# ANALYSIS OF THE DIVERSITY OF BENDING IN RIVER MEANDERING IN TERMS OF SHANNON ENTROPY

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#### EXTENDED ABSTRACT

The diversity of bending in river meandering is analyzed and calculated theoretically in this work. The diversity of bending in river meandering means not only how linearized or not a river is, but also how sufficient variations of bending sizes in its meandering a river contains.

First, we constructed the general formula for the diversity index of bending in river

meandering in terms of Shannon entropy as  $YI = \frac{\log n - (-\sum_{i=1}^{n} p_i \log p_i)}{\log n}$ , where *n* is the

number of segments on a river and  $p_i$  is given by  $p_i = \frac{s_i}{l} (s_i (i = 1, 2, \dots, n))$  is the straight

line distance between two adjacent ends of each segment). This index *YI* has the following several merits.i) *YI* satisfies the inequality  $0 \le YI \le 1$ . ii) *YI* does not depend on

the size of river nor on the number of segments. iii) When the shape of a river has a fractal

structure, the YI does not depend on the scale. iv) When values of YI for every tributary of a river are calculated, the value of YI for the whole of the river can be automatically calculated by the stratification theorem.

We calculated the bending diversity for several rivers in Hokkaido, Japan, at two different times, namely 1950 and 2000, using this index YI on Geographical Informational System (GIS). We have found that a lot of rivers in Hokkaido, Japan, have lost the bending diversity of meandering for these fifties year, and the bending diversity of meandering for several rivers are quite different from each other. And we have found from these results that the index YI can detect the difference between these rivers' meandering more highly than just an index of the rates of river bending.

We believe this index YI will serve ecological analysis including the analysis of biodiversity of hydrobiology worldwide.

**KEYWORDS:** the index *YI*, the diversity of bending in river meandering, Shannon entropy, GIS (Geographical Informational System).

### 1. INTRODUCTION

Geodiversity, including geographical diversities of various kinds, has attracted increased attention recently. Closely related to biodiversity, geodiversity is believed important in ecological research. In a previous work we (T.Y.) analyzed pedodiversity in terms of Shannon entropy. Pedodiversity is a geodiversity subspecialty dealing with the diversity of soil distribution. Now we believe that the geomorphological features of a river, that is a fluvial form, might also have an important influence on the biodiversity of hydrobiology such as various kinds of fishes. A growing number of ecologists argue that artificial linearization or channelization of a river might reduce the biodiversity in the river because the monotonous geographical environment, they believe, would not allow a lot of creatures to live in the river. Indices for measuring the rates of river bending have been proposed and used in calculation for several rivers. These indices can designate only how a river has artificially linearized or channelized. However, we believe that if a river contains sufficient variations of bending sizes in its meandering, then a wide variety of hydrobiology would be able to live in the river. Given that a river has a variety of rapids and trenches in its bending and that different kinds of hydrobiology inhabit each spot, we have supposed that not only linearization of a river but also the monotonous repetition of the same bending of a river should bring about less biodiversity. Currently, however, no methodology is available for measuring the bending diversity in the meandering of a river. In this work we propose how to quantitatively measure the diversity of river bending. The present study was conducted to construct an index for detecting the diversity of river bending in terms of Shannon entropy and to calculate the bending diversity for several rivers in Hokkaido, Japan, using this index YI on Geographical Informational System (GIS).

#### 2. FORMULATION

First, we divided a river into *n* segments having equal length *l* along the line of the river and we numbered the segments sequentially. Second, we measured the straight line distances  $s_i$  ( $i = 1, 2, \dots, n$ ).between two adjacent ends of each segment.(See Fig.1) The calculations were performed according to our following index *YI*:

$$YI = \frac{\log n - (-\sum_{i=1}^{n} p_i \log p_i)}{\log n}, \dots$$
(1)

where n is the number of segments on a river and  $p_i$  is given by the following definition;

$$p_i = \frac{s_i}{l} \dots$$
 (2)

The value of index YI satisfies the following inequality:

$$0 \le YI \le 1 \tag{3}$$

When every  $p_i$  has the same value  $\frac{1}{n}$ , Shannon entropy  $\left(-\sum_{i=1}^{n} p_i \log p_i\right)$  in the index *YI* has maximum value  $\log n$  and the index *YI* takes 0. According to the definition (1), the value of index *YI* generally depends on the segment length l. This index allows us to evaluate how much bending diversity is contained in the river meandering on a given scale (the segment length l). The index *YI* indicates 0, not only in the case of a river that does not meander, that is, where the shape of the river is a straight line, but also in the case of a river that contains all the same bending, because every  $p_i$  has the same value

 $\frac{1}{n}$  and  $-\sum_{i=1}^{n} p_i \log p_i = \log n$  in both cases. l  $S_2$  l  $S_3$  l  $S_4$  l  $S_4$  l  $S_4$  l  $S_3$ 

Figure 1. Elements for the index YI

The index *YI* has several merits for determining the diversity of a river's bending:

- 1) *YI* does not depend on the size of river nor on the number of segments, and the index will take similar values for rivers that are similar in two-dimensional shapes
- 2) When the two-dimensional shape of a river has a fractal structure, *YI* does not depend on the scale (the segment length).
- 3) When values of YI for every tributary of a river are calculated, the value of YI for the whole of the river can be automatically calculated by the stratification theorem.

