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On Trade Models with Variable Markups and Pareto-Distributed Productivity*

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Abstract

We provide more generalized properties of gains from trade and trade liberalization in monopolistic competition models featuring firm heterogeneity characterized as Pareto-distributed productivity and variable markups associated with pro-competitive effects. For a large class of utility functions that we consider, firm heterogeneity alters the nature of markup distortion that should be addressed by the pro-competitive effects. Our finding implies that the pro-competitive effects of trade are not effective in correcting such markup distortion unique to heterogeneous firm frameworks. As a result, gains from trade in our framework are characterized as consumption variety expansion and selection effects without efficiency gains. We provide rich insights into the varying impacts of trade and trade liberalization across countries with differences in factors such as market size, technology, and geography.

Keywords: Monopolistic competition, Firm heterogeneity, Efficiency, Pro-competitive effect, Gains from trade

JEL classification: D43, D61, F12, L13

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1 Introduction

Since the pioneering study by Melitz (2003), the focus of international trade researchers has shifted towards models characterized by firm heterogeneity in terms of productivity. This trend has also been encouraged by the growing availability of micro data at the firm level. For example, empirical studies have revealed that exporting firms are larger and more productive (Bernard, Jensen, Redding and Schott, 2007), markups are higher for more productive firms (De Loecker and Warzynski, 2012), and trade liberalization has a pro-competitive effect on firm-level prices (De Loecker, Goldberg, Khandelwal and Pavcnik, 2016). Thus, trade models have been developed so that they can incorporate these findings.

Building on this foundation, recent studies on monopolistic competition models of international trade have been focused on analyzing the gains from trade resulting from a link between firm heterogeneity and the pro-competitive effect, which is not incorporated in the Melitz model. The pro-competitive effects of trade, introduced by Krugman (1979), arise from the concept of variable markup pricing and refer to a reduction in prices and markups resulting from opening up to trade. Essentially, the pro-competitive gains from trade are characterized as a corrective impact on markup distortion in the domestic market. Once firm heterogeneity is taken into account, the pro-competitive gains from trade become more complex. This is because, as we discuss in more detail below, the markup distortion is attributed not only to markups set by each firm itself but also to differences in markups across firms.

For instance, in a standard monopolistic competition model with single-sector symmetric firms that set markups endogenously and require labor for both developing and producing a variety of goods (e.g., Behrens and Murata, 2012), each firm charges inefficiently high markups in a closed economy, which leads to underproduction and, simultaneously, excessive variety (or excessive firm entry) as the remaining labor is allocated to development.[1] In this context, the pro-competitive gains from trade can be characterized as a reallocation of labor: exposure to competition through opening up to trade forces these firms to reduce their prices and markups, which reallocates labor from development to production, thereby enhancing resource allocation efficiency and welfare.

Incorporating firm heterogeneity into this framework changes the nature of the markup distortion that should be addressed by the pro-competitive effects, as differences in markups across firms become an additional source of distortion. Specifically, more productive firms inefficiently underproduce by charging relatively high markups, while less productive firms inefficiently overproduce by charging relatively low markups. This also implies that the least productive firms inefficiently operate (i.e., overproduce). Such markup distortions in the domestic market—which have been observed under various settings of monopolistic competition models with pro-competitive effects (e.g., Nocco, Ottaviano and Salto, 2014; Dhingra and Morrow, 2019; Behrens, Mion, Murata and Suedekum, 2020)—are independent of the extensive margin distortion related to firm entry and can therefore be interpreted as an intensive margin

[1] This type of distortion arises when utility is additively separable and the elasticity of its subutility with respect to quantity is decreasing with quantity.

distortion unique to heterogeneous firm frameworks.[2] Hence, to evaluate the pro-competitive gains from trade, one should examine the impact of trade on resource allocation efficiency not only through changes in average or aggregate markup levels but also through changes in the distribution of markups that can affect the intensive margin distortion unique to heterogeneous firm frameworks.

The purpose of this study is to clarify the more generalized properties of gains from trade and trade liberalization in international trade models featuring monopolistic competition, firm heterogeneity, and variable markups associated with pro-competitive effects. Although several studies have analyzed international trade in such a framework (e.g., Melitz and Ottaviano, 2008; Behrens, Mion, Murata and Südekum, 2014; Simonovska, 2015; Demidova, 2017), little is known about the extent to which the results obtained in these studies depend on the specified utility functions.[3] Our analysis derives several general results that do not depend on the specified utility functions, while also presenting some new insights into the gains from trade.

For a clear and detailed analysis of the gains from trade in such a framework, we assume a Pareto distribution of productivity, as in the literature described above. This enables us to characterize in detail the markup distortion in the domestic market that the pro-competitive effects should address, without specifying a utility function. Considering that a Pareto productivity distribution has been deemed a reasonable approximation of the observed distribution, our findings in this study serve well as a benchmark for understanding how the distribution of productivity can affect the pro-competitive gains from trade.

The key findings of this study are outlined as follows. First, we identify the markup distortion in the domestic market, which can be characterized as follows: (i) the mass of entrants is optimal; (ii) firm selection is too weak; (iii) more (less) productive firms that charge relatively high (low) markups produce too little (much); (iv) the mass of consumed varieties is too much; and (v) the aggregate and average quantities of differentiated varieties are too little. Note that the last two results are in line with the markup distortion in symmetric firm settings as in Behrens and Murata (2012). However, the nature of the underlying misallocation that leads to these results differs from ours. Specifically, the excessive consumed variety and the insufficient aggregate/average production arise from the inefficient skew in labor allocation for production toward less (and the least) productive firms, as the second and third results show. Thus, markup distortion emerges as the intensive margin distortion unique to heterogeneous firm frameworks. Indeed, as the first result shows, the extensive margin distortion is absent, meaning that labor allocation for development (i.e., the mass of entrants) is optimal. Therefore, in contrast to symmetric firm settings, labor misallocation between development and production is absent.[4] This is

[2]This intensive margin distortion can also be confirmed by evaluating the efficiency of the allocation of production (or labor) to each entrant for a given mass of entrants. This approach allows us to identify the presence of the intensive margin distortion described above, which is independent of the extensive margin distortion in terms of firm entry. By contrast, when firms are homogeneous, a symmetric equilibrium in which labor is equally allocated to each entrant is efficient outcome. Since the market equilibrium can achieve this allocation, there is no intensive margin distortion that is independent of the extensive margin distortion.

[3]Strictly speaking, the utility functions we target are additively separable and therefore of a different class of utility functions than those considered by Melitz and Ottaviano (2008) and Demidova (2017).

[4]In symmetric firm settings, since the mass of consumed varieties and entrants are equal in a closed economy,

because, in our model, the underallocation of labor for production to more productive firms and its overallocation to less productive firms offset each other. Hence, our findings suggest that considering firm heterogeneity (especially when assuming a Pareto productivity distribution) mutes the extensive margin distortion in terms of firm entry by absorbing markup distortion into the intensive margin.

Second, we characterize the impact of economic integration (i.e., the impact of an increase in market size). We find that integration does not improve resource allocation efficiency as a whole. Specifically, integration does not affect the gap between the equilibrium and optimal levels of: aggregate and average quantities; mass of consumed varieties; firm selection; and, consequently, utility. This implies that the pro-competitive effects do not generate any efficiency gains. Our finding differs from that obtained within symmetric firm settings as in Behrens and Murata (2012), where the economic integration narrows the gap between the equilibrium and optimal utility. As explained above, in our model assuming a Pareto distribution of productivity, all markup distortions are absorbed into the intensive margin distortion unique to heterogeneous firm frameworks (i.e., inefficient underproduction (overproduction) by more (less) productive firms). Hence, our finding suggests that the pro-competitive effects are not effective in correcting the intensive margin distortion unique to heterogeneous firm frameworks. This implication is likely to be consistent with that provided by Arkolakis, Costinot, Donaldson and Rodríguez-Clare (2019), who predict the gains from trade under the same framework as ours and conclude that the pro-competitive gains from trade are "elusive."

Third, we identify the source of gains from trade in the market equilibrium. We show that all countries gain from trade through consumption variety expansion and selection effects. In both closed and open economies, the utility function can be expressed as being proportional to the mass of consumed varieties or inversely proportional to the domestic cost cutoff. This also implies that each country's mass of consumed varieties is inversely proportional to that country's domestic cost cutoff. Opening up to trade expands the mass of varieties available to consumers through imports and simultaneously boosts average productivity (i.e., decreases the domestic cost cutoff) by allowing more efficient firms to export, while less efficient ones exit the market. These factors contribute to raising the welfare of each country, in contrast to the Melitz model (with Pareto-distributed productivity), where, as pointed out by Feenstra (2018), gains from trade result from firm selection rather than variety expansion. These findings are also confirmed by Behrens et al. (2014) within a framework of a specified utility function and a Pareto distribution of productivity. We demonstrate that these characteristics are broadly applicable, extending beyond specific utility functions.

Lastly, we provide insights into the varying impacts of trade and trade liberalization across countries with differences in factors such as market size, technology, and geography. In a two-country economy, the gains from trade are greater in a country with a smaller market size, lower technological level, and easier access to that market. Intuitively, in such a country, the mass

the insufficient production and excessive variety are tied to the extensive margin distortion (i.e., excessive firm entry). As explained in footnote 2, if the extensive margin is not considered (i.e., if the mass of entrants is fixed), there will be no distortion in the symmetric firm settings.

of new varieties available to consumers through imports is larger, and the selection effects are greater due to exports from a more technologically advanced partner country. Both factors, as mentioned above, contribute to greater gains from trade. Furthermore, we find that the country gaining more from opening up to trade also benefits more from bilateral trade liberalization.[5] As a result, such differences in the magnitude of welfare gains across countries can lead to various scenarios in which opening up to trade and trade liberalization impact relative welfare level. We find that both opening up to trade from autarky and subsequent bilateral liberalization can result in: (i) narrowing the welfare gap, (ii) widening the welfare gap, or (iii) reversing the welfare relationship. In particular, the last scenario highlights that even if trade costs are symmetric, a country with relatively high welfare in a closed economy (or pre-liberalization) does not necessarily maintain relatively high welfare after opening up to trade (or post-liberalization). The last two scenarios emerge due to the characteristics of trade and trade liberalization, which tend to reduce the welfare gap between countries arising from difference in market size more than the welfare gap arising from difference in technological level. In other words, a smaller market size tends to contribute more to a country's welfare gains than a lower technological level. This is a key factor explaining why trade and bilateral liberalization do not necessarily lead to a monotonic reduction in the welfare gap between the two countries.

The present study is related to the literature of monopolistic competition, originating from the seminal paper by Dixit and Stiglitz (1977). Although their substantial contribution is often considered to propose an analytically tractable framework with constant elasticity of substitution (CES) preference, they also examined more general cases wherein utility function is additively separable and the elasticity of substitution is not constant. The frameworks with additively separable preferences and variable elasticity of substitution (VES) have been investigated more thoroughly by subsequent studies such as, for example, Behrens and Murata (2007) and Zhelobodko, Kokovin, Parenti and Thisse (2012) under homogeneous firms and Dhingra and Morrow (2019) and Behrens et al. (2020) under heterogeneous firms.[6] Some other studies focus on the cases in which the direct utility is not additively separable. One of the most widely used preferences that exhibit VES and non-additivity is the quadratic utility function, promoted by Ottaviano, Tabuchi and Thisse (2002) and Melitz and Ottaviano (2008).[7] As different approaches, Bertoletti and Etro (2017) examined a model wherein the indirect utility is additively separable, and Feenstra (2003) and Matsuyama and Ushchev (2023) considered non-CES homothetic

[5] We consider a bilateral liberalization scenario where both countries simultaneously reduce their trade costs by the same proportion, while allowing for asymmetric trade costs.

[6] Regardless of whether firms are homogeneous or heterogeneous, these studies agree that the CES preference yields quite special results: for instance, the equilibrium allocation coincides with the optimal one only in the CES case (Dixit and Stiglitz, 1977; Dhingra and Morrow, 2019).

[7] The quadratic utility generates a linear demand function and is usually applied in quasi-linear models (e.g., Nocco et al., 2014), while some recent studies exclude the outside good to incorporate the income effect (e.g., Bagwell and Lee, 2023).

preferences.[8][9] In the present study, we build on directly additive preferences, which are classical but most widely applied in a number of fields in economics including international trade, macroeconomics, and economic geography. In particular, we focus on the VES case with firm heterogeneity. In that sense, our study considers a similar framework to Dhingra and Morrow (2019), but is different from theirs in that (i) we exclude fixed costs for starting production, which is a prevalent setting in applications of monopolistic competition including international trade; (ii) we mainly focus on properties under Pareto-distributed productivity; and (iii) we also deal with a multi-country framework wherein trade among countries is subject to trade costs. Therefore, the present study can be viewed as a complement of Dhingra and Morrow (2019).

Our study also contributes to the literature of international trade under monopolistic competition with VES preferences (e.g., Ottaviano et al., 2002; Melitz and Ottaviano, 2008; Behrens and Murata, 2012; Bertoletti and Epifani, 2014; Kichko, Kokovin and Zhelobodko, 2014; Simonovska, 2015; Bykadorov, Gorn, Kokovin and Zhelobodko, 2015; Arkolakis et al., 2019; Slepov and Kokovin, 2023; Behrens, Kichko and Ushchev, 2024).[10] Such frameworks are often applied to examining trade policies, although most studies, including Demidova (2017) and Bagwell and Lee (2020), use a quadratic utility, partly because of analytical tractability.[11] Our results indicate that, as long as firms' productivity follows a Pareto distribution, the trade equilibrium can be characterized quite simply, irrespective of the functional form of utility function. Thus, our study can contribute to the literature by providing more generalized properties of the gains from trade and increasing the applicability of the VES frameworks to policy studies.[12] The VES models of trade are also applied in quantitative studies, such as Corcos, Del Gatto, Mion and Ottaviano (2012), Behrens et al. (2014), Bertoletti, Etro and Simonovska (2018), and Jung, Simonovska and Weinberger (2019). Therefore, our study may also be helpful for understanding the quantitative results derived from such studies.

The remainder of the paper is organized as follows. Section 2 describes the closed-economy model. Section 3 characterizes markup distortion in the domestic market and how the pro-

[8]The CES utility is the only preference that simultaneously exhibits direct additivity, indirect additivity, and homotheticity (Samuelson, 1965), and Parenti, Ushchev and Thisse (2017) have shown that the equilibrium price and output are independent of both individual income and market size only in the case with the CES utility.

[9]There are also unique and notable frameworks. For example, Bertoletti and Etro (2021, 2022) dealt with a more generalized concept of additive separability, Mrázová and Neary (2017) focused on the relationship between the elasticity and convexity of demand, and Mrázová, Neary and Parenti (2021) examined the case wherein firms' productivity and sales distributions have the same form.

[10]As pointed out by Mrázová and Neary (2020), the CES model is quite restrictive in its theoretical and empirical implications, increasing the need for using VES frameworks in policy studies.

[11]A notable exception departing from the quadratic specification is Zeng and Peng (2021), who examined the effects of distortions arising from international tax competition on the welfare under a large class of additively separable preferences.

[12]Given that the pro-competitive effects of trade do not correct the distortions arising from differences in markups across firms, policy intervention and cooperation become crucial in addressing these distortions within the framework of VES preferences and firm heterogeneity. Although we do not address this issue in this study, the tractability of our model will contribute to the literature on optimal policies. For the literature on first-best policies in a closed economy with differences in markups across firms, see Nocco et al. (2014), Tadokoro (2024), and Melitz, Ottaviano, Oshmakashvili and Suverato (2024). See also Nocco, Ottaviano and Salto (2019) and Nocco, Ottaviano, Salto and Tadokoro (2024) for insights on first-best policy coordination to achieve globally efficient resource allocation in an open economy.

competitive effects affect this distortion. Section 4 extends the model to a multi-country framework and discusses the gains from trade. Section 5 analytically explores the varying impacts of trade on countries with differences in factors such as market size, technology, and geography. Finally, Section 6 concludes.

2 The Model

Consider an economy with a measure L of workers. Each worker inelastically supplies a unit of labor and faces a wage rate w, which we normalize to one. The preferences of workers are identical and defined over a continuum of horizontally differentiated varieties.

2.1 Preferences

The preference of each worker is represented by an *additively separable* utility function as in Zhelobodko et al. (2012) and Dhingra and Morrow (2019):

$$U = \int_{\Omega} u(q(\iota))\mathrm{d}\iota, \tag{1}$$

where Ω is the set of varieties available to workers and $q(\iota)$ is the quantity of variety $\iota \in \Omega$. The subutility $u(\cdot)$ is a function of one variable defined over a convex set $X \subseteq \mathbb{R}^{+}$ with $\inf X = 0$. Moreover, it satisfies the following assumption.

Assumption 1. *The subutility $u(\cdot)$ is thrice continuously differentiable, strictly increasing, and strictly concave on its domain X. In addition, it satisfies $u(0) = 0$, $\lim_{x\to 0} u'(x) < \infty$, and $\lim_{x\to \sup X} u'(x) = 0$.*

Note that the supremum of the domain X can be either infinite or finite. In the case of $\sup X = \infty$, simple examples for the subutility satisfying Assumption 1 are the constant absolute risk aversion (CARA) function (Behrens and Murata, 2007) and the Stone-Geary function (Simonovska, 2015).[13] If $\sup X$ is finite, a typical example is a quadratic specification as in Melitz and Ottaviano (2008).[14]

Each worker maximizes their own utility subject to the budget constraint $\int_{\Omega} p(\iota)q(\iota)\mathrm{d}\iota = E$, where $p(\iota)$ is the price of variety ι and E is the income of a worker. Then, the inverse demand function can be derived as $p(\iota) = u'(q(\iota))/\lambda$, where λ is the Lagrange multiplier. Assumption 1 indicates that there exists a choke price above which the demand for a variety is zero. We let $\lim_{x\to 0} u'(x) = \bar{u}$ and then denote the choke price by $\widehat{p} \equiv \lim_{x\to 0} u'(x)/\lambda = \bar{u}/\lambda$. As a result, the inverse demand function can be rewritten as

$$p(\iota) = \frac{\widehat{p}u'(q(\iota))}{\bar{u}}. \tag{2}$$

[13] Note that the CES specification is excluded, because it violates $\lim_{x\to 0} u'(x) < \infty$.

[14] The quadratic utility function (for differentiated goods) used in Melitz and Ottaviano (2008) is not actually additively separable, but setting $\eta = 0$ (and dropping the outside good assumption) in their specification renders the utility function additively separable.

2.2 Production

Each variety is produced by a single firm in a monopolistically competitive market. For each firm to enter the market, f units of labor are required as fixed inputs to develop a new variety. The mass of entering firms is denoted by N. After the entry, each firm realizes its marginal labor requirement m, which is drawn from a known probability distribution $G(m)$ with its support given by $(0, M)$, where M represents the marginal labor requirement of the potentially most inefficient firms. We impose the following assumption on the distribution function $G(m)$.

Assumption 2. *$G(m)$ is twice continuously differentiable at any point on the support $(0, M)$.*

Let $g(m) \equiv G'(m)$ denote the probability density function of m. Then, Assumption 2 assures that $g(m)$ is differentiable at any point on $(0, M)$.

Since the wage rate is normalized to 1, the operating profit of a firm with cost draw m is expressed as

$$\pi(m) = L(p(m) - m)q(m). \tag{3}$$

Firms start production if they can expect non-negative operating profits; otherwise, they exit the market. The surviving firms choose the quantity $q(m)$ to maximize their profits $\pi(m)$.[15] For the profit-maximizing output to be uniquely determined, we impose the following assumption, which assures that a firm's revenue is strictly concave in $q(m)$.

Assumption 3. *$u(x)$ satisfies $2u''(x) + xu'''(x) < 0$.*

Because each firm faces the inverse demand function (2), the first order condition for profit maximization is given by

$$\frac{u'(q(m)) + q(m)u''(q(m))}{\bar{u}} = \frac{m}{\widehat{p}}. \tag{4}$$

Note that the most inefficient firm that actually produces its variety has the marginal cost cutoff $\widehat{m}$, which is just equal to the choke price, i.e., $\widehat{m} = \widehat{p}$. Therefore, (4) enables us to write the profit-maximizing quantity as a function of $m/\widehat{m}$:

$$q(m) = \psi\left(\frac{m}{\widehat{m}}\right), \tag{5}$$

where $\psi(\cdot)$ is defined as the inverse function of the left-hand side in (4): that is, $\psi^{-1}(x) = (u'(x)+xu''(x))/\bar{u}$ holds.[16] It readily follows from Assumption 3 that $\psi(\cdot)$ is a strictly decreasing function, implying that firms with lower costs produce higher quantities. The domain of $\psi(\cdot)$ is $(0, 1)$ and $\lim_{x\to 1} \psi(x) = 0$ is satisfied.

The condition (4) also implies that the markup charged by a firm with cost m can be written as

$$\mu(m) \equiv \frac{p(m)}{m} = \frac{1}{1 - r(q(m))}, \tag{6}$$

[15] To be precise, the quantity produced by each firm is $q^{\text{firm}}(m) \equiv Lq(m)$. However, maximizing $\pi(m)$ with respect to $q(m)$ and $q^{\text{firm}}(m)$ yields the same equilibrium outcome. Thus, we consider the optimization with respect to $q(m)$ for simplicity.

[16] The existence of the inverse function $\psi(\cdot)$ is guaranteed by Assumption 3.

where $r(q(m)) \equiv -q(m)u''(q(m))/u'(q(m))$ measures the *relative love for variety*. We impose the following assumption on $r(q(m))$.

Assumption 4. *The relative love for variety $r(x)$ satisfies $r(q(m)) < 1$ for $0 < m < \widehat{m}$ and $r'(x) > 0$.*

The first constraint, $r(q(m)) < 1$, assures a positive markup. The second one, $r'(x) > 0$, implies that firms with lower costs charge higher markups, because the derivative of (6) is calculated as $\mu'(m) = (1 - r(q(m)))^{-2} r'(q(m))q'(m) < 0$. This assumption reflects the findings of several empirical studies that more productive firms charge higher markups (De Loecker and Warzynski, 2012; De Loecker et al., 2016).

2.3 Market Equilibrium

There are two endogenous variables to be determined in the model: the marginal cost cutoff ($\widehat{m}$) and the mass of entering firms (N). It requires two equilibrium conditions to close the model.

First, ex ante average profits net of entry costs must be equal to zero, implying the following zero expected profit condition:

$$\int_0^{\widehat{m}} \pi(m) \mathrm{d}G(m) = f. \tag{7}$$

Second, the total mass of labor demanded by firms must be equal to the mass of workers in the economy, which can be written as the following labor market clearing condition:

$$N\left(L \int_0^{\widehat{m}} mq(m)\mathrm{d}G(m) + f\right) = L, \tag{8}$$

where N represents the mass of entrants. As implied by the Walras's law, combining (3), (7) and (8) results in

$$N \int_0^{\widehat{m}} p(m)q(m)\mathrm{d}G(m) = 1, \tag{9}$$

which is equivalent to the goods market clearing condition.

In order that all integrals to appear are convergent, we impose the following integrability conditions.

Assumption 5. *Each firm's labor requirement for producing goods is finite: $\int_0^{\widehat{m}} mq(m)\mathrm{d}G(m) < \infty$. Each consumer's expenditure is finite: $\int_0^{\widehat{m}} p(m)q(m)\mathrm{d}G(m) < \infty$. Each consumer's utility is finite: $\int_0^{\widehat{m}} u(q(m))\mathrm{d}G(m) < \infty$.*

Finally, for the equilibrium to exist, we impose the following assumption.

Assumption 6. *The entry cost f is small enough to satisfy $f < LM \int_0^1 \psi(t)G(Mt)\mathrm{d}t$.*

Defining the elasticities of subutility and probability distribution function as $\mathcal{E}^u(x) \equiv xu'(x)/u(x)$ and $\mathcal{E}^G(m) \equiv mg(m)/G(m)$, respectively, we obtain the following lemma that assures the existence and uniqueness of the market equilibrium.[17]

Lemma 1. *At the market equilibrium, $\widehat{m}$ and N are uniquely determined in the following two equations:*

$$\widehat{m} \int_0^1 \psi(t) G(\widehat{m}t) \mathrm{d}t = \frac{f}{L}, \tag{10}$$

$$N\widehat{m} \int_0^1 \psi(t) G(\widehat{m}t) \left(1 + \mathcal{E}^G(\widehat{m}t)\right) \mathrm{d}t = 1. \tag{11}$$

Moreover, each worker's utility can be expressed as

$$U = \frac{\bar{u}}{\widehat{m}} + N \int_0^1 \frac{u(\psi(t))(1 - \mathcal{E}^u(\psi(t)))\mathcal{E}^G(\widehat{m}t)G(\widehat{m}t)}{t} \mathrm{d}t. \tag{12}$$

Proof. See Appendix A.1. □

As shown in Appendix A.1, (10) and (11) are obtained by manipulating (7) and (9), respectively.

