Low-Complexity Implementation of Channel Estimation for ESPAR-OFDM Receiver

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Abstract—Electronically steerable parasitic array radiator (ESPAR) antenna is a novel low-cost technique to gain comparable diversity order to that of using multiple antennas while maintaining one radio frequency (RF) front-end, low power consumption, and simple wiring at the mobile receiver. It has huge potential to realize high energy efficiency for multiple and massive antennas systems. The main drawback of the ESPAR based OFDM system is that the channel estimation (CE) usually should be realized in time-domain with a huge computational complexity (CC) because the received signal to each parasitic element is overlapped each other both in frequency and time domain. This article will propose three ways to reduce the complexity of CE for the ESPAR-OFDM system. The parallel multi-column compressive sensing (CS) algorithm was first introduced to detect all locations of channel impulse response simultaneously using one small segment of sensing matrix. Furthermore, by exploiting the symmetrical properties of Digital Fourier transform (DFT), the size of matrix to decrease the CC of matrix operation. Finally, symmetrical properties of Digital Fourier transform (DFT), the size of matrix to decrease the CC of matrix operation. Finally, by selecting a small set of row vectors of sensing matrix at the receiver side, the CC involved in CS algorithm can be reduced. From the analysis and simulated results, the proposed method can obtain more than 90% CC reduction for CE of ESPAR-OFDM receiver can be confirmed.

Index Terms—Channel estimation, compressed sensing, orthogonal matching pursuit, ISDB-T system, ESPAR antenna.

I. INTRODUCTION

MULTIPLE-INPUT and multiple-output (MIMO) has become an essential element of wireless communication standards including IEEE 802.11n (Wi-Fi), IEEE 802.11ac (Wi-Fi), HSPA+ (3G), WiMAX, Long Term Evolution (4G) and 5th generation (5G) wireless communication systems [1]-[4]. MIMO technique utilizes multiple antennas to obtain the diversity or multiplexing gain. Usually, each antenna branch requires a set of radio frequency (RF) front-end devices such as analog/digital (A/D) transfer and power amplifier which consumes half of energy at each antenna unit [5]. The large energy consumption of multiple RF front-ends refrains the usage of MIMO with large number antennas into mobile devices. In addition, for massive MIMO system, one of key technique to realize target of 5G system, large number of antenna branches increases power consumption, CO2 emission and the temperature of system devices which require large air-conditioners to make it work properly [6].

To solve this issue and improve the energy efficiency of MIMO system, a novel electronically steerable parasitic array radiator (ESPAR) antenna has been proposed as one efficient method [7]. An ESPAR antenna includes one radiator element which is connected to radio frequency (RF) front-end and several parasitic elements. To gain diversity, ESPAR antenna changes the directivity of its input signal using some cheap variable reactance circuits connected to each parasitic elements. Then, all the input signal with different directivity is mixed together into one RF front-end [8]. Wireless system using ESPAR antenna can obtain the comparable diversity order to that of MIMO system while just employing only one set of device for RF front-end [9]. In addition, using multiple ESPAR antennas to configure massive MIMO system would further exploit its energy efficiency.

ESPAR antenna has been researched for many purposes such as an adaptive beamforming multi-antenna system [10]–[13] and recent spatial modulation system [14]. Some researches focus the oscillatory/unstable behavior of ESPAR antenna for signal transmission because a negative input resistance would lead to transmit signal closely approximating the actual signal that keep the antenna stable [15], [16]. For mutual coupling of ESPAR antenna which strictly harms the system performance, by exploiting the concept of constructive interference, the mutual coupling effect can be manipulated constructively to the useful signal transmission [17]. Based on these results, a joint A/D beamforming scheme that exploits this effect has been researched in [18] to further improve the system performance. The application of precoding on practical ESPAR antenna equipped with quantized load impedances has been researched in [19]. In addition, the inherent non-linear behavior of ESPAR antenna requires high computational complexity (CC), and makes the unavailability of precise mutual coupling and channel state information (CSI) for practical setups. Such issues have been researched in [20]. There have been several proposals for the implementation of ESPAR antenna into current and future generation of wireless communication systems based on orthogonal frequency.
division and multiplexing (OFDM) [21]. Fig. 1 shows an usage example of ESPAR antenna used for Integrated Services Digital Broadcasting—Terrestrial (ISDB-T) vehicular system. To improve its diversity of mobile ISDB-T receiver, multiple antennas with RF devices as Fig. 1(a) are required which reduces the power efficiency of vehicular system. Using ESPAR antenna as Fig. 1(b), such issue can be alleviated.

However, since the received signal from main antenna element and parasitic elements are overlapped with each other both in the frequency and time domain, the equalization and channel estimation (CE) become more complicated because the ESPAR-OFDM receiver cannot achieve a diagonal equivalent channel matrix in frequency domain. Therefore, the CE usually should be realized in time domain which requires a huge computational complexity (CC). One efficient method for CE of ESPAR-OFDM receiver is using compressed sensing (CS) based scheme. CS is a signal process method that enables signal acquisition and reconstruction with the sub-Nyquist sampling rate [22] when the reconstructed signal has a sparse structure. Implementation of the CS based CE is possible because the channel impulse response (CIR) has a sparse structure in the time domain [23]–[25]. There are two main kinds of CS algorithms, basis pursuit (BP), and greedy pursuit (GP) [26], [27]. In principle, BP has better performance than that of GP, but GP can provide a less CC. The core purpose of both BP and GP are common as locating the non-zero elements in the sparse signal. Usually CS acquires the inner product between sensing matrix and the sampled signal vector to estimate the locations of the CIR. The known CIR locations are used to simplify the reconstruction process. However, the sensing matrix size for the inner product computation becomes huge for the ESPAR-OFDM system especially when OFDM system has large number of subcarriers. Thus the inner product computation becomes the main bottleneck in the CS based CE for ESPAR-OFDM system.

To achieve the efficient CE and equalization process for ESPAR-OFDM system, we have proposed several schemes to solve this issue [28]–[31]. References [28], [29] have proposed a new matrix inversion method through minimum mean square error (MMSE) sparse sorted QR decomposition (SQRD) technique by exploiting the sparse structure of channel matrix. For the CE of ESPAR-OFDM system, we have investigated the CS based CE in [30], [31]. To reduce the huge CC, a simultaneous multi-channel reconstruction with multi-column orthogonal matching pursuit (OMP) has been proposed in [30]. The multi-column OMP considers that the CIR for each antenna element has the same delay spread. Using this property, the multi-column OMP algorithm in the [30] can reduce the iteration number in the OMP computation. Reference [31] has reduced both the iteration time and the matrix size for the inner product computation. But, the proposed solution results in significant performance degradation.

This article will propose a new method which can reduce both iteration time and the matrix size for the inner product computation and achieve promising system performance.

A. Contribution of This Article

There are several unique properties that we can exploit to reduce the computational complexity. The CC of CS for ESPAR-OFDM receiver will be reduced with the following three methods. The contributions of this article are briefly described as follows.

- Modified multi-column OMP (MMC-OMP): Due to that each antenna element is close to each other as smallest to 1/8 wavelength, it can be assumed that all CIRs have the same delay profile. The multi-column OMP algorithm as our references [29], [30] uses this property to reduce the number of iteration involved in CS computation. However, in our previous proposals, the inner product computation deals with a huge size of sensing matrix. We propose a modified multi-column OMP technique with a small size of sensing matrix for the CC reduction of ESPAR-OFDM system.

