A hybrid implementation mechanism of tradable network permits system:
An auction mechanism with day-to-day capacity control

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**Introduction**

- Congestion pricing (price-based regulation)
  - working effectively when a road manager can calculate an optimal toll level
  - needing **accurate** and **detailed demand information**
    - e.g., OD demands, VOT, desired arrival/departure time

- **Asymmetric information**
  - between the manager and road users
  - very difficult to obtain such private information
    - distorting toll levels
    - resulting in economic losses
What is tradable network permits scheme?

- ** Tradable Network Permits (TNP)** [Akamatsu et al. (06, 07)]
  - a right that allows a permit holder to pass through **specific bottleneck** during **pre-specified time period**
    - quantity-based regulation
      - e.g., highway booking

- **Trading market** for network permits
  - time-dependent
  - freedom of permit choice
    - **Allocation** and **prices** are determined through the markets.
**Why is the TNP scheme needed?**

- No queuing congestion
  - # of permits of each link ≤ bottleneck capacity

- No detailed information on user demand
  - manager only needs to know bottleneck capacity.
    cf. congestion pricing (or price-based regulation)

- Efficiency in general networks [Akamatsu, (07)]
  - equilibrium resource allocation pattern
    = Dynamic System Optimal (DSO) assignment
    - assumption: trading markets are perfectly competitive,
      i.e., trading processes were treated as a **black box.**
How to implement the TNP scheme?

- Markets do what they are supposed to do, however, only if they are well structured. [McMillan (02)]
  - preventing manipulating prices
  - achieving an efficient allocation

- Auction mechanism for a bottleneck [Wada & Akamatsu (10)]
  - Vickrey-Clarke-Groves (VCG) mechanism
    - strategy-proof (i.e., truthful bidding is a dominant strategy)
    - Permits allocation pattern is efficient.

- General networks
  - A naïve formulation of the problem leads to NP-hardness owing to the complex relationship between link and path.
Purpose of the study

- A novel auction mechanism to implement TNP scheme for general networks
  - enabling each user to purchase the preferred bundle of permits (path)
  - achieving a DSO allocation of network permits in a computationally efficient manner

Approach

- evolutionary (day-to-day) approach
  - a path-based auction with day-to-day capacity control
- obviating path enumeration by introducing a column generation procedure
Tradable permits for managing traffic congestion

- Quantity-based regulation + market institution
  - possibilities of using tradable permits for managing congestion [Verhoef et al., (97)]
  - none describes **time-dependent tradable permits** for eliminating **bottleneck congestion**.

- Tradable travel credit scheme [Yang and Wang (11)]
  - is superficially similar to but **fundamentally different** from tradable network permits scheme
  - is not a quantity-based regulation for **directly** reducing congestion but rather a **redistribution scheme of income**
Setting

- A discrete time DTA on general networks with multiple Origin-Destination (OD) pairs
  - point (or physical) queue model
  - $\mu_a$ - bottleneck capacity = # of permits of link $a$
    - no queuing congestion

- Behavior assumption for atomic road user
  - at most single trip per day between an OD
  - utility max.: choice rule of path and destination arrival time
  - must purchase a bundle of permits corresponding to a path
User valuation and utility

- User $i$’s valuation for path $r$ and destination arrival time $t$
  - **private information**
  - including **constant** travel time

\[
v_{i,r}(t) = [v_i]_{r,t} \geq 0
\]

- Quasi-linear utility = valuation – permit purchase cost

\[
u_{i,r}(t) = v_{i,r}(t) - p_r(t)
\]

- Allocation of bundles of permits (paths)

\[
f_{i,r}(t) = [f_i]_{r,t} \in \{0, 1\}
\]

\[
\sum_{r,t} f_{i,r} \leq 1
\]
**DSO allocation of network permits**

- **Dynamic system optimal problem [DSO] (atomic user)**

  \[
  \text{max}_{\{f_i\}} \sum_i v_i \cdot f_i \quad \text{... social surplus}
  \]

  - subject to

  \[1 \cdot f_i \leq 1 \quad \forall i \quad \text{... at most single trip per day}\]

  \[\sum_i x_i \leq \mu \quad \text{... link capacity constraint}\]

  \[x_i = \Delta_i f_i \quad \forall i \quad \text{... relationship path and link allocation}\]

- allocation of permits (Links): \(x_{i,a}(t) = [x_i]_{a,t} \in \{0, 1\}\)
- path-link incidence matrix: \(\Delta_i\)
Difficulties of solving the problem [DSO]

