International Journal of Wavelets, Multiresolution
and Information Processing
Vol. 11, No. 4 (2013) 1360001 (14 pages)

DOI: 10.1142/S0219691313600011

REDUNDANT MORPHOLOGICAL WAVELET AND LOCAL PATTERN SPECTRUM

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Received 1 March 2012
Revised 4 March 2013
Accepted 20 April 2013
Published 5 July 2013

Time-frequency analysis methods such as wavelet analysis are applied to investigate characteristic from non-stationary signals. In this study, we proposed redundant morphological wavelet analysis that was a kind of nonlinear discrete wavelet and redundant wavelet. This method analyzes a transition of shape information from signals in detail since this method keeps property of shift invariance though information of decomposition includes redundancy. Local pattern spectrum which corresponds to nonlinear short time Fourier transform is derived from this nonlinear wavelet. The characteristics of these methods were confirmed by applying to simulation data and actual data.

Keywords: Mathematical morphology; redundant wavelet; pattern spectrum; vocal signal analysis.

AMS Subject Classification: 22E46, 53C35, 57S20

1. Introduction

Fourier analysis is one of fundamental methods to extract feature from signal or image etc. This method expresses a target by linear combination of some sinusoidal functions and a spectrum is derived from these coefficients. However, Fourier analysis is not able to express local variation since this method assumes stationary of a target. Therefore, some time-frequency analysis methods are proposed to analysis a non-stationary target.

For example, short-time Fourier transform (STFT) is constructed by Fourier transform to a segment of signal that is divided to a short time. In addition, to adjust the resolution of time-frequency, wavelet transforms (WT) is developed. There is redundancy in information though WT is able to describe signals to time-frequency domain well. Therefore, discrete wavelet transforms (DWT) of the multiresolution analysis that has property of non-redundant is used in a lot of fields. However, DWT lost a property of shift invariant though this method does not need many memory.

Mallat proposed redundant wavelet transforms (RWT) that is able to extract detail information since this method gives additional information to the coefficients of decomposition information.[1] Several methods were proposed in this field. For example, stationary discrete wavelet transforms (SWT) is composed of up-sampling core functions in decomposition instead of down-sampling of signal coefficients.[2] Complex discrete wavelet transforms (CDWT) keeps shift-invariance by adding the phase information to decomposition.[3] Matching Pursuits (MP) is composed of sequentially extracting of core function that fits a target signal.[4]

On the other hand, some nonlinear methods were proposed. Morphological wavelet (MW) is composed of set operation instead of integral transformation with core function.[5] Mathematical morphology is a method for extracting form information from a target, and it has a similar framework to Fourier analysis.[6] For example, convolution corresponds to Minkowski operation. In addition, Fourier series expansion and Fourier transforms correspond to skeleton and slope transform of morphology.[7] Also, Fourier spectrum corresponds to pattern spectrum.[8] Additionally, DWT corresponds to MW that is able to decompose feature of signal to each form. MW has a property of shift variant though information of decomposition is not redundant as well as DWT.

In this study, we propose redundant morphological wavelet (RMW) and local pattern spectrum (LPS) method. RMW is one of the nonlinear RWT, and it is composed of expanding the core function in decomposition instead of down-sampling of signal coefficients. RMW has the property of shift invariant and redundancy. In addition, LPS is one of the nonlinear time-frequency analysis method that is composed of RMW. By using RMW information, it becomes difficult to receive the influence of noise compared with pattern spectrum method. The characteristics of these methods are confirmed by applying to simulation data and actual data (using Speech Corpus).

2. Mathematical Morphology

2.1. *Morphological filters*

In this section, mathematical morphology is described. As basic operations, we employ Minkowski addition $\oplus$ and Minkowski subtraction $\ominus$, which are respectively defined as follows:

$$[f \oplus g](t) = \max_{u \in G, t-u \in F} \{f(t-u) + g(u)\}, \tag{2.1}$$

$$[f \ominus g](t) = \min_{u \in G} \{f(t-u) - g(u)\}, \tag{2.2}$$

where, $f(t)$ is an original signal and $g(t)$ is the structuring function which characterizes the filter. Furthermore, F and G denote the domains of $f(t)$ and $g(t)$,

