Gradient flow representation of the 4D $\mathcal{N} = 2$ super Yang–Mills supercurrent

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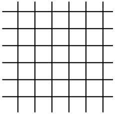
- A. Kasai, O.M. and H. Suzuki, PTEP **2018** (2018) no.11, 113B02 [arXiv:1808.07300 [hep-lat]].

Introduction

- 格子正則化による第一原理からの非摂動計算



$\updownarrow a$: 格子間隔

- 時空対称性（並進対称性、SUSY, ...）と相性が悪い
- 連続極限 $a \to 0$ でこれらの対称性は回復する（ように tuning）
  - 対応する Ward–Takahashi (WT) 関係式を満たす
- 付随する Noether カレントを構成することが困難
  - 複合演算子からの UV 発散：$a \times (1/a) \overset{a\to 0}{\to} 1$
- グラディエントフロー による「正則化に依らない構成法」
  - エネルギー運動量テンソル [Suzuki '13; Makino–Suzuki '14]
  - $\mathcal{N} = 1$ SYM スーパーカレント [Hieda-Kasai-Makino-Suzuki '17], ...

▶ 4D $\mathcal{N} = 2$ SYM スーパーカレントの構成 [Kasai–O.M.–Suzuki '18]

Gradient flow

- グラディエントフロー [Narayanan–Neuberger '06; Lüscher '10, '13]

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x) = D_\nu D_\nu B_\mu + \dots, \quad B_\mu(t=0,x) = A_\mu(x),$$
$$\partial_t \chi(t,x) = D_\mu D_\mu \chi(t,x), \quad \chi(t=0,x) = \psi(x),$$
$$\partial_t \bar{\chi}(t,x) = \bar{\chi}(t,x)\overleftarrow{D}_\mu \overleftarrow{D}_\mu, \quad \bar{\chi}(t=0,x) = \bar{\psi}(x).$$

where $G_{\mu\nu}(t,x) = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$,

$$D_\mu = \partial_\mu + [B_\mu, \cdot], \qquad \overleftarrow{D}_\mu = \overleftarrow{\partial}_\mu - [\cdot, B_\mu].$$

- 拡散方程式（拡散長 $x \sim \sqrt{8t}$）
- フローされた場 $(t > 0)$ の複合演算子は自動的に有限
  - ファルミオンは波動関数くりこみが必要：

$$\chi_R(t,x) = Z_\chi^{1/2}\chi(t,x) \qquad \bar{\chi}_R(t,x) = Z_\chi^{1/2}\bar{\chi}(t,x)$$

  - χ_R, $\bar{\chi}_R$ の複合演算子は有限
- 複合演算子の正則化に依らない表式を与える

Small flow-time expansion

- フローされた複合演算子と元々の理論の複合演算子の関係?
- Small flow-time expansion [Lüscher–Weisz 2011]

$$\tilde{\mathcal{O}}_i(t,x) = \sum_j \zeta_{ij}(t)\mathcal{O}_j(x) + O(t)$$

$$\longrightarrow \qquad \mathcal{O}_i(x) = \lim_{t\to 0}\left\{\sum_j (\zeta^{-1})_{ij}(t)\tilde{\mathcal{O}}_j(t,x)\right\}$$

- $\zeta_{ij}(t)$ はくりこみスケール μ に依らない

$$\zeta(t)[g,m;\mu] = \zeta(t)\left[\bar{g}(1/\sqrt{8t}), \bar{m}(1/\sqrt{8t}); 1/\sqrt{8t}\right]$$

- 漸近自由性 $\bar{g}(1/\sqrt{8t}) \overset{t\to 0}{\to} 0$ より摂動論で $\zeta_{ij}(t)$ が計算できる

$\underbrace{\text{非摂動計算}}_{\text{格子正則化}} \leftarrow$ フローされた複合演算子 $\rightarrow \underbrace{\text{正しい表式}}_{\text{次元正則化}}$

