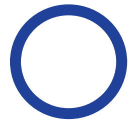
Invariance of Khovanov homology and quantum code **伊藤昇 (東大)**

ある LDPC **量子誤り訂正** code **は**, **ホモロジーとして捉えられる**.

ホモロジーの不変性は "図形" の形を判別する.  $\neq$ 

Jones 1984	**ジョーンズ多項式** $J(K)$ **を発見**
Khovanov 2000	TQFT **から** $J(K)$ **より判別が高度な** $H(K)$ **を発見**. $J(K) = \sum_j q^j \sum_i (-1)^i \dim H^{i,j}(K)$.
Viro 2004	$H(K)$ **の基底を記述**, Type I **不変性を再証明**.
発表者 2011	**続いて** Type II, Type III **不変性を再証明**.
Audoux 2014	[Viro2004, Ito2011] **を** Quantum code **へ応用**.

分野横断ワークショップ, **量子コンピュータ研究開発の現在とこれから**, **文部科学省**, 2020.1.

KHOVANOV HOMOLOGY AND QUANTUM CODE

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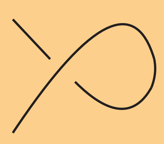
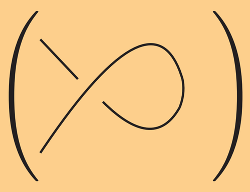
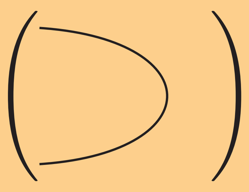
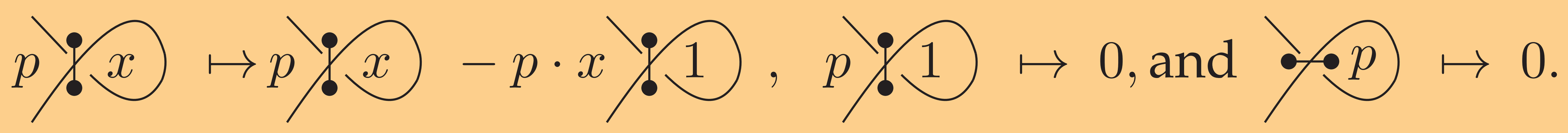
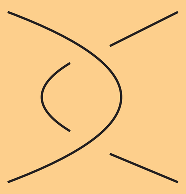
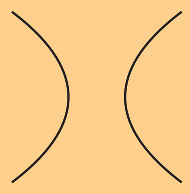
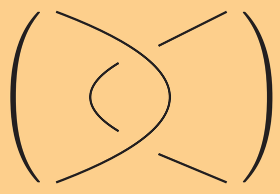
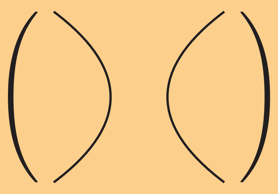
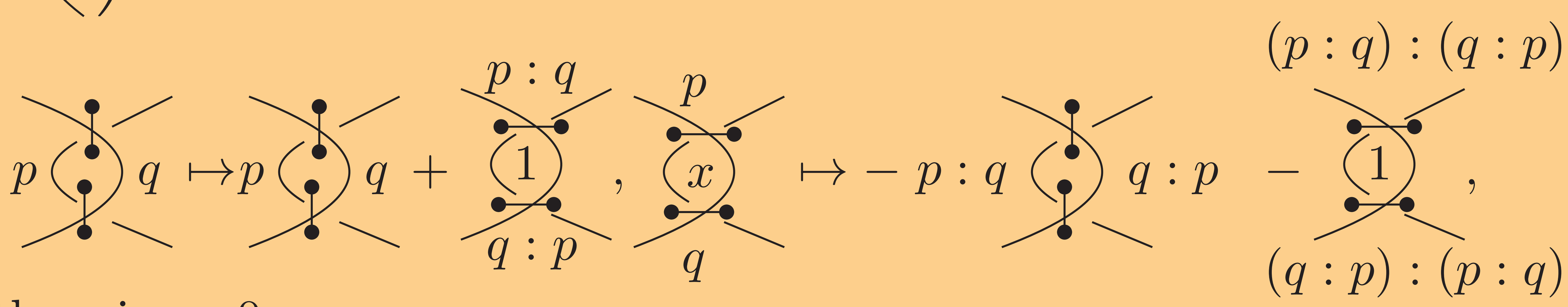
RESULTS

Theorem 1 ([Viro (2004)], [Ito (2011)]). Explicit correspondences between generators of Khovanov homology groups for each of Reidemeister moves of type I (Viro) and of types II and III (Ito) are given as follows.

