Matrix Factorization-Based RSS Interpolation for Radio Environment Prediction

Norisato Suga, Member, IEEE, Kazuto Yano, Member, IEEE, Julian Webber, Senior Member, IEEE, Yafei Hou, Senior Member, IEEE, Eiji Nii, Member, IEEE, Toshihide Higashimori, and Yoshinori Suzuki

Abstract—This letter proposes matrix factorization (MF) based interpolation of received signal strength (RSS) from a transmitter mounted on moving robot in factory environment. For realizing the reliable wireless communication, machine learning based channel prediction methods have been intensively studied in the past decade. However, some traffic models will make the observation of RSS sequence be intermittent, and the missing values must be interpolated before input to the predictor. Classical interpolation such as linear interpolation cannot appropriately estimate the missing values because the result of the interpolation depends on the observation time. In this letter, we propose to apply an MF-based interpolation technique to RSS interpolation in order to restore the true RSS variation pattern. Moreover, the basic MF-based interpolation is improved by introducing a smoothing term in an objective function to represent the smooth variation of the RSS sequence. The simulation results show that the MF-based interpolation can improve the prediction accuracy of the machine learning based channel prediction method.

Index Terms—Channel prediction, matrix factorization, received signal strength interpolation

I. INTRODUCTION

THE demand for the Internet of things (IoT) is growing in manufacturing sites such as factories and warehouses [1]. However, dynamic change of wireless communication environment in factories impedes the usage of IoT because unexpected trouble of wireless communication severely degrades the productivity of a factory. To maintain the reliability of wireless communication, it is important to predict a future quality of wireless communication in advance and to take an action to avoid serious damage to the productivity. MAC-layer scheduling and physical-layer power-control can also benefit from the prediction of wireless communication quality. In factories, there are many robots which move along the pre-determined route such as automated guided vehicle (AGV) and stacker crane [2]. In general, these robots are controlled via wireless communication because it is impractical to use a wired link. Nowadays, these robots are becoming prevalent in the factory environment, and thus the prediction of wireless communication quality such as received signal strength (RSS) of these robots is more and more important.

It is well-known that neural network (NN) can extract signal features and predict patterns. It was reported in [3] that an adaptive NN-based channel predictor can reduce the mean deviation between a measured and predicted signal to within 8 dB. Improved signal field-strength prediction was demonstrated using a carefully trained NN in [4]. We recently proposed a procedure for predicting the future RSS variation using a probabilistic NN (PNN) in [5], [6]. These NN-based methods can accurately predict the future RSS for the trained patterns of RSS variation. However, the observation of RSS can be intermittent due to sparse traffic generation of wireless applications in factories and warehouses. To appropriately perform the training and prediction, the missing RSS values must be interpolated before input to the predictor.

In factories and warehouses, there are many wireless applications with low traffic generation frequencies, for example once per second to once per minute [7]. During such a long period, the RSS complicatedly varies due to multipath fading and shadowing, which cannot be regarded as linear variation for the time lapse. Therefore, the classical interpolation methods such as linear interpolation cannot estimate the missing values in RSS. This causes deterioration of prediction accuracy. Moreover, the RSS prediction is performed in an online manner. In the classical interpolation methods, the missing values must be sandwiched by the observed RSS values to perform the interpolation. This means that the missing values after the last observation cannot be estimated until a new RSS is observed.

As mentioned above, there are many robots which repeatedly move along a pre-determined route in factories. According to the repeated movement, the RSS variations of such robots are also repeated. A matrix, which is constructed by the extraction and stacking of RSS sequence, is expected to be low-rank when the same RSS variation pattern is repeated. Hence, we propose to apply the matrix factorization (MF) technique [8] to interpolate the missing values of RSS sequence as exploiting the low rank property, and overcome the problems of the classical interpolation methods. The MF-based interpolation, which forces the rank of the interpolated matrix to a fixed value, can fill the missing values as preserving the true RSS variation pattern. Furthermore, the MF-based interpolation can estimate the missing values after the last observation because the MF-based interpolation does not depend

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N. Suga is also with Faculty of Engineering, Tokyo University of Science, Tokyo, Japan (e-mail: norisato.suga@rs.tus.ac.jp).