- **Incomplete information** on the objective function
  - Users’ private valuations are unknown

  ✔ Vickrey-Clarke-Groves (VCG) combinatorial auction
    - gives users an incentive to report their valuations truthfully
    - must solve [DSO] **exactly** in many times to calculate allocation and individual payment (i.e., marginal cost)

- [DSO] is integer multi-commodity flow problem
  - complex relationship between **link and path variables on individual allocations**
  - **NP-hard**: no polynomial time algorithm exists
Basic ideas of day-to-day auction mechanism

- Decomposition of [DSO] based on Benders’ method
  - **master-P** – adjusting # of bundles of permits (path capacity) on day-to-day basis
  - **sub-P** – assigning bundles (path capacity) to users

- Solve the sub-P by an auction mechanism
  - gives users an incentive to report their valuation truthfully
  - can obtain the efficient permit allocation with incomplete information

- Assumption: users try to maximize their current utilities, i.e., (myopic) best response dynamics
Reformulation of the problem [DSO]

- Introducing **non-individual** integer variables
  - $F$ - aggregated path variables (path-capacities)
  - $X$ - aggregated link variables

- Equivalent optimization problem to the problem [DSO]

$$\max_{\{F_w\},\{f_i\}} \sum_i v_i \cdot f_i \quad \ldots \text{social surplus}$$

- subject to
  
  $1 \cdot f_i \leq 1 \quad \forall i \quad \ldots \text{at most single trip per day}$

  $\sum_i f_i \leq F_w \quad \forall w \quad \ldots \text{Path capacity constraint for OD pair } w$

  $X \leq \mu \quad \ldots \text{link capacity constraint}$

  $X = \sum_w \Delta_w F_w \quad \ldots \text{relationship path and link allocation}$
Reformulation of the problem [DSO]

- Introduce *non-individual* integer variables
  - $F$ - non-individual path variables (path-capacities)
  - $X$ - non-individual link variables

- Equivalent optimization problem to the problem [DSO]

\[
\max \{ F_w, \{ f_i \} \} \sum_i v_i \cdot f_i
\]

- subject to

\[
1 \cdot f_i \leq 1 \quad \forall i
\]
\[
\sum_i f_i \leq F_w \quad \forall w
\]
\[
X \leq \mu
\]
\[
X = \sum_w \Delta_w F_w
\]

all constrains for individual variables are represented by path variables.
Framework of the day-to-day auction mechanism

master-P: $F$
path capacity adjustment phase

sub-P: $f_i$
auction phase

sub-P: $f_i$
auction phase

sub-P: $f_i$
auction phase

day
Decomposition of the problem [DSO]

- **Sub-problem (assignment problem): auction phase**
  - assigning bundles to users for **fixed path capacities**
  - satisfying **totally unimodularity**, i.e., LP = IP

\[
\begin{align*}
\max \{ F_w \} \sum_w S_w(F_w) &= \max \{ f_i \} \sum_i v_i \cdot f_i \\
\text{subject to} & \\
X &\leq \mu \\
X &= \sum_w \Delta_w F_w \\
1 \cdot f_i &\leq 1 \quad \forall i \\
\sum_i f_i &\leq F_w
\end{align*}
\]

unknown variables: **individual path variables**
Decomposition of the problem [DSO]

- **Dual sub-problem: auction phase**
  - providing information on prices/payoffs
  - $\pi_i$: user $i$’s payoff, $p$: bundles prices

\[
\max \{ F_w \} \sum_w S_w(F_w) = \min \{ p_w, \{ \pi_i \} \} \sum_i \pi_i + p_w \cdot F_w \\
\text{subject to}
\]

- $X \leq \mu$
- $X = \sum_w \Delta_w F_w$

\[
\pi_i 1 \geq v_i - p_w \quad \forall i
\]

unknown variables: payoff / price variables
Decomposition of the problem [DSO]

- Master problem: path capacity adjusting phase
  - adjusting path capacities based on demand information (payoffs and prices) of each day auction phase

\[
\max_{\{F_w\}} \sum_w S_w(F_w) = \min_\omega [\sum_i \pi_i^0 + p_w^0 \cdot F_w]
\]

- subject to
  \[
  X \leq \mu
  \]
  \[
  X = \sum_w \Delta_w F_w
  \]

