THE CORRIDOR PROBLEM WITH DISCRETE MULTIPLE BOTTLENECKS

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Introduction

- Departure-time choice (DTC) equilibrium
  - Single bottleneck [Vickrey, 69; Hendrickson & Kocur, 81]
    * Existence: Smith (84), Uniqueness: Daganzo (85)
  - 2 tandem bottlenecks
    * Kuwahara (90), Arnott et al. (93) Lago & Daganzo (07)

- General problem: $N$ tandem bottlenecks
  - It is almost impossible to obtain equilibrium by analytical approach
    * Corridor problem: Arnott & DePalma (11)
      – Variant of general problem: continuous entries & KW model
      – could not provide a complete equilibrium solution.
Introduction (cont.)

❖ Dynamic User Equilibrium (DUE) on general networks
  – More computational approach (see, for a review, Szeto & Wong, 11)
    • Formulations are too complex to analyze properties of equilibrium because they must handle complicated nested structure btw link & path travel times.

❖ DUE properties and solution algorithms
  • There remain many issues regarding DUE properties: existence, uniqueness and stability [Iryo, 13]
  • There is no algorithm that ensures convergence to a DUE.

▷ One of the critical reasons:
  Lack of monotonicity of path travel cost function
Purpose of the study

A transparent approach for analyzing DTC equilibria for a corridor problem with discrete multiple BNs

- Equilibrium condition is formulated in a Lagrangian-like (moving) coordinate system (cf. Eulerian)
  * It is easy to evaluate the link & path travel times.
- The condition reduces to clear and concise Liner/Nonlinear Complementarity Problems (LCP/NCP)

➤ Insights into the mathematical structure of the problem
  * demand/supply-sub models, DUE vs. DSO, Morning vs. Evening

➤ Results on existence & uniqueness of equilibria
Many-to-one networks (Morning rush)

- $N$ on-ramps (nodes) and $N$ bottlenecks
  - are numbered $i = 1, 2, \ldots, N$ from downstream to upstream
- $\mu_i$: capacity of bottleneck $i$ on link $(i-1, i)$
  - *Point-queue model* is assumed.
- $Q_i$: # of commuters in residential location $i$ (given)
  - They reach the destination during rush hour $[0, T]$
**User’s disutility**

- **Generalized transportation cost**
  
  \[ \text{Generalized transportation cost} = \text{free flow travel time} + \text{queuing delay} + \text{schedule penalty cost} \]

  - **Queuing delay at each bottleneck** \(i\):
    \[
    d_i(t) = \frac{E_i(t)}{\mu_i}
    \]
    # of vehicles at BN \(i\) at time \(t\)

  - **Schedule delay**: \(p(t)\)
    * continuous and convex
    * is caused by difference btw actual arrival time \(t\) and common work start time \(t_w\)
Departure-time choice equilibrium

- User’s departure-time choice principle
  - Homogeneous case: Disutility minimization model
  - Heterogeneous case: Random utility model

- Equilibrium conditions

  *No user could not reduce his/her disutility by changing departure (or arrival) times.*

(a) Queuing conditions at each bottleneck (point-queue model)
(b) Flow conservations in the network
(c) Arrival/departure-time choice conditions
(a) Queuing conditions

- State equation for # of vehicles:

\[
e_i(t) = \lambda_i(t) - x_i(t)\]

\[
dE_i(t)/dt \quad dA_i(t)/dt \quad dD_i(t)/dt\]

- Exit flow model:

\[
x_i(t) = \begin{cases} 
\mu_i & \text{if } E_i(t) > 0 \\
\min[\lambda_i(t), \mu_i] & \text{if } E_i(t) = 0
\end{cases}
\]

▷ Combining these with def. of queuing delay & relaxation

\[
\begin{cases} 
\dot{d}_i(t) = (\lambda_i(t)/\mu_i) - 1 & \text{if } d_i(t) > 0 \\
\dot{d}_i(t) \geq (\lambda_i(t)/\mu_i) - 1 & \text{if } d_i(t) = 0
\end{cases}
\]
(b) Flow conservations

- Origin-destination (OD) demand conservation:

\[ \int_0^T \hat{q}_i(t) dt = Q_i \]

arrival rate of the users with origin \( i \) at BN \( i \) at time \( t \)

- Flow conservation at each node:
  - Arrival flow rate at BN \( i \)
    \[ \lambda_i(t) = x_{i+1}(t - c_{i+1}) + \hat{q}_i(t), \quad \lambda_N(t) = \hat{q}_N(t) \]
(c) Departure-time choice conditions

