively. The infinite impedance leads to zero current intensity, i.e., no currents flow in the single FAC. The single FAC model would help us to understand the ground magnetic disturbances at high latitude, but we should start with the pair of FACs to understand the transmission of electromagnetic energy to low latitude.

### 6.2. Steady-State MIG Currents

We now examine whether the MIG current circuit is achieved by the TM0 mode waves in a steady state. Under the condition of the east-west symmetry in the transmission line system, the electric potential vanishes on the noon-midnight meridian which can be replaced with a perfectly conducting sheet (Figure 8). Therefore, as shown in Figure 12, the ionosphere and ground surface are connected by the conducting sheet in the noon-midnight meridian. As a result, the ionospheric currents driven by the

![Figure 12](image-url)
positive and negative potentials are connected with the ground surface currents by the currents on the conducting sheet. The sheet currents cancel each other and net currents are zero. Consequently, a steady current circuit is achieved between the ionosphere and ground. The ground surface currents are carried by the TM$_0$ mode waves but can be considered as return currents of the ionospheric currents via the conducting sheet in the noon-midnight meridian.

Sutcliffe and Lühr [2010] applied the EIW model with the daytime ionospheric condition to understand the observations that the Pi2 is observed on the ground but not by the CHAMP satellite orbiting at the altitude of 400 km. Under the condition of high ionospheric conductance to Alfvén conductance ratio, the magnetic field above the ionosphere given by (27) is much less than that observed on the ground. Their observations validate the MIG circuit model. The MIG current circuit can also be applied to understand the features of the geomagnetically induced currents (GIC) at mid-equatorial latitudes. Brändlein et al. [2012] found that the GICs at midlatitude manifest diurnal and seasonal variations. They explained these features by applying the current circuit carried by the TM$_0$ mode wave. The MIG current circuit studied in the present paper more convincingly explains that the GIC is connected with the quasi-steady ionospheric currents which undergo the diurnal and seasonal variations. Moreover, the equatorial enhancement of the GIC has been reported by Pulkkinen et al. [2012]. This result also indicates close connection between the ionospheric currents and GIC. Near the dip equator, the ionospheric currents are intensified by the Cowling effect [Hirono, 1952], which would flow into the ground through the equivalent conducting sheet in the noon-midnight meridian (Figure 12). The MIG current circuit allows both the rapidly changing and quasi-steady ionospheric currents to flow into the ground, but it should be noted that the GIC is largely affected by ground conductivity distribution [Pulkkinen et al., 2012].

6.3. Electric Field Response of the Low-Latitude F Region and Inner Magnetosphere

The rapidly changing electric fields such as the PI and MI (10 s–10 min) as well as long-lasting convection electric fields (10 min–a few hours) are potential fields in the ionosphere of which direction is opposite on the day- and nightsides. The potential electric field in the ionospheric E region is mapped upward into the F region and inner magnetosphere along the magnetic field lines with no appreciable attenuation as predicted by (28). Near-instantaneous transmission of the polar electric field to the equatorial F region via the low-latitude E region certifies the quick response of the equatorial F region plasma to the change in the polar cap potential [Kikuchi et al., 2003]. This mechanism would work during the geomagnetic storms, causing the anomalous enhancement in the total electron content [Maruyama et al., 2004] and intensification of the equatorial ionization anomaly [Mannucci et al., 2005].

Furthermore, the electric field transmits to the inner magnetosphere. Nishimura et al. [2010], using the cluster satellites, found that the PI of SC was of small magnitude (0.5 nT) compared to the PI on the ground (15 nT), but the electric field of the PI was fairly strong (2 mV/m). Nishimura et al. [2010] also found that the Poynting flux was directing upward from the ionosphere, which must be a fraction of the Poynting flux carried by the TM$_0$ mode wave in the Earth-ionosphere waveguide. Strong electric fields have been detected deep inside the magnetosphere (L = 2–3) during the main phase of geomagnetic storms [Wygant et al., 1998; Shinbōri et al., 2005; Nishimura et al., 2006]. The development of the electric field in the magnetosphere is immediate after the increase in the polar cap potential [Nishimura et al., 2009], which leads to the quick development of the partial ring current responsible for the decrease in the ground magnetic field [Hashimoto et al., 2002].

