Proceedings of the 10th International Conference on Environmental Science and Technology
Kos island, Greece, 5 – 7 September 2007

ANALYSIS OF THE ENERGY EFFICIENCY OF PHOTOSYNTHESIS IN TERMS OF ENTROPY

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EXTENDED ABSTRACT

The energy efficiency of photosynthesis is analyzed and calculated theoretically in this work. First, the actual energy efficiencies η of photosynthesis on the incident light were calculated for various plants on the basis of the quantum yield. The results of our calculation for various plants show that the energy efficiencies η of photosynthesis are in the region of ± 2 or 3% of 20%, depending on the kind of plant. Second, we constructed the general formula for the maximum energy efficiency of an engine in terms of entropy as $\eta_{MAX} = 1 - \frac{T_{out}(\Delta S_{in} - \Delta S_W)}{E_{in}}$,where η_{MAX} is the maximum energy efficiency, E_{in} is the input energy, T_{out} is the temperature of the outside of the engine, ΔS_{in} is the entropy flow into the engine through the input energy E_{in}, and ΔS_W is the entropy flow fixed in the output energy. Using this formula, we formalized, in terms of entropy, how to calculate the theoretically maximal efficiency η_{MAX} of photosynthesis on the incident sunlight whose absorption spectra data are given. In our formula, the increase of entropy in the sunlight owing to the expansion during travel of the sunlight from the sun to the earth is estimated theoretically, and maximum efficiency η_{MAX} is given by the formula $\eta_{MAX} = \frac{\eta^{Carnot} - \eta^{*}}{1 - \frac{T_{out}\Delta S_W}{W_{out}}}$,
where W_{out} is the chemical energy (enthalpy) fixed in glucose, ΔS_W is the entropy fixed in it, and T_{out} is the ambient temperature, η^{Carnot} is the Carnot efficiency, and η^* is the amount of efficiency decrease caused by expansion of the sunlight. We calculated the maximum efficiencies of various plants, and our results show that many of the plants are about 75% efficient. Therefore, it can be generalized that the photosynthesis efficiency of many present-day plants will be potentially increased from about 20% to about 75%.

Keywords: photosynthesis, entropy, actual energy efficiency η of photosynthesis, maximum efficiency η_{MAX} of photosynthesis on the incident sunlight

1. INTRODUCTION

Photosynthesis has greatly contributed to the environment of the biosphere of our planet in a twofold way. One is as a resource of energy for almost all life on earth, and the other is as a sink for carbon dioxide, which is believed to be responsible for global warming. Therefore, it is very important and useful to analyze the energy efficiency of photosynthesis quantitatively.

The present study was conducted to analyze the energy efficiency of photosynthesis and to calculate the energy efficiency of photosynthesis for a variety of plants. In this study, photosynthesis is regarded as an engine performed by a plant, in which the input energy E_{in} is the energy of absorbed light, and the output energy W_{out} is the chemical energy, that is enthalpy, fixed in glucose (see Figure 1), and the energy efficiency η is given by the formula $\eta = \frac{W_{out}}{E_{in}}$.

First, the actual energy efficiency of photosynthesis is calculated on the basis of the quantum yield.

Second, we calculated the theoretically maximal efficiency η_{MAX} of photosynthesis on the incident sunlight whose absorption spectra data are given. In this calculation, the maximum energy efficiency is analyzed not in terms of free energy but in terms of entropy, because the effect of the increased entropy in the incident sunlight owing to the expansion during travel of the sunlight from the sun to the earth should be estimated.(see Figure 2)