❖ Homogeneous case: Disutility minimization

\[
\begin{align*}
\pi_i(t) + p(t + \pi_i(t)) &= \rho_i \quad \text{if } \hat{q}_i(t) > 0 \\
\pi_i(t) + p(t + \pi_i(t)) &\geq \rho_i \quad \text{if } \hat{q}_i(t) = 0
\end{align*}
\]

path travel time \hspace{1cm} \text{equilibrium cost}

❖ Heterogeneous case: Random disutility minimization

\[
\hat{q}_i(t) = Q_i \hat{P}_i(t)
\]
\[
\hat{P}_i(t) \equiv \Pr[\hat{\vartheta}_i(t) + \hat{\epsilon}_i(t) \leq \hat{\vartheta}_i(t') + \hat{\epsilon}_i(t')]
\]

users’ idiosyncratic choices for departure times

where \(\hat{\vartheta}_i(t) \equiv \pi_i(t) + p(t + \pi_i(t))\)
Evaluating path travel time $\pi_i(t)$

Dynamic network loading problem

- **Recursive equations with time delay that are state-dependent and time-varying**

\[
\pi_i(t) = d_i(t) + c_i + \pi_{i-1}(t_{i-1})
\]
\[
= d_i(t) + c_i + d_{i-1}(t + d_i(t) + c_i) + c_{i-1} + \cdots
\]
Evaluating path travel time $\pi_i(t)$

\[ t_i \leftarrow t_i + \pi_i(t_i) \]

Formulation in an Eulerian coordinate system

- Variables are defined at position $i$ at time $t$
- Must trace the time-space path of each user in the network
  - Extreme difficulties in analyzing the properties of equilibrium
From Eulerian to moving coord. system

- **Lagrangian-like (moving) coord. system**
  - Variables are defined **at position i for user number**
    - Kuwahara (90), Kuwahara & Akamatsu (93), Akamatsu (01)
    - *T-model* in variational theory [Laval & Leclercq, 13]
  - Equilibrium concept along with the FIFO discipline of the point-queue model implies that **order of arrival at destination must be kept at any BN from origin**
    - User number = *Destination arrival time* $s$
From Eulerian to moving coord. system

- **Lagrangian-like (moving) coord. system**
  - Time points are defined **at position $i$ for dest. arrival time $s$**

  \[ \sigma_i(s) = \tau_i(s) + w_i(s), \quad \sigma_1(s) + c_1 = s \]
  
  departure time  arrival time  queuing delay

- Arrive at bottleneck $i$
- Depart from bottleneck $i$
- Arrive at bottleneck $i-1$
- Depart from bottleneck $i-1$
- Arrive at the destination
Reformulation

- Variables for a user arriving at destination at time $s$
  - Arrival flow rate at BN $i$:
    
    $$y(s) \equiv \frac{dA_i(\tau_i(s))}{ds} = \lambda_i(\tau_i(s)) \cdot \Delta \tau_i(s) \equiv \frac{d\tau_i(s)}{ds}$$

  - Queuing delay at BN $i$:
    
    $$\omega_i(s) = d_i(\tau_i(s)) \Rightarrow \Delta \omega_i(s) = \frac{d_i(\tau_i(s))}{dt} \cdot \Delta \tau_i(s) \equiv \frac{d\omega_i(s)}{ds}$$
(a) Queuing conditions

- **Eulerian coord. system:**

\[
0 \leq d_i(t) \perp \dot{d}_i(t) - [(\lambda_i(t)/\mu_i) - 1] \geq 0
\]

- **Lagrangian-like coord. system:**

  * Substitute new variables \(w(s), y(s)\) into this condition, we have

\[
0 \leq w_i(s) \perp \Delta w_i(s) - [(y_i(s)/\mu_i) - \Delta \tau_i(s)] \geq 0
\]

  * Because \(\sigma_i(s) = \tau_i(s) + w_i(s)\), this condition reduces to capacity constraint-like condition:

\[
\begin{cases}
    y_i(s) = \mu_i \Delta \sigma_i(s) & \text{if } w_i(s) > 0 \\
    y_i(s) \leq \mu_i \Delta \sigma_i(s) & \text{if } w_i(s) = 0
\end{cases}
\]

  can be interpreted as a capacity
Other conditions

(b) Flow conservations

$q_i(s)$: Entering demand rate with origin $i$ arriving at the dest. at time $s$

\[
\text{[OD]} \quad \int_S q_i(s) \, ds = Q_i
\]

\[
\text{[Node]} \quad y_i(s) = q_i(s) + y_{i+1}(s)
\]

\[
y_N(s) = q_N(s)
\]
Other conditions

(b) Flow conservations

\( q_i(s) \): Entering demand rate with origin \( i \) arriving at the dest. at time \( s \)

\[
\text{[OD]} \quad \int_s q_i(s)\,ds = Q_i
\]

\[
\text{[Node]} \quad y_i(s) = q_i(s) + y_{i+1}(s), \quad y_N(s) = q_N(s)
\]