J. Webber is also with Graduate School of Engineering Science, Osaka University, Toyonaka City, Japan.

Y. Hou is also with Graduate School of Natural Science and Technology, Okayama University, Okayama City, Japan.

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the position of the missing values. Since the RSS variation in a short period is small, we also propose to introduce a condition to smooth the interpolated RSS sequence into the optimization problem for the MF-based interpolation as in [9]. Computer simulation assuming a warehouse shows that the proposed MF-based interpolation can improve the prediction accuracy compared with various interpolation methods.

**Notations:** Matrices are expressed in uppercase boldfaced letters. \( R \) denote real number set. \( R^{M \times N} \) is an \( M \times N \) real matrix set. \( A \) is the complementary set of \( A \). \( |A| \) represents the cardinality of \( A \). \( |\cdot| \) denotes Frobenius norm, and \( (\cdot)^T \) denotes the transpose. The \( (i,j) \)-th element of matrix \( M \) is denoted as \( M_{i,j} \). \( M(a:b,c:d) \) is a matrix whose \( (i,j) \)-th element is \( \sqrt{M_{i,j}} \). \( I \) denotes the identity matrix. \( M_{a:b,c:d} \) represents a submatrix that consists of rows \( a \) to \( b \) and columns \( c \) to \( d \) columns of \( M \). \( M_{a:} \) and \( M_{:d} \) mean the \( a \)-th row and \( a \)-th column of \( M \), respectively. \( \lfloor \cdot \rfloor \) is floor function.

### II. Problem Statement

In this letter, we focus on the interpolation problem of the RSS sequence which is observed at the receiver. To perform channel prediction, the missing RSS values must be estimated before input to the predictor. Let \( r_t \) be the RSS at time \( t \). According to the traffic model, a transmitter intermittently transmits wireless frames. Therefore, at the receiver side, the observed RSS is also obtained intermittently.

Let \( T \) be the set of the time indices when the RSS is observed, and \( T_c \) the set of that when the wireless frame is not transmitted. The missing rate is defined as \( \rho = \frac{|T_c|}{|T + T_c|} \). For a high missing rate, the number of observations is small compared to the number of missing points. In general, the channel prediction is performed in an on-line manner. Therefore, the interpolation must estimate the RSS values between the current time and the last observation time. Strictly speaking, this is extrapolation which cannot be treated by the classical interpolation such as linear interpolation. Therefore, the problem treated in this letter is estimation of \( r_t \) for \( t \in T_c \) under the condition that \( \max(T) < \max(T_c) \) where \( \max(T_c) \) means the current time index.

### III. MF-based RSS Interpolation

The channel prediction needs to estimate the missing RSS values in the repeated RSS variation pattern including the values after the last observation. The MF-based interpolation satisfies this requirement, and we apply it to RSS interpolation. For the interpolation, we construct a Hankel matrix \( R \in \mathbb{R}^{N_w \times N_w} \) from the RSS sequence as illustrated in Fig.1. Each row of the Hankel matrix is obtained by the size-\( N_w \) sliding window and \( N_s \) times stacking. We denote the \((i,j)\)-th element of \( R \) as \( r_{i,k} \). Note that the RSS value \( r_{i,k} \) is identical to \( r_i \), if \( i + k - 1 = t \). Two-dimensional index set \( \Omega \) represents the observed point indices in \( R \), while index set \( \Omega_c \) is a set of the missing point indices. The indices set \( \Omega \) and \( \Omega_c \) are uniquely determined from the \( T \) and \( T_c \), respectively.

