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PAPER

Eigenvalue Based Relay Selection for XOR-Physical Layer Network Coding in Bi-Directional Wireless Relaying Networks

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SUMMARY This paper proposes relay selection techniques for XOR physical layer network coding with MMSE based non-linear precoding in MIMO bi-directional wireless relaying networks. The proposed selection techniques are derived on the different assumption about characteristics of the MMSE based non-linear precoding in the wireless network. We show that the signal to noise power ratio (SNR) is dependent on the product of all the eigenvalues in the channels from the terminals to relays. This paper shows that the best selection techniques in all the proposed techniques is to select a group of the relays that maximizes the product. Therefore, the selection technique is called "product of all eigenvalues (PAE)" in this paper. The performance of the proposed relay selection techniques is evaluated in a MIMO bi-directional wireless relaying network where two terminals with 2 antennas exchange their information via relays. When the PAE is applied to select a group of the 2 relays out of the 10 relays where an antenna is placed, the PAE attains a gain of more than 13dB at the BER of 10^{-3}.

key words: *physical layer network coding, non-linear precoding, MIMO, signal to noise power ratio, eigenvalues*

1. Introduction

Wireless communication is best suited to connect devices with the internet for internet of things (IoT) applications. Because IoT devices are scattered in the world, wider coverage area is demanded for wireless networks to support IoT applications. Wireless relaying has been considered to extend coverage areas. Wireless relays have been introduced not only in wireless networks for IoT such as wireless smart utility network (WiSUN), but also wireless local area networks (WLANs). In those wireless networks, time division duplex (TDD) is applied to send information from the source to the destination via relays [1]. When bi-directional communications are performed in the wireless relaying networks, however, the frequency utilization efficiency is reduced. Physical layer network coding (PLNC) has been considered to mitigate the drawback [2]–[5]. Many PLNC techniques have been proposed for further performance improvement [6]–[12]. Especially, XOR-PLNC has been shown to achieve the optimum performance in terms of channel capacity. Precoding techniques [13] have been proposed for XOR-PLNCs to reduce computational complexity of relays, while achieving a little bit better transmission performance [14]. The XOR-PLNC has been extended to MIMO bi-directional relaying networks [15], [16]. Moreover, some precoding techniques have been proposed for transmission performance enhancement in MIMO bi-directional relaying networks [17]–[19]. In the above investigations, relays are assumed to be pre-assigned for bi-directional communications in the networks. Since many relays may be scattered around the source terminal and the destination terminal, actually, relay selection has been investigated to improve transmission performance in one-directional relaying networks [20]–[22]. Relay selection has also been considered for bi-directional relaying network with PLNC [23]–[25]. In the literature, the transmission performance is analyzed when the relay that maximizes criteria is selected based on the min-max approach in systems where an antenna is installed on terminals and relays. Overall SNR, minimum Euclidean distance, total throughput are applied as the criteria. The same approach has been extended for MIMO bi-directional relay systems. However, the conventional techniques are complicated to find the best combination of relays even though channel coding is not taken into account.

This paper proposes techniques to select relays in order to improve the transmission performance in MIMO bi-directional relaying networks with XOR-PLNC. We introduce a simple selection criterion that is defined as the end-to-end BER performance of the relaying network. The proposed technique selects a group of the relays that maximizes the selection criterion. We apply lattice reduction-aided non-linear precoding to simplify the selection signal processing and to improve the transmission power efficiency of the transmission signals. Additionally, we use error correction coding to each stream to improve transmission performance. We show that the proposed selection technique can be implemented with almost same computational complexity as that in conventional multi-user MIMO based on the block diagonalization.

Next section describes a network model, and the proposed relay selection techniques are explained in Sect. 3. The performance of the proposed selection is evaluated in Sect. 4 and the concluding remarks are presented in Sect. 5.

Throughout the paper, $(\mathbf{A})^{-1}$, $(\mathbf{B})^{\frac{1}{2}}$, superscript T, and superscript H denote an inverse matrix of a matrix $\mathbf{A}$, a square root matrix of a real diagonal matrix $\mathbf{B}$, transpose, and Hermitian transpose of a matrix or a vector, respectively. $\mathrm{tr}[\mathbf{A}]$ denotes trace of a matrix $\mathbf{A}$. j indicates the imaginary unit. $\mathrm{E}[\zeta]$, $\Re[\alpha]$, and $\Im[\alpha]$ represent the ensemble average of a variable ζ, a real part, and an imaginary part of a com-

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2. Network Model

We assume a wireless network where two terminals communicate with each other via relays. The two terminals are called "terminal A" and "terminal B" respectively, in this paper. While N_{T} antennas are placed on the terminal, only one antenna is deployed in the relays. We assume that the N_{R} relays are scattered between the two terminals. When the PLNC is applied to the communication, those terminals transmit their own signals for the relays, simultaneously in the first slot, and the N_{S} relays out of the N_{R} relays broadcast the signals for the two terminals. However, precoding is applied to the transmitters on the terminals to reduce the complexity of the demodulation at the relays [18], [19], while the relays send their signals without any precoding.

The signal processing in the XOR-PLNC is described in detail as follows. N_{S} bit streams are encoded and the N_{S} encoded bit streams are provided to the Quaternary phase shift keying (QPSK) modulators independently, at each terminal. The modulation signals are fed to a non-linear precoder. The non-linear precoder output vector is transmitted as a transmission signal vector. Let $\mathbf{X}_{\Omega} \in \mathbb{C}^{N_{\mathrm{T}}\times 1}$ denote the transmission signal vector, where Ω takes A or B, the transmission signal vector can be defined as [26]–[28],

$$\mathbf{X}_{\Omega} = \mathbf{W}_{\Omega}\left(\mathbf{S}_{\Omega} + \mathbf{K}_{\Omega}\mathbf{M}_{\Omega}\right) \tag{1}$$

In (1), $\mathbf{W}_{\Omega} \in \mathbb{C}^{N_{\mathrm{T}}\times N_{\mathrm{S}}}$, $\mathbf{S}_{\Omega} \in \mathbb{C}^{N_{\mathrm{S}}\times 1}$, $\mathbf{K}_{\Omega} \in \mathbb{C}^{N_{\mathrm{S}}\times N_{\mathrm{S}}}$ and $\mathbf{M}_{\Omega} \in \mathbb{C}^{N_{\mathrm{S}}\times 1}$ represent a precoding matrix, a modulation signal vector, a Gaussian integer vector, and a modulus vector that is changed adaptivery [18]. The transmission signal vectors $\mathbf{X}_{\Omega}$ Ω =A, B, are transmitted from the two terminals with the N_{T} antennas, simultaneously, and are traveling in fading channels. Since the two terminals transmit their signals for the relays simultaneously, superposition of the transmission signal vectors are received at all the relays. As is described below, the N_{S} relays that are selected out of all the relays actually detect the transmission signals. Let $\mathbf{Y}_{\mathrm{R}} \in \mathbb{C}^{N_{\mathrm{S}}\times 1}$ denote a received signal vector containing the signals received at those relays, the received signal vector can be written in the following.