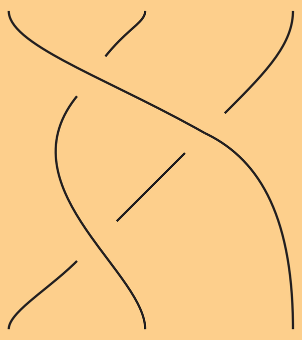
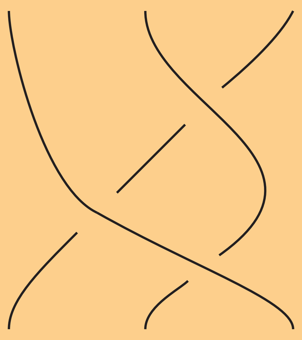
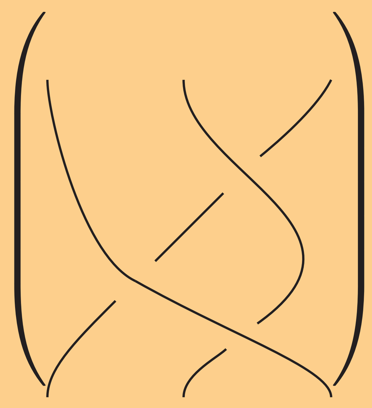
(**Type I, Viro**) Invariance under  $\sim$  is given by $\mathcal{C}$  $= \mathcal{C} \oplus \mathcal{C}_{\text{contr}} \overset{\rho_1}{\to} \mathcal{C} \overset{\text{isom}_1}{\to} \mathcal{C}$  with a chain homotopy, here ρ_1 is defined by



(**Type II, Ito**) Invariance under  $\sim$  is given by $\mathcal{C}$  $= \mathcal{C} \oplus \mathcal{C}_{\text{contr}} \overset{\rho_2}{\to} \mathcal{C} \overset{\text{isom}_2}{\to} \mathcal{C}$  with a chain homotopy, here ρ_2 is defined by

