Original Paper

Classification and Computer Generation of Necktie Patterns

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Abstract

We investigate necktie patterns according to the group theory and develop a computer graphic system specialized to the necktie patterns. We show some pictures of the necktie patterns investigated and those produced by computer graphics.

1 Introduction

Neckties seem to have a variety of patterns, but not much attention has been paid to them as patterns on a plane. Thus, in the present study, we investigate and classify the patterns of the neckties carefully and then try to generate them with computer graphics.

In section 2, we present the some of the existing necktie patterns and classify them according to the group theory. In section 3, we present a computer graphic system to generate necktie patterns and show some examples of the generated patterns. Section 4 is the summary.

2 Classification of Necktie Patterns

We have investigated patterns for 107 neckties. The samples are collected by taking pictures in department stores in Shizuoka-shi city. Some of them are shown in Fig. 1.

We classify the necktie patterns which are symmetric with respect to translation according to the theory of one- and two-dimensional space groups (Coxeter and Moser, 1965). It is well known that there are seventeen two-dimensional space groups labeled **p1**, **p2**, **pm**, **pg**, **pmm**, **pmg**, **pgg**, **cm**, **cmm**, **p4**, **p4m**, **p4g**, **p3**, **p3m1**, **p31m**, **p6** and **p6m**, and seven one-dimensional space groups labeled **t**, **tm'**, **tm"**, **tm'm"**, **t2**, **t2m"**, $\mathbf{t}_{1/2}\mathbf{m'}$ (Fushimi, 1967). Thus, the patterns symmetric with respect to translation are classified into 24 ($= 17 + 7$) groups. An example of a **p4g** necktie pattern is shown in Fig. 1(a).

We note that superposition of two one- or two-dimensional patterns is also symmetric with respect to translation, if the translation vectors satisfy

$$n\mathbf{a} + m\mathbf{b} = n'\mathbf{a}' + m'\mathbf{b}',$$

where n, m, n', m' are integers, and $\{\mathbf{a}, \mathbf{b}\}$ and $\{\mathbf{a}', \mathbf{b}'\}$ are translation vectors of the two original patterns (for a one-dimensional pattern, $\mathbf{b}$ or $\mathbf{b}'$ is zero); otherwise, the superposed pattern is not symmetric with respect to translation. Thus, it is convenient to classify such patterns, if they are apparent, to the superposed pattern other than the above 24 groups regardless of the translational symmetry. We denote the superposed pattern by a symbol '+'. For example, the superposition of **pm** and **pmg** is labeled as **pm+pmg**. Similarly, if the necktie pattern is the superposition of a finite picture on a one- or two-dimensional pattern, as shown in Fig. 1(b), we denote this by the symbol '**+a**'.

Though the necktie pattern shown in Fig. 1(c) is symmetric with respect to translation for one direction, it is homogeneous in the perpendicular direction. We denote the patterns homogeneous in one direction by **h**, separate from the above mentioned 24 space groups, and, if they have mirror symmetry lines along the homogeneous direction, we denote them by **hm**.

In Table 1, we have classified our 107 necktie patterns. In the third column, we tabulate a rough estimation of the average number of fundamental regions appearing in the photograph, which is considered to be proportional to the total number of fundamental regions included in the necktie. According to the number of fundamental regions, we further classify **p1** neckties, since about half of the neckties belong to **p1**. In Table 1, the label **p1(0)** means that the fundamental region is larger than the necktie region, though the pattern is considered to belong to the **p1** group. The label **p1(large)** and **p1(small)** means that they include less than or equal to 3 fundamental regions and more than 3 fundamental regions, respectively. The **p1** neckties with small patterns which give us a strong impression of a repetitive pattern are about 10%. Typical **p1(small)** and **p1(large)** neckties are shown in Figs. 1(d) and 1(e) for comparison. From Table 1, we can find that the neckties which have two-dimensional patterns other than **p1** are about one-forth of the our 107 samples and that the patterns belonging to the higher symmetry groups tend to have smaller fundamental regions.

For some neckties, part of the one- or two-dimensional patterns are embedded in the fundamental regions as shown in Fig. 1(f). We denote these embedded patterns by the symbol '*' in Table 1.