6.4. Time Constant for the Steady Currents

When the IMF turns southward abruptly, it takes more than 10 min for the convection electric field to achieve the quasi-steady electric field depending on the local time [Murr and Hughes, 2001; Lu et al., 2002]. Murr and Hughes [2001] indicated that the time constant is larger as the station leaves from the noon meridian toward the nightside. Based on the MI transmission line scenario, the time constant depends on the length of the field line and the ionospheric conductance (61). With d = 80,000 km and Σ = 8 mho, we have obtained the time constant of 804 s comparable to the observed values. The time constant would be larger on the nightside, where the field line is longer and the ionospheric conductivity is larger at auroral latitude than on the dayside. These conditions lead to longer time constant as the observation point moves from the noon meridian to the midnight.
The electric potential is transmitted to the low-latitude ionosphere by the TM₀ mode wave at the speed of light in the EIW or IG transmission line. As a result, the onset of the PI is nearly simultaneous at high latitude and equator [Araki, 1977; Kikuchi, 1986]. On the other hand, Kikuchi et al. [1996] reported that an impulsive magnetic increase starts simultaneously at both latitudes, but the peak is delayed at the equator by more than 10 s. Kikuchi et al. [1996] pointed out that the high conductivity of the dayside equatorial ionosphere may cause an inductive effect on the electromagnetic wave propagating from the polar ionosphere to the equator. Further study of the PI basing on high time resolution magnetometer data (1 s) indicated that the peak of the equatorial PI was delayed by 15 s at the dip equator compared with that at the off-dip equatorial station [Kikuchi and Araki, 2002]. This result indicates that the time lag in the peak amplitude is more significant closer to the dip equator and suggests again a substantial role of the high conductivity in the time lag of the peak amplitude. Similarly, the phase of the Pi2 is delayed by 34° at the dayside dip equator as reported by Shinohara et al. [1997]. They explained the phase lag by applying the LR circuit, which is certified in the present paper by showing that the IG transmission line can be replaced by the lumped-element LR circuit (section 5).

6.5. Earth’s Curvature and 3-D Transmission System

We have not focused on the curvature of the ground and ionosphere in both the waveguide and transmission line models (Figures 1 and 3). This simplification should be valid for the short distance propagation, but the polar-to-equatorial transmission is along the spherical ground and ionosphere as shown in Figure 2. We here note that the flat plane transmission line can be converted to the spherical plane transmission line by applying the conformal mapping. We remember that the electric and magnetic fields of the TM₀ mode wave are uniform behind the wavefront as seen in (21) and (22). There are no electric charges and currents between the parallel planes, leading to that both the divergent and rotation of the electric and magnetic fields are zero and the fields satisfy the Cauchy-Riemann equation. Then, the flat planes in the complex z-plane is converted to the spherical planes in the complex w-plane by applying the conformal mapping with the function, \( w = \exp[z] \). The vertical electric field lines in the flat plane transmission line are converted to the radial lines in the spherical plane transmission line and the equi-potential lines are converted to the circle lines parallel to the spherical planes. The perpendicular relation between the magnetic field and electric field holds in the conformal mapping to the spherical transmission line. Therefore, the Poynting flux is along the spherical surface, that is, the TM₀ mode waves propagate along the spherical ground and ionosphere with no changes in their properties. In the transient stage, the wavefront currents should be parallel to the electric field along the radial lines as schematically shown in Figure 2.

We solved the transmission line equations in the uniform lossy ionosphere model. In the real MIG transmission line system, the MI transmission line is connected with the two-dimensional IG transmission plane. Therefore, we need to solve the one-dimensional transmission along the MI transmission line and two-dimensional transmission in the IG transmission plane. Such complicated system may not be solved analytically, but should be solved numerically. This issue remains for the complete understanding of the MIG transmission system.