Supercurrent of the 4D $\mathcal{N}=2$ SYM

- エネルギー運動量テンソルの構成・数値計算
- グラディエントフロー → 超対称性理論
  - $\mathcal{N}=1$ SYM のフロー方程式 [Kikuchi-Onogi '14, Kadoh-Ukita '18]
  - $\mathcal{N}=1$ SYM スーパーカレント [Hieda–Kasai–Makino–Suzuki '16]
- 次のターゲット：

  4D $\mathcal{N}=2$ SYM スーパーカレントの構成 [Kasai–O.M.–Suzuki]

$$\mathcal{L} = \frac{1}{4g_0^2} F_{\mu\nu}^a F_{\mu\nu}^a + \bar{\psi}^a \not{D} \psi^a + D_\mu \varphi^{\dagger a} D_\mu \varphi^a - \frac{g_0^2}{2} f^{abc} f^{ade} \varphi^{\dagger b} \varphi^c \varphi^{\dagger d} \varphi^e + \sqrt{2} g_0 f^{abc} \bar{\psi}^a \left(P_+ \varphi^b - P_- \varphi^{\dagger b} \right) \psi^c$$

in Wess–Zumino ゲージ, where $P_\pm = \frac{1}{2}(1 \pm \gamma_5)$

  - スカラー場 φ を含む理論に拡張

- SUSY を明白に保つ正則化がない（次元正則化で計算）

  →（1 ループレベルで）正しく規格化されたスーパーカレント??

SUSY WT relation

- e.g., $\delta_{\text{SUSY}}\langle A_\nu^b(y)\bar{\psi}^c(z)\rangle = 0$

$$
\begin{aligned}
&\left\langle \left[\partial_\mu S_\mu + X_{\text{Fierz}} + X_{\text{gf}} + X_{c\bar{c}}\right](x) A_\nu^b(y)\bar{\psi}^c(z)\right\rangle \\
&= -\delta(x-y)\frac{g_0}{2}\left\langle \gamma_\nu \psi^b(y)\bar{\psi}^c(z)\right\rangle \\
&\quad - \delta(x-z)\frac{1}{4g_0}\left\langle A_\nu^b(y)\sigma_{\rho\sigma}F_{\rho\sigma}^c(z)\right\rangle \\
&\quad + \delta(x-z)\frac{g_0}{2}\left\langle A_\nu^b(y)\gamma_5 f^{cde}\varphi^{\dagger d}(z)\varphi^e(z)\right\rangle \\
&\quad + \delta(x-z)\frac{1}{\sqrt{2}}\left\langle A_\nu^b(y)\gamma_\rho\left[P_- D_\rho\varphi^c(z) - P_+ D_\rho\varphi^{\dagger c}(z)\right]\right\rangle \\
&\quad - \partial_\mu^x\delta(x-z)\frac{1}{2\sqrt{2}}\left\langle A_\nu^b(y)\gamma_\mu\left[P_-\varphi^c(z) - P_+\varphi^{\dagger c}(z)\right]\right\rangle
\end{aligned}
$$

X_{Fierz}: Fierz 恒等式の破れ, $X_{\text{gf},c\bar{c}}$: ゲージ固定・ゴーストによる SUSY の破れ

- くりこまれた量で書き直し、正しいスーパーカレントを決定
  - SUSY 変換は非線形（Wess–Zumino ゲージ）

Renormalization (1)