* The latter is the same form as flow conservation for static model

(c) Departure/arrival-time choice conditions

It is easy to evaluate travel time: \( s - \tau_i(s) = \sum_{j=1}^{i} (c_j + w_j(s)) \)

\[
\begin{cases}
  p(s) + (s - \tau_i(s)) = \rho_i & \text{if } q_i(s) > 0 \\
  p(s) + (s - \tau_i(s)) \geq \rho_i & \text{if } q_i(s) = 0
\end{cases}
\]
Overall DTC equilibrium problem

✿ Equivalent Complementarity Problem

- Homogeneous case: LCP (cf. Heterogeneous case: NCP)
  * eliminating redundant variables \(\{y(s), \tau(s)\}\)

\[
0 \leq q_i(s) \perp p(s) + \sum_{j=1}^{i} (c_j + w_j(s)) - \rho_i \geq 0, \quad \forall i, s
\]

... Departure-time choice conditions

\[
0 \leq w_i(s) \perp \mu_i \Delta \sigma_i(s) - \sum_{j=i}^{N} q_i(s) \geq 0, \quad \forall i, s
\]

... Queuing conditions

\[
0 \leq \rho_i \perp \int_S q_i(s) ds - Q_i \geq 0 \quad \forall i \quad \text{... Flow conservations}
\]

where \(\Delta \sigma(s) \equiv 1 + [I - L] \Delta w(s)\) (\(\Delta \sigma_i(s) = 1 - \sum_{j=1}^{i-1} \Delta w_i(s)\))
Overall DTC equilibrium problem

- **Equivalent Complementarity Problem**
  - Homogeneous case: LCP (cf. Heterogeneous case: NCP)
    * eliminating redundant variables \( \{y(s), \tau(s)\} \)

\[
0 \leq q(s) \perp p(s)1 + L(c + w(s)) - \rho \geq 0, \quad \forall s \in S
\]

... Departure-time choice conditions

\[
0 \leq w(s) \perp C\Delta \sigma(s) - L^T q(s) \geq 0, \quad \forall s \in S
\]

... Queuing conditions

\[
0 \leq \rho \perp \int_S q(s)ds - \bar{Q} \geq 0 \quad ... \text{Flow conservations}
\]

\[
\Delta \sigma(s) \equiv 1 + [I - L]\Delta w(s) \quad (\Delta \sigma_i(s) = 1 - \sum_{j=1}^{i-1} \Delta w_i(s))
\]
Overall DTC equilibrium problem

- **Time discretization:**
  - Finite dimensional LCP with a **skew-symmetric matrix** \( M \)

Find \( X \) such that \( 0 \leq X \perp F(X) \equiv MX + b \geq 0 \),

\[
X \equiv \begin{bmatrix} q \\ w \\ \rho \end{bmatrix}, \quad M \equiv \begin{bmatrix} I_K \otimes L & I_K \otimes L & -1_K \otimes I \\ -I_K \otimes L^T & \Delta_K \otimes C[I - L] & -1_K \otimes I \\ 1_K^T \otimes I & 1_K^T \otimes I & 1_K^T \otimes I \end{bmatrix}.
\]

- \( \Delta \sigma = \Delta_K \otimes C[I - L]w \)
- \( \Delta_K \): Backward-difference operator

▷ Non-negative queuing delay is guaranteed [Ban et al. 12]
Connections with DSO assignment

- If equilibrium value of $\Delta \sigma^*(s) (= \Delta_K \otimes C[I - L]w^*)$ is known in advance, the LCP reduces to parametric LP problem.

$$\min_{q \geq 0} \cdot \sum_{s=1}^{K} \sum_{i=1}^{N} (p(s) + \sum_{j=1}^{i} c_j) q_i(s)$$

s.t. $\sum_{j=i}^{N} q_i(s) \leq \mu_i \Delta \sigma_i^*(s) \quad \forall i, s$ ... [Capacity const.]

$$\sum_{s=1}^{K} q_i(s) = Q_i \quad \forall i \quad \ldots [OD \ flow \ conservation]$$

- It can be interpreted as a variant of DSO problem which has the capacity that is given by $C \Delta \sigma^*(s)$ rather than actual capacity $C_1$.