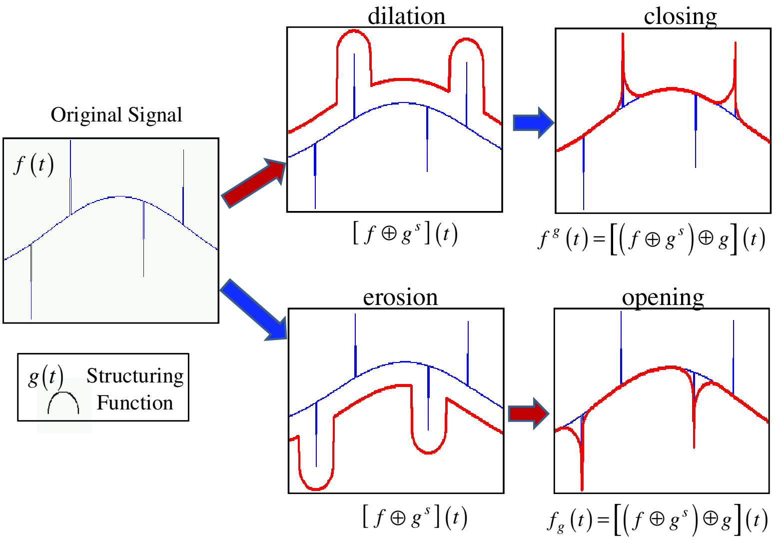


Fig. 1. Morphological operations (color online).

respectively. Conventionally, it is assumed that every signal takes the value $-\infty$ out of its domain.

Morphological filters are composed of these combinations (Fig. 1). These filters behave nonlinear low, high, band pass and band stop filter:

$$\text{low pass} : \psi_g(t) = (f_g)^g(t) \tag{2.3}$$

$$\text{high pass} : \omega_g(t) = f(t) - \psi_g(t) \tag{2.4}$$

$$\text{band pass} : p_{g_m,g_n}(t) = \psi_{g_m}(t) - \psi_{g_n}(t) \tag{2.5}$$

$$\text{band stop} : c_{g_m,g_n}(t) = f(t) - \psi_{g_m}(t) + \psi_{g_n}(t) \tag{2.6}$$

where, g_m and g_n are structuring functions of different shape and $f_g = [(f \ominus g^s) \oplus g]$, $f^g = [(f \oplus g^s) \ominus g]$ and $g^s(t) := g(-t)$. Also, f^g, f_g and $(f_g)^g$ are called opening, closing and open-closing filter respectively. The characteristic of filter is decided by the shape of structuring function. Figure 2 shows the example of morphological filter that uses structuring function of ellipse type.

2.2. *Pattern spectrum*

Pattern spectrum is size density function using morphological filter. At first, a base structuring function g is selected and morphological low pass filter is processed. Next, the same operation is processed by similar expansion of this structuring function. There is an expansion method using distributive law of morphological

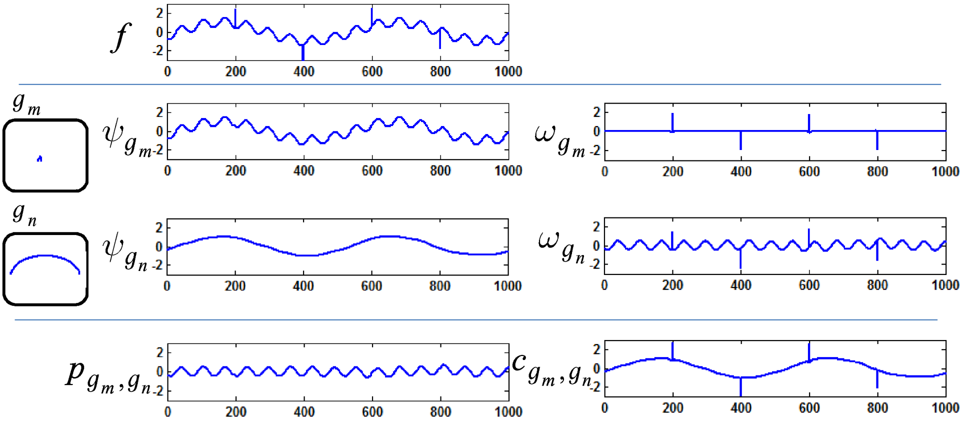


Fig. 2. Morphological filters.

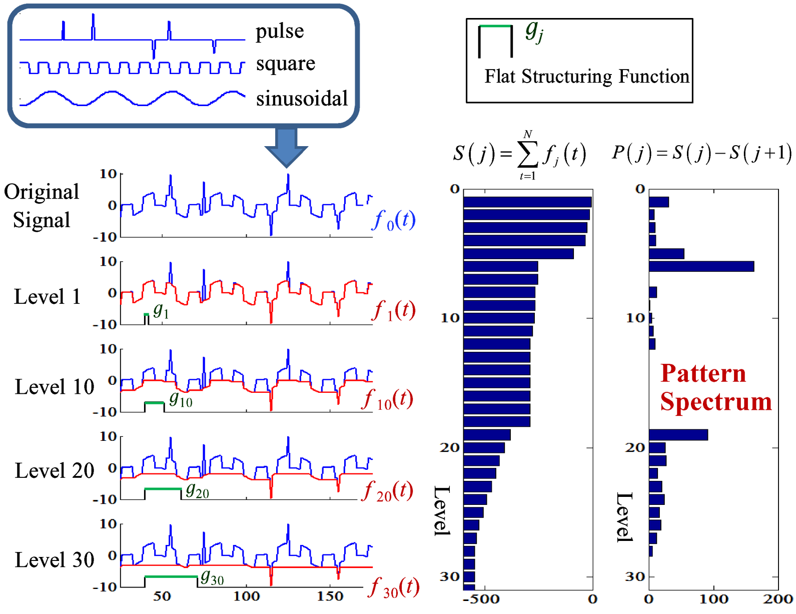


Fig. 3. Pattern spectrum of signal (color online).

