Perturbation Approach for Open Boundary Problems Based on the Equivalence Theorem

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Abstract— The perturbation method for open boundary problems based on the equivalence theorem has been proposed. Most of the open boundary techniques in electromagnetic analysis are approximate, whereas more accurate techniques require large computational resources. In order to improve the accuracy of the open boundary solution, the equivalence theorem has been employed as a perturbation correction to the electromagnetic analysis with the approximate open boundary conditions. In this manuscript, the proposed method has been verified with numerical examples.

1. INTRODUCTION

Various types of techniques have been proposed to solve electromagnetic openboundary problems [1–25]. Open boundary techniques for electromagnetic analysis are a long-standing topic which is still discussed in the recent literature [19–25]. Each technique has certain advantages and limitations. Some of them are only applicable to low-frequency problems [1, 2, 4, 5–8, 11, 22, 23], while others are high-frequency only [3, 9, 12, 15]. Most of the open boundary techniques in electromagnetic analysis are approximate, whereas more accurate techniques require large computational resources [6, 18, 25]. To improve the accuracy of the open boundary solution, the equivalence theorem has been employed as a perturbation correction to the electromagnetic analysis with the approximate open boundary conditions. In this manuscript, the proposed method has been verified with numerical examples.

2. FORMULATION

Figure 1 shows a schematic image of the electromagnetic radiation problem. The analysis domain is truncated by the exterior boundary where the open-boundary condition is employed. If the open-boundary condition is not perfect, the wave is reflected by the exterior boundary and that reflected wave is again reflected by the radiating object and re-radiate. In our proposed method, the reflected wave is considered as a first-order perturbation and the re-radiating wave as a second-order perturbation which is neglected.

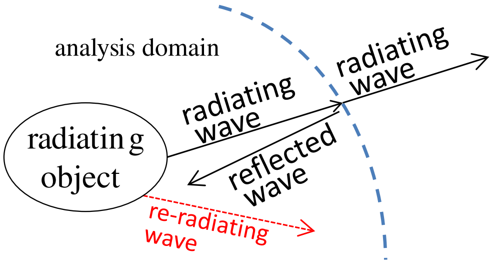


Figure 1: Schematic image of the electromagnetic radiation problem.

Figure 2 shows schematic images of the equivalence theorem. When the sources are only inside the boundary, the electric and magnetic equivalent surface currents will reproduce the radiating field outside and zero field inside. When the sources are only outside the boundary, the electric and magnetic equivalent surface currents will reproduce the incoming field inside and zero field outside. When the sources are both inside and outside, the electric and magnetic equivalent surface currents will reproduce the radiating wave outside and the negative of reflecting wave inside. We employ the last equivalent theorem and Eqs. (1) and (2) to reproduce the reflecting wave to eliminate the unexpected reflection from the boundary. The flowchart of the method is given in Fig. 3. The first step is to obtain the open boundary solution using approximate open boundary technique. The second step is to reconstruct the field using equivalence theorem. The final step is to add the reconstructed field to the former obtained field thus to make the approximated open boundary solution more accurate.

$$\mathbf{E}(\mathbf{r}) = \oiint \left[-j\omega\mu_0 \mathbf{J}\psi_0 - \mathbf{M} \times \nabla\psi_0 + \frac{1}{j\omega\varepsilon_0} \left[\mathbf{J} \cdot \nabla\right] \nabla\psi_0 \right] dS \tag{1}$$

$$\mathbf{H}(\mathbf{r}) = \oiint \left[-j\omega\varepsilon_0 M\psi_0 + \mathbf{J} \times \nabla\psi_0 + \frac{1}{j\omega\mu_0} \left[\mathbf{M} \cdot \nabla\right] \nabla\psi_0 \right] dS \tag{2}$$