$$\mathbf{Y}_{\mathrm{R}} = \mathbf{H}_{\mathrm{A}}\mathbf{X}_{\mathrm{A}} + \mathbf{H}_{\mathrm{B}}\mathbf{X}_{\mathrm{B}} + \mathbf{N}_{\mathrm{R}}, \tag{2}$$

where $\mathbf{H}_{\mathrm{A}} \in \mathbb{C}^{N_{\mathrm{S}}\times N_{\mathrm{T}}}$, $\mathbf{H}_{\mathrm{B}} \in \mathbb{C}^{N_{\mathrm{S}}\times N_{\mathrm{T}}}$, and $\mathbf{N}_{\mathrm{R}} \in \mathbb{C}^{N_{\mathrm{S}}\times 1}$ represent a channel matrix between the terminal A and the selected relays, that between the terminal B and those relays, and the additive white Gaussian noise (AWGN) vector. Those relays possibly receive superposition of the $2N_{\mathrm{T}}$ signal streams, even though only one antenna is placed on every relay. The network is regarded as one of overloaded MIMO networks. Those relays estimate an XOR-coded signal vector $\hat{\mathbf{X}}_{\Omega} \in \mathbb{C}^{N_{\mathrm{S}}\times 1}$ from the received signal vector $\mathbf{Y}_{\mathrm{R}}$ [18]. Let $\hat{\mathbf{X}}_{\Omega}$ be defined as $\hat{\mathbf{X}}_{\Omega} = (\hat{x}_{\Omega}(n_1)\cdots\hat{x}_{\Omega}(n_{N_{\mathrm{S}}}))^{\mathrm{T}}$ where $\hat{x}_{\Omega}(k) \in \mathbb{C}$ denotes a transmit signal from the kth relay to the terminal Ω. The XOR-coded signal $\hat{x}_{\mathrm{R}}(k) \in \mathbb{C}$ is defined as $\hat{x}_{\mathrm{R}}(k) = s_{\mathrm{A}}(k) \oplus s_{\mathrm{B}}(k)$, where $s_{\Omega}(k) \in \mathbb{C}$ and $\oplus$ denote a modulation signal defined a $\mathbf{S}_{\Omega}(k) = (s_{\Omega}(1)\cdots s_{\Omega}(N_{\mathrm{S}}))^{\mathrm{T}}$, and a complex XOR operation defined in the following. The complex XOR operation $\oplus$ is defined as $s_{\mathrm{A}}(k) \oplus s_{\mathrm{B}}(k) = \Re\left[s_{\mathrm{A}}(k)\right]\cdot\Re\left[s_{\mathrm{B}}(k)\right] + \mathrm{j}\Im\left[s_{\mathrm{A}}(k)\right]\cdot\Im\left[s_{\mathrm{B}}(k)\right]$.

Let $y_{\mathrm{R}}(k)$ denote the signal received at the kth relay, i.e, $\mathbf{Y}_{\mathrm{R}} = (y_{\mathrm{R}}(n_1)\cdots y_{\mathrm{R}}(n_{N_{\mathrm{S}}}))^{\mathrm{T}}$, if the precoder orthogonalizes the channels between the relays and the terminal, the soft decision signal $\bar{x}_{\mathrm{R}}(k) \in \mathbb{C}$ of the XOR-coded signal can be estimated as [14],

$$\begin{aligned}\Re\left[\bar{x}_{\mathrm{R}}(k)\right] &= \log\frac{p\left(\Re\left[\hat{x}_{\mathrm{R}}(k)=1\right]|\mathbf{Y}_{\mathrm{R}}\right)}{p\left(\Re\left[\hat{x}_{\mathrm{R}}(k)=-1\right]|\mathbf{Y}_{\mathrm{R}}\right)}\\ &\approx \log\frac{p\left(\Re\left[s_{\mathrm{A}}(k)s_{\mathrm{B}}(k)=1\right]|\Re\left[y_{\mathrm{R}}(k)\right]\right)}{p\left(\Re\left[s_{\mathrm{A}}(k)s_{\mathrm{B}}(k)=-1\right]|\Re\left[y_{\mathrm{R}}(k)\right]\right)}.\end{aligned} \tag{3}$$

In (3), $p(\alpha|\beta)$ represents a conditional probability that an event α happens when an event β occurred. The imaginary part of the soft decision signal can be estimated in the same way. In the above derivation, since the relays do not have a functionality to exchange their received signals among them[†], the soft decision symbol is estimated with only the signal received at its own relay, e.g., the kth relay in (3). The output signal $\bar{x}_{\mathrm{R}}(k)$ is provided to the channel decoder, and the output signal stream from the decoder is encoded and re-modulated in the same manner to that at the terminals. In other words, the kth selected relay decodes the XOR-coded signal stream of the kth signal streams transmitted from the terminal A and B. The decoder output stream is re-encoded and the output is re-modulated at the kth relay, independently.

In the next time slot, the selected relays transmit their re-modulated signals, which can be expressed in a vector form, i.e., $\underline{\mathbf{X}}_{\mathrm{R}} \in \mathbb{C}^{N_{\mathrm{R}}\times 1}$ defined as $\underline{\mathbf{X}}_{\mathrm{R}} = \left(\underline{x}_{\mathrm{R}}(n_1)\cdots\underline{x}_{\mathrm{R}}(n_{N_{\mathrm{S}}})\right)^{\mathrm{T}}$ where $\underline{x}_{\mathrm{R}}(n_i) \in \mathbb{C}$ denotes a re-modulated signal at the n_ith relay. The signal vector $\underline{\mathbf{X}}_{\mathrm{R}}$ is received at both the terminals as,

$$\mathbf{Y}_{\Omega} = \mathbf{H}_{\Omega}^{\mathrm{H}}\underline{\mathbf{X}}_{\mathrm{R}} + \mathbf{N}_{\Omega}, \quad \Omega = \mathrm{A\ or\ B}. \tag{4}$$

$\mathbf{Y}_{\Omega} \in \mathbb{C}^{N_{\mathrm{T}}\times 1}$ and $\mathbf{N}_{\Omega} \in \mathbb{C}^{N_{\mathrm{T}}\times 1}, \Omega = $ A or B in (4) denote a received signal vector and the AWGN vector at the terminal Ω. As is seen in (4), the network model is regarded as that in single user MIMO networks. Conventional MIMO receivers can be applied to the receiver on the terminal. The network model is illustrated in Fig. 1.