2.4 Social Optimum

To evaluate the markup distortion in the domestic market, we then derive the socially optimal resource allocation. A benevolent social planner maximizes individual welfare (U), taking as given the labor endowment (L) and the production function. The planner faces the same entry process as the market, where the unit labor requirement m for each variety is randomly drawn from the distribution $G(m)$ after f units of labor have been allocated to designing that variety. Thus, the optimization problem is given by

$$\max_{q(m), N, \widehat{m}} U = N \int_0^{\widehat{m}} u(q(m)) \mathrm{d}G(m) \quad \text{s.t. } N \left(\int_0^{\widehat{m}} mq(m) \mathrm{d}G(m) + \frac{f}{L} \right) = 1. \tag{13}$$

The first-order conditions with respect to $q(m)$, N and $\widehat{m}$ respectively yields

$$u'(q(m)) = \delta m, \tag{14}$$

$$\int_0^{\widehat{m}} u(q(m)) \mathrm{d}G(m) = \delta \left(\int_0^{\widehat{m}} mq(m) \mathrm{d}G(m) + \frac{f}{L} \right), \tag{15}$$

$$u(q(\widehat{m})) = \delta \widehat{m} q(\widehat{m}), \tag{16}$$

where δ is the Lagrange multiplier.

[17]Note that $0 < \mathcal{E}^u(x) < 1$ holds for $x \in X$ under Assumption 1. This can be shown in the following way. We take the derivative of $u(x) - xu'(x)$, resulting in $-xu''(x) > 0$. As $\lim_{x \to 0}(u(x) - xu'(x)) = 0$ holds, we have $u(x) - xu'(x) > 0$ for $x \in X$, implying that $0 < \mathcal{E}^u(x) < 1$.

Substituting (14) for $m = \widehat{m}$ into (16) yields $u(q(\widehat{m})) - u'(q(\widehat{m}))q(\widehat{m}) = 0$, which determines $q(\widehat{m})$. By Assumption 1, the left-hand side of this equation is strictly increasing with $q(\widehat{m})$ and satisfies $\lim_{q(\widehat{m})\to 0} (u(q(\widehat{m})) - u'(q(\widehat{m}))q(\widehat{m})) = 0$, implying $q(\widehat{m}) = 0$. Therefore, for $m = \widehat{m}$, (14) can be expressed as

$$\delta = \frac{\bar{u}}{\widehat{m}}, \tag{17}$$

where $\bar{u} = \lim_{x\to 0} u'(x)$. Substituting this back into (14), we get

$$\frac{u'(q(m))}{\bar{u}} = \frac{m}{\widehat{m}}. \tag{18}$$

The left-hand side of (18) is strictly decreasing with $q(m)$ and satisfies $0 < u'(q(m))/\bar{u} < 1$. Therefore, (18) implies that $m/\widehat{m} \in (0, 1)$ and $q(m)$ have one-to-one correspondence, enabling us to rewrite (18) as

$$\phi\left(\frac{m}{\widehat{m}}\right) = q(m), \tag{19}$$

where $\phi(\cdot)$ is defined as the inverse function of the left-hand side in (18), i.e., $\phi^{-1}(x) = u'(x)/\bar{u}$. As $\phi(\cdot)$ is a strictly decreasing function, firms with lower costs produce higher quantities. The domain of $\phi(\cdot)$ is $(0, 1)$ and $\lim_{x\to 1} \phi(x) = 0$ holds.

Using (17) and (19) into (15) and the resource constraint in (13), we can derive the system of equations for $\widehat{m}$ and N as expressed in the following lemma.

Lemma 2. *Social optimum values of $\widehat{m}$ and N are uniquely determined in the following two equations:*

$$\widehat{m} \int_0^1 \phi(t)G(\widehat{m}t)\mathrm{d}t = \frac{f}{L}, \tag{20}$$

$$N\widehat{m} \int_0^1 \phi(t)G(\widehat{m}t)\left(1 + \mathcal{E}^G(\widehat{m}t)\right)\mathrm{d}t = 1. \tag{21}$$

Moreover, each worker's utility can be expressed as

$$U = \frac{\bar{u}}{\widehat{m}}.$$

Proof. See Appendix A.2. □

3 Pareto Distribution

In models of monopolistic competition with heterogeneous firms, the productivity is often assumed to follow a Pareto distribution. This is because a Pareto distribution is not only analytically tractable but also regarded as a reasonable approximation of the observed distribution. For example, the Pareto-distributed productivity successfully replicates the well-known fact that the firm size distribution exhibits a power law as documented by Axtell (2001) and Gabaix (2009). Here, we also assume a Pareto distribution for productivity, providing some notable properties on the market equilibrium and its efficiency. Specifically, we let $G(m) = (m/M)^k$, where $k > 1$

is the shape parameter that governs the dispersion of productivity.

This specification enables a detailed description of the equilibrium and the markup distortion. In particular, by assuming a Pareto distribution, the distribution function has the following properties:

$$\mathcal{E}^G(m) = k \quad \text{and} \quad G(\widehat{m}t) = t^k G(\widehat{m}). \tag{22}$$

The first expression represents that the elasticity of the distribution function becomes a positive constant with the Pareto shape parameter. The second expression implies that in Lemmas 1 and 2, the integral variable $t \in (0, 1)$ can be separated from the the distribution function.

3.1 Properties of Equilibrium

We first characterize the market equilibrium under Pareto-distributed productivity. Let superscripts 'eqm' denote variables at the market equilibrium. Then, from (22) and Lemma 1, the marginal cost cutoff and the mass of entrants can each be expressed in the following closed form:

$$\widehat{m}^{\text{eqm}} = \left(\frac{fM^k}{\Psi L}\right)^{\frac{1}{k+1}} \quad \text{and} \quad N^{\text{eqm}} = \frac{1}{k+1}\frac{L}{f}, \tag{23}$$

where

$$\Psi \equiv \int_0^1 t^k \psi(t)\mathrm{d}t \tag{24}$$

is a positive constant that depends on exogenous choices of the Pareto shape parameter, k, and the functional form of subutility, $u(\cdot)$.

The mass of varieties that are actually consumed in the market, denoted by n, is expressed as $n = NG(\widehat{m})$. Then, by rewriting this expression using (23), we obtain

$$n^{\text{eqm}} = \frac{1}{(k+1)\Psi\widehat{m}^{\text{eqm}}}, \tag{25}$$

which exhibits an inversely proportional relationship between $\widehat{m}$ and n. Furthermore, from (22), (25) and Lemma 1, the equilibrium utility is expressed as

$$U^{\text{eqm}} = \left[\bar{u} + \frac{k\zeta}{(k+1)\Psi}\right]\frac{1}{\widehat{m}^{\text{eqm}}} = [(k+1)\Psi\bar{u} + k\zeta]n^{\text{eqm}}, \tag{26}$$

where

$$\zeta \equiv \int_0^1 t^{k-1}u(\psi(t))(1 - \mathcal{E}^u(\psi(t)))\mathrm{d}t \tag{27}$$

is a positive constant that depends on exogenous choices of the Pareto shape parameter, k, and the functional form of subutility, $u(\cdot)$. Therefore, under Pareto-distributed productivity, the welfare can be measured solely by either $\widehat{m}$ or n. We summarize these results in the following lemma.

Lemma 3. *Suppose that firms' productivity $(1/m)$ follows a Pareto distribution with the shape* *parameter k. Then, at the market equilibrium, the mass of consumed varieties (n) can be expressed as a constant multiple of the inverse of the marginal cost cutoff ($\widehat{m}$). Furthermore, each worker's utility (U) can be expressed as a constant multiple of the inverse of the marginal cost cutoff ($\widehat{m}$), or a constant multiple of the mass of consumed varieties (n).*

Lemma 3 shows that changes in the marginal cost cutoff or the mass of consumed varieties are sufficient statistics for welfare changes. The same results are confirmed by Behrens et al. (2014), where utility is specified as the CARA function. We find that the results hold independent of their specific utility function.

It can be readily confirmed that the pro-competitive effect is at work in our framework. Specifically, expressions in (23) and (25) show that an increase in market size (L) attracts more firms (entrants) into the market. This leads to tougher competition, reflected in lower markups and prices given by (6).[18]

3.2 Efficiency

Next, we identify the markup distortion in the domestic market under Pareto-distributed productivity. Let superscripts 'opt' denote variables at the social optimum. From (22) and Lemma 2, the optimal levels of the marginal cost cutoff and the mass of entrants are expressed as

$$\widehat{m}^{\text{opt}} = \left(\frac{fM^k}{\Phi L}\right)^{\frac{1}{k+1}} \quad \text{and} \quad N^{\text{opt}} = \frac{1}{k+1}\frac{L}{f}, \tag{28}$$

where $\Phi \equiv \int_0^1 t^k \phi(t)\mathrm{d}t$ is a positive constant that depends on exogenous choices of the Pareto shape parameter, k, and the functional form of subutility, $u(\cdot)$. Then, using (28) to rewrite the definition of n, we get

$$n^{\text{opt}} = \frac{1}{(k+1)\Phi\widehat{m}^{\text{opt}}}.$$

To describe in detail the markup distortion in the domestic market, we define the aggregate and average production. Let $Q \equiv N\int_0^{\widehat{m}} q(m)\mathrm{d}G(m)$ and $\bar{q} \equiv Q/n$ denote the aggregate and average production per consumer, respectively.[19] Then, the markup distortion in the domestic market can be summarized in the following proposition.

Proposition 1. *Suppose that firms' productivity ($1/m$) follows a Pareto distribution with the shape parameter k. Compared to the social optimum, in the market equilibrium,*

(i) the mass of entrants is optimal, $N^{\text{eqm}} = N^{\text{opt}}$;

[18] An increase in L decreases $\widehat{m}$ in (23). Then, it results in a decrease in $q(m)$ for $m \in (0, \widehat{m})$ from (5), as $\psi(\cdot)$ is a decreasing function. This decrease in $q(m)$ also leads to a decrease in $r(q(m))$ by Assumption 4, implying a decrease in $\mu(m)$ and $p(m)$ from (6). Therefore, an increase in market size leads to a decrease in markups and prices.

[19] As stated in Proposition 1, we need to impose an additional assumption that the aggregate production converges, in order that Q and $\bar{q}$ are successfully defined. Technically speaking, this additional assumption serves as a stronger restriction than Assumption 5. However, we believe that this additional assumption is not irrational, because the aggregate production is finite in the real world.

(ii) firm selection is too weak, $\widehat{m}^{\text{eqm}} > \widehat{m}^{\text{opt}}$*;*

(iii) quantity of a variety is too little (too much) for more (less) productive firms, $q^{\text{eqm}}(m) < (>) q^{\text{opt}}(m)$ *for* $m < (>) m^*$*, where* $m^* \in (0, \widehat{m}^{\text{opt}})$*;*

(iv) the mass of consumed varieties is too much, $n^{\text{eqm}} > n^{\text{opt}}$*.*

Moreover, if we additionally assume $\int_0^{\widehat{m}} q(m)\mathrm{d}G(m) < \infty$*, the following also holds:*

(v) aggregate and average quantities of varieties are too little, $Q^{\text{eqm}} < Q^{\text{opt}}$ *and* $\bar{q}^{\text{eqm}} < \bar{q}^{\text{opt}}$*.*

Proof. The first result immediately follows from comparing (23) and (28). Next, we prove the second result. Using (5) and replacing $m/\widehat{m} \in (0,1)$ with t, we can rewrite (4) as $u'(\psi(t)) + \psi(t)u''(\psi(t)) = t\bar{u}$. Similarly, from (18) and (19), we obtain $u'(\phi(t)) = t\bar{u}$. As $u(\cdot)$ is strictly increasing and strictly concave on its domain, these two expressions show that $\psi(t) < \phi(t)$ holds for $t \in (0,1)$. Therefore, we obtain $\Psi = \int_0^1 t^k \psi(t)\mathrm{d}t < \Phi = \int_0^1 t^k \phi(t)\mathrm{d}t$, meaning that $\widehat{m}^{\text{eqm}} > \widehat{m}^{\text{opt}}$ holds. Based on the first two results, the fourth result can be readily shown, because $N^{\text{eqm}} = N^{\text{opt}}$ and $\widehat{m}^{\text{eqm}} > \widehat{m}^{\text{opt}}$ imply that $n^{\text{eqm}} = N^{\text{eqm}}G(\widehat{m}^{\text{eqm}}) > N^{\text{opt}}G(\widehat{m}^{\text{opt}}) = n^{\text{opt}}$ holds. See Appendix A.3 for proofs of the third and fifth results. □

It is worth noting that the fourth and fifth results in Proposition 1 are in line with the markup distortion in symmetric firm settings as in Behrens and Murata (2012). However, the nature of the underlying misallocation that leads to these results differs from ours. In symmetric firm settings, these insufficient production and excessive variety in the market equilibrium arise from misallocation of labor used either for production or for development (to design a new variety), due to inefficiently high markups. Specifically, an insufficient production in market equilibrium is caused by inefficiently high markups set by each firm. This also means that an inefficiently small share of labor is allocated to production in relation to the total labor supply, which is equivalent to an inefficiently large share of the remaining labor allocated to development (that leads to an excessive variety).

In our heterogeneous firm settings, however, this is not necessarily the case. In the context of heterogeneous firms, an insufficient production and excessive variety in the market equilibrium are generally associated not only with labor misallocation between production and development, but also with labor misallocation within production among heterogeneous firms.[20] The first three results in Proposition 1 reveal that under Pareto-distributed productivity, the latter type of labor misallocation is pronounced. Specifically, the first result of the proposition, $N^{\text{eqm}} = N^{\text{opt}}$, shows the absence of labor misallocation between production and development, as it implies that the aggregate labor used for development (fN^{eqm}) and the remaining labor, the aggregate labor used for production, align with those in the optimal levels. Meanwhile, the second and third results of Proposition 1 highlight that, even though the market efficiently provides the

[20] The latter type of labor misallocation is absent in symmetric settings (see footnote 2).

mass of entrants and aggregate labor for production (that will be allocated to these entrants), the allocation of this labor to each entrant is inefficiently skewed toward less productive firms: more productive firms set relatively high markups and end up inefficient underproduction (i.e., more productive firms hire too little labor for production), whereas less productive firms set relatively low markups and end up inefficient overproduction (i.e., less productive firms hire too much labor for production). Therefore, the insufficient total/average production in the market equilibrium shown in the fifth result of Proposition 1 arises from the inefficient skew in labor allocation for production toward less productive firms. In addition, overproduction by these less productive firms implies that even the least productive firms with cost $m \in (\widehat{m}^{\mathrm{opt}}, \widehat{m}^{\mathrm{eqm}})$, which should not produce for efficiency reasons, are also in operation. This leads to the excessive variety in the market equilibrium shown in the fourth result of Proposition 1.

Hence, although the markup distortion in our heterogeneous firm framework shares similarities with that in symmetric firm settings—where variable markups lead to an insufficient production and excessive variety in the market equilibrium—the underlying labor misallocation causing these outcomes is completely different. The markup distortion described in Proposition 1 is independent of the extensive margin distortion related to firm entry and can thus be interpreted as the intensive margin distortion unique to heterogeneous firm frameworks.[21] Our findings suggest that the extensive margin distortion (i.e., an excess entry) caused by inefficiently high markups under symmetric firm settings may be overestimated due to the lack of consideration of differences in markups across firms. The markup differences generate underproduction by more productive firms and overproduction by less productive firms. From a labor allocation perspective, while the former can lead to excess entry as in the symmetric firm settings, the latter works rather in the direction of reducing this excess entry. Hence, the presence of markup differences across firms—observed in a framework with firm heterogeneity and variable markups—generates the intensive margin distortion among heterogeneous firms, which may reduce the extensive margin distortion compared to symmetric firm settings. Especially under Pareto-distributed productivity, the extensive margin distortion observed in symmetric firm settings is entirely transformed into the intensive margin distortion unique to heterogeneous firm frameworks. This result helps to demonstrate that the effectiveness of the pro-competitive effects can differ depending on the nature of the markup distortion, as we confirm below.

[21] The second and third results in Proposition 1, which highlight the intensive margin distortion unique to heterogeneous firm frameworks, are commonly observed in more generalized monopolistic competition models than the one that we consider. For instance, Dhingra and Morrow (2019) show that these two results hold (when the elasticities of subutility and marginal utility with respect to quantity are negative and positive, respectively) in a monopolistic competition model that includes fixed cost of production and does not specify the distribution of productivity. Behrens et al. (2020) further show that these results continue to hold even when considering multi-sector monopolistic competition model. With respect to them, we find that assuming a Pareto distribution allows us to unambiguously characterize the nature of markup distortion for aggregate/average quantity, consumed variety, and firm entry, without the need to specify a utility function.

3.3 Effects of Market Size

Our concern is whether the pro-competitive effects can eliminate the markup distortion described in Proposition 1. In a symmetric firm framework with the CARA utility, Behrens and Murata (2012) show that the pro-competitive effects raise efficiency by narrowing the gap between equilibrium and optimal utility. This efficiency-enhancing effect can be interpreted as pro-competitive gains from trade. Here, we examine whether the pro-competitive gains from trade are also present in our framework.

We confirm this by examining how changes in market size (L) affect allocation efficiency. Note that, as shown by Behrens and Murata (2012) and Dhingra and Morrow (2017), an increase in market size in a closed economy has the same consequences as integration or free trade among countries with identical levels of technology. Therefore, investigating the impact of an increase in market size on allocation efficiency is a plausible measure to assess the impact of trade on allocation efficiency.[22]

From (23), (26), (28), and Lemma 2, the welfare gap between equilibrium and optimum can be characterized as

$$\frac{U^{\text{eqm}}}{U^{\text{opt}}} = \left[1 + \frac{k\zeta}{(k+1)\Psi\bar{u}}\right] \frac{\widehat{m}^{\text{opt}}}{\widehat{m}^{\text{eqm}}} = \left[1 + \frac{k\zeta}{(k+1)\Psi\bar{u}}\right] \left(\frac{\Psi}{\Phi}\right)^{\frac{1}{k+1}}.$$

This is independent of market size L. Hence, we obtain the following proposition.

Proposition 2. *Suppose that firms' productivity ($1/m$) follows a Pareto distribution with the shape parameter k. An increase in market size (L) dose not affect the welfare gap between the equilibrium and optimum.*

Proposition 2 shows that the markup distortion is not improved by integration. We can also confirm this by the fact that the gaps between equilibrium and optimum levels of the aggregate/average quantity, the mass of consumed varieties, and the cost cutoff are not affected by integration (see Appendix A.3.1). In Section 3.1, we have confirmed that integration (i.e., an increase in market size) does indeed generate the pro-competitive effects. Therefore, Proposition 2 demonstrates that the pro-competitive effects do not generate any efficiency gains in our framework. This result differs from that of Behrens and Murata (2012) in their symmetric firm setting, where the pro-competitive effects raise efficiency.

As explained in Section 3.2, in our model assuming a Pareto distribution of productivity, the extensive margin distortion is canceled out, and all markup distortions emerge as the intensive

[22] In the context of quasi-linear preferences, as in Nocco et al. (2019, 2024), a constant and identical marginal utility of income across individuals allows for a straightforward definition of globally efficient outcome in an open economy as the maximized sum of all individuals' utilities. Comparing this result with the market equilibrium under international trade enables a consistent analysis of the effect of trade on resource allocation efficiency. However, how to define a globally efficient outcome under a class of utility function without quasi-linearity is a matter of debate. One compelling approach to analyzing the impact of trade on allocation efficiency in this context is to examine the consequences of integration (i.e., an increase in market size) on allocation efficiency, as analyzed here. Note that some studies address this issue using the same approach as in the quasi-linear preference case (Egger and Huang, 2023).

margin distortion caused by differences in markups across firms. Therefore, Proposition 2 shows that the pro-competitive effects are not effective in correcting the intensive margin distortion unique to heterogeneous firm frameworks. Hence, in the presence of markup differences across firms, the pro-competitive gains from trade may not be as large as those expected in symmetric firm settings, such as in Behrens and Murata (2012), due to the presence of the intensive margin distortion that is not improved by the pro-competitive effects. Especially under Pareto-distributed productivity, as all markup distortions emerge as this intensive margin distortion, there is no pro-competitive gains from trade: that is, the pro-competitive effects do not generate any efficiency gains. This result is likely to be consistent with that provided by Arkolakis et al. (2019), who predict the gains from trade under the same framework as ours (including the class of utility functions, the model structure, and the Pareto-distributed productivity) and conclude that the pro-competitive gains from trade are "elusive." Our results provide a theoretical explanation for their findings in terms of the absence of a corrective impact of the pro-competitive effects on markup distortion in this framework.

4 International Trade

We now extend our baseline framework to a multi-country model wherein all countries are interconnected through goods trade. Countries are indexed by $i = 1, \cdots, J$ and the mass of workers in country i is denoted by L_i. Workers are not allowed to migrate across countries, and inelastically supply a unit of labor to the country where they live. The wage rate in country i is represented by w_i.

The utility of a worker in country i can be defined as a natural extension of (1) as follows:

$$U_i = \sum_j \int_{\Omega_{ji}} u(q_{ji}(\iota))\mathrm{d}\iota,$$

where Ω_{ji} is the set of varieties produced in country j and available to workers in country i, and $q_{ji}(\iota)$ is the quantity of variety $\iota \in \Omega_{ji}$. Workers maximize their own utilities subject to the budget constraint, $\sum_j \int_{\Omega_{ji}} p_{ji}(\iota)q_{ji}(\iota)\mathrm{d}\iota = E_i$, where $p_{ji}(\iota)$ is the price of variety $\iota \in \Omega_{ji}$ and E_i is the individual expenditure of workers in country i. Then, the inverse demand function can be expressed as $p_{ji}(\iota) = \widehat{p}_i u'(q_{ji}(\iota))/\bar{u}$, where $\widehat{p}_i$ is the choke price in country i. Again, $\bar{u}$ is defined as the marginal utility of a variety evaluated at zero: $\bar{u} = \lim_{x\to 0} u'(x)$.

On the producer side, we consider a situation in which the fixed requirement for entry and the distribution function of the marginal labor requirement vary across countries. The fixed requirement and the distribution function in country i are denoted by f_i and $G_i(m)$, respectively.

Exporting goods to foreign countries is subject to trade costs. Specifically, for a firm in country i to serve a unit of good to country j, it needs to produce $\tau_{ij} \geq 1$ units. We assume that $\tau_{ii} = 1$ holds for any i, indicating that firms do not need to incur domestic trade costs. Thus, the operating profit that a firm with marginal requirement m earns by selling from country i to country j is

$$\pi_{ij}(m) = L_j(p_{ij}(m) - \tau_{ij} m w_i) q_{ij}(m).$$

We denote by $\widehat{m}_{ij}$ the marginal cost cutoff for exporting from country i to country j. Since a marginal firm charges the same price as the choke price in a destination country, it holds that $p_{ij}(\widehat{m}_{ij}) = \tau_{ij}\widehat{m}_{ij}w_i = \widehat{p}_j$. Similarly, the marginal cost cutoff for serving a domestic market satisfies $p_{jj}(\widehat{m}_{jj}) = \widehat{m}_{jj}w_j = \widehat{p}_j$. Then, the exporting cutoff $\widehat{m}_{ij}$ can be expressed by using the destination country's domestic cutoff $\widehat{m}_{jj}$ as follows:

$$\widehat{m}_{ij} = \frac{\widehat{m}_{jj} w_j}{\tau_{ij} w_i}. \tag{29}$$

As a result of profit maximization, the equilibrium quantity of a firm with $m < \widehat{m}_{ij}$ is

$$q_{ij}(m) = \psi\left(\frac{m}{\widehat{m}_{ij}}\right),$$

where the definition of $\psi(\cdot)$ is the same as in the closed-economy model.

4.1 Market Equilibrium

The zero expected profit condition for a firm in country i is given by

$$\sum_j \int_0^{\widehat{m}_{ij}} \pi_{ij}(m) \mathrm{d}G_i(m) = f_i w_i. \tag{30}$$

The labor market clearing condition in country i is expressed as

$$N_i \left(\sum_j \tau_{ij} L_j \int_0^{\widehat{m}_{ij}} m q_{ij}(m) \mathrm{d}G_i(m) + f_i \right) = L_i. \tag{31}$$

In addition to these conditions, it is also required that trade balance holds for each country. That is, the total sales earned by firms in country i have to be equal to the total expenditure spent by workers in country i. Thus, the condition for trade balance can be written as follows:

$$N_i \sum_j L_j \int_0^{\widehat{m}_{ij}} p_{ij}(m) q_{ij}(m) \mathrm{d}G_i(m) = L_i \sum_j N_j \int_0^{\widehat{m}_{ji}} p_{ji}(m) q_{ji}(m) \mathrm{d}G_j(m). \tag{32}$$

Provided that exporting cutoffs can be expressed using destination countries' domestic cutoffs as in (29), the market equilibrium is characterized by equations (30), (31), and (32), which determine $3 \times J$ endogenous variables: the wages, $\{w_i\}_{i=1}^J$; the masses of entrants, $\{N_i\}_{i=1}^J$; and the domestic cutoffs, $\{\widehat{m}_{ii}\}_{i=1}^J$.[23]

[23] To be precise, we need to take a country's labor as the numéraire.