1) and 2) can be seen right away by the definition (1), and 3) can be seen by the stratification theorem which has been found in our research.

If the index YI is calculated separately for every tributary of a river, the overall index YI of the river can be readily obtained based on what we termed the stratification theorem. The mathematical formula of this stratification theorem is given as follows. (See Fig.2)

$$YI_{total} = 1 + \frac{1}{\log(m_1 + \dots + m_n)} \left\{ \sum_{i=1}^n q_i \log q_i + \sum_{i=1}^n q_i (YI_i - 1) \log m_i \right\}, \quad \dots (4)$$

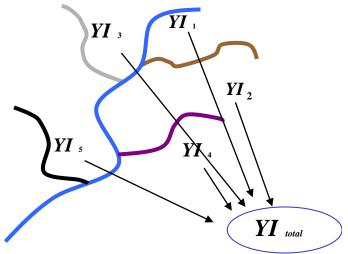


Figure 2. Stratification theorem

where  $m_i$  is the number of segments in each tributary and  $YI_i$  is the index of each tributary, and  $q_i$  is given by the following formula.

$$q_i = \frac{S_i}{\sum_{i=1}^n S_i} \quad \cdots \quad (5)$$

Where  $S_i$  is the sum of the straight line distances s in each tributary.

### 2. RESULTS

We have calculated the bending diversity for several rivers in Hokkaido, Japan, using the index YI on Geographical Informational System (GIS). Several results are shown as follows.

#### 1) Comparison between two different rivers

We calculated the index *YI* for two typical rivers in Hokkaido, Japan, namely the Kushiro River and the Chitose River. Kushiro River flows near the Shiretoko Peninsula, which has been registered as a World Heritage site, and Chitose River flows near Sapporo city, the capital of Hokkaido with a city population of about two million. Fig.3 and Fig.4 are GIS graph of both rivers. We have obtained the result that the index *YI* for whole Kushiro River is greater than that for whole Chitose River (See Fig.6), and we have found from this result that the index *YI* can detect the difference between these rivers' meandering more highly than just an index of the rates of river bending given by the following formula.(See Fig.5)

$$K = \frac{1}{n} \sum_{i=1}^{n} \frac{s_i}{l},$$
 (6)

This formula means the average of bending rates on each segment.

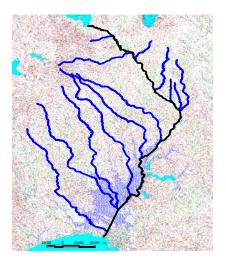


Figure 3. Kushiro River

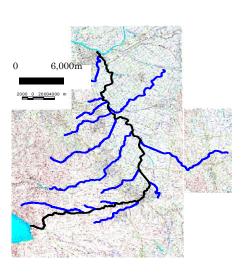
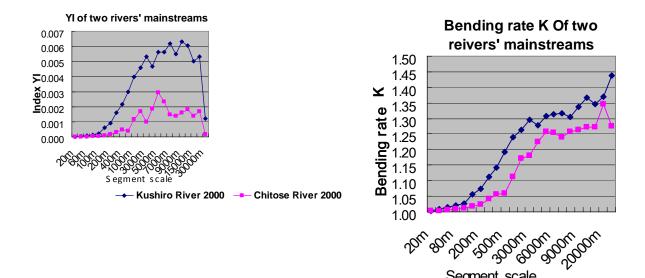


Figure 4. Chitose River

Segment scale





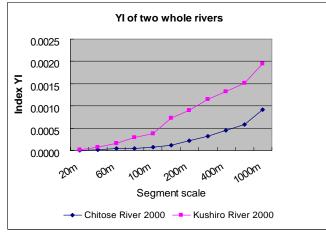


Figure 6. YI of two whole rivers

### 2) Comparison between two different times

We calculated the index YI of two different times, namely 1950 and 2000, for almost all rivers in Hokkaido, Japan. (See a sample in Fig.7) We have found from these results that most rivers in Hokkaido have lost the bending diversity of meandering for these fifties year.

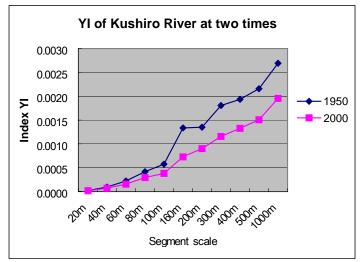


Figure 7. YI at two different times

## 4. CONCLUSION

The results of this study document that our new index YI provides a useful method for measuring the bending diversity of river meandering and suggest that rivers characterized by diverse bending would provide an ecological environment sufficiently conducive for the habitation of many creatures to live in the river.

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