Next section proposes novel relay selection techniques for the XOR-PLNC with non-linear precoding.

3. Relay Selection for Precoded XOR Physical Layer Network Coding

3.1 Non-Linear Precoding Based on MMSE

For maximizing the signal to noise power ratio (SNR) of the

[†]The need for additional time slots or frequency band to exchange reduces the frequency utilization efficiency.

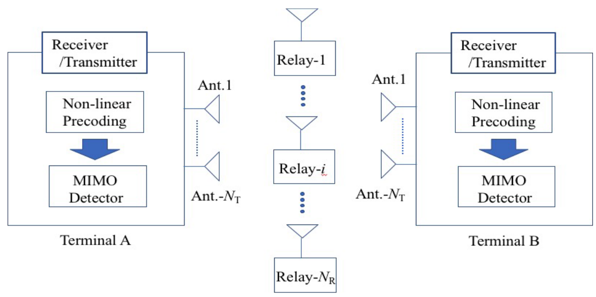

Fig. 1 Network model.

signals received at the selected relays, we apply minimum mean square error (MMSE) based precoding to the terminals [29], [30]. The precoding matrix $\mathbf{W}_\Omega$ of the MMSE based precoder is written as follows.

$$\mathbf{W}_\Omega = g_\Omega \mathbf{H}_\Omega^{\mathrm{H}} \mathbf{\Phi}_\Omega \tag{5}$$

$$\mathbf{\Phi}_\Omega = \left(\mathbf{H}_\Omega \mathbf{H}_\Omega^{\mathrm{H}} + \frac{\sigma^2 N_{\mathrm{R}}}{\sigma_{\mathrm{S}}^2} \mathbf{I}_{N_{\mathrm{S}}} \right)^{-1} \tag{6}$$

In (6), $\mathbf{\Phi}_\Omega \in \mathbb{C}^{N_{\mathrm{S}} \times N_{\mathrm{S}}}$, $g_\Omega \in \mathbb{R}$, $\sigma^2 \in \mathbb{R}$, $\sigma_{\mathrm{S}}^2 \in \mathbb{R}$, and $\mathbf{I}_{N_{\mathrm{S}}} \in \mathbb{Z}^{N_{\mathrm{S}} \times N_{\mathrm{S}}}$ denote a rectangular matrix, a normalization factor, the power of the AWGN generated in the relay, that of the transmit signal, and the N_{S} dimensional identity matrix, respectively. Because the QPSK modulator sends the modulation signal $s_\Omega(k) = \pm 1 + \pm \mathrm{j}$, σ_{S}^2 becomes 2, i.e., $\sigma_{\mathrm{S}}^2 = 2$. Although many non-linear precoding techniques have been proposed based on the MMSE, we apply a non-linear precoding [18], [19] based on Tomlinson-Harashima precoding(THP) [26], [27], because such precoding can be implemented with relatively lower computational complexity than the other techniques, such as the vector perturbation [28]. The lattice reduction is applied to improve the performance. Let $\mathbf{T}_\Omega \in \mathbb{C}^{N_{\mathrm{S}} \times N_{\mathrm{S}}}$ denote a unimodular matrix, the channel matrix is transformed with the unimodular matrix as [31],

$$\begin{pmatrix} \mathbf{H}_\Omega^{\mathrm{H}} \\ \sqrt{\frac{\sigma^2 N_{\mathrm{R}}}{\sigma_{\mathrm{S}}^2}} \mathbf{I}_{N_{\mathrm{S}}} \end{pmatrix} \mathbf{T}_\Omega = \mathbf{Q}_\Omega \mathbf{D}_\Omega^{\frac{1}{2}} \mathbf{R}_\Omega. \tag{7}$$

In (7), $\mathbf{Q}_\Omega \in \mathbb{C}^{(N_{\mathrm{S}}+N_{\mathrm{T}}) \times N_{\mathrm{S}}}$, $\mathbf{R}_\Omega \in \mathbb{C}^{N_{\mathrm{S}} \times N_{\mathrm{S}}}$, and $\mathbf{D}_\Omega \in \mathbb{C}^{N_{\mathrm{S}} \times N_{\mathrm{S}}}$ represent a part of a unitary matrix, an upper triangular matrix with 1 in all the diagonal positions, and a diagonal matrix, respectively. Let $\mathbf{B}_\Omega \in \mathbb{C}^{N_{\mathrm{S}} \times N_{\mathrm{S}}}$ represent a lower off-diagonal matrix defined as $\mathbf{B}_\Omega = \mathbf{R}_\Omega^{\mathrm{H}} - \mathbf{I}_{N_{\mathrm{S}}} \in \mathbb{C}^{N_{\mathrm{S}} \times N_{\mathrm{S}}}$, feedback filtering can be carried out with the off-diagonal matrix as follows [19].

$$\begin{aligned} v_\Omega(k) &= \mathrm{mod}\left[\mathbf{T}_\Omega^{\mathrm{H}}(k) \mathbf{S}_\Omega - \mathbf{B}_\Omega(k) \mathbf{V}_\Omega, M_\Omega(k) \right] \\ &= \mathbf{T}_\Omega^{\mathrm{H}}(k) \mathbf{S}_\Omega - \mathbf{B}_\Omega(k) \mathbf{V}_\Omega + k_\Omega(k) M_\Omega(k) \\ & \qquad k = 1, \cdots, N_{\mathrm{S}} \end{aligned} \tag{8}$$

In (8), $v_\Omega(k) \in \mathbb{C}$, $\mathbf{B}_\Omega(k) \in \mathbb{C}^{N_{\mathrm{S}} \times 1}$ and $\mathbf{T}_\Omega(k) \in \mathbb{C}$ denote a kth entry of a feedback filter output vector $\mathbf{V}_\Omega \in \mathbb{C}^{N_{\mathrm{S}} \times 1}$, the kth column vector of the matrix $\mathbf{B}_\Omega$ and the kth row vector of the unimodular vector $\mathbf{T}_\Omega$, i.e., $\mathbf{V}_\Omega = (v_\Omega(1) \cdots v_\Omega(N_{\mathrm{S}}))^{\mathrm{T}}$, $\mathbf{B}_\Omega = (\mathbf{B}_\Omega(1) \cdots \mathbf{B}_\Omega(N_{\mathrm{S}}))$ and $\mathbf{T}_\Omega^{\mathrm{H}} = \left(\mathbf{T}_\Omega(1)^{\mathrm{H}} \cdots \mathbf{T}_\Omega(N_{\mathrm{S}})^{\mathrm{H}}\right)$, respectively. In addition, $k_\Omega(k) \in \mathbb{C}$ and $M_\Omega(k)$ denote a Gaussian integer and the kth entry of the vector $\mathbf{M}_\Omega$. Let $\mathbf{F}_\Omega \in \mathbb{C}^{N_{\mathrm{T}} \times N_{\mathrm{S}}}$ denote a feedforward filter weight matrix, the feedforward filter weight matrix is defined as $\mathbf{F}_\Omega^{\mathrm{H}} \mathbf{R}_\Omega^{-\mathrm{H}} = \mathbf{H}_\Omega^{\mathrm{H}} \mathbf{\Phi}_\Omega$. If (5)~(8) are taken into account, the feedforward filter weight matrix $\mathbf{F}_\Omega^{\mathrm{H}}$ can be expressed as,

$$\mathbf{F}_\Omega^{\mathrm{H}} = \mathbf{H}_\Omega^{\mathrm{H}} \mathbf{T}_\Omega \mathbf{R}_\Omega^{-1} \mathbf{D}_\Omega^{-1}, \tag{9}$$