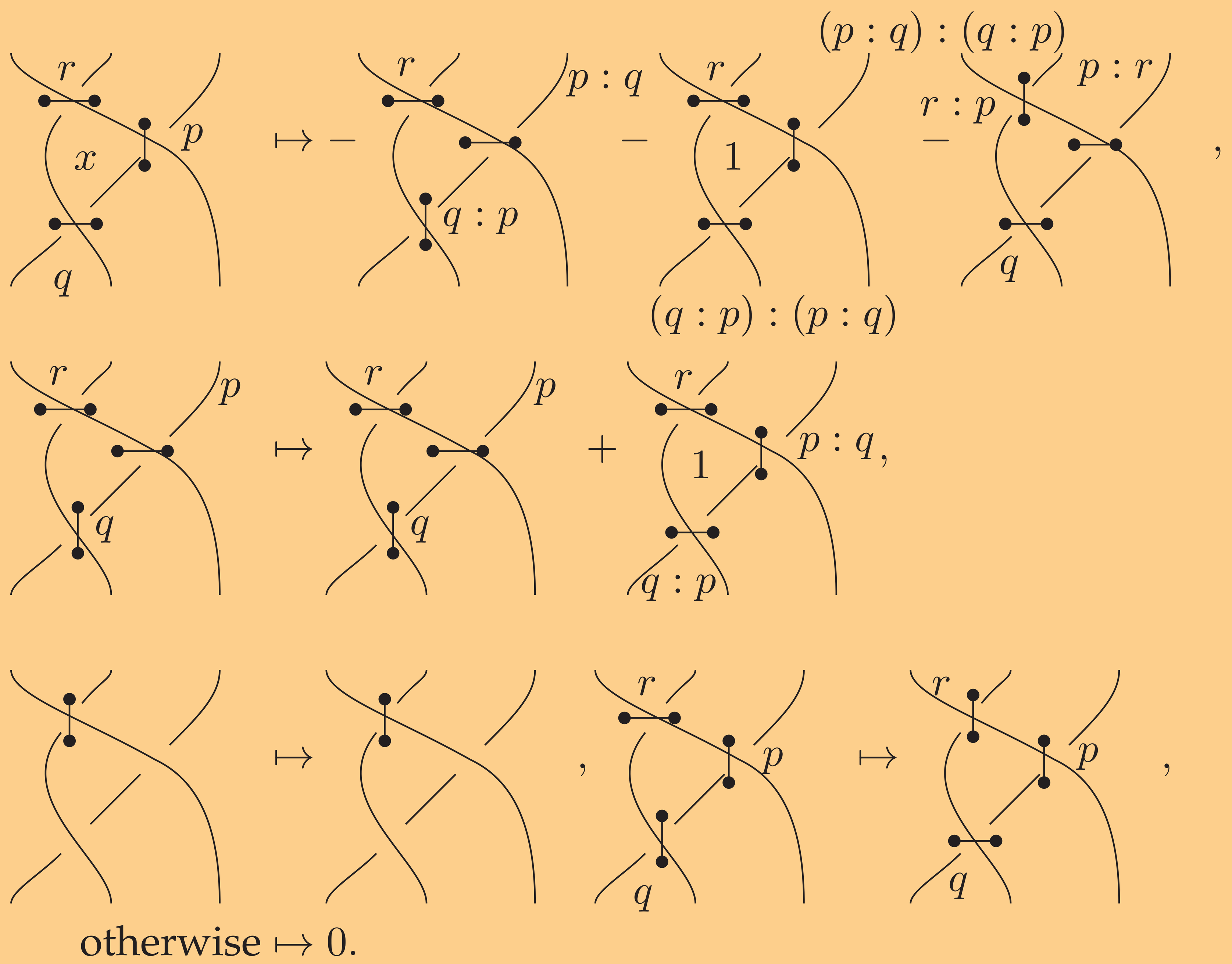


otherwise $\mapsto 0$.

(**Type III, Ito**) Invariance under  $\sim$  is given by $\mathcal{C}'$  $=$ $\mathcal{C}' \oplus \mathcal{C}'_{\text{contr}}$,

$\mathcal{C}$  $= \mathcal{C} \oplus \mathcal{C}_{\text{contr}} \overset{\rho_3}{\to} \mathcal{C} \overset{\text{isom}_3}{\to} \mathcal{C}' \overset{\text{isom}_3^{-1}}{\to} \mathcal{C} \overset{\text{in}}{\to} \mathcal{C}$ ,

and a chain homotopy, here ρ_3 is defined by



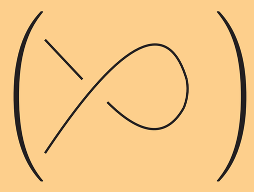
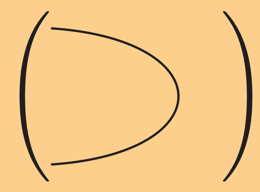
otherwise $\mapsto 0$.

QUANTUM CODE

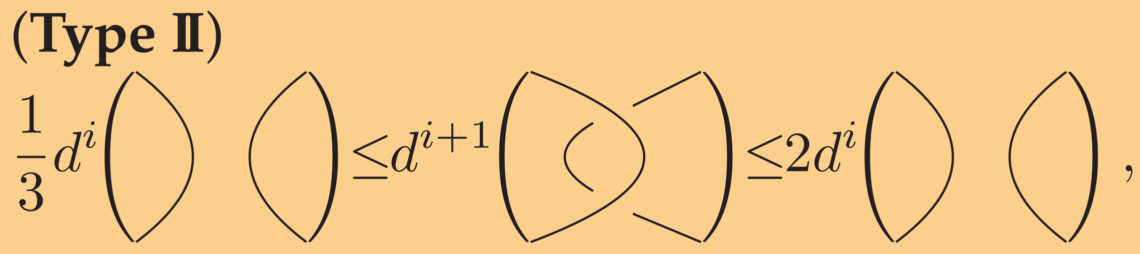
An application of Theorem 1 to quantum code is given by B. Audoux.

Theorem [Audoux (2014)] Let $d(\cdot)$ be the minimum distance of a code, i.e. $d(\cdot)$ is the minimal weight of a non detectable error that does alter codewords for a Khovanov complex of a link diagram. Then,

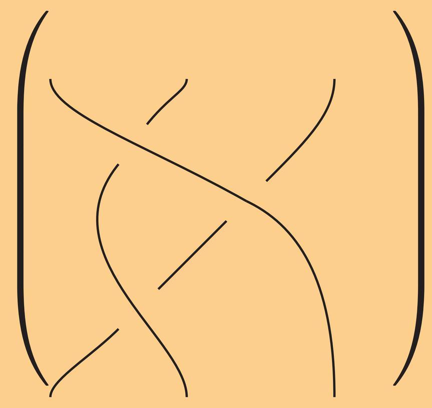
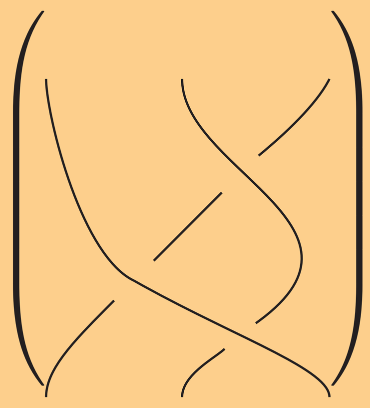
(Type I)