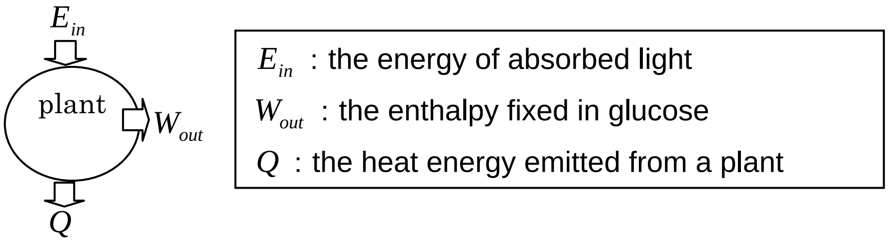


Figure 1: Engine in photosynthesis

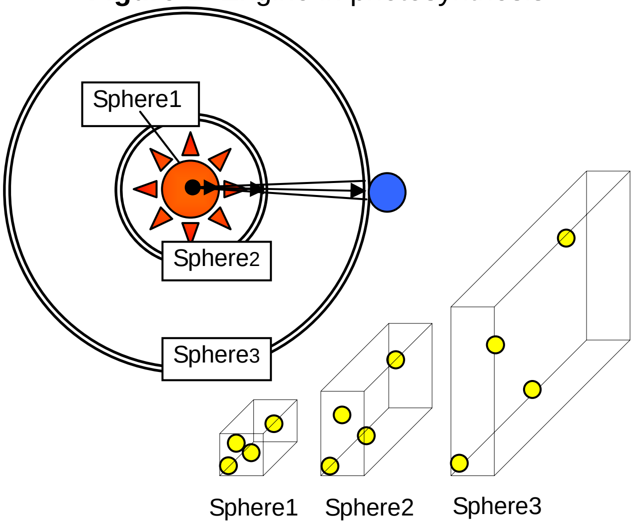


Figure 2: Increased entropy of photons owing to expansion during the sunlight travel

2. THE ACTUAL ENERGY EFFICIENCY of PHOTOSYNTHESIS

The actual energy efficiencies η_i of photosynthesis on the incident light with the wavelength λ_i were calculated for the various plants on the basis of the quantum yield. The calculations were performed according to the following formula:

$$\eta_i = \frac{\Phi_i \lambda_i \Delta H}{6 N_A hc}, \qquad (1)$$

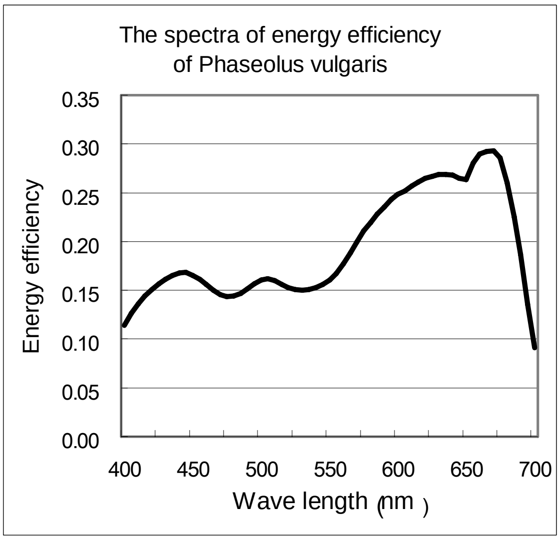
where λ_i is the wavelength of absorbed light, Φ_i is the quantum yield of the plant that absorbed the light with the wavelength λ_i, ΔH is the variation of enthalpy, N_A is the

Avogadro constant, h is the Plank constant, and c is the velocity of light . The results of our calculation for two kinds of plants are shown in Figure 3.

From the formula above, the total energy efficiency η of photosynthesis can be derived as follows:

$$\eta = \frac{\sum_i \eta_i I_i A_i}{\sum_i I_i A_i} \qquad (2)$$

where I_i is the energy of light with the wave length λ_i and A_i is the rate of absorption of light with the wave length λ_i. The results of our calculation for various plants show that total energy efficiencies η of photosynthesis are in the region ($\pm$2 or 3% of 20%), depending on the kind of plant.