With the feedback filter and the feedforward filter, the precoder weight matrix $\mathbf{W}_\Omega$ can be calculated as follows.

$$\mathbf{W}_\Omega = g_\Omega \mathbf{F}_\Omega^{\mathrm{H}} \mathbf{V}_\Omega \tag{10}$$

$$g_\Omega = \sqrt{\frac{N_{\mathrm{S}}}{\sum_{k=0}^{N_{\mathrm{S}}-1} \left|\mathbf{F}_{\Omega,k}\right|^2 \sigma_{\Omega,M}^2(k)}}, \tag{11}$$

where $\sigma_{\Omega,M}^2(k)$ represents power of the feedback filter output signals. In (11), the normalization factor g_Ω is derived to set the transmission power to N_{S}.

The next section explains our proposed relay selection for the XOR-PLNC with the above precoder.

3.2 Relay Selection

As is described above, all the relays are numbered such as a kth relay, $k = 0 \sim N_{\mathrm{R}} - 1$. We specify groups of the relays where N_{S} relays are contained as described previously. When the n_1th $\sim n_{N_{\mathrm{S}}}$th relays are contained in an nth group, the group index n is defined in the following.

$$n = \sum_{k=1}^{N_{\mathrm{S}}} n_k N_{\mathrm{R}}^{k-1} \tag{12}$$

Although the group index n was omitted for simplicity of the notations in the above sections, we hereafter add the group index, such as $\mathbf{W}_{\Omega_n}$, $\mathbf{Y}_{\mathrm{R}_n}$, $\mathbf{H}_{\Omega_n}$, $\mathbf{S}_{\Omega_n}$, and g_{Ω_n}.

Let $P_{\mathrm{A}\to\mathrm{B},n} \in \mathbb{R}$ denote the BER of the XOR-PLNC in the relaying network from the terminal A to the terminal B via the nth relay group, the BER can be written as follows.

$$\begin{aligned} P_{\mathrm{A}\to\mathrm{B},n} &= P_{\mathrm{R}_n}(1 - P_{\mathrm{R}_n\to\mathrm{B}}) + (1 - P_{\mathrm{R}_n}) P_{\mathrm{R}_n\to\mathrm{B}} \\ &\fallingdotseq P_{\mathrm{R}_n} + P_{\mathrm{R}_n\to\mathrm{B}} \end{aligned} \tag{13}$$

In (13), $P_{\mathrm{R}_n} \in \mathbb{R}$ and $P_{\mathrm{R}_n\to\mathrm{B}} \in \mathbb{R}$ represent the BER at the relays, and that from the relays to the terminal B, respectively. In the above derivation, $P_{\mathrm{R}_n} P_{\mathrm{R}_n\to\mathrm{B}}$ is neglected, because it is much smaller than the probability P_{R_n} when the E_{b}/N_0 is high.

The BER P_{R_n} depends on the performance of the input sequence provided to the channel decoder. The soft detection signals are fed to the decoder as the input sequence at the relays. When the above precoding is applied at the terminals, the soft decision signal can be rewritten in the following equation [14].

$$\Re\left[\bar{x}_{\mathrm{R}}(k)\right] = \left|\Re\left[y_{\mathrm{R}}(k)\right]\right| \left| \left((\xi_n - 1) g_{\mathrm{A}_n} - (\xi_n + 1) g_{\mathrm{B}_n} \right) + 2 g_{\mathrm{A}_n} g_{\mathrm{B}_n} \right.$$

$$= \begin{cases} g_{\mathrm{B}_n} \Re\left[s_{\mathrm{A}_n}(k) \oplus s_{\mathrm{B}_n}(k)\right] + \Re\left[n_{\mathrm{R}}(k)\right] & g_{\mathrm{A}_n} > g_{\mathrm{B}_n} \\ g_{\mathrm{A}_n} \Re\left[s_{\mathrm{A}_n}(k) \oplus s_{\mathrm{B}_n}(k)\right] + \Re\left[n_{\mathrm{R}}(k)\right] & g_{\mathrm{A}_n} \leq g_{\mathrm{B}_n} \end{cases}, \tag{14}$$

where ξ_n indicates a function defined in the following.

$$\xi_n = \begin{cases} 1 & g_{\mathrm{A}_n} > g_{\mathrm{B}_n} \\ -1 & g_{\mathrm{A}_n} \leq g_{\mathrm{B}_n} \end{cases} \tag{15}$$

As is described above, the imaginary part of the soft decision signal can be estimated in the same manner to (14). Eq. (14) implies that the SNR of the input signal sequence to the decoder is expressed with the normalization factor smaller than the other. Because the decoding performance is dependent on the SNR of input signal sequence in principle, the BER of the channel decoder output stream can be reduced to that in the worse channel. Let $P_{\mathrm{A}\to\mathrm{R}_n} \in \mathbb{R}$ and $P_{\mathrm{B}\to\mathrm{R}_n} \in \mathbb{R}$ denote the BER in the channel with the channel gain g_{A_n} and that with the gain g_{B_n}, the BER at the relay can be expressed as,

$$P_{\mathrm{R}} = \min\left(P_{\mathrm{A}\to\mathrm{R}_n}, P_{\mathrm{B}\to\mathrm{R}_n}\right). \tag{16}$$