In factories, AGVs move along a fixed route repeatedly, and similar RSS variation patterns can be observed. If similar RSS variation patterns emerge repeatedly, the rank of the Hankel matrix \( R \) is low because some rows of \( R \) are close in Euclidean norm each other. When the rank of \( R \) is \( L \), it can be factorized into two \( L \)-rank matrices \( P \in \mathbb{R}^{N_s \times L} \) and \( Q \in \mathbb{R}^{L \times N_w} \) as \( R = PQ \). Therefore, the interpolation of the missing values in \( R \) is replaced to estimation of \( P \) and \( Q \). The basic MF problem [8] is defined as

\[
\min_{P,Q} J_1(P,Q) + \lambda J_2(P,Q) \tag{1}
\]

where \( J_1(P,Q) \) is Euclidean norm between the observation and estimated value, and \( J_2(P,Q) \) is a regularization term to avoid over fitting, \( \lambda \) is a parameter which controls the extent of regularization. \( J_1(P,Q) \) and \( J_2(P,Q) \) are given by

\[
J_1(P,Q) = \sum_{(i,k) \in \Omega} |r_{i,k} - P_{i,:} Q_{:,k}|^2, \tag{2}
\]

\[
J_2(P,Q) = ||P||_F^2 + ||Q||_F^2. \tag{3}
\]

The objective function in (1) is a bi-convex problem and can be minimized by alternating minimization algorithms such as the methods presented in [10], [11].

The RSS variation in a short period is small because the moving robot locates near place. Therefore, the estimated RSS should be interpolated as a continuous shape. To realize smooth interpolation, we further propose to consider following minimization problem as described in [9].

\[
\min_{P,Q} J_1(P,Q) + \lambda J_2(P,Q) + \mu J_3(P,Q) \tag{4}
\]

where \( J_1(P,Q) \) means the RSS variation between neighboring time samples, and \( \mu \) is a parameter to control the extent of \( J_3(P,Q) \). The smoothing term \( J_3(P,Q) \) is given by

\[
J_3(P,Q) = \sum_{i=1}^{N_s} \sum_{k=1}^{N_w-1} |P_{i,:} Q_{:,k} - P_{i,:} Q_{:,k+1}|^2. \tag{5}
\]

The objective function in (4) is also bi-convex. It can iteratively be solved by an alternating minimization approach.

First, suppose \( P \) is fixed. Then, the problem becomes the minimization of (4) for \( Q \). As in the procedure of [10], we decompose the problem into \( N_w \) separate problems, i.e, the optimization of \( Q_{:,k} \) for \( k = 1, \cdots, N_w \), given as

\[
\min_{Q_{:,k}} J_1(Q_{:,k}) + \lambda J_2(Q_{:,k}) + \mu J_3(Q_{:,k}) \tag{6}
\]

This problem is convex and has the closed form solution as

\[
\hat{Q}_{:,k} = (P_p^T P_p + \lambda I + \mu P_T P)^{-1} P_p^T (x + \mu P Q_{:,k+1}) \tag{7}
\]

where \( P_p \) is a matrix constructed from some rows of \( P \), and \( x \) is a vector constructed from some elements of \( R_{:,k} \). The relation between \( P_p \) and \( P \) (similarly, \( x \) and \( R_{:,k} \)) is explicitly
Algorithm 1 MFS-based RSS interpolation

Require: \(R^{(0)}, \Omega, L, \lambda, P, N, P_{bs}, m_{\max}\)

1: \(R \leftarrow R^{(0)}\)
2: \([U, \Sigma, V] \leftarrow \text{SVD}(R)\)
3: \(P \leftarrow U[:,1:1:1:1:1:L;1]: Q \leftarrow \Sigma^{1/2}_{1:1:1:1;1:1:L}; m \leftarrow 0\)
4: \(J_2 = \|P\|^2 \Sigma + \|Q\|^2 \Sigma\)
5: Calculate (11) and \(J_3 \leftarrow N_s(N_w - 1)r_{\text{diff}}\)
6: \(\mu \leftarrow \lambda J_3\)
7: repeat
8: for all \(k \in \{1, 2, \cdots, N_w - 1\}\) do
9: Update \(Q_{k,i}\) by (7)
10: end for
11: Update \(Q_{i,:}\) by (8)
12: for all \(i \in \{1, 2, \cdots, N_s\}\) do
13: Update \(P_{i,:}\) by (10)
14: end for
15: \(R \leftarrow P Q\)
16: \(R_{e,k} \leftarrow R^{(0)}_{e,k}\) for \(i, k \in \Omega\)
17: \(m \leftarrow m + 1\)
18: until \(m_{\max} < m\)