- Feynman ゲージで、右辺をくりこまれた量で書くと

$$
\begin{aligned}
&-(1-3\Delta)\delta(x-y)\frac{g}{2}\left\langle \gamma_\nu \psi_R^b(y)\bar{\psi}_R^c(z)\right\rangle \\
&\quad -\left(1-\frac{5}{2}\Delta\right)\delta(x-z)\frac{1}{4g}\left\langle A_{\nu R}^b(y)\sigma_{\rho\sigma}\left[\partial_\rho A_{\sigma R}^c(z)-\partial_\sigma A_{\rho R}^c(z)\right]\right\rangle \\
&\quad +(1-3\Delta)\delta(x-z)\frac{g}{2}\left\langle A_{\nu R}^b(y)\gamma_5\{f^{cde}\varphi^{\dagger d}\varphi^e\}_R(z)\right\rangle \\
&\quad +\left(1-\frac{5}{2}\Delta\right)\delta(x-z)\frac{1}{\sqrt{2}}\left\langle A_{\nu R}^b(y)\gamma_\rho\left[P_-\partial_\rho\varphi_R^c(z)-P_+\partial_\rho\varphi_R^{\dagger c}(z)\right]\right\rangle \\
&\quad -(1-\Delta)\partial_\mu^x\delta(x-z)\frac{1}{2\sqrt{2}}\left\langle A_{\nu R}^b(y)\gamma_\mu\left[P_-\varphi_R^c(z)-P_+\varphi_R^{\dagger c}(z)\right]\right\rangle \\
&\qquad +(\text{higher order terms})
\end{aligned}
$$

ここで $\Delta = \frac{g^2}{(4\pi)^2}C_2(G)\times(1/\epsilon)$

Renormalization (2)

- くりこまれた演算子 $X_{\mathrm{gf}R}$, $X_{c\bar{c}R}$, $S_{\mu R}$:

$$
\begin{aligned}
&X_{\mathrm{gf}} + X_{c\bar{c}} \\
&= (1+\Delta)(X_{\mathrm{gf}R} + X_{c\bar{c}R}) \\
&\quad + \Delta\partial_\mu S_\mu + \Delta\frac{1}{8g}\partial_\mu\left[(A^a_{\nu R}\gamma_\nu\gamma_\mu + 2A^a_{\mu R})\frac{\delta S}{\delta\bar{\psi}^a}\right] + \Delta g\gamma_\mu\psi^a_R\frac{\delta S}{\delta A^a_\mu} \\
&\quad + \Delta\Bigg[\frac{3}{8g}\sigma_{\mu\nu}(\partial_\mu A^a_{\nu R} - \partial_\nu A^a_{\mu R}) - g\gamma_5\{f^{abc}\varphi^{\dagger b}\varphi^c\}_R \\
&\qquad - \frac{3}{2\sqrt{2}}\gamma_\mu(\partial_\mu\varphi^a_R P_- - \partial_\mu\varphi^{\dagger a}_R P_+)\Bigg]\frac{\delta S}{\delta\bar{\psi}^a} \\
&\quad + \Delta(-\sqrt{2})P_-\psi^a_R\frac{\delta S}{\delta\varphi^a} + \Delta\sqrt{2}P_+\psi^a_R\frac{\delta S}{\delta\varphi^{\dagger a}} + (\text{higher order terms})
\end{aligned}
$$

$$
S_\mu = S_{\mu R} + \Delta\left(-\frac{1}{8g}\right)(A^a_{\nu R}\gamma_\nu\gamma_\mu + 2A^a_{\mu R})\frac{\delta S}{\delta\bar{\psi}^a} + (\text{higher order terms})
$$

SUSY WT relation in renormalized quantities

- 有限のくりこまれた量で書かれた SUSY WT 関係式

$$\left\langle [\partial_\mu S_{\mu R} + X_{\mathrm{gf}R} + X_{c\bar{c}R}](x) A^b_{\nu R}(y) \bar{\psi}^c_R(z) \right\rangle$$
$$= -\delta(x-y) \frac{g}{2} \left\langle \gamma_\nu \psi^b_R(y) \bar{\psi}^c_R(z) \right\rangle$$
$$- \delta(x-z) \frac{1}{4g} \left\langle A^b_{\nu R}(y) \sigma_{\rho\sigma} \left[\partial_\rho A^c_{\sigma R}(z) - \partial_\sigma A^c_{\rho R}(z) \right] \right\rangle$$
$$+ \dots$$