Let $P_{\mathrm{PLNC}} \in \mathbb{R}$ denote the average BER of the two directional wireless communications, i.e., that from the terminal A to the B, and that from the terminal B to the A, the average BER is described as,

$$\begin{aligned} P &= \frac{1}{2}\left(P_{\mathrm{A}\to\mathrm{B},n} + P_{\mathrm{B}\to\mathrm{A},n}\right) = P_{\mathrm{R}} + \frac{1}{2}\left(p_{\mathrm{R}_n\to\mathrm{A}} + P_{\mathrm{R}_n\to\mathrm{B}}\right) \\ &< \begin{cases} P_{\mathrm{A}\to\mathrm{R}_n} + P_{\mathrm{R}_n\to\mathrm{A}} \simeq P_{\mathrm{A}\to\mathrm{R}_n} & P_{\mathrm{A}\to\mathrm{R}_n} > P_{\mathrm{B}\to\mathrm{R}_n} \\ P_{\mathrm{B}\to\mathrm{R}_n} + P_{\mathrm{R}_n\to\mathrm{B}} \simeq P_{\mathrm{B}\to\mathrm{R}_n} & P_{\mathrm{A}\to\mathrm{R}_n} \leq P_{\mathrm{B}\to\mathrm{R}_n} \end{cases} \end{aligned} \tag{17}$$

In the above derivation, we use the fact that the BER $P_{\Omega\to R_n}$ is worse than that of $P_{\mathrm{R}_n\to\Omega}$, because the channel from the terminal to the relays is regarded as that in an overloaded network where the 2 signal streams are simultaneously received at every relay with an antenna†. The BER P_{PLNC} is approximated by that of the channel worse that the other. In a word, the BER of the XOR-PLNC depends on that of the worse channel.

It is the optimum to select a group of the relays that makes the network achieve the minimum BER performance. In other words, it is the best to select a relay group that minimizes the BER $P_{\mathrm{A}\to\mathrm{R}_n}$ when $P_{\mathrm{A}\to\mathrm{R}_n} > P_{\mathrm{B}\to\mathrm{R}_n}$. We take a min-max approach to select a group out of all the possible groups in the channel. We propose terminal selection algorithms based on the min-max approach for the XOR-PLNC in the following section.

†Even if the non-linear precoding is applied at the terminals, the 2 signal streams are received at all the relays with one antenna in the network model. The signal reception at the relays is regarded as that in an overloaded MIMO network. On the other hand, the network model described in (4) is also regarded as one of non-overloaded MIMO channels. Therefore, the transmission performance in the channels from the relays to the terminals in the second slot is better than that in the first slot, because the maximum likelihood detection (MLD) is applied at the terminals for the optimum performance.

3.2.1 Minimum of Normalization Gains (MNG)

This paper applies the MMSE-based non-linear precoding defined in (5) and (6). When the E_{b}/N_0 is high enough, the MMSE precoding is approximated with the zero forcing (ZF) precoding. The approximation simplifies the network model in (2) to,

$$\mathbf{Y}_{\mathrm{R}} \approx g_{\mathrm{A}_n}\mathbf{S}_{\mathrm{A}_n} + g_{\mathrm{B}_n}\mathbf{S}_{\mathrm{B}_n} + \mathbf{N}_{\mathrm{R}}. \tag{18}$$

The Gaussian integer multiples of the modulus are omitted from the above equation, because they can be removed at the relays. The above equation means that the transmission performance between the terminal to the relays only depends on the normalization factor g_{Ω_n}, Ω =A or B. The BER $P_{\mathrm{A}\to\mathrm{R}_n}$ and $P_{\mathrm{B}\to\mathrm{R}_n}$ can be written as follows.

$$P_{\mathrm{A}\to\mathrm{R}_n} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{g_{\mathrm{A}_n}^2}{2\sigma^2}}\right), \quad P_{\mathrm{B}\to\mathrm{R}_n} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{g_{\mathrm{B}_n}^2}{2\sigma^2}}\right)$$

In the above equations, erfc (t) indicates the complementary error function defined as,

$$\mathrm{erfc}(t) = \frac{2}{\sqrt{\pi}}\int_t^\infty \mathrm{e}^{-y^2}\mathrm{d}y.$$

Since the BER performance is a monotonically decreasing function with respect to an input t, the BER is improved as the SNR such as $\frac{g_{\mathrm{A}_n}^2}{2\sigma^2}$ increases. Because the BER P_{R} depends on the performance in the worse channel as described above, the min-max approach is reduced to the following selection algorithm, which is expected to achieve the best performance in the channel written in (18).

$$n_{\mathrm{MNG}} = \arg\max_n \left[\min_{\mathrm{A}_n,\,\mathrm{B}_n}\left[g_{\mathrm{A}_n}^2,\ g_{\mathrm{B}_n}^2\right]\right] \tag{19}$$

The above selection is called as "minimum of normalization gains (MNG)" in this paper, because the group to maximize the minimum among the two normalization factors is selected.

3.2.2 Product of All Eigenvalues (PAE)

Although the network model is approximated in the previous section, the model can be rewritten as follows without approximation.

$$\begin{aligned} \mathbf{Y}_{\mathrm{R}} &\approx g_{\mathrm{A}_n}\mathbf{H}_{\mathrm{A}_n}\mathbf{H}_{\mathrm{A}_n}^{\mathrm{H}}\mathbf{\Phi}_{\Omega_n}\mathbf{S}_{\mathrm{A}_n} + g_{\mathrm{B}_n}\mathbf{H}_{\mathrm{B}_n}\mathbf{H}_{\mathrm{B}_n}^{\mathrm{H}}\mathbf{\Phi}_{\Omega_n}\mathbf{S}_{\mathrm{B}_n} + \mathbf{N}_{\mathrm{R}} \\ &= g_{\mathrm{A}_n}\mathbf{S}_{\mathrm{A}_n} + g_{\mathrm{B}_n}\mathbf{S}_{\mathrm{B}_n} \\ &\quad - \frac{2\sigma^2 N_{\mathrm{R}}}{\sigma_{\mathrm{S}}^2}\left(g_{\mathrm{A}_n}\mathbf{\Phi}_{\mathrm{A}_n}\mathbf{S}_{\mathrm{A}_n} + g_{\mathrm{B}_n}\mathbf{\Phi}_{\mathrm{B}_n}\mathbf{S}_{\mathrm{B}_n}\right) + \mathbf{N}_{\mathrm{R}} \\ &= g_{\mathrm{A}_n}\mathbf{S}_{\mathrm{A}_n} + g_{\mathrm{B}_n}\mathbf{S}_{\mathrm{B}_n} + \bar{\mathbf{N}}_{\mathrm{R}} \end{aligned} \tag{20}$$

The Gaussian integer multiples of the modulus are also omitted in the above equation. In (20), $\bar{\mathbf{N}}_{\mathrm{R}} \in \mathbb{C}^{N_{\mathrm{S}}\times 1}$ denotes a noise vector defined as,

$$\bar{\mathbf{N}}_{\mathrm{R}} = -\frac{2\sigma^2 N_{\mathrm{S}}}{\sigma_{\mathrm{S}}^2}\left(g_{\mathrm{A}_n}\mathbf{\Phi}_{\mathrm{A}_n}\mathbf{S}_{\mathrm{A}_n} + g_{\mathrm{B}_n}\mathbf{\Phi}_{\mathrm{B}_n}\mathbf{S}_{\mathrm{B}_n}\right) + \mathbf{N}_{\mathrm{R}}. \quad (21)$$

The kth entry of the noise vector $\bar{\mathbf{N}}_{\mathrm{R}} \in \mathbb{C}^{N_{\mathrm{S}}\times 1}$ is represented as $\bar{n}_{\mathrm{R}}(k) \in \mathbb{C}$, i.e., $\bar{\mathbf{N}}_{\mathrm{R}} = (\bar{n}_{\mathrm{R}}(1)\cdots\bar{n}_{\mathrm{R}}(N_{\mathrm{S}}))^{\mathrm{T}}$.