operation with size j,

$$jg = g \oplus g \oplus \cdots g(j-1 \text{ times} \oplus \text{operation}). \tag{2.7}$$

Next, this process is repeated to a preassigned level (size) and a summation of low frequency component of each level is taken. Finally, pattern spectrum that is size density function is derived by taking difference between the levels of this size distribution function. A non-negative pattern spectrum using opening filter and closing filter in discrete space is defined as follows:

$$P_f(j,g) = S(j) - S(j+1) = \sum_{t=1}^{N} [f_{jg}(t) - f_{(j+1)g}(t)], \quad j > 0, \tag{2.8}$$

$$P_f(-j,g) = S(j) - S(j+1) = \sum_{t=1}^{N} [f^{jg}(t) - f^{(j-1)g}(t)], \quad j > 1. \tag{2.9}$$

$P_f(j,g)$ means particular component to size j when structuring function g is used. Figure 3 shows the example of pattern spectrum that uses only opening filter.

3. Modified Methods of Discrete Wavelet

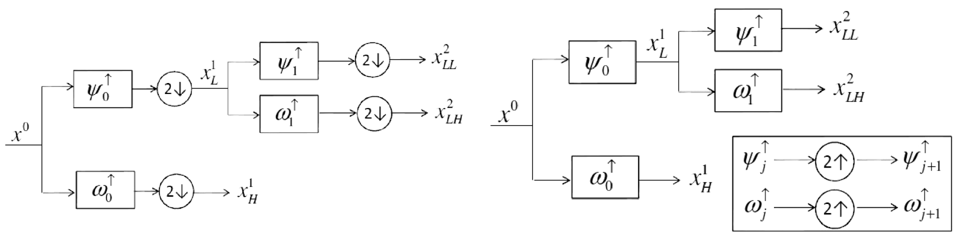
3.1. *Redundant wavelet*

In this section, modified methods of DWT are described. DWT has properties of shift variant and redundancy of information. In the case of shift variant, energy at each level in multiresolution analysis changes into small shift of the input. This phenomenon is undesirable to analyze the signal in detail since the result has no stability. In particular, this causes a serious problem about detecting the pulse and the edge. The reason of this phenomenon is down-sampling in multiresolution analysis. Therefore, there is no redundancy in information on decomposition coefficient since the number of samples of signal corresponds to the number of decomposition coefficients.

RDWT keeps property of shift-invariant by omitting down-sampling in multiresolution analysis. SWT is one of the RDWT, and it has achieved the multiresolution analysis by up-sampling of core function instead of down-sampling of signals (Fig. 4).

3.2. *Morphological wavelet*

Heijmans proposed the framework of MW that consisted of morphological operation. In morphological haar wavelet (MHW) for signals, the operations of analysis



(a) Wavelet

(b) Redundant wavelet

Fig. 4. Framework of redundant wavelet.

and synthesis are defined as follows:

$$\psi^{\uparrow}(x)(t) = \min\{x(2t), x(2t+1)\} \tag{3.1}$$

$$\omega^{\uparrow}(x)(t) = x(2t) - x(2t+1) \tag{3.2}$$

$$\psi^{\downarrow}(x)(2t) = \psi^{\downarrow}(x)(2t+1) = x(t) \tag{3.3}$$

$$\omega^{\downarrow}(y)(2t) = \max\{y(t), 0\}, \quad \omega^{\downarrow}(y)(2t+1) = -\min\{(y(t), 0\}. \tag{3.4}$$

Though MW is a nonlinear wavelet that is able to reconstruct completely, this method has a property of shift variance as well as DWT.

4. Redundant Morphological Wavelet and Local Pattern Spectrum

The important points in this study are summarized as follows:

- Nonlinear low pass filter and high pass filter concerned with form information are able to be constructed by combining morphological operations.
- Pattern spectrum expresses a form feature from signals, and that corresponds to nonlinear Fourier spectrum.
- DWT has properties of shift variance and information redundancy.
- To obtain a property of shift invariance, there is a method that expands core function instead of down-sampling of signals in decomposition.
- As a nonlinear wavelet, MHW was proposed and this method has the same problem as DWT.

In this section, we will propose RMW method that includes the frameworks of RDWT to MW. In addition, LPS that is one of nonlinear time-frequency analysis method is composed of RMW coefficients.