- DFT strength reduction (DSR): The sensing matrix used for the CS computation is a truncated DFT matrix which has symmetrical structure and consists of many identical elements. This property can be applied as the matrix power reduction (MPR) technique which has been proposed in our paper [32] for general OFDM system. This article will further design this MPR technique for the CC reduction of CS computation of ESPAR-OFDM system. The proposal can calculate the inner product result involved in multiple columns of the sensing matrix in once time which largely reduces CC of ESPAR-OFDM system.
Matrix row-vector reduction (MRR): CS based channel estimation basically can work with less measurements compared to that of the conventional channel estimation. Therefore, reducing the size of observation vector can reduce the number of row vectors of sensing matrix and then computation complexity. Based on the mutual coherence property [34], [35], this article will propose a genetic algorithm (GA) based MRR technique at the receiver side to find the smaller size of sensing matrix with the comparable BER performance.

This article is organized as follows. Section II describes the background of the ESPAR antenna, its channel estimation and compressed sensing. Our proposed methods will be described at the Section III. The Section IV shows the simulation results of the proposed method, and the Section V gives the conclusion.

B. Notation

This article uses bold letter for vector and matrix. Bold lower case is for signal or matrix in the time domain and bold capital case for signal or matrix in the frequency domain. Inner product multiplication is denoted by $\langle , \rangle$. We also use $(\cdot)H$ to denote Hermitian transpose of a matrix or vector, notation $(\cdot)^\ast$ is the conjugate of a parameter and notation diag$(\cdot)$ defines a diagonal matrix. We use notation $|\cdot|$ to denote the absolute value of a scalar and notation $[\cdot]$ for the floor operation. The set of complex number is denoted by notation $\mathbb{C}$.

II. BACKGROUND

A. ESPAR Antenna

We first introduce the fundamental knowledge about ESPAR antenna. To help easily understand the channel properties of ESPAR antenna, Here we use a 3-element ESPAR antenna for the explanation. It should be noted that each process can be used for ESPAR antenna with any elements. Fig. 2(a) shows the structure of a 3-element ESPAR antenna hardware. This antenna consists of a circular ground planar as a base, one radiator element and two parasitic elements. The radiator element is placed in the center of the ground planar, and the parasitic elements are placed around the radiator element with the same distance. A variable passive reactor (varactor) is connected to each parasitic element, and an active reactor is connected to the radiator element. The connection between each parasitic and radiator element works as an oscillator which frequency $(f_s)$ is controlled by varactor value $Z_1$ and $Z_2$ [7].

The equivalent circuit model of the ESPAR antenna is shown in Fig. 2(b). The time domain signal output ($b(t)$) from the ESPAR antenna can be described as

$$b(t) = b_0(t) + b_1(t)e^{2\pi f_s t} + b_2(t)e^{-2\pi f_s t},$$

where $b_0$, $b_1$ and $b_2$ are the time domain signal input from radiator element and parasitic element 1 and 2 respectively. Diversity can be gained by adjusting the varactor thus each pair of the parasitic elements has an alternate directivity as sine or cosine with the same $f_s$ frequency. The element with the cosine oscillator is called the positive shifted element and the negative shifted element is named for the element with sine oscillator [9].

In order to achieve one RF front-end, the antenna performs the beamforming at the analog part. This causes a more complex computation and configuration at the digital process part of RF hardware device. However, compared to the analog electronic, the computing capacity of digital system is becoming more powerful and cheaper each year.

B. OFDM System With ESPAR Antenna

The block diagram structure of the OFDM system using ESPAR antenna is shown in Fig. 3. In this article, 3-element ESPAR antenna, with one radiator element and two parasitic elements, is used for an example. The varactor at each pair of the parasitic element is adjusted so that its oscillator circuit has an alternate directivity as sine or cosine with the same $f_s$ frequency. The element with the cosine oscillator is called the positive shifted element and the negative shifted element is named for the element with sine oscillator.
It is also assumed that the transmitter utilizes the conventional antenna for OFDM system. The input bit stream is mapped by the multilevel quadrature amplitude modulation (M-QAM). Then pilots are inserted in the data stream. The transmitter uses IDFT to generate the OFDM symbol. Before the OFDM symbol is transmitted through the multi-path channel, the cyclic prefix (CP) is inserted to the OFDM symbol by copying a certain part of the symbol’s near to its front. CP has a purpose to prevent the inter-symbol interference (ISI). Moreover, CP removal at the receiver side will create a cyclic form of the channel matrix. Thus the frequency domain channel matrix can be expressed as

\[ H = F h F^H, \]  

where \( h \) is the time domain channel matrix, \( F \) is DFT matrix and \( F^H \) is the inverse DFT matrix. The \( h \) matrix is a Toeplitz structure as

\[ h = \begin{bmatrix} e_0 & 0 \\ \vdots & \vdots \\ e_{L-1} & \ldots & \ldots \\ 0 & \ldots & e_{L-1} \end{bmatrix}, \]  

(3)

The CIR \( (e \in \mathbb{C}^L) \) is the vector of \([e_0, e_1, \ldots, e_{L-1}]\) and \( e_l \) is the \( l \)-th path \( h_l e^{j \theta_l} \) consisting of amplitude \( h_l \) and phase \( \theta_l \). The \( e \) is sparse and only a small number of its elements is not 0.

Because the \( h \) matrix has a circulant Toeplitz structure, without the signal from two parasitic elements, the \( H \) matrix will be a diagonal form as

\[ H = \begin{bmatrix} H_0 & 0 \\ & H_1 & 0 \\ & & \ddots & \ddots \\ & & & 0 & H_{N-1} \end{bmatrix}. \]  

(4)

where \( N \) is the size of the DFT operation.

The received signal at each antenna element has an independent CIR. Moreover, the signal at each parasitic elements is oscillated to create the diversity gain due to a pair of \( \cos(2\pi f_s t) \) or \( \sin(2\pi f_s t) \) applied to the received signal at each parasitic element. Therefore, the received signal \( Y \) can be represented as

\[ Y = F (g_p h_p + h_0 + g_n h_n) F^H X, \]  

(5)

where vector \( X \in \mathbb{C}^N \) is the transmitted symbols, \( h_0 \in \mathbb{C}^{N \times 1}, h_p \in \mathbb{C}^{N \times N}, \text{and} \ h_n \in \mathbb{C}^{N \times N} \) are the time domain CIR of radiator, positive shifted and negative shifted elements, respectively. Matrices \( g_p \in \mathbb{C}^{N \times N} \text{and} \ g_n \in \mathbb{C}^{N \times N} \) are diagonal matrices in terms of \( \cos(2\pi f_s t) \) and \( \sin(2\pi f_s t) \) respectively. Using Eq. (2), Eq. (5) can be redefined as

\[ Y = (G_p H_p + H_0 + G_n H_n) X, \]  

(6)

where matrices \( H_0 \in \mathbb{C}^{N \times N}, H_p \in \mathbb{C}^{N \times N} \text{and} \ H_n \in \mathbb{C}^{N \times N} \) are the frequency domain channel matrices for radiator, positive shifted, and negative shifted element respectively. The \( G_p \in \mathbb{C}^{N \times N} \text{and} \ G_n \in \mathbb{C}^{N \times N} \) are shift matrices with alternate directivity of frequency \( f_s \).