- using relaxation: generating extreme points iteratively
- using heuristics: linear relaxation, box-step constraints

\( \omega \): set of all extreme points of dual constraints
unknown variables: non-individual variables
Procedure of day-to-day auction mechanism

**Day 1**
- Master problem: Path capacity adjusting
  - Initial path capacity $F^1$
- Subproblem: Auction
  - Dual: Price $p^1$, Payoff $\pi^1$
  - Primal: Allocation $f^1$

**Day 2**
- Master problem: Path capacity adjusting
  - Path capacity $F^2$
- Subproblem: Auction
  - Dual: Price $p^2$, Payoff $\pi^2$
  - Primal: Allocation $f^2$
Obtaining truthful users’ valuation

- **Ascending proxy auction** [Demange et al. (86), Parkes & Ungar (02)]
  - users report their valuations to proxy agents
  - agents bid most preferred items under the current prices
  - choosing the overdemanded set & raising prices in the set
    - **Individual rationality**: winners’ payoffs are non-negative
    - **strategy-proof**: honesty is a best strategy for users
    - **efficiency**: Pareto efficient allocation can be achieved

- Corresponding to the mathematical programming
  - the process of the auction is corresponding to the **primal-dual (Hungarian) algorithm** for solving the sub-problem
Properties of the day-to-day auction mechanism

- **Proposition 1 (Convergence property)**
  
  The day-to-day auction mechanism converges in a finite number of iterations.

- **Proposition 2 (Efficiency property)**
  
  The permits allocation pattern by the mechanism converges to the most efficient allocation when the number of users is large.

Proof:

$$\lim_{N \to \infty} \frac{S_L - S_I}{S_L} = 0$$

- $S_L$ : obj. fun. of linear relaxation of MP
- $S_I$ : obj. fun. of integer MP
Extension: obviate path enumeration

- Introducing **column generation** procedure
  1. Considering a **subset of paths** of [DSO]. For the fixed path set, executing the day-to-day auction mechanism.
  2. After convergence, **each user generates** preferred path based on the previous day-to-day auction. Go to Step 1.

- Path generation is efficient because **the numerous # of users generate paths simultaneously**
**Numerical example: convergence process**

- Sioux Falls network with 528 OD pairs
- The total number of users is 90150

![Graph showing convergence process with phases](image)

- Ratio of realized value of SS to the maximum value
- MP’s objective function value (upper bound)
- Realized value of social surplus (sub-problem)

The path generation phase

The day-to-day auction phase

Day
Conclusion

- We proposed the day-to-day auction mechanism for implementing tradable network permits scheme for general networks.

- We showed that the mechanism has desirable properties: strategy-proof; finite convergence; Pareto efficiency (DSO).

- We extended the mechanism to obviate path enumeration by introducing a column generation procedure.
Thank you for your attention

My first baby (son) was born yesterday!!

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References

References

Future works

- Applying the proposed mechanism to the management of other transportation networks
  - railway networks, freight networks, logistics networks etc.
  - Problem: a network manager aims to maximizes not the social surplus but his/her profit

- Tradable network permits under the second-best situation
  - Queuing congestion occurs at a link that is not controlled.
  - needing connect the tradable network permits scheme to a DTA problem.
Unrealistic!?

- imagine that ...

Each user’s car has “agent software”

Agent software

Agent

Bottleneck

General networks
Multi-agent system

Each agent chooses a path and arrival time using **local information only**.
Multi-agent system

Each agent deals with the cumbersome procedure of trading the bottleneck permits

Agent software
Feasibilities for implementation

❖ Technical point of views
  - Procedures for network permits
    » Dedicated short range communication (DSRC) system (e.g., Electric toll collection (ETC))
  - Trading markets
    » Internet auction markets (inexpensive!!)

❖ Institutional point of views
  - Minimal legal restrictions is needed
Assigning schemes of network permits

- Market selling scheme (In this study)
  - The road manager **sells** all the bottleneck permits to users in the trading market

- Free distribution scheme
  - The road manager distributes all the permits to users **for free** according to methods that consider the equity among users, e.g., rotation system of license plate numbers

- Remark
  - In terms of the efficiency of resource allocation, the two schemes are essentially identical.
Tradable network permits with stochastic arrivals

- Some users arrive at a bottleneck late (or fast)

- Stochastic queuing congestion can be decreased when a number of permits for the link is less than the capacity
  - about 80% [Kasahara & Akamatsu (06)]