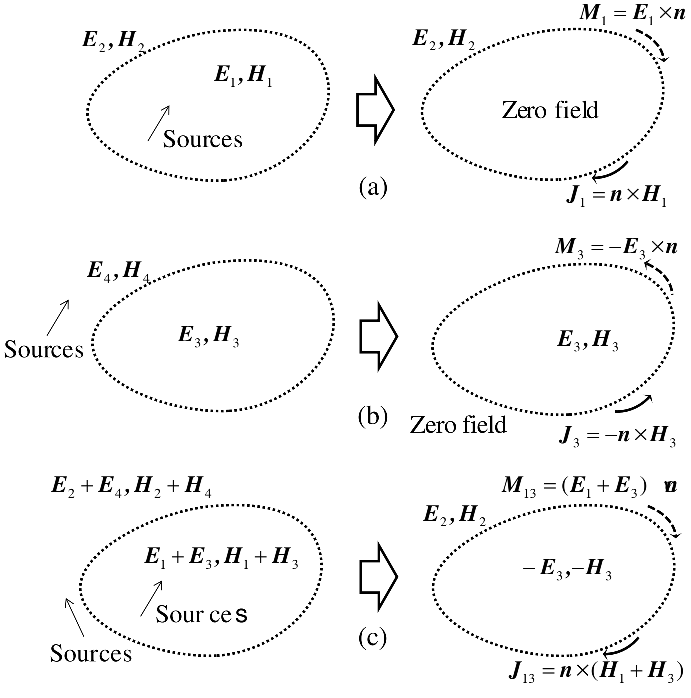


Figure 2: Schematic image of the equivalence theorem. (a) Sources are inside the boundary, (b) sources are outside the boundary, and (c) sources are both inside and outside the boundary.

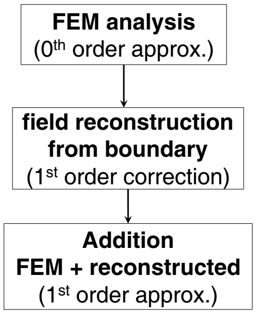


Figure 3: The flowchart of the perturbation approach proposed in this manuscript.

3. NUMERICAL RESULT

3.1. TEAM Workshop Problem 7 (Asymmetrical Conductor with a Hole)

Figure 4 shows a schematic image of the TEAM workshop problem 7 [26]. A coil is placed over the asymmetrical aluminum conductor with a hole. The size of the analysis domain is chosen to be 530 × 540 × 380 mm and the exterior boundary is a perfect conducting wall. The magnetic flux lines are shown in Fig. 5. The FEM result is obviously not an open boundary solution, however, the sum of the FEM and the reconstructed field from the surface is a good approximation to the open boundary solution.

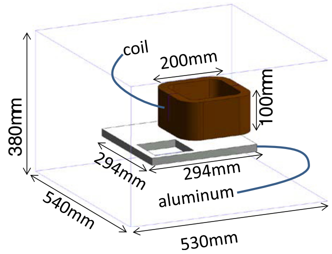
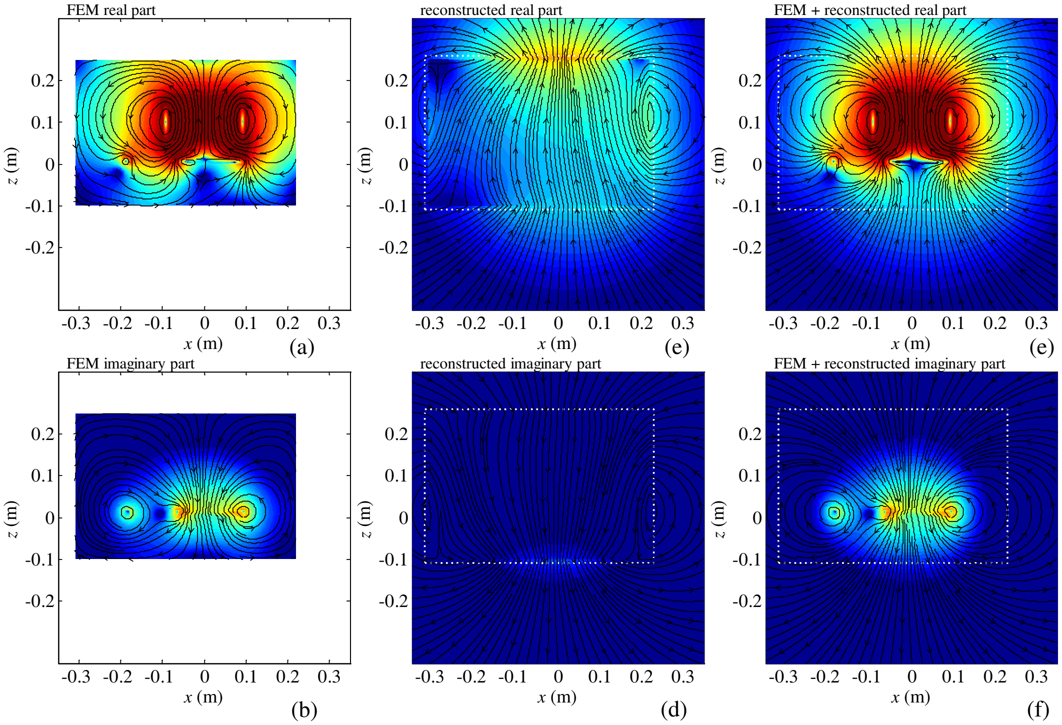