The equivalent channel matrix \( (H_{eq} \in \mathbb{C}^{N \times N}) \) in this transmission can be defined as

\[ H_{eq} = (G_p H_p + H_0 + G_n H_n). \]  

(7)

Fig. 4 shows an equivalent structure of the channel matrix. Because the matrix is not diagonal, the channel estimation with the interpolation method cannot be applied here. Moreover, to prevent the inter-carrier interference between pilots and data, the oscillator frequency \( (f_s) \) at each parasitic element is adjusted to be equal to the pilot spacing [8]. For ESPAR-OFDM system, the channel estimator needs to estimate the CIR for each antenna element. For 3-element ESPAR antenna, the CIR can be defined as

\[ a = \begin{bmatrix} e_p \\ e_0 \\ e_n \end{bmatrix}. \]  

(8)

Here \( e_0 \in \mathbb{C}^{L \times 1}, e_p \in \mathbb{C}^{L \times 1}, \text{and} \ e_n \in \mathbb{C}^{L \times 1} \) are the CIR vectors from radiator, positive and negative parasitic element respectively.
The CIRs obtained from the channel estimation are used to build the $H_{eq}$ matrix. Then, the equalization process can remove the effect of channel fading. Finally, M-QAM demapper will demodulate the sub-carrier symbols into the bit stream.

C. Compressed Sensing Based Channel Estimation

We have proposed the use of CS with OMP as a solution to the channel estimation of the ESPAR-OFDM system in [30]. The CE can be modeled as a CS based solution for an under-determined linear system, and the sensing matrix is from the multiple Fourier vectors which are decided by the number of the antenna elements. The OMP computation for CS algorithm can be divided into two main parts as inner product computation and CIR reconstruction. The inner product computation is to find the correct CIR locations. Usually the maximum value from the multiplication results between the sensing matrix and the observation vector can decide the locations of CIR. For the CIR reconstruction, usually it can be realized using least square (LS) based method. Using the estimated CIR locations, the LS calculation only utilizes the columns of sensing matrix. Then, the equalization process can greatly reduce for the CC of LS calculation. Of many practical applications is sparse, the matrix size will be greatly reduced for the CC of LS calculation.

Let us define the CS based channel estimation using the following equation as

$$\mathbf{B} = \mathbf{\Psi} \mathbf{a} + \mathbf{n},$$

where $\mathbf{B}$ is the sampled receive signal for channel estimation, $\mathbf{a}$ is the sparse reconstructed signal related to CIR, $\mathbf{n}$ is the AWGN noise and $\mathbf{\Psi}$ is the sensing matrix. The sampled signal $\mathbf{B}$ can be defined as an observation vector, which is given as

$$\mathbf{B} = \left( G_p^c \mathbf{H}_p^c + \mathbf{H}_0^c + G_n^c \mathbf{H}_n^c \right) \mathbf{P},$$

where $(\cdot)^c$ is the truncated matrix which only includes the matrix elements that are related to the pilot locations in both row and column direction. The size of each truncated matrix $(\cdot)^c$ is $T \times T$, where $T$ is the number of pilot or size of observation vector. Here, the vector $\mathbf{P} \in \mathbb{R}^{T \times 1}$ is the pilot vector. Using the unfolding technique, we can rewrite Eq. (10) as

$$\mathbf{B} = \begin{bmatrix} G_p^c \mathbf{H}_p^c, & \mathbf{H}_0^c, & G_n^c \mathbf{H}_n^c \end{bmatrix} \mathbf{P}^{\prime}. \quad (11)$$

Because the time domain channel matrix $\mathbf{h}$ has a Toeplitz structure, the frequency domain channel matrix can be defined as

$$\mathbf{H} = \mathbf{F} \text{diag}(\mathbf{e}), \quad (12)$$

where $\mathbf{e}$ is the channel impulse response from the first column of the matrix $\mathbf{h}$. Using Eq. (12), Eq. (11) can be changed to

$$\mathbf{B} = \begin{bmatrix} G_p^c \mathbf{F} \text{diag}(\mathbf{e}_p), & \mathbf{F} \text{diag}(\mathbf{e}_0), & G_n^c \mathbf{F} \text{diag}(\mathbf{e}_n) \end{bmatrix} \mathbf{P}^{\prime}. \quad (13)$$

where $\mathbf{F} \in \mathbb{R}^{T \times L}$ is a partial DFT matrix which its $T$ row vectors are from DFT matrix according to the pilot locations and each row vector just includes the first $L$ values of DFT vectors related to the CP length. Using the diagonal matrix property, we can express the channel estimation process as

$$\mathbf{B} = \begin{bmatrix} G_p^c \mathbf{F} \text{diag}(\mathbf{P}), & \mathbf{F} \text{diag}(\mathbf{P}), & G_n^c \mathbf{F} \text{diag}(\mathbf{P}) \end{bmatrix} \mathbf{a} = \mathbf{\Psi} \mathbf{a}. \quad (14)$$

Here the sensing matrix $\mathbf{\Psi} \in \mathbb{C}^{T \times 3L}$ is given as

$$\mathbf{\Psi} = \begin{bmatrix} G_p^c \mathbf{F} \text{diag}(\mathbf{P}), & \mathbf{F} \text{diag}(\mathbf{P}), & G_n^c \mathbf{F} \text{diag}(\mathbf{P}) \end{bmatrix}. \quad (15)$$

Fig. 5 shows the structure of the expected CIR reconstruction result ($\mathbf{a}$) after the channel estimation. The vector $\mathbf{a}$ consists of three $L$-length vectors from positive shifted, non-shifted and negative shifted element.

Usually, the distance among antenna elements is $\lambda/4$ or even to $\lambda/8$ [9], where $\lambda$ is the wavelength. For most multipath environments, the time delay spread among different paths is at least larger than several wavelengths. Therefore, we can assume the sparse locations on each CIR are almost the same. The reference [30] proposed a 3-columns (3C) OMP

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**Algorithm 1 3C-OMP Algorithm**

1. **input:** $\mathbf{B}$; $\mathbf{\Psi}$
2. **init:** $\mathbf{r} \leftarrow \mathbf{B}$; $t \leftarrow 0$; $\mathbf{\Phi}_0 \leftarrow [\ ]$
3. **loop:**
   4. $\alpha \leftarrow \arg \max |\langle \mathbf{\Psi}^H, \mathbf{r} \rangle |$
   5. $l \leftarrow \alpha \mod L$
   6. create index set $\Omega \leftarrow l + i \times L$, $i = 0, 1, 2$
   7. **augment:**
   8. $\mathbf{\Phi}_t \leftarrow \mathbf{\Phi}_{t-1} | \mathbf{\Psi}_t^H$
   9. **least square:**
   10. $\mathbf{a} \leftarrow \arg \min |\mathbf{b} - \mathbf{\Phi}_t \mathbf{a}|^2$
11. **update:**
   12. $\mathbf{r} \leftarrow \mathbf{b} - \mathbf{\Phi}_t \mathbf{a}$
   13. $t \leftarrow t + 1$
14. **if stopping criterion not met** goto loop.
by exploiting this property as shown in Algorithm 1. The main difference of 3C-OMP with the conventional OMP lies in the Step 4 and 5. As shown in Fig. 5, if each component has almost the same delay profile, then it only needs to get the relative delay profile and retrieve the delay profile once for calculating all the CIR components for the positive shifted, non-shifted part and negative shifted elements. In the Step 4, when a location of one non-zero element is found in a sub-set of CIR, the CS algorithm can get the relative locations when a location of one non-zero element is found in a sub-

Fig. 5. Expected CIR (a) reconstruction result.