4.1. *Redundant morphological wavelet*

An approximation $\psi_j^{\uparrow}$ in Eq. (3.1) corresponds to erosion operation with a Flat structuring function (width is 2 and value is 0). Redundant morphological haar wavelet (RMHW) is constructed to expand core function instead of down-sampling of signals in decomposition. The expansion method of core function is defined by distributive law of morphological operation (Eq. (2.7)). In level j, the approximative operation $\psi_j^{\uparrow}$ corresponds to processing an erosion operation with a Flat structuring function that width is $j+1$. The detail operation $\omega_j^{\uparrow}$ is defined by morphological high pass filter in Eq. (2.4).

$$\psi_j^{\uparrow} : x_{j+1}(t) = \epsilon_j(x_j), \tag{4.1}$$

$$\omega_j^{\uparrow} : y_{j+1}(t) = x_j(t) - x_{j+1}(t) \tag{4.2}$$

$$\epsilon_j(f) = \underbrace{(\epsilon \circ \epsilon \circ \cdots \circ \epsilon)}_{j}(f), \quad \epsilon(f)(t) = \min(f(t), f(t+1)),$$

where ϵ denotes erosion operation with Flat structuring function of size 2.

This approximative operation is able to replace it by arbitrary morphological operation. By using approximative operation $\psi_j^{\uparrow}$ to erosion, this output traces bottom of signals. Then, operation of open-closing is used to approximative operation instead of erosion as follows:

$$\psi_j^{\uparrow} : x_{j+1}(t) = \epsilon_j(\delta_j(\delta_j(\epsilon_j(x_j))))(t), \tag{4.3}$$

$$\omega_j^{\uparrow} : y_{j+1}(t) = x_j(t) - x_{j+1}(t) \tag{4.4}$$

$$\delta_j(f) = \underbrace{(\delta \circ \delta \circ \cdots \circ \delta)}_{j}(f), \quad \delta(f)(t) = \max(f(t), f(t+1)).$$

When important information is included in polarity of the amplitude of signals, information of the polarity is able to be kept by processing the opening operation and closing operation separately (Fig. 5(a)) as follows:

$$\text{opening} : \psi_j^{\uparrow} : x_{j+1}(t) = (\delta_j \circ \epsilon_j)(x_j)(t), \quad (1 \le j \le J) \tag{4.5}$$

$$\omega_j^{\uparrow} : y_{j+1}(t) = x_j(t) - x_{j+1}(t) \tag{4.6}$$

$$\text{closing} : \psi_j^{\uparrow} : x_{j-1}(t) = (\epsilon_j \circ \delta_j)(x_j)(t), \tag{4.7}$$

$$\omega_j^{\uparrow} : y_{j-1}(t) = x_j(t) - x_{j-1}(t). \tag{4.8}$$

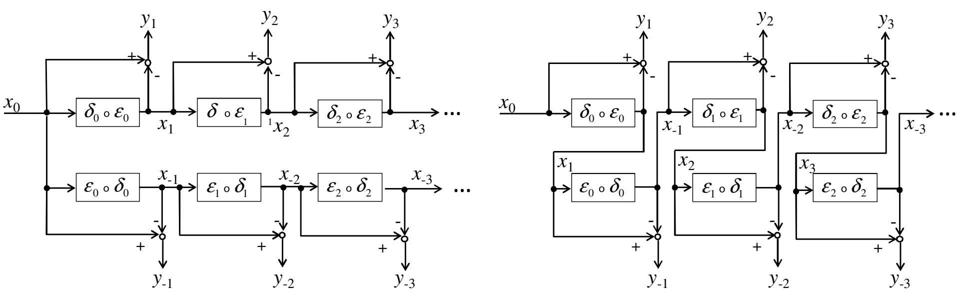
When the characteristics of operation has idempotent, the output takes influence of noise greatly since this operation corresponds to processing to the input signal x_0:

$$\begin{gathered} \delta_j(\epsilon_j(x_j))(t) = \delta_j(\epsilon_j(x_0))(t) \\ (\delta^2\epsilon^2\delta\epsilon = \delta^2\epsilon(\epsilon\delta\epsilon) = \delta^2\epsilon^2). \end{gathered} \tag{4.9}$$

Then, to avoid the characteristic of idempotent of morphological operation, it exchanges into process opening and closing sequentially (Fig. 5(b)):

$$\text{opening} : \psi_j^{\uparrow} : x_{j+1}(t) = (\delta_j \circ \epsilon_j)(x_{-j})(t), \quad (1 \le j \le J) \tag{4.10}$$

$$\omega_j^{\uparrow} : y_{j+1}(t) = x_{-j}(t) - x_{j+1}(t) \tag{4.11}$$



(a) Idempotent decomposition

(b) Sequential decomposition

Fig. 5. Multi-level decomposition of redundant morphological haar wavelet.

$$\text{closing}: \psi_j^{\uparrow} : x_{-j-1}(t) = (\epsilon_j \circ \delta_j)(x_{j+1})(t) \tag{4.12}$$

$$\omega_j^{\uparrow} : y_{-j-1}(t) = x_j(t) - x_{-j-1}(t). \tag{4.13}$$

The input signal x_0 is able to completely reconstruct by taking summation of components of high frequency y_j and the residual x_j since high frequency components are composed of the difference.