Figure 4: TEAM workshop. problem 7 (asymmetrical conductor with a hole). The exterior boundary is settled as a perfect conducting wall.


FEM real part

reconstructed real part

FEM + reconstructed real part

FEM imaginary part

reconstructed imaginary part

FEM + reconstructed imaginary part

Figure 5: Contour plots and flux lines of the magnetic field. (a) Real part obtained by FEM, (b) imaginary part obtained by FEM, (c) real part reconstructed from the surface, (d) imaginary part reconstructed from the surface, (e) real part obtained by the proposed method, and (f) imaginary part obtained by the proposed method.

3.2. Monopole Antenna

Figure 6 shows a schematic image of a monopole antenna with a ground. The length of the antenna element is a quarter of the wave length whose frequency is 300 MHz. The analysis domain is a cube and the edge length of the cube is 750 mm and the antenna is offset by (0.4 m, 0.0 m,

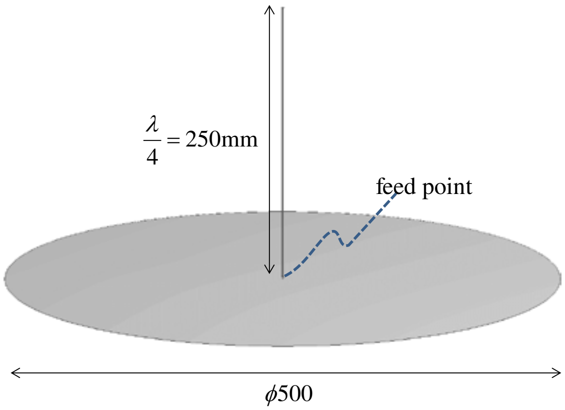
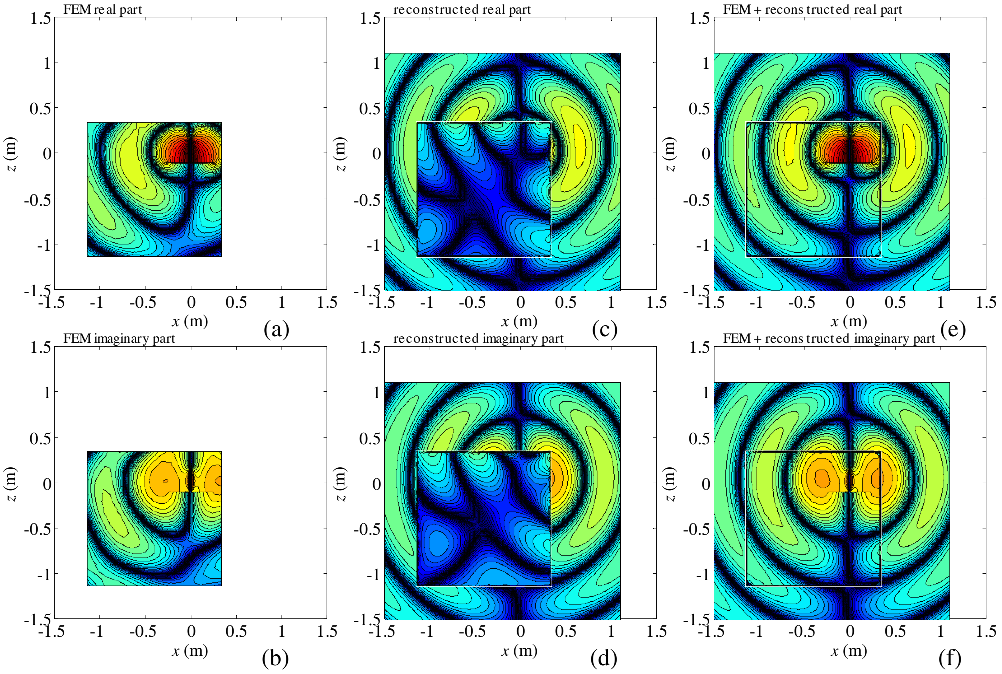


