Texture Modelling by Optimal Gray Scale Structuring Elements Using Morphological Pattern Spectrum

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Abstract

This paper proposes a novel texture modelling method based on the pattern spectrum. The pattern spectrum is a mathematical morphological method to describe the size distribution of objects contained in an image. Our method is based on the idea of obtaining a model of the elementary particles that form a texture by optimizing a gray scale structuring element to fit the shape of elementary particles. The optimization method is applied in two stages: The first stage optimizes the extent of structuring element and the second optimizes the pixel values in the extent.

1. Introduction

The pattern spectrum [1] is an application of the mathematical morphology [2][3]. The pattern spectrum extracts the size distribution of objects contained in an image by decomposing the target image into objects of various sizes whose shapes are similar to a small object called structuring element, as well as the Fourier spectrum decomposes a signal into a set of sinusoidal functions.

We investigate a method of modelling of textures using the pattern spectrum in this paper. A texture is defined as an image that is a set of particles and that is mainly characterized by shapes and density of the particles. An application of the pattern spectrum to the texture analysis has been proposed by Dougherty *et al.* [4]. They prepare structuring elements of some typical shapes, for example circle, line segment, etc., and calculate pattern spectra by each structuring element. They divide the target texture into some segments each of which belongs to a class of typical shape, by measuring characteristics of each pattern spectrum. This method is effective for segmentation of textures, however, it does not describe characteristics of textures but only classifies textures into a limited number of typical shapes that are prepared and fixed before the analysis.

Our method describes a texture without the limitation of prepared typical shapes. The basic idea of our method is finding a structuring element of a limited extent which fits the particle in the target texture best. Textures in natural scenes often contain particles of a shape at various sizes, since the shapes of particles depend on the materials of which the entities of textures are made. The spectral values of pattern spectrum are highly dependent on the shape of structuring element that is used for the calculation of the pattern spectrum. If the structuring element is similar to the particles in the target texture, the spectral value, normalized by the whole area of the particles in the texture, of each size may be almost similar. Now suppose that the pattern spectra of the texture are calculated by structuring elements of various shapes. The structuring element where the smallest variance of spectral values is obtained is almost similar to the shape of particles in the texture, and regarded as modelling the shape of particle best. Thus we optimize the shape of structuring element by the criterion of minimizing the

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variance of spectral values using the method of simulated annealing, and describe the shape of particles by the shape of the optimized structuring element.

We presented in [5] our modelling method for gray scale images and binary structuring elements. In this paper, we extended our method to gray scale structuring elements to obtain more precise results of modelling. We apply the optimization method in two stages; firstly for optimization of the extent of structuring element, and secondly for optimization of the gray scale value of structuring element.

2. Pattern Spectrum

The pattern spectrum of size n by a structuring element is defined as the pixelwise difference between the target image morphologically opened by a homothetic set of structuring element of size n and that opened by structuring element of size $n+1$. Let nB be the homothetic set of a structuring element B of size n, defined as follows:

$$nB = B \oplus B \oplus \ldots \oplus B \ \ ((n-1) \text{ additions}),\ 0B = \{\mathbf{0}\} \qquad (1)$$

where $\oplus$ denotes Minkowski set addition. Then the pattern spectrum of size n by the structuring element B for image X, denoted as $\mathrm{PS}(X, B, n)$, is defined as follows:

$$\mathrm{PS}(X, B, n) = \sum_{x \in \text{whole image}} \left\{ X_{nB}(x) - X_{(n+1)B}(x) \right\} \qquad (2)$$

where $X(x)$ denotes the pixel value of image X at position x, X_B denotes the image X opened by B, and "–" sign denotes the pixelwise difference.

Since the opening removes the portion smaller than the structuring element, the difference of the images opened by the structuring elements of size n and size $n+1$ contains the portion whose size is exactly n. The normalized pattern spectrum, which is defined as the ratio of the original pattern spectrum to the sum of the pixel values over the whole original image, is often used. The spectral values of the normalized one indicate the ratio of the portions of a size to the whole image. In the following of this paper, the normalized pattern spectrum is called the size distribution because of the analogy between the normalized pattern spectrum and the probability distribution function, and the term "spectral value" means the value of the size distribution at each size.