Of the seventeen two-dimensional patterns, only **p1**, **p2**, **pm**, **pg**, **pmm**, **pmg**, **cmm**, **p4m** and **p4g** pattern groups are found in our 107 neckties. Especially, the patterns which have trigonal rotational axes, i.e., **p3**, **p3m1**, **p31m**, **p6** and **p6m**, seem to be rare as necktie patterns.

3 Computer-Generated Necktie Patterns

We have developed a computer graphic system specialized for generating necktie patterns on the operating system, MS-Windows 3.1. Our system regards a part of an input picture as the fundamental region of the one- or two-dimensional pattern and then generates the whole pattern on the screen. The generated picture is saved to a file in the same form as the input picture. The shape of the fundamental region for our system is a rectangle for all one-dimensional patterns and for two-dimensional patterns; a parallelogram for **p1**, a triangle for **p2**, a rectangle for {**pm**, **pmm**, **pgg**}, a rhombus for **pg**, an isosceles triangle for {**cm**, **pmg**}, a right triangle for **cmm**, an isosceles right triangle for {**p4**, **p4m**, **p4g**}, an equilateral hexagon for **p3**, a 120 degree isosceles triangle for {**p3m1**, **p6**}, an equilateral triangle for **p31m**, and a bisected equilateral triangle for **p6m**.

The input and the output pictures of our system are given in the standard form of a bit map data file as defined by the operating system. Therefore, we can use any other paint program to generate the input picture or to modify the output picture. For example, in order to prepare the input picture for the embedded pattern shown in Fig. 1(f), we combine several two-dimensional patterns which are generated by our system using the bit map editor provided with the operating system. Also the two-dimensional pattern with a finite image as shown in Fig. 1(b) can be produced by superposing the two-dimensional pattern with an image from a paint program. It takes several minutes to generate a typical two-dimensional pattern on our personal computer which has an micro-processor Intel 80486/DX2.

In Fig. 2, we show six examples of the necktie patterns generated by our system. The first five patterns, Figs. 2(a.1)—2(c.2) are pure two-dimensional patterns and belong to **pg**, **p6**, **pgg**, **p1** and **p3**, respectively; and Fig. 2(d) is an example of an embedded pattern **p1*(t, tm'm")**. Fig. 2(a) is the input picture for Figs. 2(a.1) and 2(a.2), Fig. 2(b) for Fig. 2(b.1), and Fig. 2(c) for Figs. 2(c.1) and 2(c.2). Figs. 2(a) and 2(b) are manually drawn computer pictures by one of the authors (T. S.), and Fig. 2(c) is the colored Mandelbrot set (Mandelbrot, 1977). While we have used only part of the input pictures in Figs. 2(a) and 2(c), we have included the whole finite image of the input picture Fig. 2(b) in the fundamental region.

4 Summary

All necktie patterns we have investigated can be considered to be symmetric with respect to translation except finite pictures and superposition, though some of them have a larger fundamental region than the necktie region. For two-dimensional patterns, the patterns having trigonal rotational axes seem to be rare as necktie patterns.

We have developed a computer graphic system which enables us to generate one- or two-dimensional patterns from part of any input picture. Complex patterns, such as the embedded and superposed necktie patterns discussed in Section 2, can be produced by recursive use of our system or with the aid of other paint programs. We believe that our simple system covers all characteristics of necktie patterns and that it can generate patterns similar to existing neckties.

REFERENCES

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TABLES

Table 1. Classification of 107 necktie patterns. The third column shows the average number of the fundamental regions appearing in the photograph.

label		occurrence	regions
p1(0)		24	
p1(large)		19	1
p1(small)		11	20
p1	**+p1(0)**	5	2
	+p1	1	7
	***(p2, cmm)**	1	3
	***(p2, p1, t)**	1	2
	***t**	4	4
p2		5	8
	+p1(0)	1	4
pm		2	7
	+pmg	1	10
pg		1	20
pmm		3	85
	+a	1	40
pmg		1	130
cmm		4	911
p4m		2	1372
p4g		5	130
t	***t**	1	3
tm'm"	**+a**	1	28
h		4	
hm		9	