In the following explanation, we assume that the normalization factor g_{A_n} is bigger than the other g_{B_n}. As is explained, the SNR of the soft decision signal at the kth relay $\rho_{\mathrm{R},n} \in \mathbb{R}$ is written as follows.

$$\begin{aligned}\rho_{\mathrm{R},n} &= \frac{g_{\mathrm{B}_n}^2 \mathrm{E}\left[\left|s_{\mathrm{A}_n}(k)\odot s_{\mathrm{B}_n}(k)\right|^2\right]}{\mathrm{E}\left[|\bar{n}_{\mathrm{R}}(k)|^2\right]} \\ &\approx \frac{g_{\mathrm{B}_n}^2\sigma_{\mathrm{R}}^2}{2\sigma^2 + \frac{4N_{\mathrm{S}}\sigma^4}{\sigma_{\mathrm{s}}^2}\left(g_{\mathrm{A}_n}^2\Upsilon_{\mathrm{A}_n} + g_{\mathrm{B}_n}^2\Upsilon_{\mathrm{B}_n}\right)} \end{aligned} \quad (22)$$

When the MMSE criterion is used in precoders, the received signal SNR is the same among the streams. Therefore, the index k to specify the relay number is not attached to the SNR ρ_{R_n} in the above equation. In (22), $\sigma_{\mathrm{R}}^2 \in R$ and $\Upsilon_{\mathrm{B}_n} \in \mathbb{R}$ represent the power of the XOR coded signal, i.e., $\sigma_{\mathrm{R}}^2 = \mathrm{E}\left[\left|s_{\mathrm{A}_n}(k)\odot s_{\mathrm{B}_n}(k)\right|^2\right]$ and a term defined in the following.

$$\begin{aligned}\Upsilon_{\Omega_n} &= \mathrm{tr}\left[\mathbf{\Phi}_{\Omega_n}^{-2}\right] \\ &= \sum_{i=0}^{N_{\mathrm{S}}-1}\left(\gamma_{\Omega_n}(i) + \frac{2N_{\mathrm{S}}\sigma^2}{\sigma_{\mathrm{s}}^2}\right)^{-2} \\ &\geq N_{\mathrm{S}}\sqrt[N_{\mathrm{S}}]{\prod_{i=0}^{N_{\mathrm{S}}-1}\left(\gamma_{\Omega_n}(i) + \frac{2N_{\mathrm{S}}\sigma^2}{\sigma_{\mathrm{s}}^2}\right)^{-2}} \end{aligned} \quad (23)$$

In (23), $\gamma_{\Omega_n}(i)$ indicates the ith eigenvalue of the channel matrix $\mathbf{H}_{\Omega_n}$, which is defined as,

$$\begin{aligned}\mathbf{H}_{\Omega_n} &= \mathbf{U}_{\Omega_n}\Gamma_{\Omega_n}^{\frac{1}{2}}\mathbf{V}_{\Omega_n}^{\mathrm{H}} \\ \Gamma_{\Omega_n} &= \mathrm{diag}\left(\gamma_{\Omega_n}(0)\cdots\gamma_{\Omega_n}(N_{\mathrm{S}}-1)\right) \end{aligned} \quad (24)$$

When the inequality in (23) is substituted for (22), the SNR of the soft decision signals at the relays $\rho_{\mathrm{R},n}$ can be rewritten in (25). In the proposed network, the relays and the terminals transmit the QPSK modulation signals with the same power, i.e., $\sigma_{\mathrm{R}}^2 = \sigma_{\mathrm{S}}^2$. Because we have assumed $g_{\mathrm{A}_n} \geq g_{\mathrm{B}_n}$ in the network, the SNR $\rho_{\mathrm{R},n}$ at the relays can be further rewritten as follows.

$$\begin{aligned}\rho_{\mathrm{R}.n} &< \frac{\sigma_{\mathrm{s}}^4}{8N_{\mathrm{S}}^2\sigma^4}\sqrt[N_{\mathrm{S}}]{\prod_{i=0}^{N_{\mathrm{S}}-1}\left(\gamma_{\mathrm{A}_n}(i) + \frac{2N_{\mathrm{S}}\sigma^2}{\sigma_{\mathrm{s}}^2}\right)\left(\gamma_{\mathrm{B}_n}(i) + \frac{2N_{\mathrm{S}}\sigma^2}{\sigma_{\mathrm{s}}^2}\right)} \\ &\approx \frac{\sigma_{\mathrm{s}}^4}{8N_{\mathrm{S}}^2\sigma^4}\sqrt[N_{\mathrm{S}}]{\prod_{i=0}^{N_{\mathrm{S}}-1}\gamma_{\mathrm{A}_n}(i)\,\gamma_{\mathrm{B}_n}(i)} \end{aligned} \quad (26)$$

As is described above, it is the optimum to select a group that maximizes the SNR of the decoder input signals. Although the SNR expressed in (26) is upperbound, we can expect that a group to maximize the SNR defined in (26) is the optimum. Since the Nth root function is a monotonically increasing function, it is the best to select a group to maximize the product of all the eigenvalues as follows.

$$n_{\mathrm{PAE}} = \max_n\left[\prod_{i=0}^{N_{\mathrm{S}}-1}\gamma_{\mathrm{A}_n}(i)\,\gamma_{\mathrm{B}_n}(i)\right] \quad (27)$$

The above selection algorithm is called as "product of all eigenvalues (PAE)" in this paper, because the index to maximize the product of all the eigenvalues is selected.