Ensure: \(R\)

indicated by the selection matrix \(S_k\) which satisfies \(P = S_k P\) (similarly, \(x = S_k R_{e,k}\)). Each row of selection matrix \(S_k\) has only one “1” to extract rows of \(P\) corresponding to observed position in \(R_{e,k}\). Note that the smoothing term in (5) is valid for \(k = 1, \cdots, N_w - 1\). Therefore, for \(k = N_w\), the optimization in (6) reduces to ridge regression whose solution is given by

\[
\hat{Q}_{i,:} = (P^T P + \lambda I)^{-1} P^T x. \tag{8}
\]

Next, we update the matrix \(P\). Similarly, suppose \(Q\) is fixed and consider the following problem:

\[
\min_{P_{i,:}} J_1(P^T_{i,:}) + \lambda J_2(P^T_{i,:}) + \mu J_3(P^T_{i,:}). \tag{9}
\]

This is also a convex problem, and the solution is derived as

\[
\hat{P}^T_{i,:} = (Q^T_p Q_p + \lambda I + \mu D^T D)^{-1} Q^T_p y, \tag{10}
\]

where \(D = (Q_{1:1:N_w-1} - Q_{1:2:N_w})^T\), \(Q_p = S_i Q_p\), \(y = S_i R_{e,k}^T\), and \(S_i\) is a selection matrix to extract some rows of \(Q_p^T\) and elements of \(R_{e,k}^T\). The update in (10) is performed for \(i = 1, \cdots, N_s\).

In the proposed MFS-based interpolation, the additional regularization parameter \(\mu\) is introduced. This incurs a problem how to balance the two regularization parameters \(\lambda\) and \(\mu\). In this letter, we also propose the estimation method of the ratio of \(\lambda\) and \(\mu\) as following. First, we roughly estimate the amount of \(J_2\) by using the interpolation result \(R\) obtained by the conventional interpolation method, and singular value decomposition (SVD) provides the estimation \(P\) and \(Q\). From these estimations, we estimates \(J_2\) according to (3). Second, the indoor path loss model [12] is used for estimation of the amount of \(J_3\) as \(\eta_l = 20 \log_{10} f + N \log_{10} d_l + \eta_{\text{floor}} - 28\) where \(f\) is frequency, \(N\) is distance power loss coefficient, \(d_l\) is distance (m) between the base station and terminal at time \(t\), and \(\eta_{\text{floor}}\) is floor penetration loss factor. The RSS difference between \(t + 1\) and \(t\) can be approximated as \(r_{t+1} - r_t \approx \eta_{l+1} - \eta_l = N \log_{10} \frac{d_{l+1}}{d_l}\). Denote the position of a base station as \(p_s\), the terminal position at time \(t\) as \(p_t\), and the speed of the moving terminal as \(v\). This approximation is expressed as \(N \log_{10} \frac{|p_t + v e - p_{bs}|}{|p_t - p_{bs}|}\), where \(e\) is an unit direction vector defined as \(|\cos \theta, \sin \theta|^T\). Therefore, we calculate the expected value of the RSS difference by the numerical integration as

\[
r_{\text{diff}} = \frac{1}{|P|} 2\pi \int_{|P|}^{2\pi} \left|N \log_{10} \frac{|p_t + v e - p_{bs}|}{|p_t - p_{bs}|}\right|^2 d\theta \tag{11}
\]

where \(P\) is the terminal position set \(\{p_0, p_1, \cdots\}\), which is determined by the pre-determined route. Then, the amount of \(J_3\) is estimated as \(J_3 = \sum_{i=1}^{N_s} \sum_{k=1}^{N_w-1} r_{\text{diff}} = N_s (N_w - 1) r_{\text{diff}}\). Finally, we estimate that \(J_3\) contributes \(\hat{J}_3 / J_2\) times larger to the objective function. To adjust the contributions, we set \(\mu = \frac{J_3}{J_2}\).