Here \( \mathbf{e}_p^i \in \mathbb{C}^{L \times 1} \) and \( \mathbf{e}_n^i \in \mathbb{C}^{L \times 1} \) \((i = 1, \ldots, K)\) are the CIR vectors from \( 2K \) positive and negative parasitic elements respectively.

Based on the Eqs. (5)–(15), we can obtain the similar channel estimation process for ESPAR antenna with \( W \) elements as

\[
\mathbf{b}_W = \Psi_W \mathbf{a}_W. \tag{17}
\]

Here the sensing matrix \( \Psi_W \in \mathbb{C}^{T \times WL} \) is given as

\[
\Psi_W = \begin{bmatrix}
\Psi_p^K, & \mathbf{F}_{L diag}(\mathbf{p}), & \Psi_n^K
\end{bmatrix}, \tag{18}
\]

where \( \Psi_p^K \) and \( \Psi_n^K \) can be represented as

\[
\Psi_p^K = \begin{bmatrix}
\mathbf{G}_1^p \mathbf{F}_{L diag}(\mathbf{p}), & \mathbf{G}_2^p \mathbf{F}_{L diag}(\mathbf{p}), & \ldots, & \mathbf{G}_K^p \mathbf{F}_{L diag}(\mathbf{p})
\end{bmatrix}. \tag{19}
\]

\[
\Psi_n^K = \begin{bmatrix}
\mathbf{G}_1^n \mathbf{F}_{L diag}(\mathbf{p}), & \mathbf{G}_2^n \mathbf{F}_{L diag}(\mathbf{p}), & \ldots, & \mathbf{G}_K^n \mathbf{F}_{L diag}(\mathbf{p})
\end{bmatrix}. \tag{20}
\]

### III. PROPOSED METHOD

The OMP computation for the channel estimation in the ESPAR-OFDM system has a huge size of sensing matrix, and its size is also determined by the number of antenna elements. The full size of the sensing matrix is only used in the computation of inner product to detect time delay of CIR as shown in Algorithm 1. However, in the following steps, the OMP only utilizes the truncated matrix \( \Phi \) for calculation and its size equals to \( T \times W K \), where \( K \) is the number of sparsity in the CIR. Compared to the matrix \( \Psi_W \) with size as \( T \times WL \), the size of \( \Phi \) is extremely small. Therefore, the large CC of the inner product computation for the multi-column OMP in Step 4 of Algorithm 1 can be reduced.

- Firstly, because the CIR delay profile is almost the same for all antenna elements, we can modify the multi-column OMP (MMC-OMP) method using a small sensing matrix with size as \( T \times L \), smaller than the original matrix \( \Psi_W \) with size as \( T \times WL \), we call this as modified multi-column OMP which will be explained in Section III-A.

- Secondly, because of the sensing matrix is based on the DFT matrix, we can exploit its symmetrical property to further reduce the sensing matrix size to \( Q \times R \) which is named as DFT strength reduction (DSR) and will be explained in Section III-B.

- Thirdly, since the CS algorithm can work with a small number of measurements, we propose a matrix row-vector reduction (MRR) method to reduce the row size of the sensing matrix in Section III-C.

The combination of three methods can largely reduce the size of sensing matrix as shown in Fig. 6.

#### A. Modified Multi-Column OMP (MMC-OMP)

The OMP algorithm needs to estimate the related locations of CIR delay to decide the non-zeros values for CE in an iterative way. The relative location is decided from the maximum value of inner product of each element. However, the CIR delay profile at each antenna element can be almost
Algorithm 2 MMC-OMP Algorithm

1: input: \( b; \Psi_W; \Psi_N \)
2: init: \( r \leftarrow b; t \leftarrow 0; \Phi_0 \leftarrow [ ] \)
3: loop:
4: \( \alpha \leftarrow \arg \max \{ \langle \Psi^H_W, r \rangle \} \)
5: create index set \( \Pi \leftarrow \alpha + \alpha \ast L; i = 0, 1, ..., W - 1 \)
6: augment:
7: \( \Phi_i \leftarrow \Phi_i - 1 \ast |\Psi^H_W| \)
8: least square:
9: \( a \leftarrow \arg \min ||b - \Phi_a||^2 \)
10: update:
11: \( r \leftarrow b - \Phi_a \)
12: \( t \leftarrow t + 1 \)
13: if stopping criterion not met goto loop.

the same when distances among them are small as to 1/8 wavelength [9]. Therefore, the maximum value from the inner product values of each antenna element appears at the same relative locations. Based on this property, we propose a modified version of our previous conventional MC-OMP [29], [30] to reduce both iteration time and the number of multipliers. To achieve that for \( W \)-element ESPAR antenna, let us firstly introduce a new sensing matrix which is a summation of the \( W \) sub-matrices as

\[
\Psi_N = \sum_{i=1}^{K} G^i_p F_L \text{diag}(P) + F_L \text{diag}(P) + \sum_{i=1}^{K} G^i_n F_L \text{diag}(P).
\]

(21)

The size of the new sensing matrix is \( T \times L \), which is the small matrix size of the original sensing matrix \( \Psi_W \) as shown in Eq. (18). The new sensing matrix can also be used for correct detection of the location of the maximum value of CIR. The \( \Psi_N \) includes the inner product results from all \( W \) sub-matrices. Using this small sensing matrix perhaps changes the multiplication results compared with the original MC-OMP, but the locations of CIR delay will be same as in original MC-OMP.

Using the new sensing matrix, we can specify the modified multi-column OMP as shown in the Algorithm 2. In this proposed method, besides the original sensing matrix \( \Psi_W \), there will be an additional new matrix \( \Psi_N \) as the input parameters. The main difference between our proposal and MC-OMP is at Step 4, that is, the inner product of MMC-OMP uses the matrix \( \Psi_N \) instead of matrix \( \Psi_W \) used in MC-OMP. The modulo operation at Step 5 of Algorithm 1 is unnecessary here. During each iteration, the MMC-OMP adds \( W \) indices to the vector \( \Pi \). The set of indices can be obtained by adding 0, \( L \), \( \ldots \), \((W-1)L\) to the maximum projection result \( \alpha \) from Step 4. The \( W \) indices are used to extract \( W \) relative columns of the original sensing matrix \( \Psi_W \) into matrix \( \Phi_N \) at each iteration. Accordingly, the expected CIR vector \( a \) include \( W \) new values after each iteration.

The MMC-OMP keeps the advantage of the original MC-OMP as the multiple CIR locations can be detected at each iteration. Furthermore, the number of multipliers in the inner product computation can be reduced from \( WTL \) into \( TL \). Thus, the proposed method, i.e., MMC-OMP can reduce the number of iterations and the multipliers need to calculate CIR. It also brings about \((W - 1)/W\) reduction of the inner product computation. In addition, the more CC reduction can be expected when this method is applied for the ESPAR antenna with more elements.

Using the distributive property of the matrix multiplication, we can further decompose the sensing matrix \( \Psi_N \) of Eq. (21) as

\[
\Psi_N = \left( \sum_{i=1}^{K} G^i_p + \sum_{i=1}^{K} G^i_n \right) F_L \text{diag}(P).
\]

(22)

Then, the inner product computation in Step 4 of Algorithm 2 can be redefined as

\[
||\langle \Psi^H_N, r \rangle|| = \left| \left| \text{diag}(P)^H F^H_L \left( \sum_{i=1}^{K} G^i_p + \sum_{i=1}^{K} G^i_n \right)^H, r \right| \right|.
\]

(23)

Now, let us define vector \( \Theta \) as

\[
\Theta = \left( \sum_{i=1}^{K} G^i_p + \sum_{i=1}^{K} G^i_n \right)^H r.
\]

(24)

Vector \( \Theta \) is easy to be computed since it is only a combination of three shifted vectors \( r \) which consist of the value of ones and zeros. Then we can redefine the inner product computation as

\[
||\langle \Psi^H_N, r \rangle|| = ||\text{diag}(P)^H F^H_L, \Theta||.
\]

(25)

Therefore, an additional memory for storing \( \Psi_N \) is not necessary because the matrix \( F^H_L \) can be retrieved from the non-shifted element of the matrix \( \Psi_W \).