6.6. Magnetospheric Dynamos

6.6.1. Pi Dynamo

When the solar wind shock hits the magnetosphere, the magnetopause current is intensified only in the subsolar region which never closes with any currents on the magnetopause. The solar wind shock has not hit yet. However, the transient magnetopause currents close with wavefront currents of the MHD waves, i.e., the compressional waves in the equatorial plane of the magnetosphere and the Alfvén waves along the magnetic field lines. The compressional waves carry the increase in the magnetic field to low latitude, while the Alfvén waves carry the PI electric field and FACs to the magnetic latitude of 70° as deduced from the ground magnetometer observations [Nagata and Abe, 1955; Araki, 1977; Nagano et al., 1985] and 72° from the global MHD simulations [Sinker et al., 1999; Fujita et al., 2003a]. The generation of the PI electric field has been attributed to the mode conversion from the compressional mode waves [Tamao, 1964]. It is to be noted that the mode conversion theory of Tamao [1964] is based on the vector potential only. If we replace the transient magnetopause current with a short dipole, we will have the electric scalar potential in addition to the vector potential [Kraus and Carver, 1973]. The electric scalar potential will make a potential electric field of which intensity is proportional to \( r^{-2} \) (r = distance) within the radial distance, \( d_r = 0.16\lambda \) (\( \lambda \); wavelength) from the...
source current. If we assume that the period of the PI is 100 s and the Alfvén speed of 1000 km/s, then we have $d_r = 2.5 R_E$, that is, the potential electric field prevails near the magnetopause. It is interesting to note that $d_r$ roughly equals the distance of the FACs from the magnetopause, which might indicate a possible contribution of the potential electric field on the generation of the FACs.

Araki [1994] suggested that the magnetospheric convection is enhanced during the main impulse of SC by the compression of the magnetosphere after the passage of the wavefront of the compressional wave to the magnetotail. As a result, the dawn-to-dusk electric field is enhanced and drives the DP2 type currents in high latitude extending to the dayside equator. The generation of the main impulse currents has been reproduced by the global MHD simulations [Slinker et al., 1999; Fujita et al., 2003b]. Fujita et al. [2003b] indicated that the dynamo is activated around the cusp, in a similar manner to the dynamo of the Region-1 field-aligned currents (R1 FACs) [Tanaka, 1995]. The pressure of the cusp plasma increases when the magnetosphere is compressed, generating the R1-type FACs.

### 6.6.2. R1 FACs Dynamo

Global MHD simulations have reproduced the dynamo which generates the convection electric field and R1 FACs [Tanaka, 1995, 2007; Siscoe et al., 2000]. Tanaka [1995] clarified that the magnetic reconnection at the magnetopause accumulates the sheath plasma to around the cusp and mantle region in the outer magnetosphere. The increased pressure of the accumulated plasma generates diamagnetic currents of which divergence supplies the R1 FACs. Figure 13 shows a schematic diagram of the magnetosphere-ionosphere current driven by the R1 FACs dynamo.

In the dynamo, the Lorentz force due to the dynamo current $J$ balances the pressure gradient force and inertia force as expressed below [Tanaka, 2007]:

$$J \times B = \nabla p + \rho \frac{dV}{dt}$$

(82)

where $B$ is the geomagnetic field, $p$ the pressure of plasma, $\rho$ the mass density, and $V$ the velocity of plasma. If the dynamo current is generated steadily, $dV/dt$ vanishes, which leads to that the electric field is constant whatever the amount of current is. Thus, the steady state is achieved by the voltage generator with null internal resistivity. On the other hand, if there were no plasma regime around the cusp like in Dungey [1961] model, the dynamo current is generated by the time derivative of the plasma velocity, i.e., braking of plasma motion. Under such condition, the electric field continues to decrease with time as the dynamo is supplying the current onto the ionosphere. This mechanism will not be able to maintain the steady-state currents in the MIG circuit.

The negative $J \cdot E$ leads to divergence of the Poynting flux $(S$ in Figure 13) given below:

$$-J \times E = \frac{\nabla (E \times \Delta B_\perp)}{\mu_0} = \nabla S > 0$$

(83)

where $\Delta B_\perp$ is the magnetic field associated with the dynamo current [Iijima, 2000]. The Poynting flux transmits to the ionosphere with the potential difference and field-aligned currents. The convective motion of plasma is equivalent to the divergence of electric field, i.e., electric charges at the center of the convection as expressed by the following relation:

$$\left(\frac{1}{B^2}\right) \nabla \cdot E = -\frac{B}{B^2} \cdot (\nabla \times v).$$

(84)

Thus, the FACs are surrounded by the magnetospheric and ionospheric convection. It should be noted that the ionospheric convection does not reach its steady state immediately because of the reflection at the ionosphere, but the steady state is reached after the time constant determined by the length of the magnetic field lines and the ionospheric conductivity as shown in section 4.