Note also that combining (30), (31), and (32) enables us to obtain the following relationship:

$$\sum_j N_j \int_0^{\widehat{m}_{ji}} p_{ji}(m) q_{ji}(m) \mathrm{d}G_j(m) = w_i, \tag{33}$$

where the left-hand side represents the individual expenditure of a worker in country i. Thus, we know that $E_i = w_i$ holds at the equilibrium.

Let us denote the elasticity of the distribution function of marginal requirements in country i by $\mathcal{E}_i^G(m) \equiv m g_i(m)/G_i(m)$, where $g_i(m) = G_i'(m)$. Then, it is convenient to express the equilibrium system of equations as in the following lemma.

Lemma 4. *At the market equilibrium in the multi-country framework, $\{w_i\}_{i=1}^J$, $\{\widehat{m}_{ii}\}_{i=1}^J$, and $\{N_i\}_{i=1}^J$ are determined, up to a choice of numéraire, in the following equations:*

$$\sum_j \widehat{m}_{jj} w_j L_j \int_0^1 \psi(t) G_i\left(\frac{\widehat{m}_{jj} w_j t}{\tau_{ij} w_i}\right) \mathrm{d}t = f_i w_i, \tag{34}$$

$$\widehat{m}_{ii} \sum_j N_j \int_0^1 \psi(t) G_j\left(\frac{\widehat{m}_{ii} w_i t}{\tau_{ji} w_j}\right)\left(1 + \mathcal{E}_j^G\left(\frac{\widehat{m}_{ii} w_i t}{\tau_{ji} w_j}\right)\right) \mathrm{d}t = 1, \tag{35}$$

$$N_i \sum_j \widehat{m}_{jj} w_j L_j \int_0^1 \psi(t) G_i\left(\frac{\widehat{m}_{jj} w_j t}{\tau_{ij} w_i}\right)\left(1 + \mathcal{E}_i^G\left(\frac{\widehat{m}_{jj} w_j t}{\tau_{ij} w_i}\right)\right) \mathrm{d}t = w_i L_i. \tag{36}$$

Moreover, each worker's utility in country i can be expressed as

$$U_i = \frac{\bar{u}}{\widehat{m}_{ii}} + \sum_j N_j \int_0^1 \frac{u(\psi(t))(1 - \mathcal{E}^u(\psi(t)))}{t} \mathcal{E}_j^G\left(\frac{\widehat{m}_{ii} w_i t}{\tau_{ji} w_j}\right) G_j\left(\frac{\widehat{m}_{ii} w_i t}{\tau_{ji} w_j}\right) \mathrm{d}t. \tag{37}$$

Proof. See Appendix A.4. □

Note that (34), (35), and (36) are obtained by manipulating (30), (33), and (32), respectively. Using these equations, we can analytically show that the following proposition holds true.

Proposition 3. *Opening up to trade from autarky lowers each country's domestic cost cutoff.*

Proof. See Appendix A.5. □

Proposition 3 indicates that opening up to trade from autarky unambiguously increases the toughness of competition in the domestic market, which can be regarded as the pro-competitive effect of trade.

It is also noteworthy that combining (34) and (36) enables us to express each country's mass of entrants in the following way:

$$N_i = \frac{\sum_j \widehat{m}_{jj} w_j L_j \int_0^1 \psi(t) G_i\left(\frac{\widehat{m}_{jj} w_j t}{\tau_{ij} w_i}\right) \mathrm{d}t}{\sum_j \widehat{m}_{jj} w_j L_j \int_0^1 \psi(t) G_i\left(\frac{\widehat{m}_{jj} w_j t}{\tau_{ij} w_i}\right)\left(1 + \mathcal{E}_i^G\left(\frac{\widehat{m}_{jj} w_j t}{\tau_{ij} w_i}\right)\right) \mathrm{d}t} \frac{L_i}{f_i}. \tag{38}$$

Unfortunately, it is difficult to derive a clear-cut result on how trade affects the mass of entrants in each country without specifying the distribution of marginal costs. Thus, in the subsequent analysis, we focus on the case with a Pareto-distributed productivity, enabling us to obtain further insights.

4.2 Pareto Distribution

In the case with Pareto-distributed productivity, $G_i(m)$ exhibits a constant elasticity: $\mathcal{E}_i^G(m) = k_i$ for $G_i(m) = (m/M_i)^{k_i}$. Then, expression (38) reduces to a quite simple formula:

$$N_i = \frac{1}{k_i + 1}\frac{L_i}{f_i}. \tag{39}$$

This expression solely depends on domestic exogenous parameters and is exactly the same as the one derived in the closed-economy model with Pareto-distributed productivity. Therefore, we obtain the following proposition.

Proposition 4. *If firms' productivity $(1/m)$ follows a Pareto distribution, trade does not affect the mass of entrants in any country.*

Intuitively, opening up to trade has two opposing effects on firm entry in each country. The positive effect is that market expansion into foreign countries boosts expected profits, leading to increased firm entry. Conversely, the negative effect is that intensified competition among firms diminishes expected profits, thereby reducing firm entry. Interestingly, even with a Pareto distribution that varies in the shape parameter across countries, these opposing effects are canceled out, resulting in no net impact on firm entry in each country.

In what follows, we consider the case in which each country has the same Pareto shape parameter, i.e., $G_i(m) = (m/M_i)^k$. Then, we can simplify the system of equations in Lemma 4 to (39) and the following equations:

$$\Psi \sum_j \widehat{m}_{jj}^{k+1} L_j \tau_{ij}^{-k} \left(\frac{w_j}{w_i}\right)^{k+1} = f_i M_i^k, \tag{40}$$

$$(k+1)\Psi \widehat{m}_{ii} \sum_j N_j G_j \left(\frac{\widehat{m}_{ii} w_i}{\tau_{ji} w_j}\right) = 1, \tag{41}$$

where (40) and (41) follow from (34) and (35), respectively. Note that $G_j\left(\frac{\widehat{m}_{ii} w_i t}{\tau_{ji} w_j}\right) = t^k G_j\left(\frac{\widehat{m}_{ii} w_i}{\tau_{ji} w_j}\right)$ holds under Pareto distribution and Ψ is defined in the same way as in (24).

Plugging (39) into (41) and rearranging it, we obtain

$$\widehat{m}_{ii} = \left[\Psi \sum_j \frac{L_j}{f_j M_j^k} \tau_{ji}^{-k} \left(\frac{w_i}{w_j}\right)^k\right]^{-\frac{1}{k+1}}. \tag{42}$$

Using (42) into (40) to eliminate the domestic cost cutoffs, we obtain

$$\sum_j \frac{L_j \tau_{ij}^{-k} \left(\frac{w_j}{w_i}\right)^{k+1}}{\sum_h \frac{L_h}{f_h M_h^k} \tau_{hj}^{-k} \left(\frac{w_j}{w_h}\right)^k} = f_i M_i^k. \tag{43}$$

Thus, equation (43) for $i = 1, 2, \ldots, J$ determines the wages, which are independent of the functional form of subutility, $u(\cdot)$.[24]

Next, we derive the equilibrium utility under Pareto-distributed productivity. Let n_i denote the mass of consumed varieties in country i: $n_i \equiv \sum_j N_j G_j(\widehat{m}_{ji})$. Using (29) and (41), this can be expressed in terms of the domestic cost cutoff as

$$n_i = \frac{1}{(k+1)\Psi \widehat{m}_{ii}}. \tag{44}$$

We rewrite (37) using (29) and the definition of n_i to obtain $U_i = \bar{u}/\widehat{m}_{ii} + k\zeta n_i$, where ζ is defined in the same way as in (27). Hence, using (44), the equilibrium utility can be expressed as a constant multiple of the inverse of the domestic cost cutoff or a constant multiple of the mass of consumed varieties:

$$U_i = \left[\bar{u} + \frac{k\zeta}{(k+1)\Psi}\right] \frac{1}{\widehat{m}_{ii}} = [(k+1)\Psi \bar{u} + k\zeta] n_i. \tag{45}$$

Finally, we investigate the impact of opening up to trade. The equilibrium under closed economy is obtained by letting $\tau_{ij} \to \infty$ for $j \neq i$. Then, from (42), the cost cutoff in the autarkic equilibrium, denoted by $\widehat{m}_i^a$, is expressed as

$$\widehat{m}_i^a = \left(\frac{f_i M_i^k}{\Psi L_i}\right)^{\frac{1}{k+1}}, \tag{46}$$

which is equivalent to the cost cutoff in (23). Combining (42) and (46), we can derive the following relationship between domestic cost cutoffs under autarky and open economy as follows:

$$\frac{\widehat{m}_{ii}}{\widehat{m}_i^a} = \left[1 + \sum_{j \neq i} \frac{L_j}{L_i} \frac{f_i M_i^k}{f_j M_j^k} \tau_{ji}^{-k} \left(\frac{w_i}{w_j}\right)^k\right]^{-\frac{1}{k+1}} < 1. \tag{47}$$

Therefore, as shown in Proposition 3, opening up to trade lowers the domestic cost cutoffs, reflecting an increased competition in each country. For (44) and (45), since the autarkic cost

[24]Notice that from (43), assuming no trade cost ($\tau_{ji} = 1$ for all i, j) and the same technological level across countries ($f_i = f$ and $M_i = M$ for all i) results in the factor price equalization ($w_i = w$ for all i). Then, from (42), the domestic (and exporting) cost cutoffs are identical in all countries ($\widehat{m}_{ii} = \widehat{m}$ for all i) and can be expressed as $\widehat{m} = \left(fM^k/\Psi \sum_j L_j\right)^{1/(k+1)}$. Substituting this into (45) reveals that integration or free trade among countries with identical levels of technology results in the same consequences as an increase in market size in a closed economy (see (23) and (26)), as explained in Section 3.3.

cutoff is expressed as in (46), the mass of consumed varieties and utility in autarkic equilibrium, denoted by n_i^a and U_i^a, are equivalent to (25) and (26), respectively. Hence, the following relationship holds:

$$\frac{U_i}{U_i^a} = \frac{n_i}{n_i^a} = \frac{\widehat{m}_i^a}{\widehat{m}_{ii}} > 1. \tag{48}$$

This expression shows that the relative change in welfare when moving from autarky to open economy can be simply measured by the relative change in either the cost cutoff or the mass of consumed varieties. As (47) shows, opening up to trade from autarky unambiguously lowers the domestic cost cutoff in all countries. Therefore, expression (48) demonstrates that in all countries, welfare is higher and the mass of consumed varieties is larger in the open economy than in autarky. These results are summarized in the following proposition.

Proposition 5. *Suppose that firms' productivity* $(1/m)$ *in each country follows a Pareto distribution with a common shape parameter. Then, opening up to trade from autarky enhances each country's welfare through consumption variety expansion and selection effects.*

In all countries, welfare gains from trade come from consumption variety expansion and selection effects. Intuitively, opening up to trade expands the mass of varieties available to consumers through imports (the consumption variety expansion effects). Moreover, this induces the pro-competitive effects of trade due to the rise in rival firms, which is reflected in lower domestic cost cutoffs (the selection effects). These factors contribute to raising the welfare of each country.

However, as shown in Section 3.3, the pro-competitive effects in our framework do not lead to efficiency gains. This result implies that the pro-competitive effects of trade merely offset the additional domestic distortion that would arise if domestic firms' prices and markups remained unchanged after opening up to trade. As a result, the pro-competitive effects of trade may have a limited impact on welfare within our heterogeneous firm framework, as indicated by Arkolakis et al. (2019).

5 Gains from Trade: A Comparative Analysis

In the previous section, we have confirmed that all countries gain from trade. However, the extent of these benefits varies across countries depending on factors such as market size, technology, and geography. Hence, based on a two-country model, we clarify how these factors affect the magnitude of gains from trade in each country.

Consider two countries labeled H (home) and F (foreign). We first characterize the relative welfare in autarky. From (45) and (46), it can be expressed as

$$\frac{U_H^a}{U_F^a} = \frac{\widehat{m}_{FF}^a}{\widehat{m}_{HH}^a} = \left(\frac{\Theta_L}{\Theta_c}\right)^{\frac{1}{k+1}},$$

where $\Theta_L \equiv L_H/L_F$ and $\Theta_c \equiv f_H M_H^k / f_F M_F^k$ represent the relative market size and the relative cost index in home. Note that the lower Θ_c is, the higher the technological level in the home country in relative terms. Then, we have

$$\frac{U_H^a}{U_F^a} \gtreqless 1 \Leftrightarrow \Theta_L \gtreqless \Theta_c,$$

where the $U_H^a/U_F^a = 1$ line can be depicted as the 45-degree line on the (Θ_c, Θ_L) plane in Figure 1. In the figure, the home country has relatively high (low) welfare in autarky if pair (Θ_c, Θ_L) is above (below) the 45-degree line. In other words, both a larger (smaller) market size and higher (lower) technological level are factors that contribute to relatively high (low) welfare in that country.[25] For convenience, we refer to a country with higher welfare in a closed economy as an advantaged country and one with lower welfare as a disadvantaged country.[26]

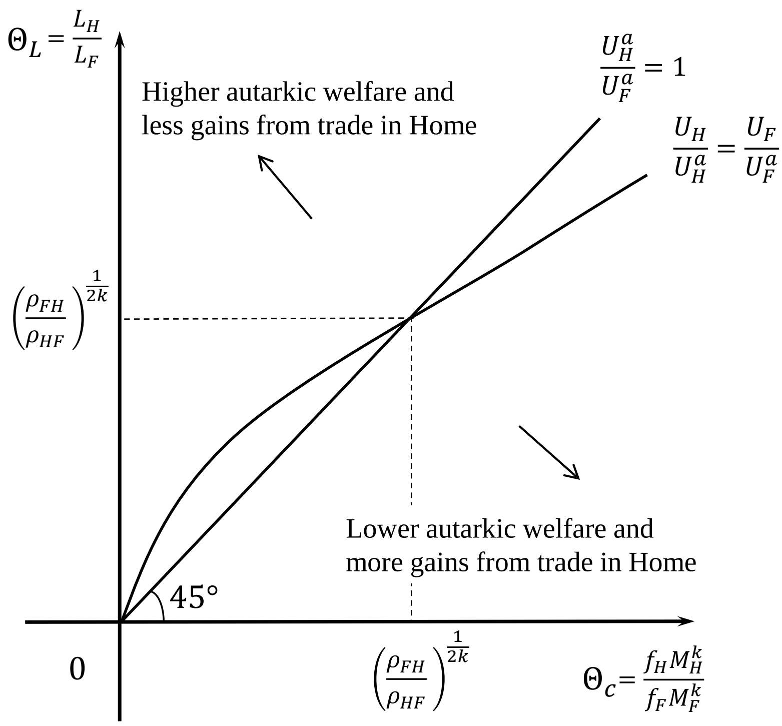

Figure 1: Relative welfare in autarky and relative welfare gains from trade

Next, we compare the gains from trade between the two countries. Let $\omega \equiv w_H/w_F$ denote the relative wage in home. As shown in Appendix B.1, from (43), the equilibrium relative wage, denoted by ω^*, is uniquely determined by

$$\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} + (\omega^*)^k - \Theta_L(\omega^*)^{k+1} - \Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1} = 0, \tag{49}$$

where $\rho_{ij} \equiv \tau_{ij}^{-k} \in (0, 1]$ represents the freeness of trade for exports from country i to country

[25] Even in absolute terms, the autarkic welfare level is higher (lower) in a country with a larger (smaller) market size and higher (lower) technological level.

[26] In other words, a country with a relatively large value of $L_i/f_i M_i^k$ is considered an advantaged country.

j. Then, using (47) and (48), gains from trade in each country can be expressed as

$$\frac{U_H}{U_H^a} = \left[1 + \frac{\Theta_c}{\Theta_L}\rho_{FH}(\omega^*)^k\right]^{\frac{1}{k+1}}, \tag{50}$$

$$\frac{U_F}{U_F^a} = \left[1 + \frac{\Theta_L}{\Theta_c}\rho_{HF}(\omega^*)^{-k}\right]^{\frac{1}{k+1}}. \tag{51}$$

Consequently, from (49), (50) and (51), gains from trade in each country are ranked as follows (see Appendix B.1):

$$\frac{U_H}{U_H^a} \gtreqless \frac{U_F}{U_F^a} \Leftrightarrow \Theta_L \lesseqgtr \left(\frac{\rho_{FH}}{\rho_{HF}}\right)^{\frac{1}{2(k+1)}} \Theta_c^{\frac{1}{k+1}}, \tag{52}$$

where the right-hand side is a concave and increasing function with respect to Θ_c. Therefore, the $U_H/U_H^a = U_F/U_F^a$ curve can be depicted as shown in Figure 1. In the figure, the home country gains more (less) from trade than the foreign country when the pair (Θ_c, Θ_L) is below (above) the $U_H/U_H^a = U_F/U_F^a$ curve. Therefore, both a smaller (larger) market size and lower (higher) technological level are factors that contribute to relatively greater (smaller) welfare gains from trade in that country.[27] This indicates that a disadvantaged country tends to gain more from trade than an advantaged country.

However, being a disadvantaged country is not a necessary or sufficient condition for the country to benefit more from trade. Specifically, Figure 1 shows that in areas below the $U_H^a/U_F^a = 1$ line and above the $U_H/U_H^a = U_F/U_F^a$ curve, the home country is a disadvantaged country ($U_H^a < U_F^a$) but gains less from trade than the foreign country ($U_H/U_H^a < U_F/U_F^a$). Conversely, in areas above the $U_H^a/U_F^a = 1$ line and below the $U_H/U_H^a = U_F/U_F^a$ curve, the home country is an advantaged country but experiences greater gains from trade. These continue to hold even if trade costs are symmetric (i.e., $\rho_{FH} = \rho_{HF}$).[28] As explained below, these results highlight the differing contributions that a smaller market size and lower technological level make to a country's welfare gains from trade.

If pair (Θ_c, Θ_L) lies on the $U_H^a/U_F^a = 1$ line in Figure 1, the autarkic welfare levels in both countries are identical. On the upper right of this line, the home country has a larger market size and lower technological level, whereas the foreign country has a smaller market size and higher technological level.[29] In this situation, the lower technological level in the home country and the smaller market size in the foreign country each become factors that contribute to more welfare gains from trade in the country. Consequently, starting from the same autarkic welfare in

[27] Even in absolute terms, welfare gains from trade is greater (smaller) in a country with a smaller (larger) market size and lower (higher) technological level (see Appendix B.1).

[28] If trade costs are symmetric, the intersection of the $U_H^a/U_F^a = 1$ line and the $U_H/U_H^a = U_F/U_F^a$ curve becomes $(\Theta_c, \Theta_L) = (1, 1)$, which represents symmetric two countries in terms of market size and technology. Thus, if the countries are symmetric in terms of market size, technology, and geography, then welfare in autarky and gains from trade are identical across countries.

[29] Note that we refer to a larger/smaller market size and higher/lower technological level with reference to the intersection point $(\Theta_c, \Theta_L) = ((\rho_{FH}/\rho_{HF})^{1/2k}, (\rho_{FH}/\rho_{HF})^{1/2k})$. Particularly, if trade costs are symmetric, $L_H > L_F$ and $f_H M_H^k > f_F M_F^k$ ($L_H < L_F$ and $f_H M_H^k < f_F M_F^k$) hold on the upper right (lower left) of the intersection point $(\Theta_c, \Theta_L) = (1, 1)$.

both countries, the foreign country experiences greater gains from trade than the home country, implying that a smaller market size contributes more to a country's welfare gains from trade than a lower technological level. Similarly, on the lower left of the $U_H^a/U_F^a = 1$ line, the home country has a smaller market size and the foreign country has a lower technological level. Then, starting from the same autarkic welfare levels, the home country experiences greater gains from trade than the foreign country.

Hence, a smaller market size contributes more to a country's welfare gains from trade than a lower technological level. This is a factor that generates the areas bounded between the $U_H^a/U_F^a = 1$ line and the $U_H/U_H^a = U_F/U_F^a$ curve in Figure 1: a disadvantaged country with a lower technological level but larger market size results in smaller welfare gains from trade, or, conversely, an advantaged country with a higher technological level but smaller market size can achieve greater welfare gains from trade.

In addition to these differences in market size and technological level, geographic differences also affect the magnitude of welfare gains from trade. Specifically, the right-hand side of (52) increases with the relative level of freeness of trade, ρ_{FH}/ρ_{HF}. Therefore, as ρ_{FH}/ρ_{HF} increases, the $U_H/U_F = U_H^a/U_F^a$ curve in Figure 1 shifts upward, implying that the region below the $U_H/U_H^a = U_F/U_F^a$ curve (i.e., the region where the home country can gain more from trade than the foreign country) expands. Therefore, a country with easier market access (i.e., a more liberalized country) can gain more from trade.

In summary, the gains from trade are relatively large in a country with a smaller market size, lower technological level, and easier access to that market. This is because these factors contribute to an increase in the mass of new varieties available to consumers through imports and a stronger firm selection due to exports from a more technologically advanced partner country, both of which enhance a country's welfare as shown in Proposition 5. In addition, a smaller market size contributes more to a country's welfare gains from trade than a lower technological level. These differences in the magnitude of gains from trade can lead to various scenarios in which opening up to trade impacts relative welfare level, as shown below.

5.1 Impact of Trade on Relative Welfare

We first characterize the relative welfare after opening up to trade. From (42) and (45), it can be expressed as

$$\frac{U_H}{U_F} = \frac{\widehat{m}_{FF}}{\widehat{m}_{HH}} = \left[\frac{\Theta_L}{\Theta_c} \frac{1 + \frac{\Theta_c}{\Theta_L} \rho_{FH} (\omega^*)^k}{1 + \frac{\Theta_L}{\Theta_c} \rho_{HF} (\omega^*)^{-k}} \right]^{\frac{1}{k+1}}, \tag{53}$$

where ω^* is determined by (49). Then, using this expression and (49), we obtain

$$\frac{U_H}{U_F} \gtreqless 1 \Leftrightarrow \Theta_L \rho_{FH}^{\frac{k}{2k+1}} \rho_{HF}^{\frac{k+1}{2k+1}} \left(\rho_{FH}^{-\frac{k}{2k+1}} \rho_{HF}^{-\frac{k+1}{2k+1}} - \Theta_c^{\frac{k}{2k+1}} \right) + \Theta_c^{\frac{k+1}{2k+1}} \left(\rho_{FH}^{\frac{k+1}{2k+1}} \rho_{HF}^{\frac{k}{2k+1}} - \Theta_c^{\frac{k}{2k+1}} \right) \gtreqless 0, \tag{54}$$

where the $U_H/U_F = 1$ curve can be added to Figure 1 and illustrated as in Figure 2 (see Appendix B.1). Figure 2 shows that the three curves/lines intersect at the point $(\Theta_c, \Theta_L) =$

$((\rho_{FH}/\rho_{HF})^{1/2k}, (\rho_{FH}/\rho_{HF})^{1/2k})$, dividing the region into six parts: D_1, D_2, D_3 ,D'_1, D'_2, and D'_3.[30] The home country has relatively high (resp. low) welfare after opening up to trade if pair (Θ_c, Θ_L) lies to the left (resp. right) of the $U_H/U_F = 1$ curve, that is, if $(\Theta_c, \Theta_L) \in D_1$, D_2, or D'_3 (resp. if $(\Theta_c, \Theta_L) \in D_3$, D'_1, or D'_2). Recall that the home country has relatively high (resp. low) welfare in autarky if pair (Θ_c, Θ_L) is above (resp. below) the $U^a_H/U^a_F = 1$ line, and that inequality $U_H/U_F > U^a_H/U^a_F$ (resp. $U_H/U_F < U^a_H/U^a_F$) holds if the pair (Θ_c, Θ_L) is below (resp. above) the $U_H/U_F = U^a_H/U^a_F$ curve.[31] Consequently, from Figure 2, we obtain the following proposition.