The size distribution $\mathrm{F}(X, B, n)$ is defined as follows:

$$\mathrm{F}(X, B, n) = \frac{\mathrm{PS}(X, B, n)}{\sum_{x \in \text{whole image}} X(x)}. \qquad (3)$$

3. Modelling of textures by optimal structuring elements

3.1 idea

Textures often contain similar-shape particles of various sizes. Our method describes the shape of particles by the structuring element optimized using the size distribution. To explain our idea, we use a texture shown by Fig. 1 (a) for an example. This image contains particles of various sizes similar to a line segment from right-top to left-bottom. Figure 1 (b) illustrates the pattern spectra calculated by binary structuring elements of the line-segments of four directions. In case of the structuring element of the line-segment from right-top to left-bottom, the particles of each size are extracted and the spectral values are almost similar. In case of the other structuring elements, however, only the spectral values of smaller sizes are high.

We get from this example that the structuring element that produces the size distribution of the smallest variance should be the most suitable structuring element of four line segments for description of the shape of particles. Here the average $\mathrm{E}[\mathrm{F}(X, B, n)]$ and variance $\mathrm{V}[\mathrm{F}(X, B, n)]$ of the size

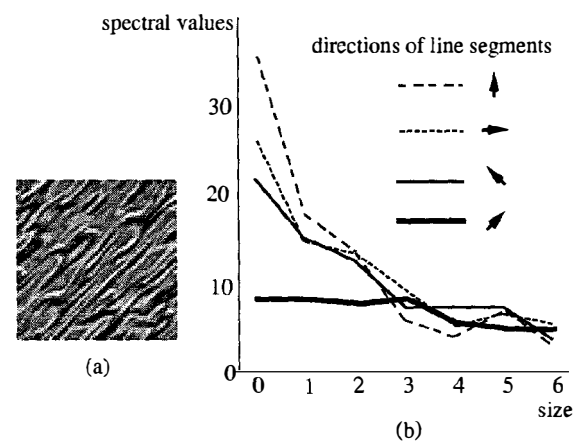


Fig. 1. (a) An example of texture. (b) Size distributions calculated by line-shape structuring elements of various directions.

distribution are defined as follows:

$$E[F(X,B,n);\,l\,] = \frac{\sum_{n=0}^{l} F(X,B,n)}{l+1}$$

$$V[F(X,B,n);\,l\,] = \frac{\sum_{n=0}^{l} \left\{F(X,B,n) - E[F(X,B,n);\,l\,]\right\}^2}{l+1} \tag{4}$$

where l denotes the upper limit of sizes for the calculation of average and variance. From this fact, we directly reach to the following idea: The structuring element which resembles the particle best is obtained by seeking the structuring element that produces the spectral values whose variance is the smallest. This is achieved by repeating a slight modification of the structuring element and the evaluation of the size distribution. Such procedure is equivalent to the optimization process of the structuring element under the criterion of the smallest variance of size distribution, and can be realized by the simulated annealing.

3.2 algorithm

Since the target texture images are gray scale ones, it is natural that the elementary particles of textures is modelled by gray scale structuring elements more precisely. We get from the definition of the morphological operations of a gray scale image by gray scale structuring element that a pixel with value zero in the extent of structuring element and a pixel out of the extent have absolutely different contributions in these operations. This means that we should optimize the extent and the pixel values separately. Thus we apply our optimization method in two stages: The first stage optimizes the extent of structuring element. The second optimizes the value of each pixel of the gray scale structuring element whose extent is optimized by the first stage.

At the first stage, we calculate at first the size distributions by the structuring elements of the line segments of fixed length and four directions, and choose an structuring element that produces the size distribution whose variance is the smallest. We use this structuring element as the initial one for the optimization process. The optimization process is described as the following set of the procedures:

1) Calculating the size distribution by the initial structuring element and the variance as the evaluation function.
2) Modifying a pixel of the structuring element. At the first stage, including or excluding a pixel within a fixed extent, for example 5x5, into / out of the structuring element.
3) Calculating the size distribution and the variance again by the modified structuring element. If the variance is smaller than that by the structuring element before modification, this modification is accepted and fixed. If the variance does not decrease, this modification is accepted with a small probability, which is determined by a function of the increment of the variance and the number of repetition. The larger increment of the variance and the larger number of iteration cause the smaller the probability of acceptance. If the modification is not accepted, it is cancelled.
4) Repeating the procedures 2) and 3) until the modification is not accepted any more.