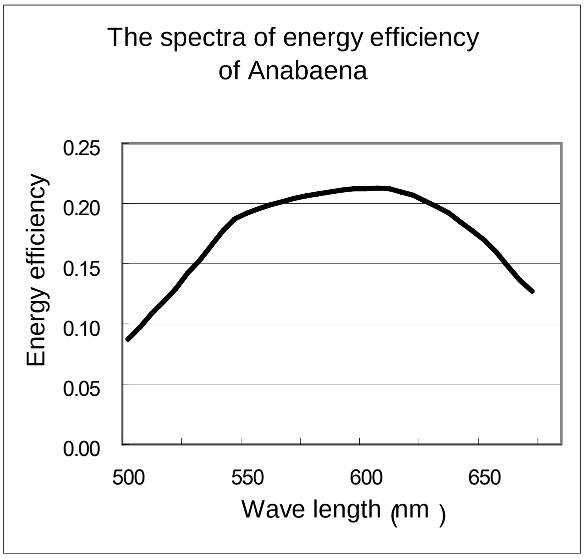


Figure 3. The spectra of energy efficiency

3. THE THEORETICALLY MAXIMAL EFFICIENCY η_{MAX} of PHOTOSYNTHESIS

In order to calculate the maximum energy efficiency of photosynthesis, we constructed the general formula for the maximum energy efficiency of an engine in terms of entropy, as follows:

$$\eta_{MAX} = 1 - \frac{T_{out}(\Delta S_{in} - \Delta S_W)}{E_{in}} \tag{3}$$

where, η_{MAX} is the maximum energy efficiency, E_{in} is the input energy, T_{out} is the temperature of the outside of the engine, ΔS_{in} is the entropy flow into the engine through the input energy E_{in}, and ΔS_W is the entropy flow fixed in the output energy. In the case of photosynthesis, E_{in} means the energy of absorbed light into a plant, ΔS_{in} is the entropy flow into the photosynthesis system via absorbed sunlight, and ΔS_W is the entropy flow fixed in the glucose, which is the output energy W_{out} in photosynthesis.
As is commonly known, the entropy in the light is estimated by the following formula on the basis of quantum physics:

$$S = k_B \sum_i G_i \{(1+f_i)\log(1+f_i) - f_i \log f_i\} , \tag{4}$$

where f_i is the average number of photons with the wavelength λ_i, and G_i is the number of the quantum states of photons with the wavelength λ_i. In the process of photosynthesis, the entropy flow ΔS_{in} contained in the photon flow with the number ΔN_i into the system with the wavelength λ_i is given by the following formula:

$$\Delta S_{in} = k_B \sum_i \log\left(1 + \frac{1}{f_i}\right) \Delta N_i . \tag{5}$$

When absorbed light in the plant is in the equilibrium with black body radiation with the temperature T_{SUN}, f_i is given by

$$f_i = \frac{1}{\exp\left(\frac{hc}{k_B T_{SUN} \lambda_i}\right) - 1} . \tag{6}$$

By using the formulae (3), (5),(6), we arrive, finally, at the following formula:

$$\eta_{MAX} = 1 - \frac{T_{out}}{T_{SUN}} . \tag{7}$$

This formula is purely Carnot efficiency.

Actually the sunlight is not in equilibrium with black body radiation with the temperature T_{SUN} and its entropy should be increased when it reaches the earth, because it is expanded in its travel from the sun to the earth. In our research, the increase of entropy in the sun light owing to the expansion has been estimated quantitatively, and its estimation has enabled us to calculate the maximum energy efficiency of photosynthesis.

We have found that the total entropy ΔS_{in} in the sunlight which is absorbed into a plant on the earth is given by the following formula:

$$\Delta S_{in} = \sum_i \frac{hc\Delta N_i}{T_{SUN} \lambda_i} + k_B \left\{ \sum_i \log\left(\frac{D^2}{R^2} - \frac{D^2 - R^2}{R^2} \exp\left(-\frac{hc}{k_B T_{SUN} \lambda_i}\right) \right) \Delta N_i \right\}, \tag{8}$$

where R and D is the radius of the sun and the distance from the sun to the earth respectively, and ΔN_i.is the absorbed photon number with the wavelength λ_i. The first term of (8) is the entropy in equilibrium with black body radiation with the temperature T_{SUN}, and the second term means the increased entropy owing to the expansion of the sun light.
The formula (8) can be rewritten in the follow form

$$\Delta S_{in} = k_B \sum_i \log\left\{1 + \frac{1}{\alpha}\left(\exp X_i - 1\right)\right\}, \tag{9}$$

with

$$\alpha \equiv \frac{R^2}{D^2} \quad \text{and} \quad X_i \equiv \frac{hc}{k_B T_{SUN} \lambda_i}. \tag{10}$$