4.2. *Local pattern spectrum*

The inside of equation of deriving pattern spectrum (Eqs. (2.8) and (2.9)) is the same as processing to RMHW using opening operation or closing operation (Eq. (4.11)). Then, the summation of coefficient of RMHW corresponds to pattern spectrum. In addition, it is able to be analyzed to non-stationary signals by summation in short time. LPS is defined as follows:

$$P(x_0, j, t_0) = \sum_{t=t_0}^{t_0+\Delta t} y_j(t) = \sum_{t=t_0}^{t_0+\Delta t} [x_j(t) - x_{j+1}(t)]. \tag{4.14}$$

If summation is processed before segmentation of signals, then the distortion is caused since the influence of the edge occurs. Additionally, the spectrum that the noise that is not largely influenced, is able to be obtained by using the sequential method of approximation (Eq. (4.11)).

5. Experiment

5.1. *Simulation*

At first, the characteristics of pattern spectrum is confirmed by applying typical signals (Fig. 6). In the case of sinusoidal signal, Fourier spectrum expresses this component at one peak. However, pattern spectrum describes this smoothness because this method extracts feature of sinusoidal little by little. This response is more clearly for saw-tooth signal. In contrast, in the case of square and pulse signal, pattern spectrum expresses this component at one peak though Fourier spectrum expresses the components over wide band.

In the second experiment, the characteristics of four kinds of multiresolution analysis methods are compared by using non-stationary signals. These signals are vowel data of Japanese that utters continuously (/a/, /i/). The result of redundant wavelet is decomposed in detail compared with haar wavelet (Fig. 7). Similarly, RMHW decomposed in detail, compared with MHW and RMHW, is able to distinguish shape components of vowel clearly.

In the third experiment, the characteristics of LPS are confirmed. In sinusoidal chirp signal, STFT shows the change of frequency accurately (Fig. 8(a)). However, LPS that derived idempotent method (Eq. (4.6)) and sequential method (Eq. (4.11)) show the component in wide band. In the case of square chirp signal, LPS shows this feature in narrow band (Fig. 8(b)). When pulse noise is added to chirp signal, it

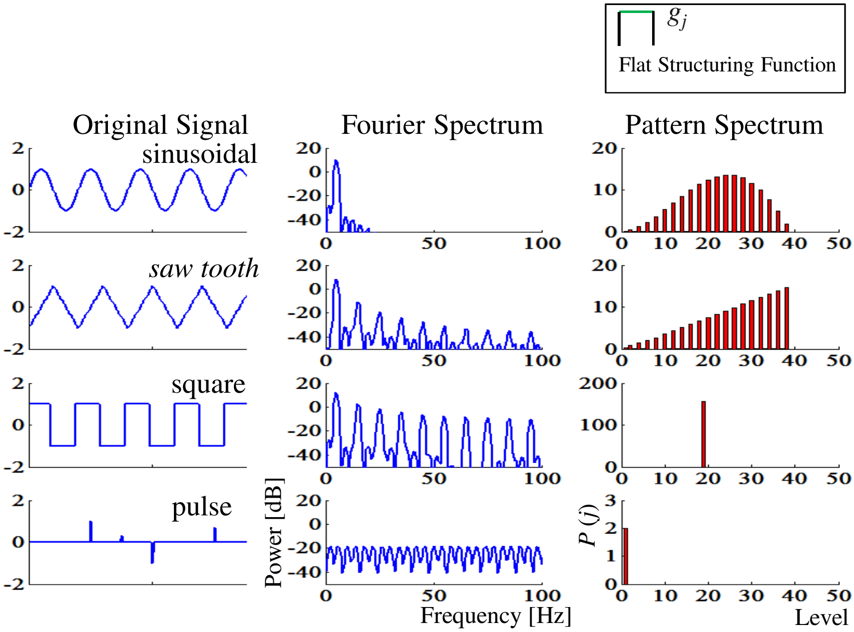


Fig. 6. Pattern spectrum (color online).

is confirmed that the idempotent method received noise influence largely compared with sequential method (Fig. 8(c)).

Finally, the necessary measurement condition is confirmed. Figure 9 shows the influence of sampling rate. It is confirmed that features are reduced with the decreasing of sampling rate (down-sampling). Then, LPS required high sampling rate.

5.2. *Implementation to vocal signal analysis*

A vowel of speech signals in short time can be discriminated by extracting three formant frequencies (F_1, F_2, F_3).[9] In general, LPC (Linear Predictive Coding) and cepstrum analysis are used to extract these features, since harmonic frequency components of Fourier analysis cause misidentification. LPC makes Fourier power spectrum smooth according to model order which determines the number of peaks. These peak frequencies (formant) are extracted by using peak extracting method such as peak picking. In this study, we implemented pattern spectrum to extract formant information from speech signals.