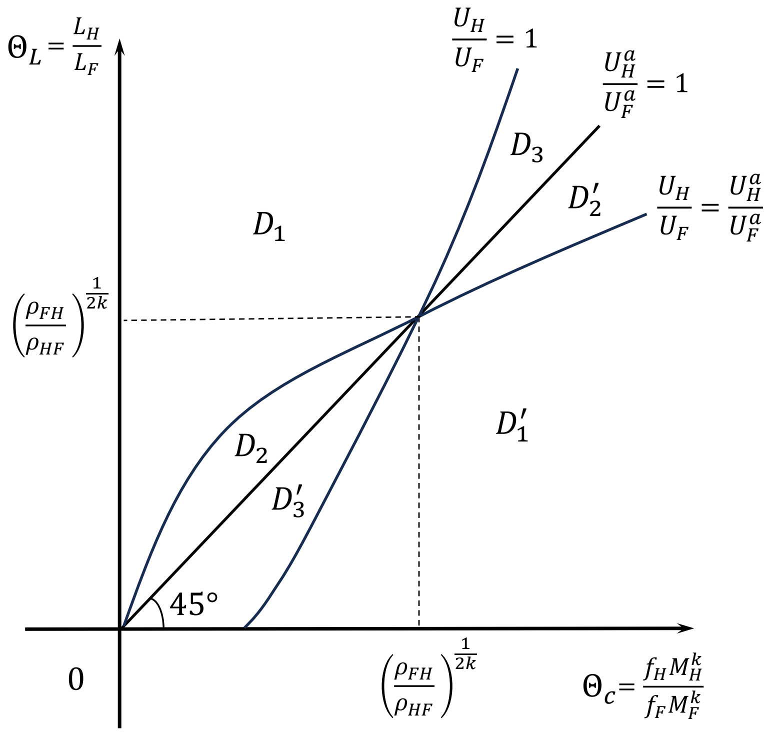

Figure 2: The impact of opening up to trade on relative welfare

Proposition 6. *Consider a two-country economy where firms' productivity $(1/m)$ in each country follows a Pareto distribution with a common shape parameter. Then, the welfare impact of opening up to trade can be divided into the following scenarios:*

(i) If $(\Theta_c, \Theta_L) \in D_1$ (resp. $(\Theta_c, \Theta_L) \in D'_1$), then $1 < U_H/U_F < U^a_H/U^a_F$ (resp. $U^a_H/U^a_F < U_H/U_F < 1$) holds: opening up to trade fills the welfare gap.

(ii) If $(\Theta_c, \Theta_L) \in D_2$ (resp. $(\Theta_c, \Theta_L) \in D'_2$), then $1 < U^a_H/U^a_F < U_H/U_F$ (resp. $U_H/U_F < U^a_H/U^a_F < 1$) holds: opening up to trade widens the welfare gap.

[30] At the intersection point, the countries have the same welfare level in both actarky and open economy, with identical gains from trade. If trade costs are symmetric, the intersection becomes $(\Theta_c, \Theta_L) = (1, 1)$, representing the symmetric two-country case in terms of market size, technology, and geography.

[31] It is straightforward to show that $U_H/U^a_H \gtreqless U_F/U^a_F \Leftrightarrow U_H/U_F \gtreqless U^a_H/U^a_F$. Therefore, the $U_H/U^a_H = U_F/U^a_F$ curve in Figure 1 is equivalent to the $U_H/U_F = U^a_H/U^a_F$ curve in Figure 2.

(iii) If $(\Theta_c, \Theta_L) \in D_3$ (resp. $(\Theta_c, \Theta_L) \in D'_3$), then $U_H/U_F < 1 < U_H^a/U_F^a$ (resp. $U_H^a/U_F^a < 1 < U_H/U_F$) holds: opening up to trade reverses the welfare relationship between the two countries.

Proposition 6 continues to hold even if trade costs are symmetric, $\rho_{FH} = \rho_{HF}$. Therefore, the varying impacts of opening up to trade on the relative welfare level are primarily due to differences in market size and technology across countries.

In Figure 2, region D_1 represents the situation where the home and foreign countries are typical advantaged and disadvantaged countries, respectively: the home country has a larger market size and higher technological level, whereas the foreign country has a smaller market size and lower technological level. Therefore, as the disadvantaged foreign country gains more from trade than the advantaged home country, opening up to trade reduces the welfare gap between the countries. Note that region D'_1 represents the situation where the relationship between the home country and the foreign country is replaced.

In region D_2, the home country is an advantaged country with a higher technological level but smaller market size, whereas the foreign country is a disadvantaged country with a lower technological level but larger market size. As explained above, a smaller market size contributes more to a country's welfare gains from trade than a lower technological level. As a result, the advantaged home country gains more from trade than the disadvantaged foreign country, thus opening up to trade increases the welfare gap between the countries.[32] In region D'_2, the relationship between the home country and the foreign country is replaced.

Finally, in region D_3, the home country is an advantaged country with a larger market size but lower technological level, whereas the foreign country is a disadvantaged country with a smaller market size but higher technological level. Since a smaller market size contributes more to a country's welfare gains from trade than a lower technological level, the disadvantaged foreign country gains more from trade than the advantaged home country, similar to the situation where pair (Θ_c, Θ_L) is located in region D_1. However, unlike the situation in region D_1, the autarkic welfare gap between the countries is not sufficiently large because the advantaged home country has a lower technological level and the disadvantaged foreign country has a higher technological level. Consequently, opening up to trade leads to higher welfare in the disadvantaged foreign country than in the advantaged home country. In region D'_3, the relationship between the home country and the foreign country is replaced.

5.2 Trade Liberalization

Next, we assess the impact of trade liberalization by analyzing how changes in $\rho_{ij} (= \tau_{ij}^{-k})$ affect welfare in each country. As shown in Appendix B.2, the rate of change in the welfare levels

[32] Note that, in the lower left areas of D_1, although the advantaged home country has a smaller market size, as in D_2, the welfare gains are larger in the disadvantaged foreign country with a lower technological level. This result stems from the fact that the gap in technology is sufficiently large relative to the gap in market size. A larger gap in technological level (or market size) leads to greater gains from trade for a country with a lower technological level (or smaller market size).

brought about by changes in trade costs can be expressed as

$$\mathrm{d}\ln U_H = A_H \left\{ \left[\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} + (\omega^*)^k + k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1} \right] \mathrm{d}\ln\rho_{FH} + k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}\mathrm{d}\ln\rho_{HF} \right\}, \tag{55}$$

$$\mathrm{d}\ln U_F = A_F \left\{ k\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1}\mathrm{d}\ln\rho_{FH} + \left[(k+1)\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} + (\omega^*)^k \right] \mathrm{d}\ln\rho_{HF} \right\}, \tag{56}$$

where

$$A_H \equiv \frac{\frac{\Theta_c}{\Theta_L}\rho_{FH}(\omega^*)^k}{(k+1)\left[1+\frac{\Theta_c}{\Theta_L}\rho_{FH}(\omega^*)^k\right]\left[(k+1)\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} + (\omega^*)^k + k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}\right]} > 0,$$

$$A_F \equiv \frac{\frac{\Theta_L}{\Theta_c}\rho_{HF}(\omega^*)^{-k}}{(k+1)\left[1+\frac{\Theta_L}{\Theta_c}\rho_{HF}(\omega^*)^{-k}\right]\left[(k+1)\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} + (\omega^*)^k + k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}\right]} > 0.$$

From (55) and (56), we can immediately see that any liberalization scenario ($\mathrm{d}\ln\rho_{FH} > 0$ and $\mathrm{d}\ln\rho_{HF} = 0$; $\mathrm{d}\ln\rho_{FH} = 0$ and $\mathrm{d}\ln\rho_{HF} > 0$; or $\mathrm{d}\ln\rho_{FH} > 0$ and $\mathrm{d}\ln\rho_{HF} > 0$) can raise welfare in both countries. Moreover, from (55), the impacts of unilateral trade liberalization by each country on welfare in the home country can be unambiguously ranked as follows:

$$\left.\frac{\mathrm{d}\ln U_H}{\mathrm{d}\ln\rho_{FH}}\right|_{\mathrm{d}\ln\rho_{HF}=0} - \left.\frac{\mathrm{d}\ln U_H}{\mathrm{d}\ln\rho_{HF}}\right|_{\mathrm{d}\ln\rho_{FH}=0} = A_H\left[\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} + (\omega^*)^k\right] > 0.$$

This shows that, for the same proportional reduction in trade costs, unilateral liberalization in the home country (an increase in ρ_{FH}) contributes more to welfare gains in the home country than liberalization in the foreign country (an increase in ρ_{HF}). The same can be true for the foreign country from (56). Hence, we obtain the following proposition.

Proposition 7. *Consider a two-country economy where firms' productivity ($1/m$) in each country follows a Pareto distribution with a common shape parameter. Then, any liberalization scenario raises welfare in both countries. Moreover, a country's own unilateral liberalization brings greater welfare gains to that country than the unilateral liberalization of its partner country.*

Having established that trade liberalization unambiguously raises both countries' welfare, the next step is to examine the impact of trade liberalization on the relative welfare level. We consider a scenario in which both countries simultaneously lower their trade costs by the same proportion, that is, we assume $\mathrm{d}\ln\rho_{FH} = \mathrm{d}\ln\rho_{HF} \equiv \mathrm{d}\ln\rho > 0$. Note that this setting allows for asymmetric trade costs.

From (49), (55), and (56), the difference in the rate of increase in welfare levels brought about by the bilateral trade liberalization can be expressed as follows (see Appendix B.2):

$$\frac{\mathrm{d}\ln U_H}{\mathrm{d}\ln\rho} \gtreqless \frac{\mathrm{d}\ln U_F}{\mathrm{d}\ln\rho} \Leftrightarrow \Theta_L \lesseqgtr \left(\frac{\rho_{FH}}{\rho_{HF}}\right)^{\frac{1}{2(k+1)}} \Theta_c^{\frac{1}{k+1}}. \tag{57}$$

For convenience, let U_i^{pre} and U_i^{post} represent the welfare levels in country i before and after liberalization, respectively. Then, we can rewrite (57) as

$$\frac{U_H^{\text{post}}}{U_H^{\text{pre}}} \gtreqless \frac{U_F^{\text{post}}}{U_F^{\text{pre}}} \Leftrightarrow \Theta_L \lesseqgtr \left(\frac{\rho_{FH}}{\rho_{HF}}\right)^{\frac{1}{2(k+1)}} \Theta_c^{\frac{1}{k+1}}. \tag{58}$$

This condition is equivalent to $U_H/U_H^a \gtreqless U_F/U_F^a$ in (52). Therefore, the $U_H^{\text{post}}/U_H^{\text{pre}} = U_F^{\text{post}}/U_F^{\text{pre}}$ curve in Figure 3 is the same as the $U_H/U_H^a = U_F/U_F^a$ curve shown in Figure 1 (or, equivalently, the $U_H/U_F = U_H^a/U_F^a$ curve shown in Figure 2).[33] In Figure 3, the home country gains more (less) from bilateral trade liberalization than the foreign country when the pair (Θ_c, Θ_L) is below (above) the curve. Note that the relative level of freeness of trade ρ_{FH}/ρ_{HF} in the right-hand side of (58) is not affected under our liberalization scenario ($\mathrm{d}\ln(\rho_{FH}/\rho_{HF})/\mathrm{d}\ln\rho = 0$). This implies that the country gaining more from opening up to trade can also gain more from the bilateral liberalization.

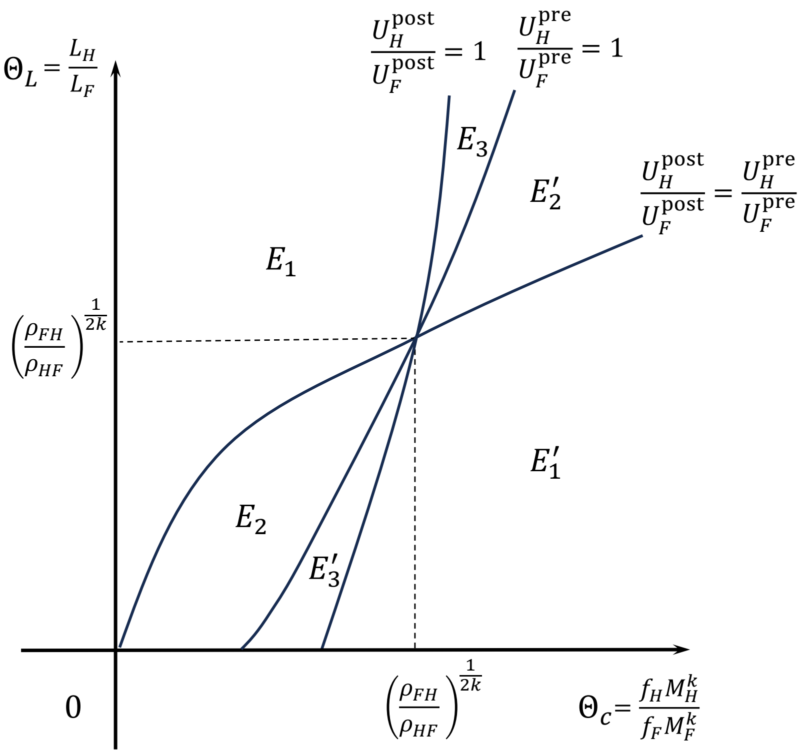

Figure 3: The impact of bilateral trade liberalization on relative welfare

The welfare relationship between the two countries before liberalization is given by (54). Thus, the $U_H^{\text{pre}}/U_F^{\text{pre}} = 1$ curve can be depicted as shown in Figure 3, which is equivalent to the $U_H/U_F = 1$ curve in Figure 2. Thus, the home country has relatively high (low) welfare before bilateral liberalization if pair (Θ_c, Θ_L) lies to the left (right) of the $U_H^{\text{pre}}/U_F^{\text{pre}} = 1$ curve.

[33] As $U_H^{\text{post}}/U_H^{\text{pre}} \gtreqless U_F^{\text{post}}/U_F^{\text{pre}} \Leftrightarrow U_H^{\text{post}}/U_F^{\text{post}} \gtreqless U_H^{\text{pre}}/U_F^{\text{pre}}$, the $U_H^{\text{post}}/U_H^{\text{pre}} = U_F^{\text{post}}/U_F^{\text{pre}}$ curve is equivalent to the $U_H^{\text{post}}/U_F^{\text{post}} = U_H^{\text{pre}}/U_F^{\text{pre}}$ curve in Figure 3.

After liberalization, the curve representing equal welfare levels in both countries shifts to the right below the point $(\Theta_c, \Theta_L) = ((\rho_{FH}/\rho_{HF})^{1/2k}, (\rho_{FH}/\rho_{HF})^{1/2k})$ and to the left above this point, as shown in Figure 3 (see Appendix B.2). The home country has relatively high (low) welfare after bilateral liberalization if pair (Θ_c, Θ_L) lies to the left (right) of the $U_H^{\text{post}}/U_F^{\text{post}} = 1$ curve.

Hence, as expression (58) shows that inequality $U_H^{\text{post}}/U_F^{\text{post}} > U_H^{\text{pre}}/U_F^{\text{pre}}$ ($U_H^{\text{post}}/U_F^{\text{post}} < U_H^{\text{pre}}/U_F^{\text{pre}}$) holds if the pair (Θ_c, Θ_L) is below (above) the $U_H^{\text{post}}/U_F^{\text{post}} = U_H^{\text{pre}}/U_F^{\text{pre}}$ curve, we obtain the following proposition from Figure 3.

Proposition 8. *Consider a two-country economy where firms' productivity ($1/m$) in each country follows a Pareto distribution with a common shape parameter. If both countries simultaneously lower their trade costs by the same proportion ($\mathrm{d}\ln\rho_{FH} = \mathrm{d}\ln\rho_{HF} = \mathrm{d}\ln\rho > 0$), the welfare impact of the bilateral trade liberalization can be divided into the following scenarios:*

(i) If $(\Theta_c, \Theta_L) \in E_1$ (resp. $(\Theta_c, \Theta_L) \in E_1'$), then $1 < U_H^{\text{post}}/U_F^{\text{post}} < U_H^{\text{pre}}/U_F^{\text{pre}}$ (resp. $U_H^{\text{pre}}/U_F^{\text{pre}} < U_H^{\text{post}}/U_F^{\text{post}} < 1$) holds: bilateral liberalization fills the welfare gap.

(ii) If $(\Theta_c, \Theta_L) \in E_2$ (resp. $(\Theta_c, \Theta_L) \in E_2'$), then $1 < U_H^{\text{pre}}/U_F^{\text{pre}} < U_H^{\text{post}}/U_F^{\text{post}}$ (resp. $U_H^{\text{post}}/U_F^{\text{post}} < U_H^{\text{pre}}/U_F^{\text{pre}} < 1$) holds: bilateral liberalization widens the welfare gap.

(iii) If $(\Theta_c, \Theta_L) \in E_3$ (resp. $(\Theta_c, \Theta_L) \in E_3'$), then $U_H^{\text{post}}/U_F^{\text{post}} < 1 < U_H^{\text{pre}}/U_F^{\text{pre}}$ (resp. $U_H^{\text{pre}}/U_F^{\text{pre}} < 1 < U_H^{\text{post}}/U_F^{\text{post}}$) holds: bilateral liberalization reverses the welfare relationship between the two countries.

Proposition 8 continues to hold even if trade costs are symmetric, implying that the various scenarios of bilateral trade liberalization arise primarily from differences in market size and technology between countries. Specifically, similar to the gains from opening up to trade, the gains from bilateral trade liberalization are greater in a country with a smaller market size and lower technological level, where the smaller market size contributes more than the lower technological level.[34] Therefore, the results of Proposition 8 can be interpreted in the same way as those of Proposition 6, which describes the various impacts of opening up to trade on relative welfare.

Combining the results of Propositions 6 and 8, the following can be summarized. Regardless of whether the economy is closed or open, differences in market size and technological level between countries each generate the difference in welfare level, with a country that has a relatively large market size and high technological level generally having relatively high welfare. International trade and bilateral trade liberalization not only enhance welfare of both countries

[34] As can be seen from the $U_H^{\text{post}}/U_F^{\text{post}} = U_H^{\text{pre}}/U_F^{\text{pre}}$ curve in Figure 3, both a smaller market size and lower technological level are factors that contribute to relatively greater welfare gains from bilateral liberalization in that country. If pair (Θ_c, Θ_L) lies on the $U_H^{\text{pre}}/U_F^{\text{pre}} = 1$ curve in Figure 3, the welfare levels in both countries are identical before liberalization. Then, starting from the same welfare in both countries, the bilateral liberalization leads to $U_H^{\text{post}} > U_F^{\text{post}}$ (resp. $U_H^{\text{post}} < U_F^{\text{post}}$) on the lower left (resp. upper right) of the $U_H^{\text{pre}}/U_F^{\text{pre}} = 1$ curve, where the home country has a smaller (resp. larger) market size and higher (resp. lower) technological level. In this sense, a smaller market size contributes more to a country's welfare gains from bilateral trade liberalization than a lower technological level.

but also offer greater gains to a country with a smaller market sizes and lower technological level. In other words, they help reduce the welfare difference stemming from both differences in market size and technological level. However, the welfare difference stemming from the difference in market size tends to be reduced more than the welfare difference stemming from the difference in technological level. As a result, trade and bilateral liberalization do not necessarily lead to a monotonic reduction in the welfare gap between the two countries. For example, in a situation where one country has a larger market size and lower technological level and the other country has a smaller market size and higher technological level, the difference in technological levels is a factor that puts the former country at a disadvantage, while the difference in market size is a factor that puts the latter country at a disadvantage. In such a case, trade and bilateral liberalization tends to provide greater welfare gains to the latter country. If this country already has relatively high welfare than the other country, opening up to trade or bilateral trade liberalization could widen the welfare gap between the two countries. Conversely, if this country has relatively low welfare before trade or liberalization, the welfare gap between the two countries would narrow, or the welfare relationship could even be reversed. In the event of a reversal, further liberalization widens the welfare gap. In this way, at the final stage where liberalization has fully advanced, the welfare difference stemming from the difference in technological level, which cannot be entirely eliminated by trade or liberalization, determines which country has relatively high welfare. Specifically, under free trade, welfare is relatively high in a country with a relatively high technological level.[35]

6 Conclusion

This study provides general properties of the gains from trade in monopolistic competition models featuring firm heterogeneity characterized as Pareto-distributed productivity and variable markups associated with pro-competitive effects. Our framework encompasses trade models widely used in the trade literature that incorporate firm heterogeneity and variable markups, and successfully derives a number of properties without specifying utility functions.

The main findings of this study are as follows. First, we identify the markup distortions in the domestic market and the impact of trade (integration) on them through the pro-competitive effects, highlighting the differences from those predicted in symmetric firm frameworks. In particular, we find that the markup distortions unique to heterogeneous firm frameworks are not ameliorated by the pro-competitive effects. This result implies that the pro-competitive effects of trade merely offset the additional domestic distortion that would arise if domestic firms' prices and markups remained unchanged after opening up to trade. Our finding is likely to explain the findings of Arkolakis et al. (2019), who conclude that the pro-competitive gains from trade are "elusive."

Second, we show that the trade equilibrium can be characterized by quite simple formula,

[35] Under free trade ($\rho_{FH} = \rho_{HF} = 1$), expression (54) becomes $U_H/U_F \gtreqless 1 \Leftrightarrow \Theta_c \lesseqgtr 1$, implying that a country with a relatively high technological level has relatively high welfare.

irrespective of the functional form of utility function. This allows, for instance, a straightforward description of a country's gains from trade as an increase in the number of consumed varieties (or a decrease in the domestic cost cutoff) when moving from autarky to an open economy. The tractability of our model will contribute to the literature on quantitative and policy studies within the framework of firm heterogeneity and variable markups, which has gained increasing attention in recent years.

Lastly, we illustrate how the magnitude of gains from trade and trade liberalization differs across countries with differences in market size, technology, and geography. In a two-country economy, we find that gains from trade and trade liberalization are greater in a country with a smaller market size, lower technological level, and easier access to that market. Additionally, a smaller market size contributes more to a country's welfare gains than a lower technological level, implying that trade and trade liberalization tend to reduce the welfare difference between countries stemming from the difference in market size more than the welfare difference stemming from the difference in technological level. Consequently, these differences in the magnitude of welfare gains across countries can lead to various scenarios in which opening up to trade and trade liberalization impact relative welfare level. Specifically, opening up to trade and trade liberalization can result in: (i) narrowing the welfare gap, (ii) widening the welfare gap, or (iii) reversing the welfare relationship.

Throughout this study, we assume an unbounded Pareto distribution of productivity and no fixed costs of production. While these are standard assumptions in both quantitative and theoretical trade literature analyzing monopolistic competition models with firm heterogeneity and variable markups, assuming a bounded Pareto distribution or fixed production costs could alter the nature of markup distortions and the impact of trade on them. Our findings will serve as a valuable benchmark for understanding how and to what extent these factors affect the results within this framework.

References

Arkolakis, Costas, Arnaud Costinot, Dave Donaldson, and Andrés Rodríguez-Clare, "The Elusive Pro-Competitive Effects of Trade," *Review of Economic Studies*, 2019, *86* (1), 46–80.

Axtell, Robert L., "Zipf Distribution of U.S. Firm Sizes," *Sciece*, 2001, *293* (5536), 1818–1820.

Bagwell, Kyle and Seung Hoon Lee, "Trade Policy under Monopolistic Competition with Firm Selection," *Journal of International Economics*, 2020, *127*, 103379.

_ **and** _ , "Monopolistic Competition and Efficiency under Firm Heterogeneity and Nonadditive Preferences," *American Economic Journal: Microeconomics*, 2023, *15* (4), 208–267.

Behrens, Kristian and Yasusada Murata, "General Equilibrium Models of Monopolistic Competition: A New Approach," *Journal of Economic Theory*, 2007, *136* (1), 776–787.

— **and** — , "Trade, Competition, and Efficiency," *Journal of International Economics*, 2012, *87* (1), 1–17.

— **, Giordano Mion, Yasusada Murata, and Jens Südekum**, "Trade, Wages, and Productivity," *International Economic Review*, 2014, *55* (4), 1305–1348.

— **,** — **,** — **, and Jens Suedekum**, "Quantifying the Gap between Equilibrium and Optimum under Monopolistic Competition," *Quarterly Journal of Economics*, 2020, *135* (4), 2299–2360.

— **, Sergei Kichko, and Philip Ushchev**, "ε-CES Preferences and Trade," *Review of International Economics*, 2024, *32* (4), 1567–1586.

Bernard, Andrew B., J. Bradford Jensen, Stephen J. Redding, and Peter K. Schott, "Firms in International Trade," *Journal of Economic Perspectives*, 2007, *21* (3), 105–130.

Bertoletti, Paolo and Federico Etro, "Monopolistic Competition When Income Matters," *Economic Journal*, 2017, *127* (603), 1217–1243.

— **and** — , "Monopolistic Competition with Generalized Additively Separable Preferences," *Oxford Economic Papers*, 2021, *73* (2), 927–952.

— **and** — , "Monopolistic Competition, as You Like It," *Economic Inquiry*, 2022, *60* (1), 293–319.

— **and Paolo Epifani**, "Monopolistic Competition: CES Redux?," *Journal of International Economics*, 2014, *93* (2), 227–238.

— **, Federico Etro, and Ina Simonovska**, "International Trade with Indirect Additivity," *American Economic Journal: Microeconomics*, 2018, *10* (2), 1–57.

Bykadorov, Igor, Alexey Gorn, Sergey Kokovin, and Evgeny Zhelobodko, "Why Are Losses from Trade Unlikely?," *Economics Letters*, 2015, *129*, 35–38.

Corcos, Gregory, Massimo Del Gatto, Giordano Mion, and Gianmarco I.P. Ottaviano, "Productivity and Firm Selection: Quantifying the 'New' Gains from Trade," *Economic Journal*, 2012, *122* (561), 754–798.

De Loecker, Jan and Frederic Warzynski, "Markups and Firm-Level Export Status," *American Economic Review*, 2012, *102* (6), 2437–2471.