3.2.3 Product of Minimum Eigenvalues (PME)

As is explained above, we apply the precoder based on a linear spatial filtering assisted by the lattice reduction. Although the SNR upperbound is expressed in (26), the transmission performance is often dependent only on the minimum eigenvalue, especially, when linear spatial filtering is used in wireless channels. A technique to select a group of the relays that maximizes the product of the minimum eigenvalues is named as "product of minimum eigenvalues (PME)" in this paper, which is defined as,

$$n_{\mathrm{PME}} = \arg\max_n\left[\min_i\left(\gamma_{\mathrm{A}_n}(i)\right),\ \min_i\left(\gamma_{\mathrm{B}_n}(i)\right)\right]. \quad (28)$$

As is seen from Sect. 3.2.1 to Sect. 3.2.3, the proposed selection techniques only need the gains of the proposed precoders and the eigenvalues of the channel matrices between the relays and the terminals, even though the performance of the channel coding is taken into account. Although most conventional techniques for MIMO bidirectional relaying networks take the min-max approach as our proposed techniques do, they explicitly search the transmission signal vector that maximizes the end-to-end performance in the system without error correction coding. The proposed selection is much less complex than conventional techniques.

When the proposed selection is implemented in wireless communication systems, the proposed selection needs a controller to select the relays. The channel matrix between the relays and the terminal Ω is estimated at the terminal Ω and the estimated matrix is sent to the controller where the best relays are selected with the proposed selection techniques. The controller informs the terminals the relays to send the signals. We assume that the controller is implemented in base stations or access points of wireless access networks.

The channel state information (CSI) between the terminals and all the relays is necessary to collect, and the eigenvalue decomposition is performed at the controller, which needs some overhead in communication and complexity. On the other hand, similar overhead and complexity is needed for a controller in conventional multi-user MIMO based on the block diagonalization. The proposed selection needs almost as same overhead and complexity as that in conventional multi-user MIMO based on the block diagonalization.

$$\rho_{\mathrm{R},n} \leq \frac{g_{\mathrm{B}_n}^2 \sigma_{\mathrm{R}}^2}{2\sigma^2 + \frac{4N_{\mathrm{S}}}{\sigma_{\mathrm{s}}^2}\left(g_{\mathrm{A}_n}^2 \sum_{i=0}^{N_{\mathrm{S}}-1}\left(\frac{\gamma_{\mathrm{A}_n}(i)}{\sigma^2} + \frac{2N_{\mathrm{S}}}{\sigma_{\mathrm{s}}^2}\right)^{-2} + g_{\mathrm{B}_n}^2 \sum_{i=0}^{N_{\mathrm{S}}-1}\left(\frac{\gamma_{\mathrm{B}_n}(i)}{\sigma^2} + \frac{2N_{\mathrm{S}}}{\sigma_{\mathrm{s}}^2}\right)^{-2}\right)}$$
$$\leq \frac{g_{\mathrm{B}_n}^2 \sigma_{\mathrm{R}}^2}{\frac{8N_{\mathrm{S}}^2\sigma^4}{\sigma_{\mathrm{s}}^2} g_{\mathrm{A}_n} g_{\mathrm{B}_n}\left(\sqrt[N_{\mathrm{S}}]{\prod_{i=0}^{N_{\mathrm{S}}-1}\left(\gamma_{\mathrm{A}_n}(i) + \frac{2N_{\mathrm{S}}\sigma^2}{\sigma_{\mathrm{s}}^2}\right)^{-1}\left(\gamma_{\mathrm{B}_n}(i) + \frac{2N_{\mathrm{S}}\sigma^2}{\sigma_{\mathrm{s}}^2}\right)^{-1}}\right)} \leq \frac{\sigma_{\mathrm{s}}^2\sigma_{\mathrm{R}}^2}{8N_{\mathrm{S}}^2\sigma^4}\frac{g_{\mathrm{B}_n}}{g_{\mathrm{A}_n}} \sqrt[N_{\mathrm{S}}]{\prod_{i=0}^{N_{\mathrm{S}}-1}\left(\gamma_{\mathrm{A}_n}(i) + \frac{2N_{\mathrm{S}}\sigma^2}{\sigma_{\mathrm{s}}^2}\right)\left(\gamma_{\mathrm{B}_n}(i) + \frac{2N_{\mathrm{S}}\sigma^2}{\sigma_{\mathrm{s}}^2}\right)} \tag{25}$$

4. Simulation

The performance of the proposed selection techniques is verified by computer simulation. As is described above, the QPSK modulation is applied, and the rate half convolutional code with the constraint length of 3 is used [32]. The number of the antennas on the terminals is 2, i.e., $N_{\mathrm{T}} = 2$. The proposed techniques select a group of 2 relays among 10 relays, i.e., $N_{\mathrm{S}} = 2$ and $N_{\mathrm{R}} = 10$. The channel between the terminal and the relay is modeled with independent and identically distributed (i.i.d.) Rayleigh fading based on Jakes' model [33][†]. Simulation parameters are listed in Table 1. The performance of the proposed selection is compared with that of the random selection in the following, because the random selection is regarded as one of conventional techniques. However, since most conventional techniques do not explicitly take account of error correction coding, they are not compared with the proposed selection.

4.1 SNR Distribution at Relays

Figure 2 shows the cumulative distribution function (CDF) with respect to the SNR at the relays. In the figure, the proposed selection techniques are compared. The performance of the random selection is added as a reference. The PAE achieves the best SNR performance, while the SNR of the PME is slightly worse than that of the PAE. Besides, the SNR of the MNG is much worse than the PME and the random selection is the worst in all the selection techniques. As is discussed above, while the MNG is derived on the assumption of the MMSE precoding coming close to the zero forcing (ZF) precoding when the E_{b}/N_0 is high, the PAE is derived without that assumption. The above performance gap between the MNG and the PAE infers that the transmission performance is greatly affected by the second term in the right hand side of (21), which has to be taken into account when the network performance is discussed.

[†]This means that all the relays are located at the same distance from the terminals, which is the best situation in terms of the diversity gain. The proposes selection is expected to achieve the potential performance, i.e., the best performance. If the configuration is changed, the proposed selection degrades. For instance, since the PAE uses the product of all the eigenvalues in the channels of the two links such as the terminal A to the relays and the terminal B to the relays, if the eigenvalues of the one link are attenuated, the performance is deteriorated. Since overall performance depends on the characteristics of the two links in one-directional relaying networks, however, similar performance degradation will happen if the characteristics are changed. In other words, when the distance between the terminal and the relay is different from each other, the performance of the proposed selection is degraded as that of any selection techniques gets worse.

Table 1 Simulation parameters.