B. DFT Strength Reduction (DSR)

The main complexity for the inner product of Eq. (25) is from the matrix \( F^H_L \) since the matrix \( P \) only consists of known pilot symbols such as 1 and -1. The matrix \( F^H_L \) is a truncated DFT matrix in both column and row vectors. Because of that, the FFT algorithm cannot give an efficient hardware implementation. However, some symmetrical properties of DFT matrix can be remained in this truncated DFT matrix, and its size is smaller than a full DFT matrix. Then the matrix strength reduction (MSR) technique can be implemented in
the matrix $F_L^H$ multiplication to achieve the lower complexity than that of using a full DFT matrix.

The measurement matrix has some unique properties which inherited from DFT matrix. Let us describe the measurement matrix $\Omega$ as the Hermitian of $\Psi$ matrix represented as

$$\Omega = F_L^H = \begin{bmatrix}
\omega^0 & \omega^0 & \omega^0 & \cdots & \omega^0 \\
\omega^0 & \omega^p & \omega^{2p} & \cdots & \omega^{(T-1)p} \\
\omega^0 & \omega^{3p} & \omega^{4p} & \cdots & \omega^{3(T-1)p} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\omega^0 & \omega^{9p} & \omega^{10p} & \cdots & \omega^{9(T-1)p}
\end{bmatrix},$$

(26)

where $\omega = e^{-2\pi j/N}$ from $N$-size DFT and $p$ is the equal spacing period of pilots. The $\Omega$ matrix elements are well-structured with that each element at $k$-th row and $l$-th column can be represented as

$$\Omega(k, l) = \omega^{(plk)}.$$  

(27)

Since the elements in $\Omega$ are exponent function, any element of matrix can be obtained with multiplication of two elements as

$$\Omega(k_1 + k_2, l) = \Omega(k_1, l) \times \Omega(k_2, l) = \omega^{(k_1plk_2pl)}.$$  

(28)

The matrix $\Omega$ has the periodicity and symmetry properties derived from DFT matrix. The $k$ and $l$ of Eq. (27) decide the phase angle of $\omega$. Because the phase angle has periodic property with a modulo of $2\pi$, there exists periodic elements at several rows of the sensing matrix. We can express these elements in Eq. (27) as

$$\Omega(k, l) = e^{j\text{mod} \left( \frac{2\pi plk}{N}, 2\pi \right)} = e^{2\pi j \text{mod} \left( \frac{plk}{N}, 1 \right)},$$

(29)

where $\text{mod}(a, b)$ defines the modulo operation of dividend $a$ and divisor $b$. It is easy to conclude from Eq. (29) that, for $k$-th row vector, when $k$ is a greatest common divisor of $N$, there exist periodic elements at that row vector.

To show the relationship among row vectors of $\Omega$, we separate the $\Omega$ into $C$ equal-sized sub-matrices as $\Omega_i$ ($i = 0, \ldots, C - 1$) and $C$ is a number from powers of 2. The $k$-th vector of $i$-th sub-matrix is expressed as $\Omega^k_i$, and the first vector of each sub-matrix $\Omega^0_i$ is $\Omega(iL/C)$ as shown in Fig. 7. Therefore, two following correlation properties among sub-matrices and vectors can be easily found as

$$\Omega_i = \Omega_0 \times \text{diag}(\Omega^0_i);$$

(30)

$$\Omega^k_i = \Omega^0_i \times \text{diag}(\Omega^k_0).$$

(31)

From Eqs. (30) and (31), we can find that using the first sub-matrix $\Omega_0$ and the first row vectors $\Omega^0_0$ ($i = 1, \ldots, C - 1$) of other sub-matrices, all vectors of $\Omega$ can be calculated. To show the details, we first build a new matrix $\Gamma \in \mathbb{C}^{C \times T}$ which consists all $C$ first row vectors $\Omega^0_i$ ($i = 0, \ldots, C - 1$) as

$$\Gamma = \left[\Omega[0], \Omega[\frac{L}{C}], \Omega[\frac{2L}{C}], \ldots, \Omega[\frac{(C-1)L}{C}]\right]^T.$$  

(32)

Therefore, the $k$th vectors of all sub-matrices can be calculated as

$$\begin{bmatrix}
\Omega^k_0 \\
\Omega^k_1 \\
\vdots \\
\Omega^k_{C-1}
\end{bmatrix} = \Gamma \text{diag}(\Omega^k_0).$$

(33)

Therefore, the results of inner product multiplication $g_k$ of $\langle \Psi^k, r \rangle$ in Step 4 of Algorithm 2 with $g_k = [g[k], g[k + L/C], \ldots, g[k + L(C - 1)/C]]^T$ can be obtained after $L/C$ time of calculations using

$$g_k = \Gamma \text{diag}(\Omega^k_0) r, \quad k = 0, 1, \ldots, L/C - 1.$$  

(34)

The main idea is to further reduce the complexity by sharing the same values of the matrix $\Gamma$ and decreasing the number of full complex multiplication using symmetry property appeared on each row of $\Gamma$.

Firstly, for each row of the matrix $\Gamma$, the values appear periodic property. The number of different value $U_r$ of $k$-th row can be calculated as

$$U_k = \frac{N}{\text{GCD}(pk, N)},$$

(35)

where $\text{GCD}(a, b)$ is the operation of finding the greatest common divisor (GCD) among $a$ and $b$. Therefore, the total of multipliers for the matrix $\Gamma$ is calculated as follows

$$U = \sum_{i=1}^{C-1} U_i = \sum_{i=1}^{C-1} \frac{N}{\text{GCD}(ipL/C, N)}.$$  

(36)

Moreover, for $U$ different multipliers, we can utilize the relationship among phase angles or symmetry property in the matrix $\Gamma$ to further reduce its requirement of multiplications. The elements of matrix $\Gamma$ can be represented using complex unit circle and these elements will divide this complex unit circle evenly. Thus, by using only the elements in the first quadrant of complex unit circle (phase less than $\pi/2$), we can obtain all the different elements located on other quadrants.

Let us suppose there is a phase value in the first quadrant at the $k$-th row and $l$-th column of $\Gamma$ which represented as

$$\Omega^0_l = \Omega[0] \times \text{diag}(\Omega^0_l);$$

Fig. 7. C unique rows in $\Omega$ matrix.