- 1 BN case: The parametric LP reduces to LP because $\Delta \sigma_1(s) = 1.$ (LP-based DTC equilibrium formulation by Iryo & Yoshii (07))
Connections with DSO assignment

- If equilibrium value of $\Delta \sigma^*(s) = \Delta K \otimes C[I - L]w^*$ is known in advance, the LCP reduces to parametric LP problem.

\[
\begin{align*}
\min_{q \geq 0} & \cdot \sum_{s=1}^{K} (p(s)1 + Lc) \cdot q(s) \\
\text{s.t. } & L^T q(s) \leq C \Delta \sigma^*(s) \quad \forall s \in S \quad \text{... [Capacity const.]} \\
\sum_{s=1}^{K} q(s) &= Q \quad \text{... [OD flow conservation]}
\end{align*}
\]

- It can be interpreted as a variant of DSO problem which has the capacity that is given by $C \Delta \sigma^*(s)$ rather than actual capacity $C1$.
- 1 BN case: The parametric LP reduces to LP because $\Delta \sigma_1(s) = 1$. (LP-based DTC equilibrium formulation by Iryo & Yoshii (07))
Properties of the equilibrium problem

- **Non-monotonicity of the LCP mapping**
  - Queuing (supply) sub-model $w(q)$ is not generally monotone.
    - Overall LCP is not monotone.
  - Queuing sub-model has **P-property (uniqueness).**
    - However, overall LCP does not have such a useful property.

- **Intuitive explanation of non-uniqueness of equilibrium:**
  - The solution of the (parametric) LP is not necessarily unique, which implies the equilibrium OD flow pattern **under homogeneous case** is not necessarily unique *.
    - * even if the equilibrium cost pattern is unique.
Existence of equilibria

- Assumption: Schedule delay function $p(s)$ satisfies
  \[
  \Delta p(s) \geq -1, \text{ for homogeneous case}
  \]
  \[
  \Delta p(s) \geq -1 + (1/\theta), \text{ for logit (heterogeneous) case}
  \]

- Theorem (for both homogeneous and heterogeneous cases)
  
  * If the assumption holds, the equilibrium exists for both cases of homogeneous and heterogeneous users.

- Overview of the proof: Kakutani’s fixed point theorem
  * Construct a fixed point problem for OD demands $q$
  * The mapping is upper hemi-continuous
  * The feasible set of $q$ is non-empty, compact and convex.
Uniqueness of equilibrium

- Theorem (for only heterogeneous case)

  \textbf{For heterogeneous user case, suppose that the assumption holds. Then, the equilibrium is unique.}

- Overview of the proof: Poincaré-Hopf’s index theorem for complementarity problems [e.g., Simsek et al., 07]
  * Nonlinear complementarity problem (Heterogeneous case)
  * Consider the principal sub matrix of the Jacobian matrix of the NCP mapping corresponding to the indices that satisfy strict complementarity condition.
  * The positivity of determinant of this matrices at equilibrium points is proved.
Numerical example: 3 bottlenecks

- Equilibrium vs. system optimal assignment
  - There exists a case that *user’s cost and aggregate cumulative departure curves* are same for both assignments.
    - DUE equilibrium cost
      \[ = \text{FFTT + SDC + queuing delay} \]
    - DSO user’s cost
      \[ = \text{FFTT + SDC + opt. time-dependent congestion toll} \]
Numerical example: 3 bottlenecks

Equilibrium vs. system optimal assignment

- There exists a case that user’s cost and aggregate cumulative departure curves are same for both assignments.
- Pareto improvement can be achieved by an optimal policy.

* Aggregate DUE pattern (red: arrival, blue: departure)
Numerical example: 3 bottlenecks

Equilibrium vs. system optimal assignment

- There exists a case that *costs and aggregate cumulative departure curves* are same for both assignments.
- *Pareto improvement can be achieved by an optimal policy.*
  - Aggregate DSO pattern (green: arrival/departure)
Equilibrium vs. system optimal assignment

- However, *disaggregate cumulative curves are different* for two assignments.
- For users who have residential location $N = 3$
  * Disaggregate DUE pattern (red: arrival, blue: departure)
  * Disaggregate DSO pattern (green: arrival/departure)
Concluding remarks

- Present a transparent approach to the analysis of equilibrium for the corridor problem with discrete multiple BNs.
  - The equilibrium condition was formulated in a Lagrangian-like coordinate system.
  - The equilibrium condition reduces to clear and concise Linear/Nonlinear Complementarity Problems.