— **, Pinelopi K. Goldberg, Amit K. Khandelwal, and Nina Pavcnik**, "Prices, Markups, and Trade Reform," *Econometrica*, 2016, *84* (2), 445–510.

Demidova, Svetlana, "Trade Policies, Firm Heterogeneity, and Variable Markups," *Journal of International Economics*, 2017, *108*, 260–273.

Dhingra, Swati and John Morrow, "Efficiency in Large Markets with Firm Heterogeneity," *Research in Economics*, 2017, *71* (4), 718–728.

_ **and** _ , "Monopolistic Competition and Optimum Product Diversity under Firm Heterogeneity," *Journal of Political Economy*, 2019, *127* (1), 196–232.

Dixit, Avinash K. and Joseph E. Stiglitz, "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 1977, *67* (3), 297–308.

Egger, Peter and Ruobing Huang, "Market versus Optimum Diversity in Open Economies: Theory and Quantitative Evidence," 2023. CEPR Discussion Paper No.18089.

Feenstra, Robert C., "A Homothetic Utility Function for Monopolistic Competition Models, without Constant Price Elasticity," *Economics Letters*, 2003, *78* (1), 79–86.

_ , "Restoring the Product Variety and Pro-Competitive Gains from Trade with Heterogeneous Firms and Bounded Productivity," *Journal of International Economics*, 2018, *110*, 16–27.

Gabaix, Xavier, "Power Laws in Economics and Finance," *Annual Review of Economics*, 2009, *1*, 255–294.

Jung, Jae Wook, Ina Simonovska, and Ariel Weinberger, "Exporter Heterogeneity and Price Discrimination: A Quantitative View," *Journal of International Economics*, 2019, *116*, 103–124.

Kichko, Sergey, Sergey Kokovin, and Evgeny Zhelobodko, "Trade Patterns and Export Pricing under Non-CES Preferences," *Journal of International Economics*, 2014, *94*, 129–142.

Krugman, Paul R., "Increasing Returns, Monopolistic Competition, and International Trade," *Journal of International Economics*, 1979, *9* (4), 469–479.

Matsuyama, Kiminori and Philip Ushchev, "Selection and Sorting of Heterogeneous Firms through Competitive Pressures," 2023. CEPR Discussion Paper DP17092.

Melitz, Marc J., "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 2003, *71* (6), 1695–1725.

_ **and Gianmarco I.P. Ottaviano**, "Market Size, Trade, and Productivity," *Review of Economic Studies*, 2008, *75* (1), 295–316.

_ **,** _ **, Mikhail Oshmakashvili, and Davide Suverato**, "Markup Distortions and Optimal Non-Discriminatory Industrial Policy," 2024. mimeo.

Mrázová, Monika and J. Peter Neary, "Not So Demanding: Demand Structure and Firm Behavior," *American Economic Review*, 2017, *107* (12), 3835–3874.

_ **and** _ , "IO for Exports(s)," *International Journal of Industrial Organization*, 2020, *70*, 102561.

_ **,** _ **, and Mathieu Parenti**, "Sales and Markup Dispersion: Theory and Empirics," *Econometrica*, 2021, *89* (4), 1753–1788.

Nocco, Antonella, Gianmarco I.P. Ottaviano, and Matteo Salto, "Monopolistic Competition and Optimum Product Selection," *American Economic Review*, 2014, *104* (5), 304–309.

_ **,** _ **, and** _ , "Geography, Competition, and Optimal Multilateral Trade Policy," *Journal of International Economics*, 2019, *120*, 145–161.

_ **,** _ **,** _ **, and Atsushi Tadokoro**, "Levelling the Global Playing Field through Optimal Non-Discriminatory Corporate Taxes and Subsidies," 2024. CESifo Working Paper No.11060.

Ottaviano, Gianmarco I.P., Takatoshi Tabuchi, and Jacques-François Thisse, "Agglomeration and Trade Revisited," *International Economic Review*, 2002, *43* (2), 409–436.

Parenti, Mathieu, Philip Ushchev, and Jacques-François Thisse, "Toward a Theory of Monopolistic Competition," *Journal of Economic Theory*, 2017, *167*, 86–115.

Samuelson, Paul A., "Using Full Duality to Show that Simultaneously Additive Direct and Indirect Utilities Implies Unitary Price Elasticity of Demand," *Econometrica*, 1965, *33* (4), 781–796.

Simonovska, Ina, "Income Differences and Prices of Tradables: Insights from an Online Retailer," *Review of Economic Studies*, 2015, *82* (4), 1612–1656.

Slepov, Fedor and Sergey Kokovin, "Equilibrium Existence and Uniqueness in Additive Trade Models," 2023. HSE Working Papers WP BRP 262/EC/2023, National Research University Higher School of Economics.

Tadokoro, Atsushi, "Optimal Policy against Distortions Caused by Monopolistic Competition and Variable Markup Pricing," *Economics Letters*, 2024, *237*, 111627.

Zeng, Dao-Zhi and Shin-Kun Peng, "Symmetric Tax Competition and Welfare with Footloose Capital," *Journal of Regional Science*, 2021, *61* (2), 472–491.

Zhelobodko, Evgeny, Sergey Kokovin, Mathieu Parenti, and Jacques-François Thisse, "Monopolistic Competition: Beyond the Constant Elasticity of Substitution," *Econometrica*, 2012, *80* (6), 2765–2784.

Appendix for
"On Trade Models with Variable Markups and Pareto-Distributed Productivity"

A Omitted Proofs

In preparation for proofs, we show several lemmas that are useful for proving some propositions. First, we prove the following lemma related to the integrability of a function.

Lemma A.1. *Let $\Upsilon(m)$ be a positive-valued and continuously differentiable function that decreases in m when m is close to zero and satisfies $\lim_{m\to 0}\Upsilon(m)=\infty$. If $\int_0^T \Upsilon(m)\mathrm{d}G(m)<\infty$ holds for any $T\in(0,M)$, then it holds that $\lim_{m\to 0}\Upsilon(m)G(m)=0$.*

Proof. First, we take some $\widetilde{T}\in(0,T]$ such that $\Upsilon'(m)<0$ holds for $0<m<\widetilde{T}$. Then, the inverse function of $\Upsilon(m)$ can be defined over $0<m<\widetilde{T}$, and it is a decreasing function because $\Upsilon(m)$ decreases in m when $0<m<\widetilde{T}$. Provided the existence of this inverse function, we consider the following conditional probability:

$$\begin{aligned}\Pr(\Upsilon(m)<\upsilon\mid m<\widetilde{T}) &= \Pr(m>\Upsilon^{-1}(\upsilon)\mid m<\widetilde{T})\\ &= \frac{\Pr(\Upsilon^{-1}(\upsilon)<m<\widetilde{T})}{\Pr(m<\widetilde{T})}\\ &= 1-\frac{G(\Upsilon^{-1}(\upsilon))}{G(\widetilde{T})}\equiv G_{\Upsilon}(\upsilon\mid\upsilon>\widetilde{\upsilon}),\end{aligned}$$

where we let $\widetilde{\upsilon}\equiv\Upsilon(\widetilde{T})$. It can be readily verified that the corresponding density function is

$$g_{\Upsilon}(\upsilon\mid\upsilon>\widetilde{\upsilon})=-\frac{\mathrm{d}\Upsilon^{-1}(\upsilon)}{\mathrm{d}\upsilon}\frac{g(\Upsilon^{-1}(\upsilon))}{G(\widetilde{T})}.$$

In what follows, we show the statement of the lemma by proving its contraposition: if $\Upsilon(m)G(m)$ diverges or converges to a positive constant as $m\to 0$, then the integral $\int_0^T\Upsilon(m)\mathrm{d}G(m)$ diverges.

Suppose that $\lim_{m\to 0}\Upsilon(m)G(m)=A>0$ holds, where we admit the case of $A=\infty$. By letting $m=\Upsilon^{-1}(\upsilon)$, the limit can be rewritten as

$$\lim_{m\to 0}\Upsilon(m)G(m)=\lim_{\upsilon\to\infty}\upsilon G(\Upsilon^{-1}(\upsilon))=G(\widetilde{T})\lim_{\upsilon\to\infty}\upsilon\left(1-G_{\Upsilon}(\upsilon\mid\upsilon>\widetilde{\upsilon})\right),$$

and thus $\lim_{\upsilon\to\infty}\upsilon\left(1-G_{\Upsilon}(\upsilon\mid\upsilon>\widetilde{\upsilon})\right)=\widetilde{A}$ holds, where $\widetilde{A}\equiv A/G(\widetilde{T})$. Take some $\widetilde{A}'\in(0,\widetilde{A})$, and then we have

$$\lim_{\upsilon\to\infty}\frac{1-G_{\Upsilon}(\upsilon\mid\upsilon>\widetilde{\upsilon})}{\widetilde{A}'/\upsilon}=\frac{\widetilde{A}}{\widetilde{A}'}>1.$$

Applying L'Hopital's rule, we obtain

$$\lim_{\upsilon \to \infty} \frac{g_\Upsilon(\upsilon \mid \upsilon > \widetilde{\upsilon})}{\widetilde{A}'/\upsilon^2} = \frac{\widetilde{A}}{\widetilde{A}'} > 1.$$

This implies that there exists a sufficiently large $\bar{\upsilon}$ ($> \widetilde{\upsilon}$) such that $g_\Upsilon(\upsilon \mid \upsilon > \widetilde{\upsilon}) > \widetilde{A}'/\upsilon^2$ holds for any $\upsilon > \bar{\upsilon}$. Consequently, the divergence of the integral $\int_0^T \Upsilon(m)\mathrm{d}G(m)$ can be shown as follows:

$$\begin{aligned}
\int_0^T \Upsilon(m)\mathrm{d}G(m) &= G(\widetilde{T}) \int_0^T \Upsilon(m) \frac{g(m)}{G(\widetilde{T})}\mathrm{d}m \\
&> G(\widetilde{T}) \int_0^{\widetilde{T}} \Upsilon(m) \frac{g(m)}{G(\widetilde{T})}\mathrm{d}m \\
&= G(\widetilde{T}) \int_{\widetilde{\upsilon}}^{\infty} \upsilon g_\Upsilon(\upsilon \mid \upsilon > \widetilde{\upsilon})\mathrm{d}\upsilon \\
&> G(\widetilde{T}) \int_{\bar{\upsilon}}^{\infty} \upsilon g_\Upsilon(\upsilon \mid \upsilon > \widetilde{\upsilon})\mathrm{d}\upsilon \\
&> G(\widetilde{T}) \int_{\bar{\upsilon}}^{\infty} \frac{\widetilde{A}'}{\upsilon}\mathrm{d}\upsilon = \infty,
\end{aligned}$$

where we change the variable as $\upsilon = \Upsilon(m)$ in the third row, and the inequalities in the second, fourth, and fifth rows follow from $\widetilde{T} < T$, $\bar{\upsilon} > \widetilde{\upsilon}$, and $g_\Upsilon(\upsilon \mid \upsilon > \widetilde{\upsilon}) > \widetilde{A}'/\upsilon^2$, respectively. □

A.1 Proof of Lemma 1

By plugging (2) into the left-hand side of (9), we obtain

$$N\widehat{m} \int_0^{\widehat{m}} \frac{q(m)u'(q(m))}{\bar{u}}\mathrm{d}G(m) = 1. \tag{A.1}$$

The integral included in the left-hand side of (A.1) is calculated as

$$\begin{aligned}
\int_0^{\widehat{m}} \frac{q(m)u'(q(m))}{\bar{u}}\mathrm{d}G(m) &= \int_0^{\widehat{m}} \int_0^{q(m)} \frac{u'(z) + zu''(z)}{\bar{u}} g(m)\mathrm{d}z\mathrm{d}m \\
&= \int_0^{\widehat{m}} \int_0^{\psi(m/\widehat{m})} \psi^{-1}(z) g(m)\mathrm{d}z\mathrm{d}m \\
&= \int_0^{\widehat{m}} \int_{m/\widehat{m}}^{1} t(-\psi'(t)) g(m)\mathrm{d}t\mathrm{d}m \\
&= \int_0^{1} \int_0^{\widehat{m}t} t(-\psi'(t)) g(m)\mathrm{d}m\mathrm{d}t \\
&= \int_0^{1} t(-\psi'(t)) G\left(\widehat{m}t\right)\mathrm{d}t,
\end{aligned}$$

where the third equality follows from changing the variable as $z = \psi(t)$, and the integrability enables us to apply Fubini's theorem in the fourth equality. Substituting this back into (A.1) yields

$$N\widehat{m}\int_0^1 t(-\psi'(t))G(\widehat{m}t)\,\mathrm{d}t = 1. \tag{A.2}$$

Moreover, by using integration by parts, the integral included in the left-hand side of (A.2) can be rewritten as

$$\int_0^1 t(-\psi'(t))G(\widehat{m}t)\,\mathrm{d}t = -\left[t\psi(t)G(\widehat{m}t)\right]_0^1 + \int_0^1 \psi(t)(\widehat{m}tg(\widehat{m}t) + G(\widehat{m}t))\mathrm{d}t. \tag{A.3}$$

It is straightforward to verify that $\lim_{t\to 1} t\psi(t)G(\widehat{m}t) = 0$ holds and, if $\lim_{t\to 0}\psi(t) < \infty$ is satisfied, $\lim_{t\to 0} t\psi(t)G(\widehat{m}t) = 0$ also holds. If $\lim_{t\to 0}\psi(t) = \infty$ holds, we have $\lim_{t\to 0} t\psi(t) = \widehat{m}^{-1}\lim_{m\to 0} mq(m) = \infty$, and it can be shown that $mq(m)$ decreases in m when m is close to zero.[A.1] Then, Assumption 5 and Lemma A.1 imply that $\lim_{t\to 0} t\psi(t)G(\widehat{m}t) = 0$ holds even when $\lim_{t\to 0}\psi(t) = \infty$. Hence, the surface term in the right-hand side of (A.3) vanishes, resulting in

$$\int_0^1 t(-\psi'(t))G(\widehat{m}t)\,\mathrm{d}t = \int_0^1 \psi(t)G(\widehat{m}t)\left(1 + \mathcal{E}^G(\widehat{m}t)\right)\mathrm{d}t, \tag{A.4}$$

where $\mathcal{E}^G(m) = mg(m)/G(m)$ represents the elasticity of the distribution function $G(m)$. Consequently, (A.2) and (A.4) imply that (9) can be transformed into (11).

The left-hand side of the zero expected profit condition (7) can be rewritten as follows:

$$\int_0^{\widehat{m}} \pi(m)\mathrm{d}G(m) = L\int_0^{\widehat{m}} p(m)q(m)\mathrm{d}G(m) - L\int_0^{\widehat{m}} mq(m)\mathrm{d}G(m). \tag{A.5}$$

It is already shown that the first integral in the right-hand side of (A.5) can be calculated as $\int_0^{\widehat{m}} p(m)q(m)\mathrm{d}G(m) = \widehat{m}\int_0^1 t(-\psi'(t))G(\widehat{m}t)\mathrm{d}t$. The second integral is computed as

$$\begin{aligned}
\int_0^{\widehat{m}} mq(m)\mathrm{d}G(m) &= \int_0^{\widehat{m}} m\psi\left(\frac{m}{\widehat{m}}\right)g(m)\mathrm{d}m \\
&= \widehat{m}^2\int_0^1 t\psi(t)g(\widehat{m}t)\mathrm{d}t \\
&= \widehat{m}\underbrace{\left[t\psi(t)G(\widehat{m}t)\right]_0^1}_{=0} - \widehat{m}\int_0^1 (t\psi'(t) + \psi(t))G(\widehat{m}t)\mathrm{d}t \\
&= \widehat{m}\int_0^1 t(-\psi'(t))G(\widehat{m}t) - \widehat{m}\int_0^1 \psi(t)G(\widehat{m}t)\mathrm{d}t,
\end{aligned}$$

where we change the variable as $m = \widehat{m}t$ in the second line, and the surface term that appears in

[A.1] This can be shown by contradiction as follows. If we assume $mq(m)$ is non-decreasing in m when m is close to zero, there exists a positive ε such that $q(\varepsilon) = \infty$, which violates the continuity of $q(m)$.

the third line has already shown to be zero. As a result, (A.5) becomes

$$\int_0^{\widehat{m}} \pi(m)\mathrm{d}G(m) = L\widehat{m}\int_0^1 \psi(t)G(\widehat{m}t)\mathrm{d}t,$$

indicating that (7) can be transformed into (10). Therefore, $\widehat{m}$ and N are determined in two equations (10) and (11).

Next, we prove the uniqueness of the equilibrium. We define a function $\mathcal{F}(x)$ by $\mathcal{F}(x) \equiv x\int_0^1 \psi(t)G(xt)\mathrm{d}t$. Then, (10) implies that the marginal cost cutoff at the equilibrium is given by a solution to the equation $\mathcal{F}(x) = f/L$.

For $x > 0$, $\mathcal{F}(x)$ is a strictly increasing function because

$$\mathcal{F}'(x) = \int_0^1 \psi(t)G(xt)\mathrm{d}t + x\int_0^1 t\psi(t)g(xt)\mathrm{d}t > 0,$$

where the interchangeability of differentiation and integration is guaranteed by differentiability of $G(\cdot)$ and integrability. Furthermore, it is readily verified that $\lim_{x\to 0}\mathcal{F}(x) = 0$ holds, and Assumption 6 indicates that $\lim_{x\to M}\mathcal{F}(x) = M\int_0^1 \psi(t)G(Mt)\mathrm{d}t > f/L$. Thus, the equation $\mathcal{F}(x) = f/L$ has a unique solution in $(0, M)$, implying that $\widehat{m}$ is uniquely determined by (10). Given the value of $\widehat{m}$, N is also uniquely determined by (11).

Finally, each worker's utility (1) can be computed as

$$\begin{aligned}
U &= N\int_0^{\widehat{m}} u\left(\psi\left(\frac{m}{\widehat{m}}\right)\right)g(m)\mathrm{d}m \\
&= N\int_0^{\widehat{m}}\int_0^{\psi(m/\widehat{m})} u'(z)g(m)\mathrm{d}z\mathrm{d}m \\
&= N\int_0^{\widehat{m}}\int_{m/\widehat{m}}^1 u'(\psi(t))(-\psi'(t))g(m)\mathrm{d}t\mathrm{d}m \\
&= N\int_0^1\int_0^{\widehat{m}t} u'(\psi(t))(-\psi'(t))g(m)\mathrm{d}m\mathrm{d}t \\
&= -N\int_0^1 u'(\psi(t))\psi'(t)G(\widehat{m}t)\mathrm{d}t \\
&= -\bar{u}N\int_0^1 t\psi'(t)G(\widehat{m}t)\mathrm{d}t + N\int_0^1 \psi(t)u''(\psi(t))\psi'(t)G(\widehat{m}t)\mathrm{d}t \\
&= \frac{\bar{u}}{\widehat{m}} + N\int_0^1 \psi(t)u''(\psi(t))\psi'(t)G(\widehat{m}t)\mathrm{d}t
\end{aligned} \tag{A.6}$$

where the third equality follows from changing the variable as $z = \psi(t)$; the integrability enables us to apply Fubini's theorem in the fourth equality; the sixth equality follows from the relationship $u'(\psi(t)) = \bar{u}t - \psi(t)u''(\psi(t))$, which is derived from (4) and (5); and the last equality follows from (A.2). By applying integration by parts, the integral included in the right-hand side of

(A.6) can be transformed as

$$\int_0^1 \psi(t)u''(\psi(t))\psi'(t)G(\widehat{m}t)\mathrm{d}t = [(\psi(t)u'(\psi(t)) - u(\psi(t)))G(\widehat{m}t)]_0^1 + \int_0^1 (u(\psi(t)) - \psi(t)u'(\psi(t)))\widehat{m}g(\widehat{m}t)\mathrm{d}t. \tag{A.7}$$

It is straightforward to verify that $\lim_{t\to 1}\psi(t)u'(\psi(t))G(\widehat{m}t) = \lim_{t\to 1}u(\psi(t))G(\widehat{m}t) = 0$ holds. Note also that $\lim_{t\to 0}\psi(t)u'(\psi(t)) = \bar{u}\widehat{p}^{-1}\lim_{m\to 0}p(m)q(m)$ and $\lim_{t\to 0}u(\psi(t)) = \lim_{m\to 0}u(q(m))$ hold. If $\lim_{m\to 0}p(m)q(m) < \infty$ and $\lim_{m\to 0}u(q(m)) < \infty$ are satisfied, we readily obtain $\lim_{t\to 0}\psi(t)u'(\psi(t))G(\widehat{m}t) = \lim_{t\to 0}u(\psi(t))G(\widehat{m}t) = 0$. Furthermore, even in the cases wherein $\lim_{m\to 0}p(m)q(m) = \infty$ or $\lim_{m\to 0}u(q(m)) = \infty$ hold, Assumption 5 and Lemma A.1 assures that $\lim_{t\to 0}\psi(t)u'(\psi(t))G(\widehat{m}t) = \lim_{t\to 0}u(\psi(t))G(\widehat{m}t) = 0$ holds true because both $p(m)q(m)$ and $u(q(m))$ can be shown to decrease in m. Hence, the surface term in the right-hand side of (A.7) vanishes, enabling us to rewrite (A.6) as

$$U = \frac{\bar{u}}{\widehat{m}} + N\int_0^1 \frac{u(\psi(t))(1-\mathcal{E}^u(\psi(t)))\mathcal{E}^G(\widehat{m}t)G(\widehat{m}t)}{t}\mathrm{d}t,$$

where we let $\mathcal{E}^u(x) \equiv xu'(x)/u(x)$ represent the elasticity of the subutility. Therefore, the equilibrium utility can be transformed into (12).

A.2 Proof of Lemma 2

We first derive (20) and (21). Substituting (17) into (15) yields

$$\frac{\widehat{m}}{\bar{u}}\int_0^{\widehat{m}} u(q(m))\mathrm{d}G(m) - \int_0^{\widehat{m}} mq(m)\mathrm{d}G(m) = \frac{f}{L}. \tag{A.8}$$

Using (18) and (19), the first term in the left-hand side can be rewritten as

$$\begin{aligned}
\frac{\widehat{m}}{\bar{u}}\int_0^{\widehat{m}} u(q(m))\mathrm{d}G(m) &= \frac{\widehat{m}}{\bar{u}}\int_0^{\widehat{m}}\int_0^{q(m)} u'(z)g(m)\mathrm{d}z\mathrm{d}m \\
&= \widehat{m}\int_0^{\widehat{m}}\int_0^{\phi(m/\widehat{m})} \phi^{-1}(z)g(m)\mathrm{d}z\mathrm{d}m \\
&= \widehat{m}\int_0^{\widehat{m}}\int_{m/\widehat{m}}^1 t(-\phi'(t))g(m)\mathrm{d}t\mathrm{d}m \\
&= \widehat{m}\int_0^1\int_0^{\widehat{m}t} t(-\phi'(t))g(m)\mathrm{d}m\mathrm{d}t \\
&= \widehat{m}\int_0^1 t(-\phi'(t))G(\widehat{m}t)\mathrm{d}t,
\end{aligned} \tag{A.9}$$

where the third equality follows from changing the variable as $z = \phi(t)$, and the integrability enables us to apply Fubini's theorem in the fourth equality. The second term in the left-hand side of (A.8) can be transformed as

$$
\begin{aligned}
\int_0^{\widehat{m}} mq(m)\mathrm{d}G(m) &= \int_0^{\widehat{m}} \widehat{m}\phi^{-1}(q(m))q(m)g(m)\mathrm{d}m \\
&= \widehat{m}\int_0^{\widehat{m}}\int_0^{q(m)} \left(z\frac{\mathrm{d}\phi^{-1}(z)}{\mathrm{d}z} + \phi^{-1}(z)\right) g(m)\mathrm{d}z\mathrm{d}m \\
&= \widehat{m}\int_0^{\widehat{m}}\int_0^{\phi(m/\widehat{m})} z\frac{\mathrm{d}\phi^{-1}(z)}{\mathrm{d}z} g(m)\mathrm{d}z\mathrm{d}m + \widehat{m}\int_0^{\widehat{m}}\int_0^{\phi(m/\widehat{m})} \phi^{-1}(z)g(m)\mathrm{d}z\mathrm{d}m \\
&= -\widehat{m}\int_0^{\widehat{m}}\int_{m/\widehat{m}}^{1} \phi(t)g(m)\mathrm{d}t\mathrm{d}m + \widehat{m}\int_0^{\widehat{m}}\int_{m/\widehat{m}}^{1} t(-\phi'(t))g(m)\mathrm{d}t\mathrm{d}m \\
&= -\widehat{m}\int_0^{1}\int_0^{\widehat{m}t} \phi(t)g(m)\mathrm{d}m\mathrm{d}t + \widehat{m}\int_0^{1}\int_0^{\widehat{m}t} t(-\phi'(t))g(m)\mathrm{d}m\mathrm{d}t \\
&= -\widehat{m}\int_0^{1} \phi(t)G(\widehat{m}t)\mathrm{d}t + \widehat{m}\int_0^{1} t(-\phi'(t))G(\widehat{m}t)\mathrm{d}t,
\end{aligned}
\tag{A.10}
$$

where, again, the fourth equality follows from changing the variable as $z = \phi(t)$, and the integrability enables us to apply Fubini's theorem in the fifth equality. Thus, (A.8) can be transformed into (20).