### 7. Conclusion

The present paper constructed the magnetosphere-ionosphere-ground (MIG) transmission line system, which explains the transmission of electromagnetic energy from the magnetospheric dynamo to the
ionosphere at mid-equatorial latitudes. The MI and IG transmission lines in the MIG transmission line system replace the pair of magnetic field lines (FACs) and the Earth-ionosphere waveguide, respectively. The MI transmission line is made of a finite-length conducting parallel planes, and the IG transmission line is a finite-length lossy parallel planes. With the MIG transmission line system, we addressed the following issues critical for understanding the equatorial magnetic disturbances which are well correlated with high-latitude disturbances:

1. The instantaneous transmission of the polar electric field to the equator inferred from the simultaneous onset of the equatorial PI [Araki, 1977] has been explained by means of the TM₀ (TEM) mode waves propagating at the speed of light in the Earth-ionosphere waveguide [Kikuchi et al., 1978; Kikuchi and Araki, 1979b]. The excitation mechanism of the TM₀ mode waves remained an issue, but in this paper we clarified that a fraction of the electric and magnetic fields are transmitted from the MI to IG transmission lines through the polar ionosphere and excite the TM₀ mode wave. The source magnetic field for the TM₀ mode wave is given by the FACs below the ionosphere, contradicting with the Fukushima’s theorem [Fukushima, 1976]. The Fukushima’s theorem states that the FACs make no magnetic fields on the ground, but a pair of FACs provides magnetic fields which propagates far to the equator.

Figure 13. Schematic diagram for the relationship between the Region-1 field-aligned currents and the plasma convection in the magnetosphere and ionosphere. The dynamo currents are diamagnetic currents flowing at the surface of the high-pressure plasma (shaded region), the divergence of which drives the R1 FACs into the polar ionosphere [Tanaka, 2007].

ionosphere at mid-equatorial latitudes.
2. The TM₀ mode wave carries the ionospheric and ground surface currents which are connected by the wavefront current in the transient stage [Kikuchi et al., 1978]. On the other hand, steady ionospheric currents should be supplied by the field-aligned currents which satisfy the current continuity equation in the global ionosphere [Maekawa and Maeda, 1978; Nopper and Carovillano, 1978; Tsunomura and Araki, 1984]. By assuming the east-west symmetry of the MIG transmission line, we succeeded to give a boundary condition on the far end of the finite-length IG transmission line and clarified that consecutive propagation of the TM₀ mode waves in the finite-length lossy transmission line achieves the steady ionospheric currents with the time constant of 1–10 s. The ionospheric currents are proportional to the ionospheric conductivity, consistent with the high current density in the dayside equatorial ionosphere causing the equatorial enhancement of the PI and MI of SC, DP₂, and so on.

3. The gradual increase in the ionospheric currents well explains the time lag in the peak value of the magnetic impulse reported by Kikuchi et al. [1996], while the onset of the magnetic disturbances is nearly instantaneous at all latitudes because the onset time is determined by the arrival of the first TM₀ mode wave. Actually, the observable onset time largely depends on the amplitude of the initial change of the disturbances. The high temporal resolution magnetometer data enables us to recognize the simultaneous onset and delayed peak of the PI as observed at the dip equator and off-equatorial stations [Kikuchi and Araki, 2002].

4. The magnetosphere-ionosphere (MI) current circuit should be completed by connecting with the ionosphere-ground (IG) current circuit. The combined MI current circuit well explains the diurnal and seasonal variations of the geomagnetically induced currents (GIC) at midlatitude [Brändlein et al., 2012] and the anomalous intensification of the GIC at the equator [Pulkkinen et al., 2012]. The GICs at mid-equatorial latitudes are connected with the ionospheric currents via the equivalent conducting sheet in the noon-midnight meridian of the MIG transmission line system (Figure 12).

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