Correlation among sub-matrices /vectors

$$\begin{bmatrix}
\Omega^0_0 \\
\Omega^0_1 \\
\vdots \\
\Omega^0_{C-1}
\end{bmatrix} = \Gamma \text{diag}(\Omega^0_0).$$

(33)

$$\Gamma = \left[\Omega[0], \Omega[L/C], \Omega[2L/C], \ldots, \Omega[L(C-1)/C]\right]^T.$$
\[ \Gamma_{q1}(\theta_{k,i}) \text{, the values on other quadrants can be obtained as} \]
\[
\Gamma(k, l) = \begin{cases} 
\Gamma_{q1}(\theta_{k,i}) = e^{-j\theta_{k,i}}, & 0 < \theta_{k,l} \leq \frac{\pi}{2} \\
\Gamma_{q2}(\theta_{k,i}) = -j\Gamma_{q1}(mod(\theta_{k,i}, \frac{\pi}{2})), & \frac{\pi}{2} < \theta_{k,l} \leq \pi \\
\Gamma_{q3}(\theta_{k,i}) = -\Gamma_{q1}(mod(\theta_{k,i}, \frac{\pi}{2})), & \pi < \theta_{k,l} \leq \frac{3\pi}{2} \\
\Gamma_{q4}(\theta_{k,i}) = j\Gamma_{q1}(mod(\theta_{k,i}, \frac{\pi}{2})), & \frac{3\pi}{2} < \theta_{k,l} \leq 2\pi \end{cases} 
\] (37)

Therefore, the multiplications with \(-1, j\) and \(-j\) are needed to build all the elements in other quadrants. However, the multiplication with \(-1, j\) and \(-j\) can be negligible compared to a full complex multiplication. By exploiting this property, the Eq. (36) can be redefined as

\[ U = \sum_{i=1}^{C-1} N \left[ \frac{1}{4GCD(ipL/C, N)} \right]. \] (38)

Therefore, the DSR can be treated as decomposing matrix \( \Gamma \) into three matrices as

\[ \Gamma = \Gamma_1\Gamma_2\Gamma_3. \] (39)

The first matrix \( \Gamma_1 \) only consists of 1, \(-j\), \(-1\) and \(j\). This matrix creates the quadrant relationship of symmetry property as shown in Eq. (37). The structure of \( \Gamma_1 \in \mathbb{C}^{U \times T} \) matrix can be one example as

\[ \Gamma_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\
1 & 0 & -j & 0 & -1 & \cdots & 0 \\
0 & 1 & 0 & -j & 0 & \cdots & j \\
\vdots \\
0 & j & 0 & -1 & 0 & \cdots & 1 \end{bmatrix}. \] (40)

Then we can multiply the output from \( \Gamma_1 \) with the \( U \) different coefficients in the first quadrant, which represented by \( \Gamma_2 \in \mathbb{C}^{U \times U} \) matrix with one example as

\[ \Gamma_2 = \begin{bmatrix} e^{-j\frac{\pi}{2U}} & 0 & 0 & 0 & 0 \\
0 & e^{-j\frac{2\pi}{U}} & 0 & 0 & 0 \\
0 & 0 & e^{-j\frac{3\pi}{U}} & 0 & 0 \\
0 & 0 & 0 & \cdot & 0 \\
0 & 0 & 0 & 0 & e^{-j\frac{\pi}{U}} \end{bmatrix}. \] (41)

The diagonal of \( \Gamma_2 \) matrix is filled with \( U \) different elements in \( \Gamma \) matrix, which values are in range of \( e^{-j\frac{\pi}{2U}} \) to \( e^{-j\frac{\pi}{U}} \). Finally, the permutation matrix \( \Gamma_3 \) is used to rebuild the original matrix \( \Gamma \) using the relationship between the unique \( U \) different values and their locations. This operation can be described by the \( \Gamma_3 \in \mathbb{C}^{U \times U} \) matrix such as

\[ \Gamma_3 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 1 & 1 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & 1 & \cdots & 0 \\
\vdots \\
0 & 0 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix}. \] (42)

From three matrices, we can find that only the multiplication with matrix \( \Gamma_2 \) matrix requires a full complex number multipliers. The total number of multipliers required to compute the inner product of \( \Omega \) matrix is

\[ U_{tot} = (T + U)^L \frac{C}{\mathcal{G}} \] (43)

We can also consider this method as a column reduction where the DSR technique decreases the size of matrix column from \( L \) to \( \mathcal{Q} \) as shown in Fig. 6. Here \( \mathcal{Q} \) is defined as

\[ \mathcal{Q} = \frac{L}{\mathcal{C}} + \frac{U}{\mathcal{T}}. \] (44)

The value \( \mathcal{C} \) which decides the CC reduction is related to system parameters such as the length of guard interval, the number of OFDM subcarriers. But the optimal \( \mathcal{C} \) can be found in an off-line way.

C. Matrix Row-Vector Reduction (MRR)

There are several criteria to find the minimum required size of measurements or observation vector for the CS algorithms with good performance. The most common one is based on the restricted isometric property (RIP) [40]. RIP considers a matrix that behaves as an orthonormal matrix when dealing with sparse signal. For a channel estimation problem as shown in Eq. (9), the sensing matrix \( (\Psi) \) is RIP if there exists a constant \( \delta \) \((0 < \delta < 1)\) for all sparse vectors \( (a) \) which satisfy the below equation as

\[ (1 - \delta)||a||^2_2 \leq ||\Psi a||^2_2 \leq (1 + \delta)||a||^2_2. \] (45)

If the sensing matrix has the RIP, the minimum size of measurements, then the number of row vectors of sensing matrix required for the CS computation is \( 2\kappa \), where \( \kappa \) is the sparsity level. However, an RIP matrix condition can be perfectly achieved only when OFDM system allocates the subcarriers for pilot symbols in random locations [36].

Mutual incoherence property (MIP) is another criterion used for deciding the number of row vectors of sensing matrix for the CS computation. Different with the RIP, every matrix has a certain level of MIP. The MIP level of a sensing matrix \( A \) is defined as

\[ \mu(A) = \max_{1 \leq l, k < M, l \neq k} \frac{|a_l^T a_k|}{||a_l||_2 ||a_k||_2}. \] (46)

where \( a_l \) is the \( l \)-th column of matrix \( A \). The selection of row vectors of sensing matrix can adjust the value of \( a_l \) and then the value of \( \mu(A) \). The mutual coherence will define the maximum sparse level \( \kappa_{\text{max}} \) of matrix \( A \) as

\[ \kappa_{\text{max}} = \left[ \frac{1}{2} \left( \frac{1}{\mu(A)} + 1 \right) \right]. \] (47)

From Eq. (46), we can specify the maximum MIP level to recover a certain signal with sparsity \( \kappa_{\text{max}} \) as

\[ \mu_{\text{max}} = \frac{1}{2\kappa_{\text{max}} - 1}. \] (48)

We can see that the MIP level \( \mu_{\text{max}} \) is inversely proportional to the \( \kappa_{\text{max}} \). For the channel estimation of OFDM system, basically, the \( T \) pilots are allocated with a constant spacing among subcarriers. For such case, aliasing effect will occur when the \( T \) row
vectors of sensing matrix are selected from the corresponding pilot locations with a constant frequency spacing [37]. It means that using all \( T \) row vectors perhaps cannot ensure the minimum MIP level for CS based CE algorithm because random selection of row vectors from a sensing matrix usually has a smaller MIP level. Therefore, we propose a matrix row-vector reduction (MRR) method to select the smaller \( R \) from \( T \) corresponding row vectors of sensing matrix using a genetic algorithm (GA).

GA is an optimization method [35], [36] which is similar to the natural selection [38]. The \( R \times L \) sensing matrix chosen by GA will have a non-uniform frequency spacing which can avoid the aliasing effect. The proposed method can make the \( \mu_{\text{max}} \) enough for the CS recovery process. The MRR is operated before OMP process as shown in Fig. 8. Let us use the \( T \times L \) measurement matrix \( \Omega \) of Eq. (26) and \( L \times 1 \) pilot symbol \( P \) for explanation. The sensing matrix can be represented as

\[
A = \Omega \times \text{diag}(P).
\]  

(49)

The purpose of MRR is to select \( R \) from \( T \) corresponding row vectors from \( A \) which has minimum MIP level. It should be noted that some algorithms use the measurement matrix \( \Omega \) for search process which is essentially the same. Since \( A \) is known, the MRR process can be operated in an off-line way.