Plugging (A.10) into the resource constraint in (13), we get

$$
\begin{aligned}
& N\left[\widehat{m}\int_0^1 t(-\phi'(t))G(\widehat{m}t)\mathrm{d}t - \widehat{m}\int_0^1 \phi(t)G(\widehat{m}t)\mathrm{d}t + \frac{f}{L}\right] = 1 \\
\Leftrightarrow\ & N\widehat{m}\int_0^1 t(-\phi'(t))G(\widehat{m}t)\mathrm{d}t = 1
\end{aligned}
\tag{A.11}
$$

where the sum of last two terms in the square brackets is zero by (20). Using integration by parts, we can rewrite the integral included in the left-hand side of (A.11) as

$$
\int_0^1 t(-\phi'(t))G(\widehat{m}t)\mathrm{d}t = -\left[t\phi(t)G(\widehat{m}t)\right]_0^1 + \int_0^1 \phi(t)(\widehat{m}tg(\widehat{m}t) + G(\widehat{m}t))\mathrm{d}t \tag{A.12}
$$

It is straightforward to verify that $\lim_{t\to 1} t\phi(t)G(\widehat{m}t) = 0$ holds and, if $\lim_{t\to 0}\phi(t) < \infty$ is satisfied, $\lim_{t\to 0} t\phi(t)G(\widehat{m}t) = 0$ also holds. If $\lim_{t\to 0}\phi(t) = \infty$ holds, we have $\lim_{t\to 0} t\phi(t) = \widehat{m}^{-1}\lim_{m\to 0} mq(m) = \infty$, and it can be shown that $mq(m)$ decreases in m when m is close to zero (see footnote A.1). Then, Assumption 5 and Lemma A.1 imply that $\lim_{t\to 0} t\phi(t)G(\widehat{m}t) = 0$ holds even when $\lim_{t\to 0}\phi(t) = \infty$. Hence, the surface term in the right-hand side of (A.12) vanishes, resulting in

$$
\int_0^1 t(-\phi'(t))G(\widehat{m}t)\mathrm{d}t = \int_0^1 \phi(t)G(\widehat{m}t)\left(1 + \mathcal{E}^G(\widehat{m}t)\right)\mathrm{d}t,
$$

where we let $\mathcal{E}^G(\widehat{m}t) = g(\widehat{m}t)\widehat{m}t/G(\widehat{m}t)$. Consequently, the resource constraint in (13) can be transformed into (21).

Next, for the derived two equations (20) and (21), which determine the social optimum values of $\widehat{m}$ and N, we show the existence of a unique solution. A similar argument in Appendix A.1 shows that the left-hand side of (20) is strictly increasing in $\widehat{m}$ and converges to zero as $\widehat{m} \to 0$. Therefore, (20) has a unique solution for $\widehat{m}$ in $(0, M)$ if and only if

$$M \int_0^1 \phi(t) G(Mt) \mathrm{d}t > \frac{f}{L}. \tag{A.13}$$

We then confirm that Assumption 6 ensures (A.13) to hold. As shown in the proof of Proposition 1, the following relationship holds:

$$\psi(t) < \phi(t) \quad \text{for} \quad t \in (0, 1).$$

This implies that

$$\widehat{m} \int_0^1 \psi(t) G(\widehat{m}t) \mathrm{d}t < \widehat{m} \int_0^1 \phi(t) G(\widehat{m}t) \mathrm{d}t$$

holds for any given $\widehat{m} > 0$. Combining this inequality (for $\widehat{m} = M$) with Assumption 6 shows that (A.13) holds, meaning that the social optimum level of $\widehat{m}$ is uniquely determined by (20). Consequently, the social optimum level of N is also uniquely determined by (21).

Finally, using (A.9) and (A.11), each worker's utility can be expressed as

$$\begin{aligned} U &= N \int_0^{\widehat{m}} u(q(m)) \mathrm{d}G(m) \\ &= \bar{u} N \int_0^1 t(-\phi'(t)) G(\widehat{m}t) \mathrm{d}t \\ &= \frac{\bar{u}}{\widehat{m}}. \end{aligned}$$

A.3 Proof of (iii) and (v) in Proposition 1

Proof of (iii). The equilibrium and optimal values of $q(m)$ are determined by (4) and (18), respectively. From (4) and (18), for the same level of m, we have

$$\begin{aligned} & \frac{u'(q^{\text{eqm}}(m)) + q^{\text{eqm}}(m) u''(q^{\text{eqm}}(m))}{u'(q^{\text{opt}}(m))} = \frac{\widehat{m}^{\text{opt}}}{\widehat{m}^{\text{eqm}}} \\ \Leftrightarrow\ & \frac{u'(q^{\text{eqm}}(m))}{u'(q^{\text{opt}}(m))} (1 - r(q^{\text{eqm}}(m))) = \frac{\widehat{m}^{\text{opt}}}{\widehat{m}^{\text{eqm}}} \\ \Leftrightarrow\ & \frac{u'(q^{\text{eqm}}(m))}{u'(q^{\text{opt}}(m))} = \frac{\widehat{m}^{\text{opt}}}{\widehat{m}^{\text{eqm}}} \mu(m) \end{aligned}$$

where $r(q^{\text{eqm}}(m)) = -q^{\text{eqm}}(m) u''(q^{\text{eqm}}(m)) / u'(q^{\text{eqm}}(m))$ and $\mu(m) = 1/(1 - r(q^{\text{eqm}}(m)))$. Since $u'(\cdot)$ is a strictly decreasing function, the above expression implies

$$q^{\text{eqm}}(m) \lesseqqgtr q^{\text{opt}}(m) \Leftrightarrow \mu(m) \gtreqqless \frac{\widehat{m}^{\text{eqm}}}{\widehat{m}^{\text{opt}}}. \tag{A.14}$$

Therefore, varieties with the markup above $\widehat{m}^{\mathrm{eqm}}/\widehat{m}^{\mathrm{opt}}$ are underproduced, whereas those with markup below $\widehat{m}^{\mathrm{eqm}}/\widehat{m}^{\mathrm{opt}}$ are overproduced. As the markup $\mu(m)$ strictly decreases with m, there can be at most one value of m for which inequality (A.14) holds with equality. Let m^* denote m such that $\mu(m^*) = \widehat{m}^{\mathrm{eqm}}/\widehat{m}^{\mathrm{opt}}$. Note that the relationship $\widehat{m}^{\mathrm{eqm}} > \widehat{m}^{\mathrm{opt}}$ implies $q^{\mathrm{eqm}}(\widehat{m}^{\mathrm{opt}}) > q^{\mathrm{opt}}(\widehat{m}^{\mathrm{opt}}) = 0$. Therefore, from (A.14), we have $\mu(\widehat{m}^{\mathrm{opt}}) < \widehat{m}^{\mathrm{eqm}}/\widehat{m}^{\mathrm{opt}}$, meaning that m^* must be smaller than $\widehat{m}^{\mathrm{opt}}$.

We then show that $m^* > 0$ holds, meaning that some firms with cost $m \in (0, m^*)$ inefficiently underproduce in the market equilibrium. From (23) and (28), the mass of entrants in the market equilibrium is aligned with the optimal level: $N^{\mathrm{eqm}} = N^{\mathrm{opt}}$. Then, noting that $m^* < \widehat{m}^{\mathrm{opt}}$, the labor market clearing condition (8) requires the following relationship to hold:

$$\begin{aligned}
&\int_0^{\widehat{m}^{\mathrm{eqm}}} m q^{\mathrm{eqm}}(m) \mathrm{d}G(m) = \int_0^{\widehat{m}^{\mathrm{opt}}} m q^{\mathrm{opt}}(m) \mathrm{d}G(m) \\
\Leftrightarrow &\int_0^{m^*} m \left(q^{\mathrm{opt}}(m) - q^{\mathrm{eqm}}(m)\right) \mathrm{d}G(m) \\
&\quad = \int_{m^*}^{\widehat{m}^{\mathrm{opt}}} m \left(q^{\mathrm{eqm}}(m) - q^{\mathrm{opt}}(m)\right) \mathrm{d}G(m) + \int_{\widehat{m}^{\mathrm{opt}}}^{\widehat{m}^{\mathrm{eqm}}} m q^{\mathrm{eqm}}(m) \mathrm{d}G(m)
\end{aligned} \tag{A.15}$$

where, from (A.14) and the definition of m^*, $q^{\mathrm{opt}}(m) - q^{\mathrm{eqm}}(m)$ is positive for $m \in (0, m^*)$ and negative for $m \in (m^*, \widehat{m}^{\mathrm{eqm}})$. Assume $m^* \leq 0$ (i.e., all varieties are overproduced in the market equilibrium). Then, the relationship (A.15) no longer holds because the left- and right-hand sides of (A.15) are zero and positive, respectively. This contradicts the fact that the labor market clearing condition holds in both the market equilibrium and social optimum. Therefore, m^* must be positive, meaning that $\mu(m) = \widehat{m}^{\mathrm{eqm}}/\widehat{m}^{\mathrm{opt}}$ holds for $m = m^* \in (0, \widehat{m}^{\mathrm{opt}})$. As $\mu(m)$ is a strictly decreasing function of m, we can rewrite (A.14) as

$$q^{\mathrm{eqm}}(m) \lesseqqgtr q^{\mathrm{opt}}(m) \Leftrightarrow m \lesseqqgtr m^*, \quad m^* \in (0, \widehat{m}^{\mathrm{opt}}). \tag{A.16}$$

Proof of (v). Regarding each term in (A.15), Cauchy's mean value theorem implies that there exist m_1, m_2, and m_3 such that $0 < m_1 < m^* < m_2 < \widehat{m}^{\mathrm{opt}} < m_3 < \widehat{m}^{\mathrm{eqm}}$ and

$$\begin{aligned}
&\int_0^{m^*} m \left(q^{\mathrm{opt}}(m) - q^{\mathrm{eqm}}(m)\right) \mathrm{d}G(m) = m_1 \int_0^{m^*} \left(q^{\mathrm{opt}}(m) - q^{\mathrm{eqm}}(m)\right) \mathrm{d}G(m), \\
&\int_{m^*}^{\widehat{m}^{\mathrm{opt}}} m \left(q^{\mathrm{eqm}}(m) - q^{\mathrm{opt}}(m)\right) \mathrm{d}G(m) = m_2 \int_{m^*}^{\widehat{m}^{\mathrm{opt}}} \left(q^{\mathrm{eqm}}(m) - q^{\mathrm{opt}}(m)\right) \mathrm{d}G(m), \\
&\int_{\widehat{m}^{\mathrm{opt}}}^{\widehat{m}^{\mathrm{eqm}}} m q^{\mathrm{eqm}}(m) \mathrm{d}G(m) = m_3 \int_{\widehat{m}^{\mathrm{opt}}}^{\widehat{m}^{\mathrm{eqm}}} q^{\mathrm{eqm}}(m) \mathrm{d}G(m),
\end{aligned}$$

where (A.16) indicates that all the integrands are positive. Substituting these back into (A.15), we obtain

$$\int_0^{m^*} \left(q^{\mathrm{opt}}(m) - q^{\mathrm{eqm}}(m)\right) \mathrm{d}G(m) = \frac{m_2}{m_1} \int_{m^*}^{\widehat{m}^{\mathrm{opt}}} \left(q^{\mathrm{eqm}}(m) - q^{\mathrm{opt}}(m)\right) \mathrm{d}G(m)$$

$$
\begin{aligned}
&+\frac{m_3}{m_1}\int_{\widehat{m}^{\rm opt}}^{\widehat{m}^{\rm eqm}} q^{\rm eqm}(m){\rm d}G(m)\\
&>\int_{m^*}^{\widehat{m}^{\rm opt}}\left(q^{\rm eqm}(m)-q^{\rm opt}(m)\right){\rm d}G(m)\\
&+\int_{\widehat{m}^{\rm opt}}^{\widehat{m}^{\rm eqm}} q^{\rm eqm}(m){\rm d}G(m),
\end{aligned}
$$

where the second line follows from $m_2/m_1 > 1$ and $m_3/m_1 > 1$. Rearranging this inequality yields

$$
\int_0^{\widehat{m}^{\rm eqm}} q^{\rm eqm}(m){\rm d}G(m) < \int_0^{\widehat{m}^{\rm opt}} q^{\rm opt}(m){\rm d}G(m). \tag{A.17}
$$

The aggregate production per consumer is expressed as $Q = N\int_0^{\widehat{m}} q(m)dG(m)$. Then, (A.17) and $N^{\rm eqm} = N^{\rm opt}$ imply $Q^{\rm eqm} < Q^{\rm opt}$. Moreover, the average production per consumer, $\bar{q} = Q/n$, can also be ranked as $\bar{q}^{\rm eqm} < \bar{q}^{\rm opt}$ since $Q^{\rm eqm} < Q^{\rm opt}$ and $n^{\rm eqm} > n^{\rm opt}$.

A.3.1 The Effect of Market Size on Allocative Efficiency

Although not directly related to the proof, it is worth noting that, under a Pareto distribution, the gaps between the equilibrium and optimum levels of the cost cutoff, mass of consumed varieties, and aggregate/average quantity are independent of market size L. These can be shown as follows.

From (23) and (28), we get

$$
\frac{\widehat{m}^{\rm eqm}}{\widehat{m}^{\rm opt}} = \left(\frac{\Phi}{\Psi}\right)^{\frac{1}{k+1}} > 1,
$$

which is independent of L because $\Psi = \int_0^1 t^k\psi(t){\rm d}t$ and $\Phi = \int_0^1 t^k\phi(t){\rm d}t$ are positive constants that depend on exogenous choices of the Pareto shape parameter and the functional form of subutility. Then, the gap between the equilibrium and optimum levels of mass of varieties can be expressed as

$$
\frac{n^{\rm eqm}}{n^{\rm opt}} = \frac{N^{\rm eqm}G(\widehat{m}^{\rm eqm})}{N^{\rm opt}G(\widehat{m}^{\rm opt})} = \left(\frac{\Phi}{\Psi}\right)^{\frac{k}{k+1}} > 1, \tag{A.18}
$$

where $N^{\rm eqm} = N^{\rm opt}$. Thus, the gap is independent of L.

Let us impose an additional assumption described in the statement of Proposition 1, i.e., the aggregate production converges: $\int_0^{\widehat{m}} q(m){\rm d}G(m) < \infty$. Then, as for the aggregate and average quantities, we rewrite the term in the left-hand side of (A.17) as

$$
\begin{aligned}
\int_0^{\widehat{m}^{\rm eqm}} q^{\rm eqm}(m){\rm d}G(m) &= \int_0^{\widehat{m}^{\rm eqm}} \psi\left(\frac{m}{\widehat{m}^{\rm eqm}}\right)g(m){\rm d}m\\
&= \int_0^{\widehat{m}^{\rm eqm}}\int_0^{\psi(m/\widehat{m}^{\rm eqm})} g(m){\rm d}z{\rm d}m\\
&= \int_0^{\widehat{m}^{\rm eqm}}\int_{m/\widehat{m}^{\rm eqm}}^1 (-\psi'(t))\,g(m){\rm d}t{\rm d}m
\end{aligned}
$$

$$= \int_0^1 \int_0^{\widehat{m}^{\text{eqm}} t} (-\psi'(t))\, g(m) \mathrm{d}m \mathrm{d}t$$

$$= G(\widehat{m}^{\text{eqm}}) \int_0^1 (-\psi'(t)) t^k \mathrm{d}t,$$

where the third equality follows from changing the integration variable as $z = \psi(t)$, and the integrability enables us to apply Fubini's theorem in the fourth equality. Similarly, the right-hand side of (A.17) can be expressed as

$$\int_0^{\widehat{m}^{\text{opt}}} q^{\text{opt}}(m) \mathrm{d}G(m) = G(\widehat{m}^{\text{opt}}) \int_0^1 (-\phi'(t)) t^k \mathrm{d}t.$$

Thus, using these expressions, the gap between the equilibrium and optimum levels of Q becomes

$$\frac{Q^{\text{eqm}}}{Q^{\text{opt}}} = \frac{N^{\text{eqm}} G(\widehat{m}^{\text{eqm}}) \int_0^1 (-\psi'(t)) t^k \mathrm{d}t}{N^{\text{opt}} G(\widehat{m}^{\text{opt}}) \int_0^1 (-\phi'(t)) t^k \mathrm{d}t} = \left(\frac{\Phi}{\Psi}\right)^{\frac{k}{k+1}} \frac{\int_0^1 (-\psi'(t)) t^k \mathrm{d}t}{\int_0^1 (-\phi'(t)) t^k \mathrm{d}t} < 1, \tag{A.19}$$

which is independent of L because Ψ, Φ, $\int_0^1 (-\psi'(t)) t^k \mathrm{d}t$, and $\int_0^1 (-\phi'(t)) t^k \mathrm{d}t$ are positive constants that depend on exogenous choices of the Pareto shape parameter and the functional form of subutility. Finally, from (A.18) and (A.19), the gap between the equilibrium and optimum levels of $\bar{q} = Q/n$ can be expressed as

$$\frac{\bar{q}^{\text{eqm}}}{\bar{q}^{\text{opt}}} = \frac{\int_0^1 (-\psi'(t)) t^k \mathrm{d}t}{\int_0^1 (-\phi'(t)) t^k \mathrm{d}t} < 1,$$

which is also independent of L.

A.4 Proof of Lemma 4

As the choke price in country i satisfies $\widehat{p}_i = \tau_{ji} \widehat{m}_{ji} w_j$, the inverse demand in country j can be written as $p_{ji}(m) = \tau_{ji} \widehat{m}_{ji} w_j u'(q_{ji}(m))/\bar{u}$. Substituting this into (33), we obtain

$$\sum_j \tau_{ji} \widehat{m}_{ji} w_j N_j \int_0^{\widehat{m}_{ji}} \frac{q_{ji}(m) u'(q_{ji}(m))}{\bar{u}} \mathrm{d}G_j(m) = w_i. \tag{A.20}$$

The integral included in the left-hand side of (A.20) is calculated as

$$\int_0^{\widehat{m}_{ji}} \frac{q_{ji}(m) u'(q_{ji}(m))}{\bar{u}} \mathrm{d}G_j(m) = \int_0^{\widehat{m}_{ji}} \int_0^{q_{ji}(m)} \frac{u'(z) + z u''(z)}{\bar{u}} g_j(m) \mathrm{d}z \mathrm{d}m$$

$$= \int_0^{\widehat{m}_{ji}} \int_0^{\psi(m/\widehat{m}_{ji})} \psi^{-1}(z) g_j(m) \mathrm{d}z \mathrm{d}m$$

$$= \int_0^{\widehat{m}_{ji}} \int_{m/\widehat{m}_{ji}}^1 t(-\psi'(t)) g_j(m) \mathrm{d}t \mathrm{d}m$$

$$= \int_0^1 \int_0^{\widehat{m}_{ji}t} t(-\psi'(t)) g_j(m) \mathrm{d}m \mathrm{d}t$$
$$= \int_0^1 t(-\psi'(t)) G_j\left(\widehat{m}_{ji}t\right) \mathrm{d}t,$$

where the third equality follows from changing the variable as $z = \psi(t)$, and the integrability enables us to apply Fubini's theorem in the fourth equality. Plugging this back into (A.20) yields

$$\sum_j \tau_{ji} \widehat{m}_{ji} w_j N_j \int_0^1 t(-\psi'(t)) G_j\left(\widehat{m}_{ji}t\right) \mathrm{d}t = w_i. \tag{A.21}$$

Moreover, by using integration by parts, the integral included in the left-hand side of (A.21) can be rewritten as

$$\int_0^1 t(-\psi'(t)) G_j\left(\widehat{m}_{ji}t\right) \mathrm{d}t = -\left[t\psi(t) G_j(\widehat{m}_{ji}t)\right]_0^1 + \int_0^1 \psi(t)(\widehat{m}_{ji} t g_j(\widehat{m}_{ji}t) + G_j(\widehat{m}_{ji}t)) \mathrm{d}t. \tag{A.22}$$

It is straightforward to verify that $\lim_{t\to 1} t\psi(t) G_j(\widehat{m}_{ji}t) = 0$ holds and, if $\lim_{t\to 0} \psi(t) < \infty$ is satisfied, $\lim_{t\to 0} t\psi(t) G_j(\widehat{m}_{ji}t) = 0$ also holds. If $\lim_{t\to 0} \psi(t) = \infty$ holds, we have $\lim_{t\to 0} t\psi(t) = \widehat{m}_{ji}^{-1} \lim_{m\to 0} m q_{ji}(m) = \infty$, and it can be shown that $m q_{ji}(m)$ decreases in m when m is close to zero (see footnote A.1). Then, Assumption 5 and Lemma A.1 imply that $\lim_{t\to 0} t\psi(t) G_j(\widehat{m}_{ji}t) = 0$ holds even when $\lim_{t\to 0} \psi(t) = \infty$. Hence, the surface term in the right-hand side of (A.22) vanishes, resulting in

$$\int_0^1 t(-\psi'(t)) G_j\left(\widehat{m}_{ji}t\right) \mathrm{d}t = \int_0^1 \psi(t) G_j(\widehat{m}_{ji}t) \left(1 + \mathcal{E}_j^G(\widehat{m}_{ji}t)\right) \mathrm{d}t, \tag{A.23}$$

where $\mathcal{E}_j^G(m) = m g_j(m)/G_j(m)$ represents the elasticity of the distribution function $G_j(m)$. Consequently, (A.21), (A.23), and (29) imply that (33) can be transformed into (35).

The left-hand side of (30) can be written as

$$\sum_j \int_0^{\widehat{m}_{ij}} \pi_{ij}(m) \mathrm{d}G_i(m) = \sum_j L_j \int_0^{\widehat{m}_{ij}} p_{ij}(m) q_{ij}(m) \mathrm{d}G_i(m)$$
$$- \sum_j \tau_{ij} w_i L_j \int_0^{\widehat{m}_{ij}} m q_{ij}(m) \mathrm{d}G_i(m). \tag{A.24}$$

It is already shown that the integral included in the first term of the right-hand side of (A.5) can be calculated as $\int_0^{\widehat{m}_{ij}} p_{ij}(m) q_{ij}(m) \mathrm{d}G_i(m) = \tau_{ij} \widehat{m}_{ij} w_i \int_0^1 t(-\psi'(t)) G_i(\widehat{m}_{ij}t) \mathrm{d}t$. The integral included in the second term is computed as

$$\int_0^{\widehat{m}_{ij}} m q_{ij}(m) \mathrm{d}G_i(m) = \int_0^{\widehat{m}_{ij}} m\psi\left(\frac{m}{\widehat{m}_{ij}}\right) g_i(m) \mathrm{d}m$$
$$= \widehat{m}_{ij}^2 \int_0^1 t\psi(t) g_i(\widehat{m}_{ij}t) \mathrm{d}t$$

$$
\begin{aligned}
&= \widehat{m}_{ij} \underbrace{\left[t\psi(t)G_i(\widehat{m}_{ij}t)\right]_0^1}_{=0} - \widehat{m}_{ij} \int_0^1 (t\psi'(t) + \psi(t))G_i(\widehat{m}_{ij}t)\mathrm{d}t \\
&= \widehat{m}_{ij} \int_0^1 t(-\psi'(t))G_i\left(\widehat{m}_{ij}t\right) - \widehat{m}_{ij} \int_0^1 \psi(t)G_i(\widehat{m}_{ij}t)\mathrm{d}t,
\end{aligned}
$$

where we change the variable as $m = \widehat{m}_{ij}t$ in the second line, and the surface term that appears in the third line has already shown to be zero. As a result, (A.24) becomes

$$
\sum_j \int_0^{\widehat{m}_{ij}} \pi_{ij}(m)\mathrm{d}G_i(m) = \sum_j \tau_{ij}\widehat{m}_{ij}w_iL_j \int_0^1 \psi(t)G_i(\widehat{m}_{ij}t)\mathrm{d}t.
$$

After substituting (29) into the right-hand side of this equation, (30) can be transformed into (34).