Modulation	QPSK/Single Carrier
Channel model	Rayleigh fading
No. of antennas on every terminal	2
No. of relays N_{R}	10
No. of antennas on every relay	1
No. of selected relays N_{S}	2
Lattice reduction	LLL algorithm [34], [35] with $\delta = 0.75$
Signal detection on terminals	MLD
Error correction code	Convolutional code with $R = 1/2, K = 3$
Decoding at relays	Soft input Viterbi algorithm
Decoding at terminals	Hard input Viterbi algorithm

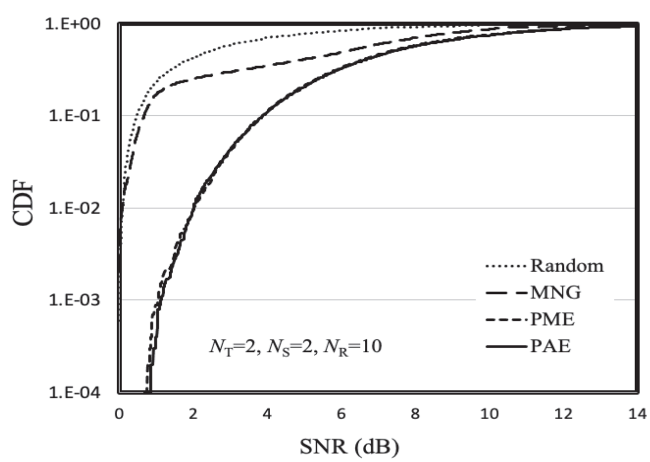

Fig. 2 Cumulative distribution function of SNR at relays.

4.2 BER Performance at Relays

Figure 3 confirms the BER performance of the proposed relay selection. The performance of the random selection is also added in the figure. As is indicated by Fig. 2, the PAE achieves the best BER performance in all the selection tech-

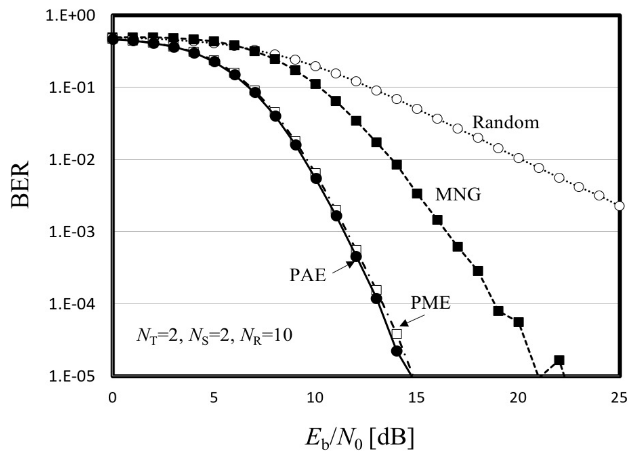

Fig. 3 BER at relays.

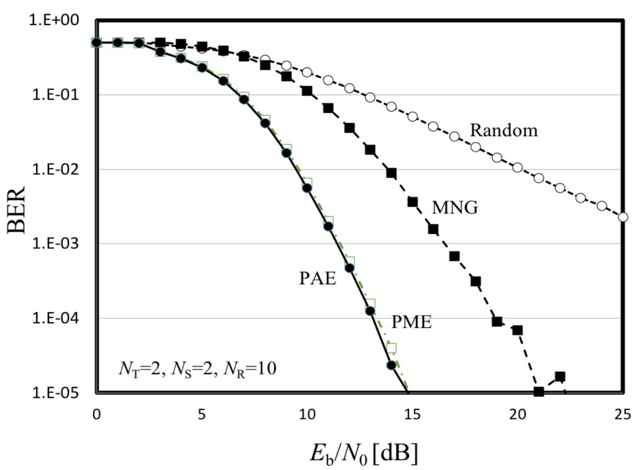

Fig. 4 BER at terminals.

niques, and the PME is a little bit inferior to the PAE†. The BER performance gap of the MNG from the PAE is about 5dB at the BER of 10^{-3}. The PAE attains more than 13dB better BER performance at the BER of 10^{-3} than the random selection technique.

4.3 BER Performance at Terminals

Figure 4 evaluated the BER performance of the XOR-PLNC with the proposed relay selection techniques at the relays. The random selection technique is added in this figure. As is described in Sect. 3.2, the transmission performance from the terminal to the relays is worse than that from the relays to the terminal. The explanation is supported by the figure that the BER performance of the XOR-PLNC is almost the same to that shown in Fig. 3.

†As is described in Sect. 3.2.2, the PAE is derived as the best selection technique in all the proposed selection techniques. However, the PAE and the PME achieve similar performance, which infers that the product of the minimum eigenvalues dominates the characteristics of the product of all the eigenvalues. In a word, the other bigger eigenvalues are not influential to the selection performance.

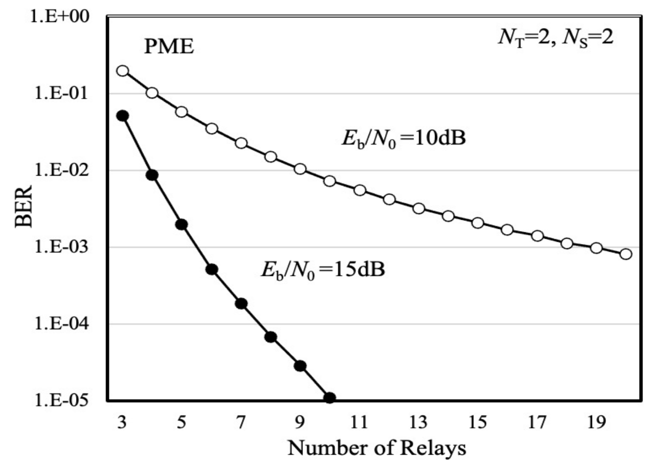

Fig. 5 BER v.s. number of relays.

4.4 BER Performance at Terminals with Respect to the Number of Relays

Figure 5 shows the BER performance at the terminals with respect to the number of the relays. Since the performance of the PAE is almost the same to that of the PME, the performance of the PME is shown in the figure. The performance of the proposed selection gets better as the number of the relays increases, because more diversity gain can be obtained as the number of the relays increases. Especially, since we apply the i.i.d. Rayleigh fading on the assumption of all the relays placed at the same distance from the terminal, the proposed selection achieves that superior performance as the number of the relays increases.

5. Conclusion

This paper has proposed relay selection techniques for the XOR-PLNC with MMSE based non-linear precoding in MIMO bi-directional two hop relaying networks. We have shown that the transmission performance of the XOR-PLNC is dependent on that from the terminal to the relays. Therefore, the proposed techniques select a group of the relays that maximizes the transmission performance. In a word, a group of the relays to maximize the SNR of the received signals at those relays is selected in the proposed techniques. This paper derives three selection techniques on different assumptions about the characteristics of the MMSE-based precoding. We show that the SNR at the relays can be characterized by the product of all the eigenvalues in the channels from the two terminals to the relays. One of the proposed techniques applies the product of all the eigenvalues as a selection metric, which is called "product of all eigenvalues(PAE)". The performance of the proposed selection techniques is confirmed by computer simulation in a situation where two terminals with 2 antennas surround 10 relays with one antenna. When 2 relays are selected, the proposed

PAE achieves the best transmission performance. The PAE attains a gain of more than 13dB at the BER of 10^{-3}.

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