After MRR, the \( T \times L \) sensing matrix will be shrank as the \( R \times L \) matrix which further reduces the complexity reduction during the CS computation. This method needs no change at transmitter side, and aims to reduce the size of measurements at receiver side but keep the comparable BER performance.

The detail of GA based MRR is shown in Algorithm 3 and its major process is given in Fig. 9. Each selected \( R \) row vectors from the sensing matrix is treated as an individual. The row indexes of these \( R \) vectors are considered as one chromosome. Each generation contains \( n_d \) individuals. The first generation is created in a brute force way to select \( R \) row vectors randomly from the sensing matrix for each individual. In each generation, we calculate its MIP level as fitness value for each individual. The recommendation probability of each individual is inversely proportional to its MIP level. In order to create the next generation, we first select \( n_d/2 \) pair individuals from all individuals in current generation, and the individuals will be selected according to their recommendation probability \( w \). For each pair, we firstly find their common row indexes from their chromosomes. Then the different row indexes will

**Algorithm 3 Genetic Algorithm Based MRR Algorithm**

**Inputs:**
1. Full \( T \times L \) matrix \( A \); \( n_d, n_g \)
2. Crossover probability \( p_c \); Mutation probability \( p_m \)

**Process:**
1. Create \( n_d \) individuals randomly selected from sensing matrix \( A \).
2. while stopping criterion not met do
3. Calculate fitness value (MIP level) of all individuals in current generation.
4. Calculate recommendation probability \( w \) according to the fitness values of \( n_d \) individuals.
5. for each pair of individual 1 to \( n_d/2 \) do
6. Select two individuals based on their values of \( w \).
7. For different values of two chromosomes, exchange them with a probability as \( p_c \).
8. For each chromosome in an individual, perform perform mutation operation with a probability as \( p_m \).
9. Get two new individuals of the next generation.
10. end for
12. end while
13. Obtain the best individual and row indexes in the final generation.
TABLE I
ESTIMATED COMPUTATIONAL COST FOR THE PROPOSED OMP PER ITERATION

<table>
<thead>
<tr>
<th>No</th>
<th>Term</th>
<th>x</th>
<th>/</th>
<th>CORDIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \mathbf{g} = (\Psi^H \mathbf{r}) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Conventional</td>
<td>( WTL )</td>
<td></td>
<td>/</td>
</tr>
<tr>
<td></td>
<td>method 1</td>
<td>( TL )</td>
<td></td>
<td>/</td>
</tr>
<tr>
<td></td>
<td>method 1+2</td>
<td>( (T + U)LC/O )</td>
<td></td>
<td>/</td>
</tr>
<tr>
<td></td>
<td>method 1+2+3</td>
<td>( (R + U)LC/O )</td>
<td></td>
<td>/</td>
</tr>
<tr>
<td>2</td>
<td>( \alpha = \arg \max</td>
<td>g_i</td>
<td>)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Conventional</td>
<td>( WL )</td>
<td></td>
<td>/</td>
</tr>
<tr>
<td></td>
<td>method 1</td>
<td>( L )</td>
<td></td>
<td>/</td>
</tr>
<tr>
<td></td>
<td>method 1+2</td>
<td>( L )</td>
<td></td>
<td>/</td>
</tr>
<tr>
<td></td>
<td>method 1+2+3</td>
<td>( L )</td>
<td></td>
<td>/</td>
</tr>
<tr>
<td>3</td>
<td>( \mathbf{H} = \alpha + \alpha/L )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>( \Phi_1 = \Phi_{i-1} \Psi_1 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>( \mathbf{b}_1 = \Phi^H \mathbf{b} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Conventional</td>
<td>( WT )</td>
<td></td>
<td>/</td>
</tr>
<tr>
<td></td>
<td>method 1</td>
<td>( WT )</td>
<td></td>
<td>/</td>
</tr>
<tr>
<td></td>
<td>method 1+2</td>
<td>( WT )</td>
<td></td>
<td>/</td>
</tr>
<tr>
<td></td>
<td>method 1+2+3</td>
<td>( WR )</td>
<td></td>
<td>/</td>
</tr>
<tr>
<td>6</td>
<td>( \mathbf{S} = \Phi^H \Phi )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>( \mathbf{S}^{-1} = \text{inv}(\mathbf{S}) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>( \mathbf{a} = \mathbf{S}^{-1} \mathbf{b} )</td>
<td></td>
<td>((W^2 \kappa^2 - W \kappa)/2 )</td>
<td>( W \kappa )</td>
</tr>
<tr>
<td>9</td>
<td>( \mathbf{r} = \mathbf{b} - \Phi \mathbf{a} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Conventional</td>
<td>( WT )</td>
<td></td>
<td>/</td>
</tr>
<tr>
<td></td>
<td>method 1</td>
<td>( WT )</td>
<td></td>
<td>/</td>
</tr>
<tr>
<td></td>
<td>method 1+2</td>
<td>( WT )</td>
<td></td>
<td>/</td>
</tr>
<tr>
<td></td>
<td>method 1+2+3</td>
<td>( WR )</td>
<td></td>
<td>/</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Conventional</td>
<td>( WTL + WL + 2WL + \frac{(W^2 \kappa^2 - W \kappa)/2}{W \kappa} )</td>
<td>( \frac{(W^2 \kappa^2 - W \kappa)/2}{W \kappa} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>method 1</td>
<td>( TL + L + 2WL + \frac{(W^2 \kappa^2 - W \kappa)/2}{W \kappa} )</td>
<td>( \frac{(W^2 \kappa^2 - W \kappa)/2}{W \kappa} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>method 1+2</td>
<td>( (T + U)L ) + ( (R + U)L ) + ( 2WL + \frac{(W^2 \kappa^2 - W \kappa)/2}{W \kappa} )</td>
<td>( \frac{(W^2 \kappa^2 - W \kappa)/2}{W \kappa} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>method 1+2+3</td>
<td>( (R + U) ) + ( (U) ) + ( 2WL + \frac{(W^2 \kappa^2 - W \kappa)/2}{W \kappa} )</td>
<td>( \frac{(W^2 \kappa^2 - W \kappa)/2}{W \kappa} )</td>
<td></td>
</tr>
</tbody>
</table>

be exchanged as crossover process with a probability \( p_c \) and both chromosomes are operated with a process of mutation with a probability \( p_m \). The mutation process will keep their common row indexes (good gene) and change the left row indexes with new values. The process of GA will be stopped after the \( n_g \)-th generation has been created. Moreover the algorithm will be stopped if the new generation is identical to its previous one. Finally, the individual with the smallest MIP level at the last generation will be chosen as the final result. The corresponding row indexes are used for the observation vector to select a \( R \times 1 \) vector for the OMP process.