The left-hand side of (32) can be rewritten as

$$
N_i \sum_j L_j \int_0^{\widehat{m}_{ij}} p_{ij}(m)q_{ij}(m)\mathrm{d}G_i(m) = N_i \sum_j \tau_{ij}\widehat{m}_{ij}w_iL_j \int_0^1 \psi(t)G_i(\widehat{m}_{ij}t)\left(1 + \mathcal{E}_i^G(\widehat{m}_{ij}t)\right)\mathrm{d}t, \tag{A.25}
$$

where we use $\int_0^{\widehat{m}_{ij}} p_{ij}(m)q_{ij}(m)\mathrm{d}G_i(m) = \tau_{ij}\widehat{m}_{ij}w_i \int_0^1 \psi(t)G_i(\widehat{m}_{ij}t)\left(1 + \mathcal{E}_i^G(\widehat{m}_{ij}t)\right)\mathrm{d}t$, which has already been shown to hold. The right-hand side of (32) can be transformed as

$$
\begin{aligned}
&L_i \sum_j N_j \int_0^{\widehat{m}_{ji}} p_{ji}(m)q_{ji}(m)\mathrm{d}G_j(m) \\
&= L_i \sum_j \tau_{ji}\widehat{m}_{ji}w_jN_j \int_0^1 \psi(t)G_j(\widehat{m}_{ji}t)\left(1 + \mathcal{E}_j^G(\widehat{m}_{ji}t)\right)\mathrm{d}t \\
&= w_iL_i\widehat{m}_{ii} \sum_j N_j \int_0^1 \psi(t)G_j\left(\frac{\widehat{m}_{ii}w_it}{\tau_{ji}w_j}\right)\left(1 + \mathcal{E}_j^G\left(\frac{\widehat{m}_{ii}w_it}{\tau_{ji}w_j}\right)\right)\mathrm{d}t \\
&= w_iL_i,
\end{aligned} \tag{A.26}
$$

where we use $\int_0^{\widehat{m}_{ji}} p_{ji}(m)q_{ji}(m)\mathrm{d}G_j(m) = \tau_{ji}\widehat{m}_{ji}w_j \int_0^1 \psi(t)G_j(\widehat{m}_{ji}t)\left(1 + \mathcal{E}_j^G(\widehat{m}_{ji}t)\right)\mathrm{d}t$ in the second line; we substitute (29) in the third line; and the last equality follows from (35). Then, (A.25), (A.26), and (29) imply that (32) can be transformed into (36). Therefore, $\{w_i\}_{i=1}^J$, $\{\widehat{m}_{ii}\}_{i=1}^J$, and $\{N_i\}_{i=1}^J$ are determined in the system of equations (34), (35), and (36).

Each worker's utility in country i can be computed as

$$
\begin{aligned}
U_i &= \sum_j N_j \int_0^{\widehat{m}_{ji}} u\left(\psi\left(\frac{m}{\widehat{m}_{ji}}\right)\right)g_j(m)\mathrm{d}m \\
&= \sum_j N_j \int_0^{\widehat{m}_{ji}} \int_0^{\psi(m/\widehat{m}_{ji})} u'(z)g_j(m)\mathrm{d}z\mathrm{d}m
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_j N_j \int_0^{\widehat{m}_{ji}} \int_{m/\widehat{m}_{ji}}^1 u'(\psi(t))(-\psi'(t)) g_j(m) \mathrm{d}t\mathrm{d}m \\
&= \sum_j N_j \int_0^1 \int_0^{\widehat{m}_{ji}t} u'(\psi(t))(-\psi'(t)) g_j(m) \mathrm{d}m\mathrm{d}t \\
&= -\sum_j N_j \int_0^1 u'(\psi(t))\psi'(t) G_j(\widehat{m}_{ji}t)\mathrm{d}t \\
&= -\bar{u} \sum_j N_j \int_0^1 t\psi'(t) G_j(\widehat{m}_{ji}t)\mathrm{d}t + \sum_j N_j \int_0^1 \psi(t) u''(\psi(t))\psi'(t) G_j(\widehat{m}_{ji}t)\mathrm{d}t \\
&= \bar{u} \sum_j N_j \int_0^1 \psi(t) G_j\left(\frac{\widehat{m}_{ii} w_i t}{\tau_{ji} w_j}\right)\left(1 + \mathcal{E}_j^G\left(\frac{\widehat{m}_{ii} w_i t}{\tau_{ji} w_j}\right)\right)\mathrm{d}t \\
&\quad + \sum_j N_j \int_0^1 \psi(t) u''(\psi(t))\psi'(t) G_j(\widehat{m}_{ji}t)\mathrm{d}t \\
&= \frac{\bar{u}}{\widehat{m}_{ii}} + \sum_j N_j \int_0^1 \psi(t) u''(\psi(t))\psi'(t) G_j(\widehat{m}_{ji}t)\mathrm{d}t
\end{aligned}
\tag{A.27}
$$

where the third equality follows from changing the variable as $z = \psi(t)$; the integrability enables us to apply Fubini's theorem in the fourth equality; the sixth equality follows from the relationship $u'(\psi(t)) = \bar{u}t - \psi(t)u''(\psi(t))$, which is derived from (4) and (5); the seventh equality follows from (A.23) and (29); and the last equality follows from (35). Similar to the case wherein we calculated (A.7) in Appendix A.1, applying integration by parts enables us to transform the integral included in the right-hand side of (A.27) into

$$
\int_0^1 \psi(t) u''(\psi(t))\psi'(t) G_j(\widehat{m}_{ji}t)\mathrm{d}t = \int_0^1 (u(\psi(t)) - \psi(t)u'(\psi(t)))\widehat{m}_{ji} g_j(\widehat{m}_{ji}t)\mathrm{d}t.
$$

Substituting this back into (A.27), we obtain

$$
U_i = \frac{\bar{u}}{\widehat{m}_{ii}} + \sum_j N_j \int_0^1 \frac{u(\psi(t))(1 - \mathcal{E}^u(\psi(t)))\mathcal{E}_j^G(\widehat{m}_{ji}t) G_j(\widehat{m}_{ji}t)}{t}\mathrm{d}t, \tag{A.28}
$$

where we let $\mathcal{E}^u(x) \equiv xu'(x)/u(x)$ represent the elasticity of the subutility. After plugging (29) into (A.28), the equilibrium utility can be transformed into (37).

A.5 Proof of Proposition 3

Divided by $w_i L_i$, (34) can be rewritten as

$$
\widehat{m}_{ii} \int_0^1 \psi(t) G_i(\widehat{m}_{ii}t)\mathrm{d}t + \sum_{j \neq i} \widehat{m}_{jj} \frac{w_j L_j}{w_i L_i} \int_0^1 \psi(t) G_i\left(\frac{\widehat{m}_{jj} w_j t}{\tau_{ij} w_i}\right)\mathrm{d}t = \frac{f_i}{L_i},
$$

indicating that $\widehat{m}_{ii} \int_0^1 \psi(t) G_i(\widehat{m}_{ii}t)\mathrm{d}t < f_i/L_i$ holds.

Let $\widehat{m}_i^a$ represent the cost cutoff in the autarkic equilibrium. Then, (10) implies that $\widehat{m}_i^a$ satisfies

$$\widehat{m}_i^a \int_0^1 \psi(t) G_i(\widehat{m}_i^a t) \mathrm{d}t = \frac{f_i}{L_i},$$

from which the following inequality follows:

$$\widehat{m}_{ii} \int_0^1 \psi(t) G_i(\widehat{m}_{ii} t) \mathrm{d}t < \widehat{m}_i^a \int_0^1 \psi(t) G_i(\widehat{m}_i^a t) \mathrm{d}t. \tag{A.29}$$

If we let $\mathcal{F}_i(x) \equiv x \int_0^1 \psi(t) G_i(xt) \mathrm{d}t$, (A.29) can be expressed as $\mathcal{F}_i(\widehat{m}_{ii}) < \mathcal{F}_i(\widehat{m}_i^a)$. As already shown in Appendix A.1, $\mathcal{F}_i(x)$ is strictly increasing for $x > 0$. Thus, (A.29) indicates that $\widehat{m}_{ii} < \widehat{m}_i^a$ holds.

B Two-Country Model

B.1 Gains from Opening Up to Trade

First, we prove that the relative wage, $\omega = w_H/w_F$, is uniquely determined by (49). In a two-country economy, equation (43) for $i = H$ becomes

$$\begin{aligned}
&\frac{L_H}{\frac{L_H}{f_H M_H^k} + \frac{L_F}{f_F M_F^k} \tau_{FH}^{-k} \left(\frac{w_H}{w_F}\right)^k} + \frac{L_F \tau_{HF}^{-k} \left(\frac{w_F}{w_H}\right)^{k+1}}{\frac{L_F}{f_F M_F^k} + \frac{L_H}{f_H M_H^k} \tau_{HF}^{-k} \left(\frac{w_F}{w_H}\right)^k} = f_H M_H^k \\
\Leftrightarrow\ &\frac{1}{1 + \frac{f_H M_H^k}{L_H} \frac{L_F}{f_F M_F^k} \tau_{FH}^{-k} \left(\frac{w_H}{w_F}\right)^k} + \frac{\frac{f_F M_F^k}{f_H M_H^k} \tau_{HF}^{-k} \left(\frac{w_F}{w_H}\right)^{k+1}}{1 + \frac{f_F M_F^k}{L_F} \frac{L_H}{f_H M_H^k} \tau_{HF}^{-k} \left(\frac{w_F}{w_H}\right)^k} = 1 \\
\Leftrightarrow\ &\frac{1}{1 + \frac{\Theta_c}{\Theta_L} \rho_{FH} \omega^k} + \frac{\frac{1}{\Theta_c} \rho_{HF} \omega^{-(k+1)}}{1 + \frac{\Theta_L}{\Theta_c} \rho_{HF} \omega^{-k}} = 1 \\
\Leftrightarrow\ &\frac{1 + \frac{\Theta_c}{\Theta_L} \rho_{FH} \omega^k}{1 + \frac{\Theta_L}{\Theta_c} \rho_{HF} \omega^{-k}} = \frac{\Theta_c^2}{\Theta_L} \frac{\rho_{FH}}{\rho_{HF}} \omega^{2k+1} && \text{(B.1)} \\
\Leftrightarrow\ &\frac{\Theta_L}{\Theta_c} \rho_{FH}^{-1} + \omega^k - \Theta_L \omega^{k+1} - \Theta_c \rho_{HF}^{-1} \omega^{2k+1} = 0, && \text{(B.2)}
\end{aligned}$$

where $\Theta_L = L_H/L_F$, $\Theta_c = f_H M_H^k / f_F M_F^k$, and $\rho_{ij} = \tau_{ij}^{-k} \in (0, 1]$. Note that we can derive (B.2) from (43) even for $i = F$.

We let $Z(\omega)$ denote the left-hand side of (49) (or (B.2)). The differentiation of $Z(\omega)$ yields

$$Z'(\omega) = \omega^{k-1} \underbrace{\left[k - (k+1)\Theta_L \omega - (2k+1)\Theta_c \rho_{HF}^{-1} \omega^{k+1}\right]}_{\equiv \widetilde{Z}(\omega)},$$

where $\widetilde{Z}(\omega)$, defined as the term in the brackets, is a strictly decreasing function. Because $\lim_{\omega \to 0} \widetilde{Z}(\omega) = k > 0$ and $\lim_{\omega \to \infty} \widetilde{Z}(\omega) = -\infty$ hold, a solution to the equation $\widetilde{Z}(\omega) = 0$ can be uniquely determined, and we denote the solution by $\widetilde{\omega}$. Then, we have

$$Z'(\omega) \gtreqless 0 \Leftrightarrow \widetilde{Z}(\omega) \gtreqless 0 \Leftrightarrow \omega \lesseqgtr \widetilde{\omega} \quad \text{for} \quad \omega > 0. \tag{B.3}$$

Therefore, $Z(\omega)$ is positive at $\omega = 0$, increases in ω for $\omega \in (0, \widetilde{\omega})$, and decreases for $\omega > \widetilde{\omega}$. This implies that a solution for equation (B.2), $Z(\omega) = 0$, must be larger than $\widetilde{\omega}$. We further define two values as $\underline{\omega} \equiv \Theta_c^{-1/(k+1)} \rho_{HF}^{1/(k+1)}$ and $\overline{\omega} \equiv \Theta_c^{-1/(k+1)} \rho_{FH}^{-1/(k+1)}$, where $\underline{\omega} \leq \overline{\omega}$ because $\rho_{ij} \in (0, 1]$. Noting that $Z'(\underline{\omega}) = -\underline{\omega}^{k-1}(k+1)(1 + \Theta_L \underline{\omega}) < 0$ holds, we have $\widetilde{\omega} < \underline{\omega} (\leq \overline{\omega})$ from (B.3). Substituting $\underline{\omega}$ and $\overline{\omega}$ into $Z(\omega)$ respectively yields

$$Z(\underline{\omega}) = \frac{\Theta_L}{\Theta_c} \left(\frac{1}{\rho_{FH}} - \rho_{HF} \right) \geq 0 \quad \text{and} \quad Z(\overline{\omega}) = -\left(\frac{1}{\rho_{HF}\rho_{FH}} - 1 \right) \overline{\omega}^k \leq 0, \tag{B.4}$$

where the equality holds true only when $\rho_{FH} = \rho_{HF} = 1$ in both inequalities. Hence, as $Z(\omega)$ is a decreasing function for $\omega > \widetilde{\omega}$ and the relationship $\widetilde{\omega} < \underline{\omega} \leq \overline{\omega}$ holds, the expressions in (B.4) imply that the solution for (49) (or (B.2)) lies in $[\underline{\omega}, \overline{\omega}]$ and is uniquely determined. We denote the equilibrium relative wage by ω^*. Note that if there is no trade cost (i.e., $\rho_{FH} = \rho_{HF} = 1$), we have $\underline{\omega} = \overline{\omega} = \Theta_c^{-1/(k+1)}$ and $Z(\Theta_c^{-1/(k+1)}) = 0$, meaning that the equilibrium relative wage becomes $\omega^* = \Theta_c^{-1/(k+1)}$ for $\rho_{FH} = \rho_{HF} = 1$.

We then show that gains from trade in each country are ranked as in (52). At the equilibrium, (50) and (51) indicate the following relationship:

$$\begin{aligned} \frac{U_H}{U_H^a} \gtreqless \frac{U_F}{U_F^a} &\Leftrightarrow 1 + \frac{\Theta_c}{\Theta_L} \rho_{FH} (\omega^*)^k \gtreqless 1 + \frac{\Theta_L}{\Theta_c} \rho_{HF} (\omega^*)^{-k} \\ &\Leftrightarrow \frac{\Theta_c^2}{\Theta_L} \frac{\rho_{FH}}{\rho_{HF}} (\omega^*)^{2k+1} \gtreqless 1 \\ &\Leftrightarrow \omega^* \gtreqless \left(\frac{\rho_{HF}}{\rho_{FH}} \right)^{\frac{1}{2k+1}} \left(\frac{\Theta_L}{\Theta_c^2} \right)^{\frac{1}{2k+1}} \equiv \omega_1, \end{aligned}$$

where we use (B.1) at the second line. As it is already shown that $\omega \gtreqless \omega^* \Leftrightarrow Z(\omega) \lesseqgtr 0$, this inequality can be written as

$$\omega^* \gtreqless \omega_1 \Leftrightarrow Z(\omega_1) \gtreqless 0 \Leftrightarrow \Theta_L \lesseqgtr \left(\frac{\rho_{FH}}{\rho_{HF}} \right)^{\frac{1}{2(k+1)}} \Theta_c^{\frac{1}{k+1}}.$$

Therefore, we obtain (52):

$$\frac{U_H}{U_H^a} \gtreqless \frac{U_F}{U_F^a} \Leftrightarrow \Theta_L \lesseqgtr \left(\frac{\rho_{FH}}{\rho_{HF}} \right)^{\frac{1}{2(k+1)}} \Theta_c^{\frac{1}{k+1}},$$

which implies that both a smaller (larger) market size and lower (higher) technological level are factors that contribute to relatively greater (smaller) welfare gains from trade in that country.

Even in absolute terms, welfare gains from trade is greater (smaller) in a country with a smaller (larger) market size and lower (higher) technological level. Noting that the relative wage ($\omega = w_H/w_F$) depends on the relative market size ($\Theta_L = L_H/L_F$) and relative cost index ($\Theta_c = f_H M_H^k / f_F M_F^k$) from (49) (or (B.2)), expressions (50) and (51) reveal that a country's gains from trade can also be expressed in terms of Θ_L and Θ_c. From (50), the effects of these components on the magnitude of gains from trade in the home country are

$$\frac{\mathrm{d}(U_H/U_H^a)}{\mathrm{d}\Theta_L} = \frac{1}{k+1}\left[1+\frac{\Theta_c}{\Theta_L}\rho_{FH}(\omega^*)^k\right]^{\frac{1}{k+1}-1}\frac{\Theta_c}{\Theta_L^2}\rho_{FH}(\omega^*)^k\left(-1+\frac{k\Theta_L}{\omega^*}\frac{\mathrm{d}\omega^*}{\mathrm{d}\Theta_L}\right), \tag{B.5}$$

$$\frac{\mathrm{d}(U_H/U_H^a)}{\mathrm{d}\Theta_c} = \frac{1}{k+1}\left[1+\frac{\Theta_c}{\Theta_L}\rho_{FH}(\omega^*)^k\right]^{\frac{1}{k+1}-1}\frac{\rho_{FH}(\omega^*)^k}{\Theta_L}\left(1+\frac{k\Theta_c}{\omega^*}\frac{\mathrm{d}\omega^*}{\mathrm{d}\Theta_c}\right). \tag{B.6}$$

Applying the implicit function theorem to (B.2), we obtain

$$\frac{\mathrm{d}\omega^*}{\mathrm{d}\Theta_L} = -\frac{\frac{1}{\Theta_c}\rho_{FH}^{-1} - (\omega^*)^{k+1}}{k(\omega^*)^{k-1} - (k+1)\Theta_L(\omega^*)^k - (2k+1)\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k}}, \tag{B.7}$$

$$\frac{\mathrm{d}\omega^*}{\mathrm{d}\Theta_c} = -\frac{-\frac{\Theta_L}{\Theta_c^2}\rho_{FH}^{-1} - \rho_{HF}^{-1}(\omega^*)^{2k+1}}{k(\omega^*)^{k-1} - (k+1)\Theta_L(\omega^*)^k - (2k+1)\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k}}. \tag{B.8}$$

Using (B.2), the denominators in (B.7) and (B.8) can be rewritten as

$$\begin{aligned} & k(\omega^*)^{k-1} - (k+1)\Theta_L(\omega^*)^k - (2k+1)\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k} \\ &= (\omega^*)^{-1}\Big\{(k+1)\underbrace{\left[(\omega^*)^k - \Theta_L(\omega^*)^{k+1} - \Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}\right]}_{=-\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} \text{ by (B.2)}} - (\omega^*)^k - k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}\Big\} \\ &= -(\omega^*)^{-1}\left[(k+1)\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} + (\omega^*)^k + k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}\right] < 0. \end{aligned} \tag{B.9}$$

Moreover, as for the sign of the numerator in (B.7), we have

$$\mathrm{sgn}\left\{\frac{1}{\Theta_c}\rho_{FH}^{-1} - (\omega^*)^{k+1}\right\} = \mathrm{sgn}\Big\{\underbrace{(\rho_{FH}\Theta_c)^{-\frac{1}{k+1}}}_{=\overline{\omega}} - \omega^*\Big\}.$$

Noting that $\omega^* \leq \overline{\omega}$ from (B.4) (where $\omega^* = \overline{\omega}$ holds only when $\rho_{FH} = \rho_{HF} = 1$), the numerator in (B.7) is non-negative:

$$\frac{1}{\Theta_c}\rho_{FH}^{-1} - (\omega^*)^{k+1} \geq 0. \tag{B.10}$$

Hence, from (B.7), (B.8), (B.9) and (B.10), we obtain

$$\frac{\mathrm{d}\omega^*}{\mathrm{d}\Theta_L} = \frac{\frac{1}{\Theta_c}\rho_{FH}^{-1} - (\omega^*)^{k+1}}{(\omega^*)^{-1}\left[(k+1)\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} + (\omega^*)^k + k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}\right]} \geq 0, \tag{B.11}$$

$$\frac{\mathrm{d}\omega^*}{\mathrm{d}\Theta_c} = \frac{-\frac{\Theta_L}{\Theta_c^2}\rho_{FH}^{-1} - \rho_{HF}^{-1}(\omega^*)^{2k+1}}{(\omega^*)^{-1}\left[(k+1)\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} + (\omega^*)^k + k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}\right]} < 0. \tag{B.12}$$

Expression (B.11) shows that the larger the home country's market size (or the smaller the foreign country's market size), the higher the home country's wage in relative terms. Similarly, expression (B.12) shows that the higher the home country's technological level (or the lower the foreign country's technological level), the higher the home country's wage in relative terms. The same can be true for the foreign country.

Consequently, substituting (B.11) and (B.12) into (B.5) and (B.6), respectively, we obtain

$$\begin{aligned}
\frac{\mathrm{d}(U_H/U_H^a)}{\mathrm{d}\Theta_L} &= -\frac{1}{k+1}\left[1+\frac{\Theta_c}{\Theta_L}\rho_{FH}(\omega^*)^k\right]^{\frac{1}{k+1}-1}\frac{\Theta_c}{\Theta_L^2}\rho_{FH}(\omega^*)^k \\
&\quad\times\frac{k\left[\Theta_L(\omega^*)^{k+1}+\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}\right]+\left[\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1}+(\omega^*)^k\right]}{(k+1)\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1}+(\omega^*)^k+k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}} \\
&= -\left[1+\frac{\Theta_c}{\Theta_L}\rho_{FH}(\omega^*)^k\right]^{\frac{1}{k+1}-1}\frac{\Theta_c}{\Theta_L^2}\rho_{FH}(\omega^*)^k \\
&\quad\times\frac{\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1}+(\omega^*)^k}{(k+1)\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1}+(\omega^*)^k+k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}} < 0,
\end{aligned} \tag{B.13}$$

$$\begin{aligned}
\frac{\mathrm{d}(U_H/U_H^a)}{\mathrm{d}\Theta_c} &= \frac{1}{k+1}\left[1+\frac{\Theta_c}{\Theta_L}\rho_{FH}(\omega^*)^k\right]^{\frac{1}{k+1}-1}\frac{\rho_{FH}(\omega^*)^k}{\Theta_L} \\
&\quad\times\frac{\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1}+(\omega^*)^k}{(k+1)\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1}+(\omega^*)^k+k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}} > 0.
\end{aligned} \tag{B.14}$$

where we use (B.2) to get (B.13). Therefore, expression (B.13) shows that the smaller the home country's market size (or the larger the foreign country's market size), the greater the home country's gains from trade in absolute terms. Similarly, expression (B.14) shows that the lower the home country's technological level (or the higher the foreign country's technological level), the greater the home country's gains from trade in absolute terms. The same can be true for the foreign country.