In the proposed MFS-based interpolation, (7), (8), and (10), are iteratively performed until the solutions of \(P\) and \(Q\) converge. The whole algorithm, denoted as MF with smoothing (MFS), is summarized in Algorithm 1. As the initial value of \(P\) and \(Q\), matrices which consist of singular vectors corresponding top \(L\) largest singular values are calculated by singular value decomposition.

To determine the appropriate rank \(L\), we use validation technique as in algorithm 2. In this algorithm, \([|T| : \nu]\) validation indexes are randomly chosen from \(T\), and the validation index set \(T_{\text{valid}}\) are created. The parameter \(\nu\) indicates the ratio of the number of validation samples to \(|T|\). Using the samples corresponding \(T - T_{\text{valid}}\), the MF and MFS-based interpolation are performed with the rank candidates \(l \in \{1, 2, \cdots, L_{\max}\}\). For each candidate, mean absolute error (MAE) \(e_l = \frac{1}{|T_{\text{valid}}|} \sum_{t \in T_{\text{valid}}} |r_t - \hat{r}_t|\) is calculated, and the rank \(L\) is determined as \(l\) corresponding to the minimum \(e_l\).

IV. Evaluation

A. Experiment setting

In this section, we evaluate the performance of the proposed method through a computer simulation assuming a warehouse where an AGV moves along some pre-determined routes, which is one of the typical use case of industrial IoT [7].
For the evaluation of the proposed interpolation, we conducted ray-tracing simulation [6] to obtain RSS sequence. In the warehouse, an AGV is controlled via wireless local area network (WLAN) [13] in 5 GHz band. For the ray-tracing simulation, we set the transmitted power to 14 dBm assuming an omni antenna equipped with the AGV and access point (AP). The number of scatter points [6] are 30, which can sufficiently realize the effect of the fading. Figure 2(a) illustrates the assumed warehouse and pre-determined routes. The area size of the warehouse is 18 m × 26 m. There are six metal shelves (12 m × 2 m) which interrupt the rays emitted from the transmitter. The AGV randomly selects its route from route-1 to route-6 with same probability. Each route length equals to 80 m. The speed of AGV is 1.7 m/s. The sampling interval of RSS sequence is 1 second. Thus, the AGV returns to the start point with 80/1.7 ≈ 47 samples and moves to the next round. The computer used for the RT simulation was Intel Core i9 3.30 GHz CPU with 64 GB of memory. Figures 2(b) and 2(c) illustrate the snapshot of the obtained RSS sequence and its auto-correlation function (ACF).

For missing type, we assume random missing as usual for the evaluation of interpolation method. Furthermore, we also assume periodic missing and bursty missing according to 5G traffic model for industrial use cases [14]. In the random missing, the missing point is randomly determined according to the missing rate ρ. In the periodic missing, the RSS values are continuously missed at an interval. In the bursty missing, the RSS missing starts at random, and it continues for 20 seconds. Under these assumptions, the RSS sequence observed at the AP is fed into the proposed interpolation unit.

As a benchmark, the performances of an average filling method, a linear interpolation, and Gaussian process regression (GPR) [15] are compared. We also demonstrate the performance of the MF-based interpolation [11] and that of the MFS-based interpolation. The parameters used in the algorithms are set to \( N_s = 1000, \ N_c = 1001, \ \lambda = 10^{-3}, \) and \( m_{\text{max}} = 200. \) In the validation for the rank estimation, \( \nu \) and \( L_{\text{max}} \) are set to 0.05 and 20, respectively. The initial value \( R^{(0)} \) is constructed from the result of linear interpolation. We further evaluate the performance of the future value predictions with the above interpolation methods. As the prediction, we adopt the PNN-based RSS prediction presented in [6] and long short-term memory (LSTM) based prediction [16]. Based on these prediction methods, the RSS value 10 seconds ahead is predicted. The performance measure is MAE calculated by \( \text{MAE} = \frac{1}{T} \sum_{t} |\hat{r}_t - r_t| \). The number of RSS samples \(|T + T_c|\) is set to 5,000.