The second stage performs almost similarly to the first stage, however, pixels are modified in different manner in the procedure of simulated annealing. In this stage, the initial structuring element is the gray scale one whose extent is optimized by the first stage and whose pixel values are all zero. The structuring element is modified by selecting one pixel and increasing the value of this pixel by one.

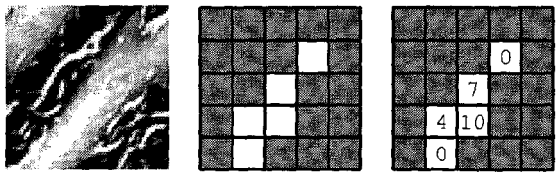
3.3 experimental results

In our experiment for optimization of gray scale structuring element we use the evaluation function ER defined as follows:

$$ER = V[F(X,B,n);\,3\,] \,/\, E[F(X,B,n);\,3\,] \tag{5}$$

in both stages. The function to determine the probability of acceptance of the modification, in step 3), denoted P(ΔER), is defined as follows:

$$\begin{cases} P(\Delta ER) = 1 & \text{if } \Delta ER < 0 \\ P(\Delta ER) = \dfrac{1}{1+\exp\left(\dfrac{\Delta ER}{T_i}\right)} & \text{if } \Delta ER \geq 0 \end{cases},$$



(a) (b) (c)

Fig. 2. Modelling of textures by the optimal gray scale structuring element. (a) example of texture image. (b) optimal binary structuring element, (c) optimal gray scale structuring element.

Table 1. Comparison of variances.

	variance
line segment	4.6101
optimal binary S. E. (Fig. 2(b))	0.2882
optimal gray scale S. E. (Fig. 2(c))	0.07116

and the "temperature" is defined as follows:

$$T_0 = 10^8,\ \ T_i = 0.998T_{i-1}\ (i \geq 1)\ . \qquad (6)$$

Figure 2 shows an example texture and the optimized gray scale structuring element of 5x5 pixels. Table 1 shows the value of $V[F(X, B, n); 3]$, which is the variance of size distributions defined in Eq. (4), for the line segment of top-right to bottom-left (the initial structuring element for the first stage), and the binary structuring element that is the intermediate result of optimization by the first stage, and the gray scale structuring element that is the final result. Figure 2(a) shows the target texture. Figure 2(b) shows the result of the first stage, i. e. optimization of the extent of structuring element. This structuring element yielded the smallest variance of our 10 trials of this stage. Figure 2(c) shows the result of the second stage, i. e. optimization of the value of structuring element, using Fig. 2(b) as the extent of the structuring element. This structuring element yielded the smallest variance of our 7 trials of this stage.

We get from Table 1 that the resultant gray scale structuring element achieves significantly smaller value of variance than the binary one. It means that the particle of texture is modelled more precisely by the gray scale one.

The range of the variances at the first stage for our 10 trials was 0.2882 – 1.073, and the output with the smallest variance was yielded twice. It means that a reliable result can be obtained by repeating the optimization several times and choosing the output with the smallest variance. The range of the variances of final results for our 7 trials was quite smaller, 0.07116 – 0.07857. The values of 7 resultant structuring elements were similar in brief, however, there were some difference in their exact values. It suggests that another criterion of optimization may be needed to obtain more precise result.

4. Conclusions

We have proposed in this paper a novel texture modelling method based on the pattern spectrum. Our method optimizes the structuring element to fit the particles under the criterion of reducing the variance of the size distribution. We have applied our method to the optimization of gray scale structuring element, and have shown experimentally that the gray scale structuring element describes the particle of textures more precisely than binary one.

Since a homothetic set nB is open respect to B if and only if B is convex [1], the extracted structuring elements should be restricted to convex ones if the value n is treated as "size." We admit non-convex structuring elements since our aim is modelling particles of textures, but this restriction is considerable for efficient optimization. Incorporation of another criterion to the optimization and applying our method to nonuniform textures, which contain particles of two or more kinds of shapes, are also open problems.

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