Finally, we drive (54). At the equilibrium, (53) indicates that

$$\begin{aligned}
\frac{U_H}{U_F} \gtreqless 1 &\Leftrightarrow \frac{\Theta_L}{\Theta_c}\frac{1+\frac{\Theta_c}{\Theta_L}\rho_{FH}(\omega^*)^k}{1+\frac{\Theta_L}{\Theta_c}\rho_{HF}(\omega^*)^{-k}} \gtreqless 1 \\
&\Leftrightarrow \Theta_c\frac{\rho_{FH}}{\rho_{HF}}(\omega^*)^{2k+1} \gtreqless 1 \\
&\Leftrightarrow \omega^* \gtreqless \left(\frac{\rho_{HF}}{\rho_{FH}}\right)^{\frac{1}{2k+1}}\Theta_c^{-\frac{1}{2k+1}} \equiv \omega_2,
\end{aligned}$$

where (B.1) is used at the second line. As it is already shown that $\omega \gtreqless \omega^* \Leftrightarrow Z(\omega) \lesseqgtr 0$, this inequality can be written as

$$\begin{aligned}
&\omega^* \gtreqless \omega_2 \Leftrightarrow Z(\omega_2) \gtreqless 0 \\
&\quad \Leftrightarrow \frac{\rho_{FH}^{-1}}{\Theta_c}\left(\Theta_L + \rho_{FH}^{\frac{k+1}{2k+1}}\rho_{HF}^{\frac{k}{2k+1}}\Theta_c^{\frac{k+1}{2k+1}} - \Theta_L\rho_{FH}^{\frac{k}{2k+1}}\rho_{HF}^{\frac{k+1}{2k+1}}\Theta_c^{\frac{k}{2k+1}} - \Theta_c\right) \gtreqless 0 \\
&\quad \Leftrightarrow \Theta_L\rho_{FH}^{\frac{k}{2k+1}}\rho_{HF}^{\frac{k+1}{2k+1}}\left(\rho_{FH}^{-\frac{k}{2k+1}}\rho_{HF}^{-\frac{k+1}{2k+1}} - \Theta_c^{\frac{k}{2k+1}}\right) + \Theta_c^{\frac{k+1}{2k+1}}\left(\rho_{FH}^{\frac{k+1}{2k+1}}\rho_{HF}^{\frac{k}{2k+1}} - \Theta_c^{\frac{k}{2k+1}}\right) \gtreqless 0.
\end{aligned}$$

Therefore, we obtain (54):

$$\frac{U_H}{U_F} \gtreqless 1 \Leftrightarrow \Theta_L\rho_{FH}^{\frac{k}{2k+1}}\rho_{HF}^{\frac{k+1}{2k+1}}\left(\rho_{FH}^{-\frac{k}{2k+1}}\rho_{HF}^{-\frac{k+1}{2k+1}} - \Theta_c^{\frac{k}{2k+1}}\right) + \Theta_c^{\frac{k+1}{2k+1}}\left(\rho_{FH}^{\frac{k+1}{2k+1}}\rho_{HF}^{\frac{k}{2k+1}} - \Theta_c^{\frac{k}{2k+1}}\right) \gtreqless 0.$$

Noting that $\rho_{FH}, \rho_{HF} \leq 1$, if there is no trade cost (i.e., $\rho_{FH} = \rho_{HF} = 1$), then we have

$$\frac{U_H}{U_F} \gtreqless 1 \Leftrightarrow 1 - \Theta_c^{\frac{k}{2k+1}} \gtreqless 0 \Leftrightarrow \Theta_c \lesseqgtr 1.$$

Otherwise (i.e., if $\rho_{FH} < 1$ or $\rho_{HF} < 1$), it is straightforward to show that

$$\frac{U_H}{U_F}\begin{cases} > 1 & \text{for} \quad \Theta_c^{\frac{k}{2k+1}} \leq \rho_{FH}^{\frac{k+1}{2k+1}}\rho_{HF}^{\frac{k}{2k+1}}\left(\Leftrightarrow \Theta_c \leq \rho_{FH}^{\frac{k+1}{k}}\rho_{HF}\right) \\ < 1 & \text{for} \quad \Theta_c^{\frac{k}{2k+1}} \geq \rho_{FH}^{-\frac{k}{2k+1}}\rho_{HF}^{-\frac{k+1}{2k+1}}\left(\Leftrightarrow \Theta_c \geq \rho_{FH}^{-1}\rho_{HF}^{-\frac{k+1}{k}}\right) \end{cases}. \tag{B.15}$$

For $\Theta_c^{\frac{k}{2k+1}} \in (\rho_{FH}^{\frac{k+1}{2k+1}}\rho_{HF}^{\frac{k}{2k+1}}, \rho_{FH}^{-\frac{k}{2k+1}}\rho_{HF}^{-\frac{k+1}{2k+1}})$ (or, for $\Theta_c \in (\rho_{FH}^{(k+1)/k}\rho_{HF}, \rho_{FH}^{-1}\rho_{HF}^{-(k+1)/k})$), the first and second terms in the left-hand side of (54) are positive and negative, respectively. Thus, we obtain

$$\frac{U_H}{U_F} \gtreqless 1 \Leftrightarrow \Theta_L \gtreqless \frac{\Theta_c^{\frac{k+1}{2k+1}}\left(\Theta_c^{\frac{k}{2k+1}} - \rho_{FH}^{\frac{k+1}{2k+1}}\rho_{HF}^{\frac{k}{2k+1}}\right)}{\rho_{FH}^{\frac{k}{2k+1}}\rho_{HF}^{\frac{k+1}{2k+1}}\left(\rho_{FH}^{-\frac{k}{2k+1}}\rho_{HF}^{-\frac{k+1}{2k+1}} - \Theta_c^{\frac{k}{2k+1}}\right)} \quad \text{for} \quad \Theta_c \in \left(\rho_{FH}^{\frac{k+1}{k}}\rho_{HF}, \rho_{FH}^{-1}\rho_{HF}^{-\frac{k+1}{k}}\right), \tag{B.16}$$

where the right-hand side of (B.16) is strictly increasing with $\Theta_c \in (\rho_{FH}^{(k+1)/k}\rho_{HF}, \rho_{FH}^{-1}\rho_{HF}^{-(k+1)/k})$. Thus, the $U_H/U_F = 1$ curve is depicted within the range $\Theta_c \in (\rho_{FH}^{(k+1)/k}\rho_{HF}, \rho_{FH}^{-1}\rho_{HF}^{-(k+1)/k})$, as shown in Figure 2. From (B.15) and (B.16), the home country has relatively high (low) welfare after opening up to trade if pair (Θ_c, Θ_L) lies to the left (right) of the $U_H/U_F = 1$ curve in Figure 2. The level of Θ_c at the intersection of this curve with the $U_H^a/U_F^a = 1$ line in Figure 2 is

$$\Theta_c = \frac{\Theta_c^{\frac{k+1}{2k+1}}\left(\Theta_c^{\frac{k}{2k+1}} - \rho_{FH}^{\frac{k+1}{2k+1}}\rho_{HF}^{\frac{k}{2k+1}}\right)}{\rho_{FH}^{\frac{k}{2k+1}}\rho_{HF}^{\frac{k+1}{2k+1}}\left(\rho_{FH}^{-\frac{k}{2k+1}}\rho_{HF}^{-\frac{k+1}{2k+1}} - \Theta_c^{\frac{k}{2k+1}}\right)} \Leftrightarrow \Theta_c = \left(\frac{\rho_{FH}}{\rho_{HF}}\right)^{\frac{1}{2k}} \in (\rho_{FH}^{\frac{k+1}{k}}\rho_{HF}, \rho_{FH}^{-1}\rho_{HF}^{-\frac{k+1}{k}}).$$

Therefore, the $U_H/U_F = 1$ curve and the $U_H^a/U_F^a = 1$ line in Figure 2 intersects at $(\Theta_c, \Theta_L) = ((\rho_{FH}/\rho_{HF})^{1/2k}, (\rho_{FH}/\rho_{HF})^{1/2k})$.

B.2 Trade Liberalization

Totally differentiating both sides of (49) (or (B.2)) with respect to ω^*, ρ_{FH}, and ρ_{HF}, we obtain

$$\mathrm{d}\ln\omega^* = \frac{-\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1}\mathrm{d}\ln\rho_{FH} + \Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}\mathrm{d}\ln\rho_{HF}}{(k+1)\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} + (\omega^*)^k + k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}}, \tag{B.17}$$

where $\mathrm{d}\ln x = \mathrm{d}x/x$, and we use (B.9) to rewrite the denominator in the right-hand side. Then, for the impact of unilateral trade liberalization, we obtain

$$\frac{\mathrm{d}\ln\omega^*}{\mathrm{d}\ln\rho_{FH}} = \frac{-\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1}}{(k+1)\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} + (\omega^*)^k + k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}} < 0 \quad \text{for} \quad \mathrm{d}\ln\rho_{HF} = 0,$$
$$\frac{\mathrm{d}\ln\omega^*}{\mathrm{d}\ln\rho_{HF}} = \frac{\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}}{(k+1)\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} + (\omega^*)^k + k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}} > 0 \quad \text{for} \quad \mathrm{d}\ln\rho_{FH} = 0.$$

Recalling that $\omega = w_H/w_F$, these expressions show that wage in the liberalized country falls in relative terms.

Next, we derive (55) and (56). From (42), the expression for the cost cutoff in H and F are

$$\widehat{m}_{HH} = \left(\frac{f_H M_H^k}{L_H\Psi}\right)^{\frac{1}{k+1}}\left[1 + \frac{\Theta_c}{\Theta_L}\rho_{FH}(\omega^*)^k\right]^{-\frac{1}{k+1}} \quad \text{and} \quad \widehat{m}_{FF} = \left(\frac{f_F M_F^k}{L_F\Psi}\right)^{\frac{1}{k+1}}\left[1 + \frac{\Theta_L}{\Theta_c}\rho_{HF}(\omega^*)^{-k}\right]^{-\frac{1}{k+1}} \tag{B.18}$$

Totally differentiating the first equation in (B.18) with respect to $\widehat{m}_{HH}$, ρ_{FH}, and ω^* yields

$$\begin{aligned}
\mathrm{d}\widehat{m}_{HH} &= -\frac{\widehat{m}_{HH}}{k+1}\frac{\frac{\Theta_c}{\Theta_L}}{1 + \frac{\Theta_c}{\Theta_L}\rho_{FH}(\omega^*)^k}\left[(\omega^*)^k\mathrm{d}\rho_{FH} + \rho_{FH}k(\omega^*)^{k-1}\mathrm{d}\omega^*\right] \\
\Leftrightarrow \mathrm{d}\ln\widehat{m}_{HH} &= -\frac{1}{k+1}\frac{\frac{\Theta_c}{\Theta_L}\rho_{FH}(\omega^*)^k}{1 + \frac{\Theta_c}{\Theta_L}\rho_{FH}(\omega^*)^k}(\mathrm{d}\ln\rho_{FH} + k\,\mathrm{d}\ln\omega^*) \\
&= -A_H\left\{\left[\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} + (\omega^*)^k + k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}\right]\mathrm{d}\ln\rho_{FH} + k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}\mathrm{d}\ln\rho_{HF}\right\},
\end{aligned} \tag{B.19}$$

where $\mathrm{d}\ln\omega^*$ is given by (B.17) and

$$A_H \equiv \frac{\frac{\Theta_c}{\Theta_L}\rho_{FH}(\omega^*)^k}{(k+1)\left[1 + \frac{\Theta_c}{\Theta_L}\rho_{FH}(\omega^*)^k\right]\left[(k+1)\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} + (\omega^*)^k + k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}\right]} > 0. \tag{B.20}$$

Similarly, totally differentiating the second equation in (B.18) with respect to $\widehat{m}_{FF}$, ρ_{HF}, and ω^*

yields

$$\begin{aligned}\mathrm{d}\ln\widehat{m}_{FF} &= -\frac{1}{k+1}\frac{\frac{\Theta_L}{\Theta_c}\rho_{HF}(\omega^*)^{-k}}{1+\frac{\Theta_L}{\Theta_c}\rho_{HF}(\omega^*)^{-k}}\left(\mathrm{d}\ln\rho_{HF} - k\,\mathrm{d}\ln\omega^*\right)\\ &= -A_F\left\{k\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1}\mathrm{d}\ln\rho_{FH} + \left[(k+1)\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} + (\omega^*)^k\right]\mathrm{d}\ln\rho_{HF}\right\}, \end{aligned} \tag{B.21}$$

where

$$A_F \equiv \frac{\frac{\Theta_L}{\Theta_c}\rho_{HF}(\omega^*)^{-k}}{(k+1)\left[1+\frac{\Theta_L}{\Theta_c}\rho_{HF}(\omega^*)^{-k}\right]\left[(k+1)\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} + (\omega^*)^k + k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}\right]} > 0. \tag{B.22}$$

From (45), the welfare in a country is expressed as $U_i = [\bar{u} + k\zeta/(k+1)\Psi]\,\widehat{m}_{ii}^{-1}$, where the terms in the square brackets are positive constants. Totally differentiating both sides with respect to U_i and $\widehat{m}_{ii}$, we get $\mathrm{d}\ln U_i = -\mathrm{d}\ln\widehat{m}_{ii}$. Substituting (B.19) and (B.21) into this, we obtain (55) and (56), respectively:

$$\mathrm{d}\ln U_H = A_H\left\{\left[\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} + (\omega^*)^k + k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}\right]\mathrm{d}\ln\rho_{FH} + k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}\mathrm{d}\ln\rho_{HF}\right\}, \tag{B.23}$$

$$\mathrm{d}\ln U_F = A_F\left\{k\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1}\mathrm{d}\ln\rho_{FH} + \left[(k+1)\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} + (\omega^*)^k\right]\mathrm{d}\ln\rho_{HF}\right\}. \tag{B.24}$$

Then, we examine the impact of bilateral trade liberalization. We consider a scenario in which both countries simultaneously lower their trade costs by the same proportion, that is, we assume $\mathrm{d}\ln\rho_{FH} = \mathrm{d}\ln\rho_{HF} \equiv \mathrm{d}\ln\rho > 0$. Then, from (B.23) and (B.24), gains from bilateral trade liberalization in each country can be expressed as

$$\frac{\mathrm{d}\ln U_H}{\mathrm{d}\ln\rho} = A_H\left[\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} + (\omega^*)^k + 2k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}\right], \tag{B.25}$$

$$\begin{aligned}\frac{\mathrm{d}\ln U_F}{\mathrm{d}\ln\rho} &= A_F\left[(2k+1)\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} + (\omega^*)^k\right]\\ &= A_H\Theta_L\omega^*\left[(2k+1)\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1} + (\omega^*)^k\right]. \end{aligned} \tag{B.26}$$

where, from (B.20) and (B.22),

$$A_F = A_H\frac{\frac{\Theta_L}{\Theta_c}\rho_{HF}(\omega^*)^{-k}}{\frac{\Theta_c}{\Theta_L}\rho_{FH}(\omega^*)^{k}}\underbrace{\frac{1+\frac{\Theta_c}{\Theta_L}\rho_{FH}(\omega^*)^{k}}{1+\frac{\Theta_L}{\Theta_c}\rho_{HF}(\omega^*)^{-k}}}_{=\frac{\Theta_c^2}{\Theta_L}\frac{\rho_{FH}}{\rho_{HF}}(\omega^*)^{2k+1}\text{ by (B.1)}} = A_H\Theta_L\omega.$$

The gap between (B.25) and (B.26) becomes

$$
\begin{aligned}
&\frac{\mathrm{d}\ln U_H}{\mathrm{d}\ln\rho}-\frac{\mathrm{d}\ln U_F}{\mathrm{d}\ln\rho}\\
&=A_H\Bigg[\underbrace{\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1}+(\omega^*)^k-\Theta_L(\omega^*)^{k+1}}_{=\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}\text{ by (B.2)}}+2k\Theta_c\rho_{HF}^{-1}(\omega^*)^{2k+1}-(2k+1)\frac{\Theta_L^2}{\Theta_c}\rho_{FH}^{-1}\omega^*\Bigg]\\
&=A_H(2k+1)\Theta_c\rho_{HF}^{-1}\omega^*\left[(\omega^*)^{2k}-\frac{\rho_{HF}}{\rho_{FH}}\left(\frac{\Theta_L}{\Theta_c}\right)^2\right].
\end{aligned}
$$

Thus, we obtain

$$
\frac{\mathrm{d}\ln U_H}{\mathrm{d}\ln\rho}\gtreqless\frac{\mathrm{d}\ln U_F}{\mathrm{d}\ln\rho}\Leftrightarrow\omega^*\gtreqless\left(\frac{\rho_{HF}}{\rho_{FH}}\right)^{\frac{1}{2k}}\left(\frac{\Theta_L}{\Theta_c}\right)^{\frac{1}{k}}\equiv\omega_3.
$$

As it is already shown that $\omega\gtreqless\omega^*\Leftrightarrow Z(\omega)\lesseqgtr 0$, this inequality can be written as

$$
\begin{aligned}
&\omega\gtreqless\omega_3\Leftrightarrow Z(\omega_3)\gtreqless 0\\
&\quad\Leftrightarrow\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1}+\left(\frac{\rho_{HF}}{\rho_{FH}}\right)^{\frac{1}{2}}\frac{\Theta_L}{\Theta_c}-\Theta_L\left(\frac{\rho_{HF}}{\rho_{FH}}\right)^{\frac{k+1}{2k}}\left(\frac{\Theta_L}{\Theta_c}\right)^{\frac{k+1}{k}}-\Theta_c\rho_{HF}^{-1}\left(\frac{\rho_{HF}}{\rho_{FH}}\right)^{\frac{2k+1}{2k}}\left(\frac{\Theta_L}{\Theta_c}\right)^{\frac{2k+1}{k}}\gtreqless 0\\
&\quad\Leftrightarrow\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1}\left[1+(\rho_{FH}\rho_{HF})^{\frac{1}{2}}\right]-\Theta_L^{\frac{2k+1}{k}}\Theta_c^{-\frac{k+1}{k}}\rho_{HF}^{-1}\left(\frac{\rho_{HF}}{\rho_{FH}}\right)^{\frac{2k+1}{2k}}\left[(\rho_{FH}\rho_{HF})^{\frac{1}{2}}+1\right]\gtreqless 0\\
&\quad\Leftrightarrow\frac{\Theta_L}{\Theta_c}\rho_{FH}^{-1}\left[1+(\rho_{FH}\rho_{HF})^{\frac{1}{2}}\right]\left[1-\Theta_L^{\frac{k+1}{k}}\Theta_c^{-\frac{1}{k}}\left(\frac{\rho_{HF}}{\rho_{FH}}\right)^{\frac{1}{2k}}\right]\gtreqless 0\\
&\quad\Leftrightarrow\Theta_L^{\frac{k+1}{k}}\lesseqgtr\left(\frac{\rho_{FH}}{\rho_{HF}}\right)^{\frac{1}{2k}}\Theta_c^{\frac{1}{k}}\\
&\quad\Leftrightarrow\Theta_L\lesseqgtr\left(\frac{\rho_{FH}}{\rho_{HF}}\right)^{\frac{1}{2(k+1)}}\Theta_c^{\frac{1}{k+1}}.
\end{aligned}
$$

Therefore, we obtain (57):

$$
\frac{\mathrm{d}\ln U_H}{\mathrm{d}\ln\rho}\gtreqless\frac{\mathrm{d}\ln U_F}{\mathrm{d}\ln\rho}\Leftrightarrow\Theta_L\lesseqgtr\left(\frac{\rho_{FH}}{\rho_{HF}}\right)^{\frac{1}{2(k+1)}}\Theta_c^{\frac{1}{k+1}}.
$$

Finally, we confirm how the welfare relationship between the two countries changes after bilateral liberalization. From (B.16), we have

$$
\frac{U_H}{U_F}\gtreqless 1\Leftrightarrow\Theta_L\gtreqless\frac{\Theta_c^{\frac{k+1}{2k+1}}\left(\Theta_c^{\frac{k}{2k+1}}-\rho_{FH}^{\frac{k+1}{2k+1}}\rho_{HF}^{\frac{k}{2k+1}}\right)}{\rho_{FH}^{\frac{k}{2k+1}}\rho_{HF}^{\frac{k+1}{2k+1}}\left(\rho_{FH}^{-\frac{k}{2k+1}}\rho_{HF}^{-\frac{k+1}{2k+1}}-\Theta_c^{\frac{k}{2k+1}}\right)}\equiv\text{RHS}
$$

for $\Theta_c \in (\rho_{FH}^{(k+1)/k}\rho_{HF}, \rho_{FH}^{-1}\rho_{HF}^{-(k+1)/k})$. We can examine the changes in the welfare relationship by examining the changes in RHS due to bilateral liberalization ($\mathrm{d}\ln\rho_{FH} = \mathrm{d}\ln\rho_{HF} = \mathrm{d}\ln\rho > 0$). Totally differentiating RHS with respect to ρ_{FH} and ρ_{HF} yields

$$\begin{aligned} \mathrm{dRHS} &= \frac{\partial \mathrm{RHS}}{\partial \rho_{FH}}\mathrm{d}\rho_{FH} + \frac{\partial \mathrm{RHS}}{\partial \rho_{HF}}\mathrm{d}\rho_{HF} \\ \Leftrightarrow \mathrm{d}\ln \mathrm{RHS} &= \frac{\rho_{FH}}{\mathrm{RHS}}\frac{\partial \mathrm{RHS}}{\partial \rho_{FH}}\mathrm{d}\ln\rho_{FH} + \frac{\rho_{HF}}{\mathrm{RHS}}\frac{\partial \mathrm{RHS}}{\partial \rho_{HF}}\mathrm{d}\ln\rho_{HF} \\ \Leftrightarrow \frac{\mathrm{d}\ln \mathrm{RHS}}{\mathrm{d}\ln\rho} &= \frac{\partial \ln \mathrm{RHS}}{\partial \ln \rho_{FH}} + \frac{\partial \ln \mathrm{RHS}}{\partial \ln \rho_{HF}}. \end{aligned} \tag{B.27}$$

By taking the logarithm of the defining equation of RHS, we get

$$\ln \mathrm{RHS} = \ln \Theta_c^{\frac{k+1}{2k+1}} + \ln\left(\Theta_c^{\frac{k}{2k+1}} - \rho_{FH}^{\frac{k+1}{2k+1}}\rho_{HF}^{\frac{k}{2k+1}}\right) - \ln\left(1 - \rho_{FH}^{\frac{k}{2k+1}}\rho_{HF}^{\frac{k+1}{2k+1}}\Theta_c^{\frac{k}{2k+1}}\right)$$

Thus, we obtain

$$\frac{\partial \ln \mathrm{RHS}}{\partial \ln \rho_{FH}} = \frac{-\frac{k+1}{2k+1}\rho_{FH}^{\frac{k+1}{2k+1}}\rho_{HF}^{\frac{k}{2k+1}}}{\Theta_c^{\frac{k}{2k+1}} - \rho_{FH}^{\frac{k+1}{2k+1}}\rho_{HF}^{\frac{k}{2k+1}}} + \frac{\frac{k}{2k+1}\rho_{FH}^{\frac{k}{2k+1}}\rho_{HF}^{\frac{k+1}{2k+1}}\Theta_c^{\frac{k}{2k+1}}}{1 - \rho_{FH}^{\frac{k}{2k+1}}\rho_{HF}^{\frac{k+1}{2k+1}}\Theta_c^{\frac{k}{2k+1}}},$$

$$\frac{\partial \ln \mathrm{RHS}}{\partial \ln \rho_{HF}} = \frac{-\frac{k}{2k+1}\rho_{FH}^{\frac{k+1}{2k+1}}\rho_{HF}^{\frac{k}{2k+1}}}{\Theta_c^{\frac{k}{2k+1}} - \rho_{FH}^{\frac{k+1}{2k+1}}\rho_{HF}^{\frac{k}{2k+1}}} + \frac{\frac{k+1}{2k+1}\rho_{FH}^{\frac{k}{2k+1}}\rho_{HF}^{\frac{k+1}{2k+1}}\Theta_c^{\frac{k}{2k+1}}}{1 - \rho_{FH}^{\frac{k}{2k+1}}\rho_{HF}^{\frac{k+1}{2k+1}}\Theta_c^{\frac{k}{2k+1}}}.$$

Substituting these into (B.27) yields

$$\begin{aligned} \frac{\mathrm{d}\ln \mathrm{RHS}}{\mathrm{d}\ln\rho} &= \frac{-\rho_{FH}^{\frac{k+1}{2k+1}}\rho_{HF}^{\frac{k}{2k+1}}\left(1 - \rho_{FH}^{\frac{k}{2k+1}}\rho_{HF}^{\frac{k+1}{2k+1}}\Theta_c^{\frac{k}{2k+1}}\right) + \rho_{FH}^{\frac{k}{2k+1}}\rho_{HF}^{\frac{k+1}{2k+1}}\Theta_c^{\frac{k}{2k+1}}\left(\Theta_c^{\frac{k}{2k+1}} - \rho_{FH}^{\frac{k+1}{2k+1}}\rho_{HF}^{\frac{k}{2k+1}}\right)}{\left(\Theta_c^{\frac{k}{2k+1}} - \rho_{FH}^{\frac{k+1}{2k+1}}\rho_{HF}^{\frac{k}{2k+1}}\right)\left(1 - \rho_{FH}^{\frac{k}{2k+1}}\rho_{HF}^{\frac{k+1}{2k+1}}\Theta_c^{\frac{k}{2k+1}}\right)} \\ &= \frac{-\rho_{FH}^{\frac{k+1}{2k+1}}\rho_{HF}^{\frac{k}{2k+1}} + \rho_{FH}^{\frac{k}{2k+1}}\rho_{HF}^{\frac{k+1}{2k+1}}\Theta_c^{\frac{2k}{2k+1}}}{\left(\Theta_c^{\frac{k}{2k+1}} - \rho_{FH}^{\frac{k+1}{2k+1}}\rho_{HF}^{\frac{k}{2k+1}}\right)\left(1 - \rho_{FH}^{\frac{k}{2k+1}}\rho_{HF}^{\frac{k+1}{2k+1}}\Theta_c^{\frac{k}{2k+1}}\right)} \\ &= \frac{-\left(\frac{\rho_{FH}}{\rho_{HF}}\right)^{\frac{1}{2k+1}} + \Theta_c^{\frac{2k}{2k+1}}}{\left(\Theta_c^{\frac{k}{2k+1}} - \rho_{FH}^{\frac{k+1}{2k+1}}\rho_{HF}^{\frac{k}{2k+1}}\right)\left(\rho_{FH}^{-\frac{k}{2k+1}}\rho_{HF}^{-\frac{k+1}{2k+1}} - \Theta_c^{\frac{k}{2k+1}}\right)}. \end{aligned}$$

As the denominator is positive for $\Theta_c \in (\rho_{FH}^{(k+1)/k}\rho_{HF}, \rho_{FH}^{-1}\rho_{HF}^{-(k+1)/k})$, we obtain

$$\frac{\mathrm{d}\ln \mathrm{RHS}}{\mathrm{d}\ln\rho} \gtreqless 0 \Leftrightarrow \Theta_c^{\frac{2k}{2k+1}} \gtreqless \left(\frac{\rho_{FH}}{\rho_{HF}}\right)^{\frac{1}{2k+1}} \Leftrightarrow \Theta_c \gtreqless \left(\frac{\rho_{FH}}{\rho_{HF}}\right)^{\frac{1}{2k}}.$$

Therefore, with bilateral trade liberalization, the $U_H/U_F = 1$ curve in Figure 2 shifts downward (to the right) for $\Theta_c < (\rho_{FH}/\rho_{HF})^{1/2k}$ and upward (to the left) for $\Theta_c > (\rho_{FH}/\rho_{HF})^{1/2k}$, while it remains unchanged at $\Theta_c = (\rho_{FH}/\rho_{HF})^{1/2k}$.