5.2.1. *Experimental data*

In this experiment, we used actual data observed in real environment GSR(A) "Regional Difference in Spoken Japanese Dialects" Spoken Japanese Dialect Corpus). 133 subjects participated in this experiment and they speak /a, i, u,

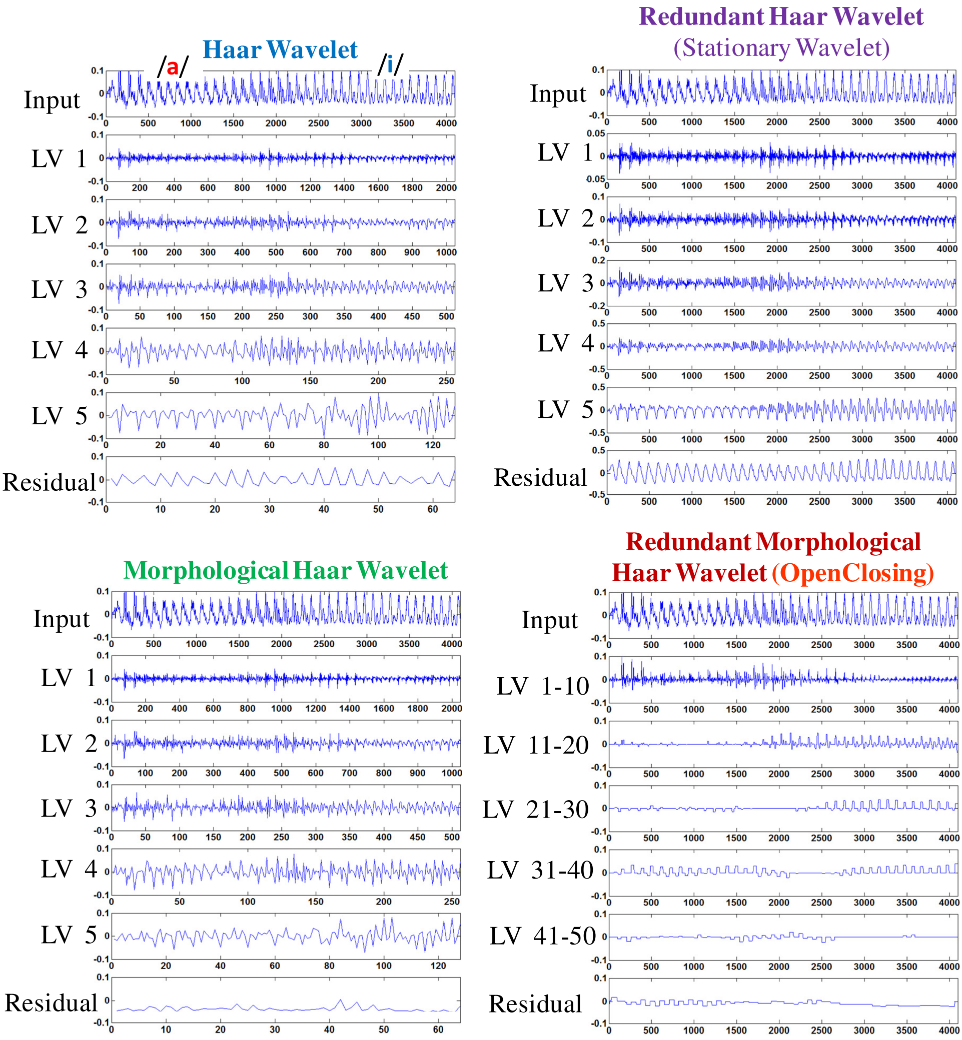


Fig. 7. Multiresolution analysis methods (color online).

e, o/ continuously. The sampling frequency was 16k [Hz] and measured with 16 [bit] A/D. In this study, speech signals were divided into 8000 points (0.5 [sec]) manually.

5.2.2. *Feature extraction and pattern recognition*

At first, pattern spectrum was calculated for each vowel signal. Next, this spectrum was smoothed by weighted moving average (window: 15 [pt]). Peak levels of pattern spectrum were detected by using peak-picking method. Feature vector