- Establish several results on the properties of equilibrium
  - Properties of queuing-sub model, relations with DSO assignment, Non-monotonicity of the problem were obtained.
  - Existence of equilibria for both cases was proved under an assumption on schedule delay function.
  - Uniqueness of equilibrium for hetero. case was proved.
References


de Palma, A., Lindsey, R., 2002. Comparison of morning and evening commutes in the Vickrey bottleneck model. Transportation Research Record 1807, 26–33.


Point-queue model

• Original Vickrey’s point queue model:

\[
\dot{d}_i(t) = \begin{cases} 
(\lambda_i(t)/\mu_i) - 1 & \text{if } d_i(t) > 0 \\
\max [0, (\lambda_i(t)/\mu_i) - 1] & \text{if } d_i(t) = 0 
\end{cases}
\]

* Time discretization (both forward and backward difference schemes) leads to a negative queuing delay.

▷ Relaxation of the \(\max\{\cdot, \cdot\}\) operation

\[
\begin{cases} 
\dot{d}_i(t) = (\lambda_i(t)/\mu_i) - 1 & \text{if } d_i(t) > 0 \\
\dot{d}_i(t) \geq (\lambda_i(t)/\mu_i) - 1 & \text{if } d_i(t) = 0 
\end{cases}
\]

* Non-negative queuing delay is guaranteed by using backward difference scheme [Akamatsu, 01; Ban et al. 12].
Overall DTC equilibrium problem

❖ Time discretization:

- Finite LCP with a **skew-symmetric matrix** $M$

  Find $X \equiv [q, w, \rho]^T$ such that $0 \leq X \perp F(X) \equiv MX + b \geq 0$,

  
  \[
  M \equiv \begin{bmatrix}
  \mathbf{I} & -\mathbf{I} \otimes \mathbf{L}^T & \mathbf{1}^T \otimes \mathbf{I} \\
  -\mathbf{I} \otimes \mathbf{L} & \Delta \mathbf{K} \otimes \mathbf{C} [\mathbf{I} - \mathbf{L}] & -\mathbf{1} \otimes \mathbf{I} \\
  \mathbf{1} \otimes \mathbf{I} & \Delta \mathbf{K} \otimes \mathbf{C} [\mathbf{I} - \mathbf{L}] & \mathbf{I} \\
  \end{bmatrix},
  \]

  ▶ can be convert into a **VI problem with only cost variables**:

  Find $(w^*(s), \rho^*) \in \Omega$ such that

  \[
  \sum_{s=1}^{K} (w(s) - w^*(s)) \cdot \Delta \sigma^*(s) - (\rho - \rho^*) \cdot Q \geq 0 \quad \forall (w(s), \rho) \in \Omega
  \]

  where $\Delta \sigma^*(s) \equiv 1 + [\mathbf{I} - \mathbf{L}] \Delta w^*(s)$,
Remark on the LCP formulation

- Cum. curves at each BN should not be backward-bending.
  - In our formulation, \( \Delta \sigma_i(s), \Delta \tau_i(s) \geq 0 \)
  - From the condition: 
    \[
    C \Delta \sigma(s) \geq L^T q(s) \geq 0
    \]
    we have \( \Delta \sigma_1(s), \ldots, \Delta \sigma_N(s) \geq 0, \Delta \tau_1(s), \ldots, \Delta \tau_{N-1}(s) \geq 0 \)
  - **Consistency condition** \( \Delta \tau_N \geq 0 \) should be satisfied.
    * Fortunately, this condition is always satisfied when \( \Delta p(s) \geq -1 \).
      This is consistent with Smith’s (84) existence condition.
An alternative formulation

- Incorporating the consistency condition explicitly
  - $\tau_i(s)$ is defined from *upstream* rather than downstream.

\[
\begin{align*}
\tau_i(s) &= \tau_N(s) + \sum_{j=i+1}^{N} w_j(s) \\
\tau_0(s) &= \tau_N(s) + \sum_{j=1}^{N} w_j(s) = s
\end{align*}
\]