Whereas the radiation intensity is given as follows, by the Plank Radiation Law:

$$I(\lambda_i, T) = \frac{8\pi hc}{\lambda_i^5 \left\{\exp\left(\frac{hc}{k_B T \lambda_i}\right) - 1\right\}}, \tag{11}$$

where $I(\lambda_i, T)$ is the radiation intensity of the photon with the wavelength λ_i in the temperature T. We can suppose that when the sunlight reaches the earth, the radiation intensity of the sunlight is attenuated at $\frac{R^2}{D^2}$ times. Therefore we can obtain the following formula:

$$I(\lambda_i, T_{Di}) = \frac{R^2}{D^2} I(\lambda_i, T_{SUN}), \tag{12}$$

where T_{Di} is the effective temperature of the sunlight on the earth.

From (12) we can obtain the following formula of the effective temperature T_{Di}:

$$T_{Di} = \frac{X_i}{\log\left\{1 + \frac{1}{\alpha}(\exp X_i - 1)\right\}} T_{SUN}. \tag{13}$$

From (9) and (13), the following formula is obtained:

$$\Delta S_{in} = \sum_i \frac{h \frac{c}{\lambda_i} \Delta N_i}{T_{Di}}, \tag{14}$$

which can be rewritten as

$$\Delta S_{in} = \sum_i \frac{E_i}{T_{Di}}. \tag{15}$$

where $E_i = h\frac{c}{\lambda}\Delta N_i$ is the sunlight energy with the wavelength λ_i.

The formulation (15) has coincided with the thermodynamic definition of entropy.

From the formula (3) and (8) and by a little calculation, the theoretically maximum energy efficiency of photosynthesis is given by the following formula:

$$\eta_{MAX} = \frac{\eta^{Carnot} - \eta^{*}}{1 - \frac{T_{out} \Delta S_{out}}{W_{out}}}, \tag{16}$$

where W_{out} is the output energy fixed in glucose, η^{Carnot} is the Carnot efficiency, and η^{*} is the decreasing part of efficiency owing to the expansion of the sunlight, which is given by the following formula:

$$\eta^{*} = \frac{k_B T_{out}}{hc} \cdot \frac{1}{\sum_i \frac{\Delta N_i}{\lambda_i}} \sum_i \Delta N_i \left\{\log\left(\frac{D^2}{R^2} - \frac{D^2 - R^2}{R^2} \exp\left(-\frac{hc}{k_B T_{SUN} \lambda_i}\right)\right)\right\}. \tag{17}$$

From the formula (16),(17) with the several necessary data, we have obtained the values of the maximum energy efficiencies of photosynthesis of several plants. The results of our calculation for various plants show that the theoretically maximum energy efficiencies η_{Max} of photosynthesis are in the region ($\pm$ 2 or 3 % of 75%), depending on the kind of plant.

4. CONCLUSSION

In our research, we have calculated the efficiencies of photosynthesis. Firstly, we calculated the actual efficiencies of various plants by using the data of quantum yield and we have got the result that they are about 20% for many plants. Secondly, we formalized, in terms of entropy, how to calculate the theoretically maximal efficiency of photosynthesis and calculated the maximum efficiencies of various plants, and our results show that many of the plants are about 75% maximum efficient. Therefore, it can be generalized that the photosynthesis efficiency of many present-day plants will be potentially increased from about 20% to about 75%. (see Figure 4)

	Actual energy efficiency	**Maximum energy efficiency**
Phaseolus vulgaris	0.230	0.761
Several plants average	0.164	0.762
Anabaena	0.182	0.764

Figure 4: The calculation results of Actual energy efficiencies and Maximum energy efficiencies

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