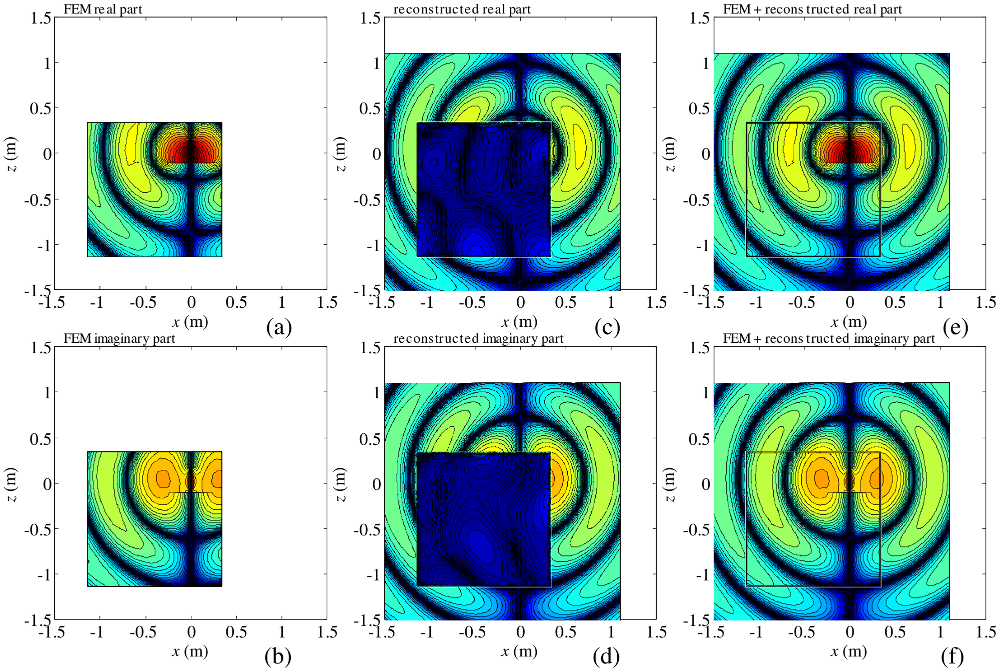
Figure 6: A monopole antenna with a ground disk.

0.4 m). Figs. 7 and 8 show the contour plots of the magnetic field when the exteriorboundary is the radiation boundary condition [9] and the PML [12], respectively. In Fig. 7, a significant reflection is observed whereas a small reflection occurs at the boundary in Fig. 8. However, the corrected results (proposed method) are almost identical in both cases.



(a) (c) (e) (b) (d) (f)

Figure 7: Contour plots of the magnetic field in a logarithmic scalewith the radiation boundary condition. (a) Real part obtained by FEM, (b) imaginary part obtained by FEM, (c) real part reconstructed from the surface, (d) imaginary part reconstructed from the surface, (e) real part obtained by the proposed method, and (f) imaginary part obtained by the proposed method.



(a) (c) (e) (b) (d) (f)

Figure 8: Contour plots of the magnetic field in a logarithmic scalewith the PML (a) real part obtained by FEM, (b) imaginary part obtained by FEM, (c) real part reconstructed from the surface, (d) imaginary part reconstructed from the surface, (e) real part obtained by the proposed method, and (f) imaginary part obtained by the proposed method.

4. CONCLUSION

In this manuscript, the perturbation correction method for open boundary problems based on the equivalence theorem has been proposed and numerical examples are demonstrated. Even when the open boundary technique is approximate, more accurate solutions can be obtained with this method. Since we only consider the first-order perturbation, the obtained results are still approximate. However, the additional computation time is rather small because it only requires the forward integration given in Eqs. (1) and (2), thus, no matrix inversions. Another advantage of this method is to visualize of the performance of the open boundary technique. The examples in this manuscript visualize explicitly how better the PML is compared with the radiation boundary condition.

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