- Equivalent Mixed Complementarity Problem

Find $X \equiv [q, w, \rho]^T$ and $\Delta \tau_N$ such that

\[
0 \leq X \perp F(X) \equiv MX + b \geq 0, \text{ and } \Delta \tau_N + \sum_{j=1}^{N} \Delta w_j = 1
\]

\[
M \equiv \begin{bmatrix}
-I_K \otimes L^T & I_K \otimes L & -I_K \otimes I \\
-I_K \otimes L^T & \Delta_K \otimes CL^T & -I_K \otimes L^T \\
1_K \otimes I & 1_K \otimes I & I_K \otimes I
\end{bmatrix},
\]
One-to-many problem

- One-to-many networks (Evening rush)

- $N$ off-ramps (nodes) and $N$ bottlenecks
  - are numbered $i = 1, 2, \ldots, N$ from upstream to downstream
- $\mu_i$: capacity of bottleneck $i$ on link $(i-1, i)$
  - *Point-queue model* is assumed.
- $Q_i$: # of commuters in residential location $i$ (given)
  - They leave from the origin during rush hour $[0, T]$
One-to-many problem

- Lagrangian-like (moving) coord. system
  - Times are defined at position $i$ for origin depart. time $s$

\[
\sigma_i(s) = \tau_i(s) + w_i(s), \quad \tau_1(s) = s + c_1
\]

departure time, arrival time, queuing delay

Depart from bottleneck $i$
Arrive at bottleneck $i$

Depart from bottleneck $i-1$
Arrive at bottleneck $i-1$

Depart from the origin
One-to-many problem

- **Equivalent Complementarity Problem**
  - Homogeneous case: LCP (cf. Heterogeneous case: NCP)
    * eliminating redundant variables \{y(s), \tau(s)\}
  
  \[
  0 \leq q(s) \perp p(s)1 + L(c + w(s)) - \rho \geq 0, \quad \forall s \in S
  \]
  ... Departure-time choice conditions

  \[
  0 \leq w(s) \perp C\Delta\sigma(s) - L^Tq(s) \geq 0, \quad \forall s \in S
  \]
  ... Queuing conditions

  \[
  0 \leq \rho \perp \int_S q(s)ds - \bar{Q} \geq 0 \quad \text{... Flow conservations}
  \]

  where $\Delta\sigma(s) \equiv 1 + L\Delta w(s)$ ($\Delta\sigma_i(s) = 1 + \sum_{j=1}^i \Delta w_i(s)$)
Remark on the LCP formulation: O2M

- Cum. curves at each BN should not be **backward-bending**.
  - In our formulation, \( \Delta \sigma_i(s), \Delta \tau_i(s) \geq 0 \)
  - From the condition:
    \[
    C \Delta \sigma(s) \geq L^T q(s) \geq 0
    \]
    we have \( \Delta \sigma_1(s), \ldots, \Delta \sigma_N(s) \geq 0, \Delta \tau_2(s), \ldots, \Delta \tau_N(s) \geq 0 \)
  - **Consistency condition** \( \Delta \tau_1 \geq 0 \) should be satisfied.
    - Fortunately, this condition is **always satisfied** because the arrival time at the most upstream BN is equal to departure time at origin (reference time): \( \tau_1(s) = \sigma_0(s) = s \Rightarrow \Delta \tau_1(s) = 1 \).
One-to-many problem

❖ Schedule delay function:
  • Schedule delay cost associated with deviation from wished departure time from the origin.
Solution method

- **ReSNA** (**R**egularized **S**moothing **N**ewton Algorithm)
  - was originally developed for solving the second-order cone complementarity problems (SOCCP) [Hayashi et al., 05]
    - SOCCP involves the LCP/NCP as a subclass
  - **Global convergence** of the algorithm is proved under the $P_0$ assumption (a weaker condition than monotonicity) for NCP.
  - **Quadratic convergence** of the algorithm is also proved under monotonicity assumption.

▷ Our experiments have successfully obtained equilibrium solutions in most cases (although our problem is not $P_0$).

★ Please try to use ReSNA:
  * [http://www.plan.civil.tohoku.ac.jp/opt/hayashi/ReSNA/](http://www.plan.civil.tohoku.ac.jp/opt/hayashi/ReSNA/)
Numerical example

Many-to-one vs. One-to-many OD demands

First row: Aggregate cumulative curves
Second row: Cumulative curves for users with origin 1
Third row: Cumulative curves for users with origin 2
Fourth row: Cumulative curves for users with origin 3
Numerical example

- Many-to-one vs. One-to-many OD demands

First row: Aggregate cumulative curves
Second row: Cumulative curves for users with destination 1
Third row: Cumulative curves for users with destination 2
Fourth row: Cumulative curves for users with destination 3

Desired departure time