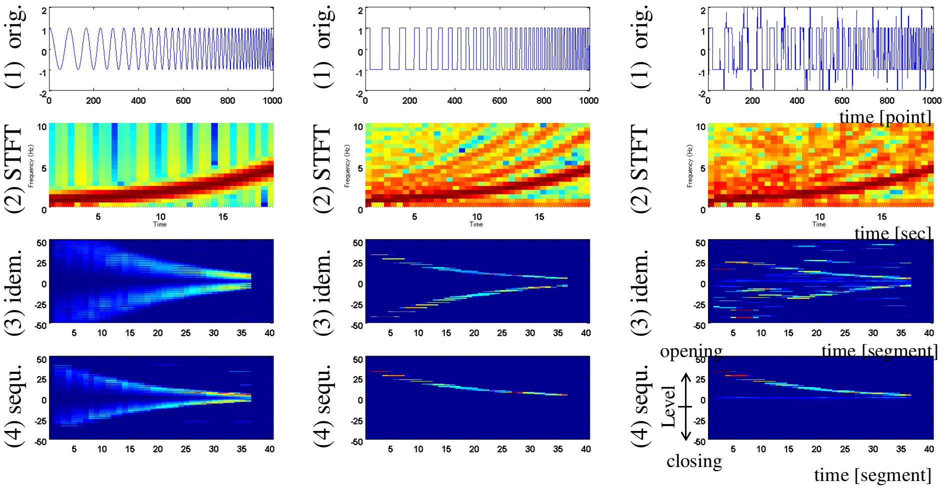


Fig. 8. Local pattern spectrum.

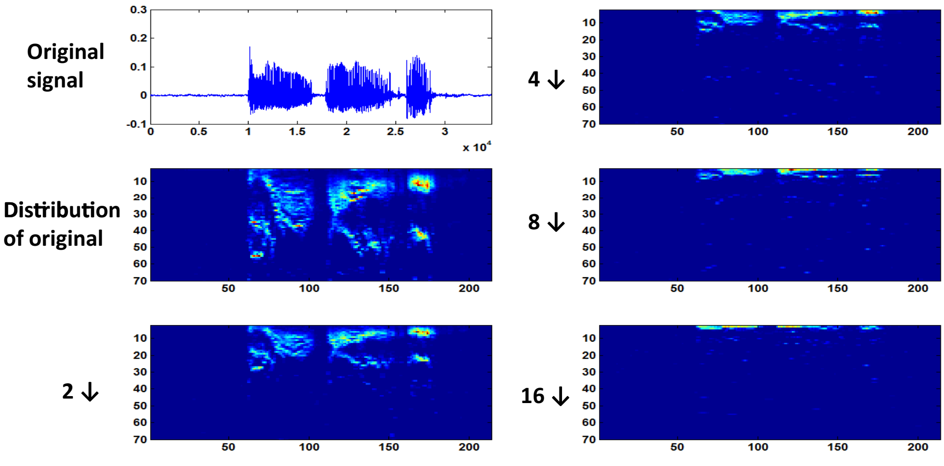


Fig. 9. Influence of sampling rate (down-sampling).

$\mathbf{s} = [s_1, s_2, s_3]^T$ was set as follows:

$$\begin{cases} s_1 = \text{1st peak of pattern spectrum} \\ s_2 = \text{2nd peak of pattern spectrum} \\ s_3 = \text{value of level 1 of pattern spectrum.} \end{cases}$$

Finally, Bayes classifier was adopted as the pattern recognition method with assumption that feature value was random variable with normal distribution. The

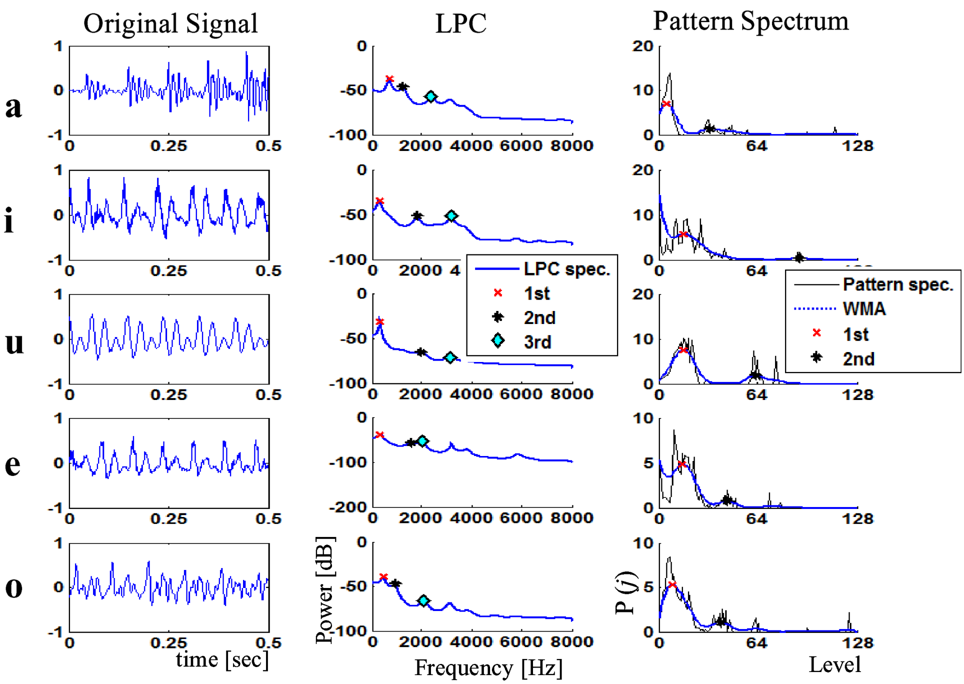


Fig. 10. Pattern spectra of vowel signals (color online).