IV. RESULTS

A. Computational Complexity Analysis

The computational complexity of the steps for Algorithm 2 is given in the Table I. In this table, we only consider the CC of multiplication, division, and coordinate rotation digital computer (CORDIC), and the CC of adder and subtractor operation are negligible here. Some steps do not require any of these operation resources. Step 3 is an index augmentation for the vector \( \Omega \) which only requires an addition operation and Step 4 is an augmentation for the matrix \( \Phi \) which can be implemented by a memory fetcher. The matrix \( (\Phi^H \Phi) \) in Step 6 can be obtained by taking the row and column of the matrix \( (\Psi^H \Psi) \) that correspond to the locations of sparse elements. Thus, we can compute it in an off-line computation and save it into the memory. In this article, we realize the matrix inversion using QR based decomposition with given rotation technique to solve the least squares problem. In addition, the given rotations process is realized by an array of CORDIC [39], and a CORDIC only consists of a look up table (LUT) and shifters.

We have compared four approaches of implementing the inner product computation of sensing matrix, the conventional method, the MMC-OMP, the combination of MMC-OMP and DSR, and the combinations of all three proposed methods of MMC-OMP, DSR and MRR. The proposed methods not only affect the inner product calculation but also other steps in the OMP computation. The search of maximum value from Step 2 will deal with the smaller vector due to the usage of the MMC-OMP method. The MRR will reduce the size of the matrices and vectors involved in the multiplications at Step 5 and 9.

To show the effectiveness of our proposal, we simulate our system with the parameters as shown in the Table II. We have set two kind of guard interval (GI) length to assess the impact of value \( L \) on the sensing matrix. The corresponding parameters of sensing matrix for the different GI length are shown in the Table III. The inner product complexity for the conventional method is \( 3L \) which are 98, 304 and 49, 152 multipliers when GI length equals to 1/8 and 1/16 of OFDM symbol duration. The proposed MMC-OMP can reduce the inner product complexity to 33%, where the total of required multiplier is \( TL \) which are about 32, 768 and 16, 384 multipliers when GI length equals to 1/8 and 1/16 of OFDM symbol duration, respectively.

In order to implement the DSR, firstly, we need to find the value of parameter \( C \) which can offer the lowest complexity. We calculate the mean of the multiplier number per row with different \( C \) as shown in Fig. 10. As shown in Fig. 10, it will be the lowest number of multipliers when \( C \) equals to 32 for both kind of GI length. From Eq. (38), we can find the value of parameter \( U \) is 87 and 171 when GI length
Fig. 10. Mean number of multipliers per row with different $C$.

Fig. 11. Multipliers reduction for inner product computation for 3-element ESPAR ($W = 3$).

equals to 1/8 and 1/16 of one OFDM symbol duration, respectively. Furthermore, the total number of multipliers for the inner product become lower to 1720 and 1196 when GI length equals to 1/8 and 1/16 of one OFDM symbol duration, respectively. Finally, we apply the MRR to reduce the $T$ value from 128 to 64.

The comparison of the computational complexity between the conventional method and proposed three approaches is shown in Fig. 11. By reducing the size of row vectors into 64 using MRR, we can gain an additional complexity reduction by 30%. The MRR method reduces the number of multipliers from 2,897 into 2,001 when GI length equals to 1/8 of one OFDM symbol duration and from 2,245 into 1,518 when GI length equals to 1/16 of one OFDM symbol duration, respectively. Finally, we can achieve more than 90% of the multiplier reduction using the combination of all three proposed methods. When the GI length equal to 1/8 of one OFDM symbol duration, the proposed overall methods can reduce the number of multipliers from 99,993 into 2001. While for GI length as 1/16 of one OFDM symbol duration, the total number of multipliers can be reduced from 50,457 into 1,158.

B. BER Performance

We compare the BER performance of the proposed methods with that of conventional approach. The ITU-6 typical urban channel with six pathes is chosen as the channel model for simulation. Due to the 3-element ESPAR antenna, the channel estimation needs to estimate the values of 18 channel impulses response. The pilots are set with 16 frequency spacing, and FFT size equals to 2048. The CP length is set as 1/8 of one OFDM symbol duration. Therefore, the sensing matrix has a size of 128 $\times$ 768.

The first simulation is to compare the BER performance between the conventional OFDM system and 3-element ESPAR-OFDM system using OMP as shown in Fig. 12. Here we used QPSK modulation. It can be seen that the 3-element ESPAR antenna using one set of RF chain can obtain significantly large diversity or better BER performance than that of the conventional OFDM system. In addition, we also provide the simulated BER performance of the ESPAR-OFDM system with an OMP based channel estimation in our previous work [30]. The performance of CS based channel estimation has about 1 dB gap of BER performance degradation compared to that of the ESPAR-OFDM system with perfect CSI.

Before we can apply the matrix row-vector reduction method, we need to obtain the individual and the row indexes of these row vectors that corresponds to the lowest MIP level. For comparison, we also provide the results of the method using random selection (RS) for row vectors. In such case, the possible combinations for that selecting the 64 from 128 row vectors is in order of $10^{37}$, which makes the random selection might not provide an optimum solution for the MRR. Here, we run the GA with the parameters as $n_g = 100$ and $n_s = 100$. Each pair exchanges its chromosome with a possibility as $p_c = 0.5$, and the mutation is performed with much less probability as $p_m = 0.005$. The GA method based MRR can obtain the convergence of MIP level compared to that of...
RS based MRR method as shown in Fig. 13. Here, to compare the MIP level of the RS based MRR with proposed GA based MRR, we simulated the MIR level with 100 random selections for showing its variation of MIR level. The average value of MIR level with 10,000 random selections is about 0.1663 which also provided in Fig. 13. As the results, we can find that the proposed GA based MRR can reduce the MIP level from 0.153 to 0.126. We also compare the BER performance of the GA based and RS based MRR method for 3-element ESPAR-OFDM in Fig. 14. From the results, we can find that when EbNo value is larger than 12 dB, GA based MRR can have about 1 dB BER gain improvement than that of RS based MRR method.

It should be noted that, since the GA based MRR is operated in an offline mode, it can be operated with many different parameters to find the best one. In this article, we just show that MRR can reduce half number of pilots with our parameters. We believe the further reduction can be achieved. But such topic will be explained in our ongoing research.

Finally, we can compare the performance of 4 approaches for 3-element ESPAR-MIMO system in Fig 15 for QPSK modulation and in Fig 16 for 16-QAM modulation, respectively. In these figures, it can be seen that the system using MMC-OMP and DSR methods have almost the same performance compared to that of using the conventional OMP method. Moreover, from the Fig. 11, it can be confirmed that the using both MMC-OMP and DSR can provide more than 90% complexity reduction. When system utilizes MRR to further reduce its computational complexity, it appears about 1 dB BER performance loss for QPSK modulation when EbNo is over 20 dB. But such performance loss will be disappeared for the system with 16-QAM modulation. The implementation of MRR method can further reduce 30% complexity compared to that of using both MMC-OMP and DSR methods. Therefore, it is inferable that the proposed methods will further largely reduce the complexity for the hardware implementation of ESPAR antenna with more elements for different usage.

V. CONCLUSION

In this article, we present a solution to reduce the computational complexity in the CS based channel estimation for
OFDM system with ESPAR antenna. We make use of the similarity of the CIR location for each antenna to propose the MMC-OMP method. Furthermore, we exploit the truncated DFT symmetrical property to perform the matrix size reduction named as DSR method. Finally, we utilize GA based MRR method to reduce the sensing matrix row size but prevent aliasing to achieve promising BER performance. These proposed methods can achieve more than 90% reduction in the number of multipliers with almost the same BER performance. Methods can achieve more than 90% reduction in the number of multipliers with almost the same BER performance. These proposed methods can achieve more than 90% reduction in the number of multipliers with almost the same BER performance. These proposed methods can achieve more than 90% reduction in the number of multipliers with almost the same BER performance.

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