FIGURE CAPTIONS

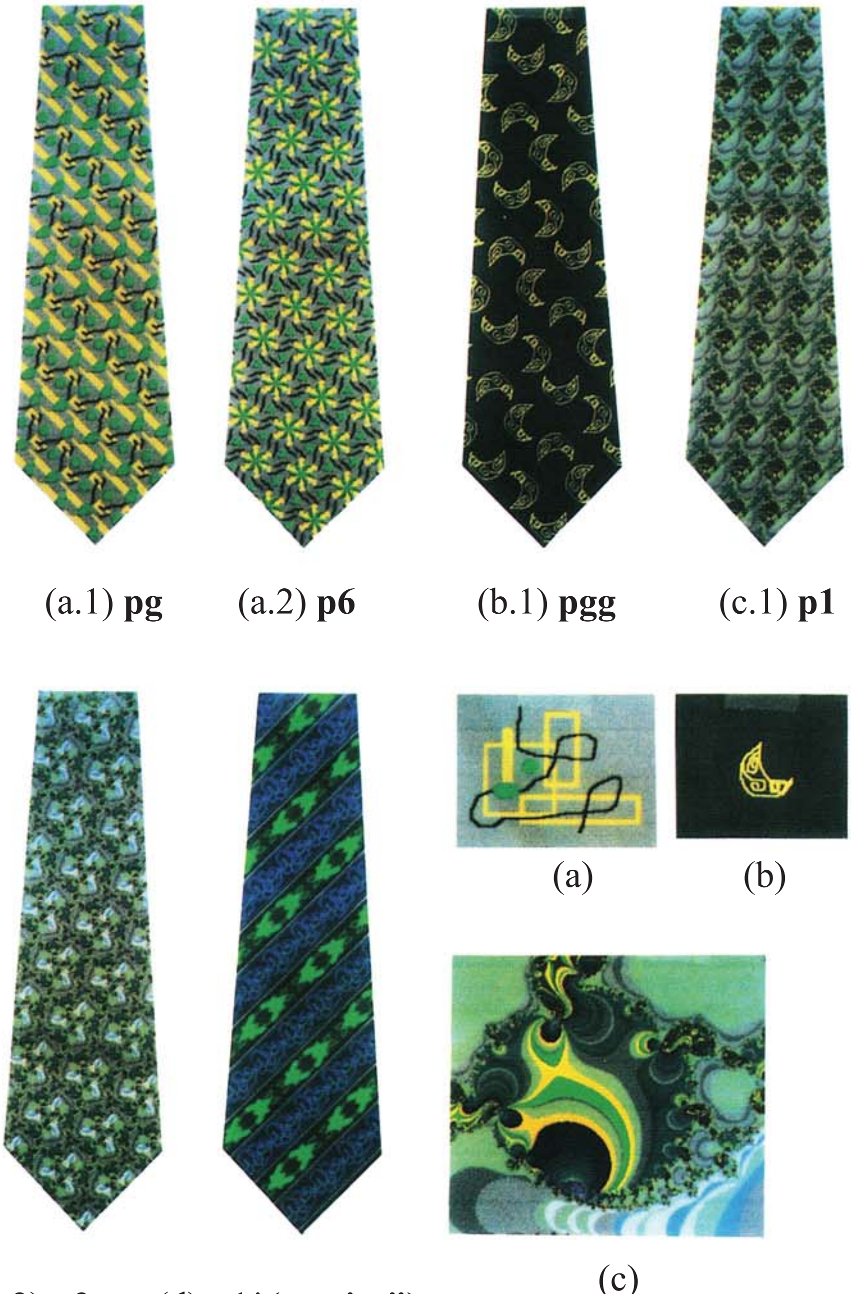
Fig. 1. Examples of the necktie patterns we have investigated.

Fig. 2. Examples of the patterns generated by our graphic system. (a) is the input picture for (a.1) and (a.2), (b) is the input for (b.1), and (c) is the input for (c.1) and (c.2).

(a) (b) (c)

(d) (e) (f)

Fig. 1



(a.1) **pg** (a.2) **p6** (b.1) **pgg** (c.1) **p1**

(a) (b)

(c)

(c.2) **p3** (d) **p1*(t,tm'm")**

Fig. 2