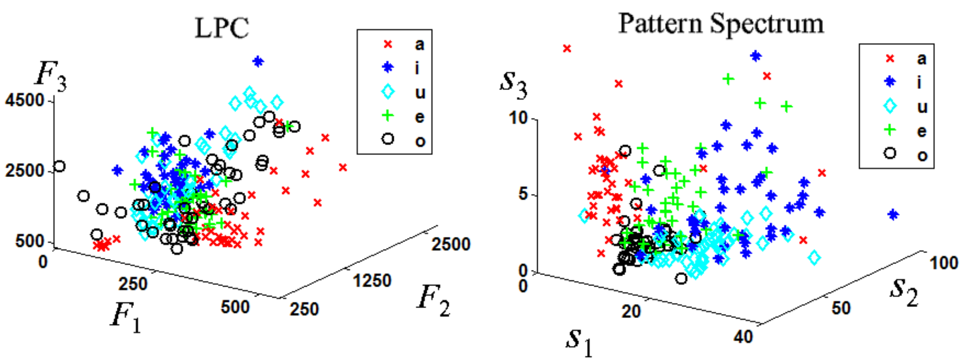


Fig. 11. Distribution of features (male) (color online).

discrimination function is as follows.

$$k^* = \arg\max_k \Pr(\omega_k \mid \mathbf{s})$$

$$= \arg\min_k \left\{ \frac{1}{2} \ln |\Sigma_k| + \frac{1}{2} (\mathbf{s} - \mathbf{m}_k)^T \Sigma_k^{-1} (\mathbf{s} - \mathbf{m}_k) - \ln \Pr(\omega_k) \right\}, \qquad (5.1)$$

Table 1. Pattern recognition results.

	5 Class (/a i u e o/)		3 Class (/a u o/)		3 Class (/i u e/)	
	Male (%)	Female (%)	Male (%)	Female (%)	Male (%)	Female (%)
PS ($[s_1, s_2, s_3]^T$)	59.4	55.9	84.3	88.0	61.4	53.5
PS ($[s_1, s_2]^T$)	50.3	47.0	79.0	84.6	<u>48.8</u>	<u>42.8</u>
LPC	53.9	55.9	66.7	81.9	60.7	53.3
PS & LPC (dim: 6)	64.3	64.2	83.3	89.1	62.1	58.3

where, $\mathbf{s}$ is feature vector composed of levels of peaks from pattern spectrum. Also, $\mathbf{m}_k$ and Σ_k are mean and variance of $\mathbf{s}$.

5.2.3. *Experimental result*

In case of pattern spectrum of speech signals, a decomposition level was set to 128. Figure 10 shows examples of pattern spectrum of vowel signals. Since these vowel signals are composed of two or three forms, pattern spectrum has the same number of peaks. The order of LPC was set to 20. High frequency component of LPC spectrum corresponds to low levels of pattern spectrum. Figure 11 depicts distribution of features and formant frequencies F_1, F_2 and F_3. In distribution, peaks of each vowel concentrated as well as LPC spectrum. These features are evaluated by cross validation method (ratio of training data to test data $= 1:1$). Table 1 shows classification accuracies using two or three features of pattern spectrum and LPC. In case of pattern spectrum, the recognition accuracy of three features was higher than the accuracy of two features. In particular, s_3 affects influence of the distinction of vowels /i u e/. Also, the recognition accuracy was improved, when features of LPC were combined ($\mathbf{s} = [s_1, s_2, s_3, F_1, F_2, F_3]^T$). It seems that pattern spectrum extracts information different from Formant.

6. Conclusion

In this study, we proposed a nonlinear signal analysis methods. RMHW had a property of shift-invariant though information of decomposition was redundant. This method can extract form information from signal. Also, LPS was derived according to the coefficients of RMHW. This method had a resistance of noise compared with existing pattern spectrum method by using framework of sequential decomposition.

In addition, the characteristics of morphological method were confirmed to adapt vowel signals. It was confirmed that another feature of vowel was extracted by using pattern spectrum different from linear method though it needs high sampling rate.

Mathematical morphology has a similar framework to Fourier analysis. For example, pattern spectrum corresponds to Fourier spectrum. In this study, we filled in the frameworks of RDWT and STFT by RMHW and LPS. The correspondence table between mathematical morphology and Fourier analysis will be achieved by advancing this research further. For example, it is thought that there exists a morphological method corresponding to matching pursuits, and the core function (structuring function and its range of effect) that fits the signal according to forms is estimated sequentially. A new type of signal identification method using information on polarity of signals and distribution of local pattern spectrum obtained by the proposed method is under construction.

Acknowledgment

This work was supported by JSPS Grant-in-Aid for Scientific Research(B) 21300208.

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