### B. Evaluation results

The interpolated RSS sequences of the interpolation methods with \( \rho = 0.9 \) are shown in Fig. 3. MAE value is also shown in a round bracket. For all missing types, the average filling (denoted as “Ave.”) method cannot properly interpolate the RSS sequence. Its MAE is around 6.4 dB regardless the missing type. Although, the linear interpolation works well for the periodic missing, it cannot recover the pattern for the random missing and the bursty missing. Compared with these interpolation methods, the MF and MFS-based interpolation methods can recover the pattern of RSS variation especially for the random and bursty missing. For the bursty missing, MF and MFS-based interpolation work even though there are no observations in a long-range. The reason of this is following. The size of the Hankel matrix was set to 1,001 × 1,000, and the repetition period of the AGV movement is approximately 47 samples. Therefore, there are many similar rows in the Hankel matrix, which introduces the low rank property in the matrix. Although the RSS values are continually lost in the period illustrated in Fig. 3(c), in the other similar row, RSS values can be observed. This contributes to recover nonlinear variation in the MF and MFS-based interpolation by the rank limitation. For the periodic missing, the performance of the normal MF-based method (denoted as “MF”) is worst due...
to the over fitting. A way to avoid this is to set a larger value to the regularization parameter λ. However, it impedes the accurate interpolation because the second term in (1) is minimized by the all zero solution. On the other hand, the proposed MFS-based interpolation shows better performance than the normal MF-based interpolation due to the smoothing term in (4).

The comparison of the simulation time of the interpolation methods is also presented in Table I. For MF and MFS, the simulation time with \( L = 10, 20 \) are shown (without validation). The time of the average filling, linear, and GPR interpolation are less than 1 second. On the other hand, the MF and MFS-based interpolation require high calculation cost due to the alternation nature. This is a drawback of MF and MFS-based interpolation and a subject for the future research.

In Fig. 4, the prediction accuracy of the PNN and LSTM-based prediction method with different interpolation methods for various missing rates is illustrated. For comparison, the prediction accuracy of the case without the RSS missing is also shown (denoted as “No Missing”). The prediction accuracy of the proposed MFS-based interpolation basically outperform the other methods for the random and bursty missing. For periodic missing, the predictions coupled with MF-based interpolation cannot accurately predict the future RSS due to the poor interpolation accuracy caused by the over fitting as shown in Fig. 3(b). Remarkably, the accuracy of the PNN and LSTM-based predictions with MFS interpolation is drastically improved by introducing the smoothing term.

V. CONCLUSION

In this letter, we proposed MF-based interpolation of RSS sequence for the channel prediction in the factory environment. For smooth interpolation, we also proposed to introduce a smoothing term into the minimization problem to suppress the drastic variation. For evaluating the effectiveness of MF-based and MFS-based interpolation, we simulated the RSS variation assuming a WLAN communication in a warehouse where an AGV moves along the pre-determined routes. The obtained RSS sequences with three types of missing, i.e., random, periodic, and bursty missing, are interpolated and trained for the future value prediction based on PNN and LSTM. The evaluation result showed the proposed MF-based and MFS-based interpolation methods can estimate the RSS sequence with better accuracy than the conventional methods for the random and bursty missing. Moreover, the proposed MF and MFS interpolation can improve the prediction accuracy of the NN-based method compared with conventional interpolation method for the random and bursty missing.

As a drawback, the MF and MFS-based interpolation take high computational cost, and it should be reduced in the future work. The evaluation using real data is also important subject for the application to the industrial use.

### REFERENCES