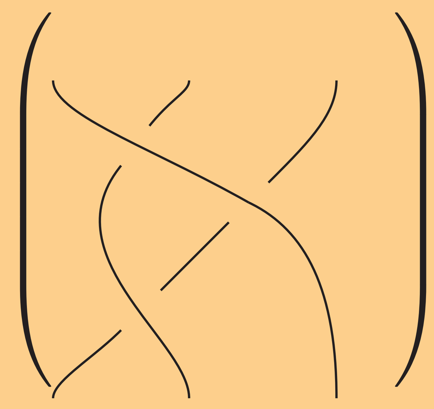
d^i  $= 2d^i$ ,

(Type II)



(Type III)

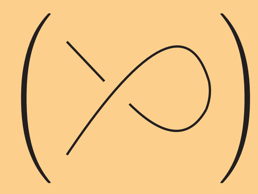
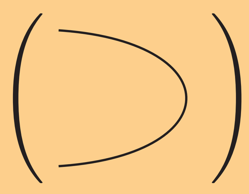
$\frac{1}{8}d^i$  $\leq d^i$ 

$\leq 8d^i$ .

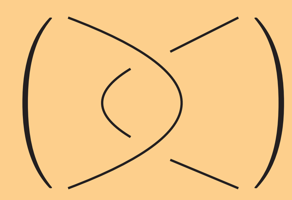
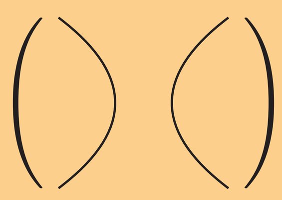
KHOVANOV HOMOLOGY

Khovanov homology is a bigraded homology and a link invariant such that

(Type I)

$\mathcal{H}^{i,j}$  $\simeq \mathcal{H}^{i,j}$ ,

(Type II)

$\mathcal{H}^{i,j}$  $\simeq \mathcal{H}^{i,j}$ ,

(Type III)

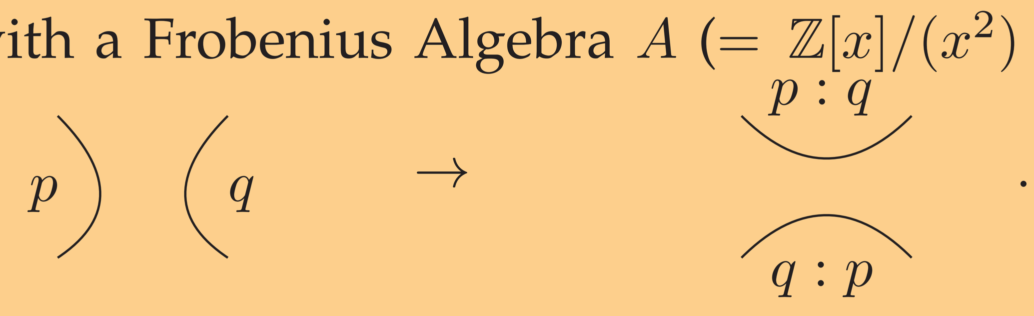
$\mathcal{H}^{i,j}$  $\simeq \mathcal{H}^{i,j}$ ,

and inducing Jones polynomial

$$V_L(q+q^{-1}) = \sum_j q^j \sum_i (-1)^i \operatorname{rank} \mathcal{H}^{i,j}(L).$$

FORMULATION

The differential of $\mathcal{H}^i$ is indued by a saddle for TQFT : $\mathbf{Cob}_2 \to \mathbf{Mod}_k$ associated with a Frobenius Algebra A $(= \mathbb{Z}[x]/(x^2))$.



REFERENCES

[Viro (2004)] O. Viro, Khovanov homology, its definitions and ramifications, *Fund. Math.* 184 (2004), 317–342.

[Ito (2011)] N. Ito, Chain homotopy maps for Khovanov homology, *J. Knot Theory Ramifications* **20** (2011) 127–139.

[Audoux (2014)] B. Audoux, An application of Khovanov homology to quantum codes, *Ann. Inst. Henri Poincaré D* **1** (2014), 185–223.