- 同様に $\delta\langle \varphi^b(y) \bar{\psi}^c(z) \rangle$, $\delta\langle \varphi^{\dagger b}(y) \bar{\psi}^c(z) \rangle$, $\delta\langle \bar{\psi}^b(y) c^c(z) \bar{c}^d(w) \rangle = 0$.
- ゲージ不変な演算子の on-shell 相関関数で、正しく規格化されたスーパーカレント

$$S_{\mu R} = -\frac{1}{4g_0} \sigma_{\rho\sigma} \gamma_\mu \psi^a F^a_{\rho\sigma}$$
$$+ \frac{1}{2\sqrt{2}} \left(\frac{1}{3} \sigma_{\mu\nu} - \delta_{\mu\nu} \right) \left(P_+ D_\nu \psi^a \varphi^a - P_- D_\nu \psi^a \varphi^{\dagger a} \right)$$
$$- \frac{1}{\sqrt{2}} \left(\frac{1}{3} \sigma_{\mu\nu} - \delta_{\mu\nu} \right) \left(P_+ \psi^a D_\nu \varphi^a - P_- \psi^a D_\nu \varphi^{\dagger a} \right)$$

Gradient flow representation of the supercurrent

- $S_{\mu R}$ をフローされた場で書き直す
  - スカラー場のグラディエントフロー [Capponi *et al.* '15, ...]

$$\partial_t \phi(t,x) = D_\mu D_\mu \phi(t,x), \qquad \phi(t=0,x) = \varphi(x)$$

- 波動関数くりこみを避けるために "Ringed field"
  $\mathring{\chi}$ [Makino–Suzuki '14], $\mathring{\phi}$ [Makino–O.M.–Suzuki '18] を導入

$$\mathring{\chi} \propto \chi / \sqrt{\langle \bar{\chi}^a \overleftrightarrow{\not{D}} \chi^a \rangle}, \qquad \mathring{\phi} \propto \phi / \sqrt{\langle \phi^{\dagger a} \phi^a \rangle}$$

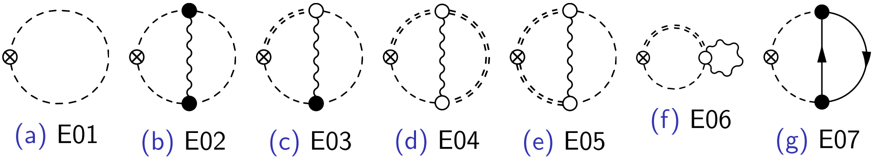
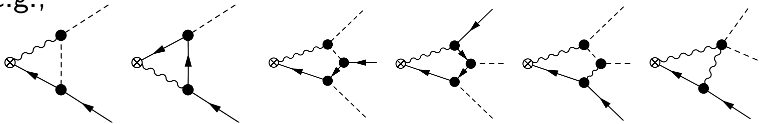


Figure: $\langle \phi^{\dagger a} \phi^a \rangle$

One-loop (flow) Feynman diagrams

- ゲージ場とフェルミオン（とゴースト）のみが関わるダイアグラムの計算 [Hieda–Kasai–Makino–Suzuki '17]
- スカラー場が関わるダイアグラムの計算が必要
- e.g.,



(a) A01 (b) A02 (c) A03 (d) A04 (e) A05 (f) A06

$$
\begin{aligned}
\frac{1}{g_0}\chi^a(t,x)G^a_{\mu\nu}(t,x) &= \left[1-\frac{2}{D-4}\xi(t)\right]\frac{1}{g_0}\psi^a F^a_{\mu\nu} \\
&\quad + \xi(t)\left\{\frac{2}{(D-4)(D-2)}\frac{1}{g_0}\left[\gamma_\mu\gamma_\rho\psi^a F^a_{\rho\nu}-\gamma_\nu\gamma_\rho\psi^a F^a_{\rho\mu}\right]\right. \\
&\qquad \left. + \frac{4}{(D-4)(D-2)D}\frac{1}{g_0}\sigma_{\rho\sigma}\sigma_{\mu\nu}\psi^a F^a_{\rho\sigma}\right\} + \cdots + \mathcal{O}(t)
\end{aligned}
$$

$$\xi(t) = \frac{g_0^2}{(4\pi)^2}C_2(G)(8\pi t)^{2-D/2}$$

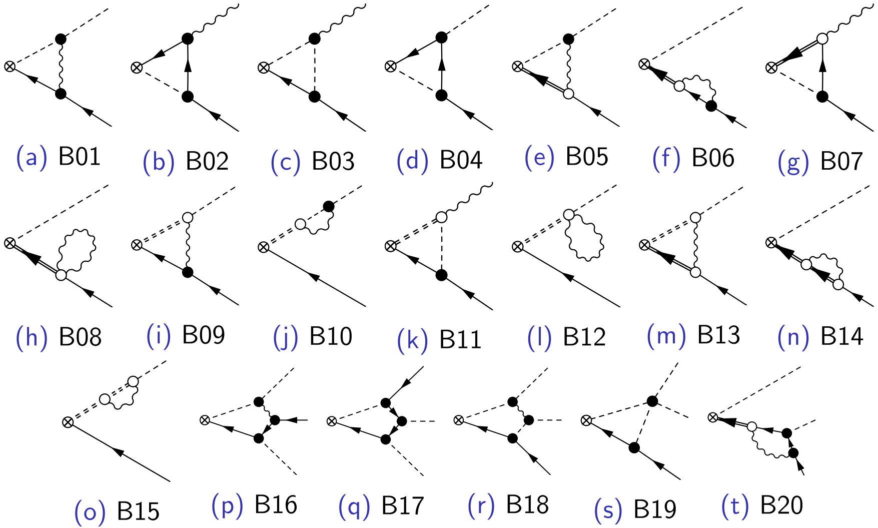
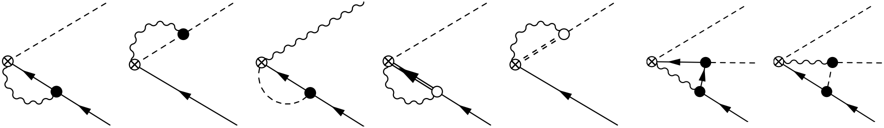
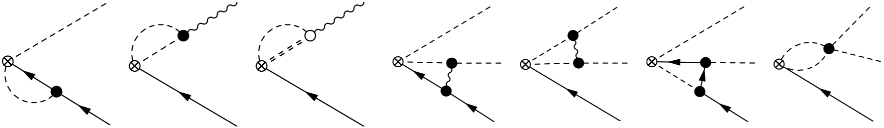


Figure: $\chi^a \partial_\mu \phi^a$

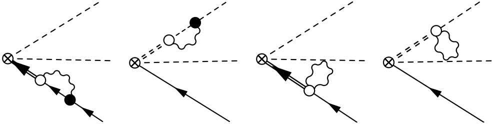


(a) C01 (b) C02 (c) C03 (d) C04 (e) C05 (f) C06 (g) C07

Figure: $f^{abc}\chi^a B^b_\mu \phi^c$



(a) D01 (b) D02 (c) D03 (d) D04 (e) D05 (f) D06 (g) D07



(h) D08 (i) D09 (j) D10 (k) D11

Figure: $f^{abc}\chi^a \phi^{\dagger b}\phi^c$

4D $\mathcal{N} = 2$ SYM supercurrent in terms of the gradient flow

$$
\begin{aligned}
S_\mu(x) = &-\frac{1}{4\bar{g}(1/\sqrt{8t})}\left\{1+\frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}C_2(G)\left[-\ln\pi-\frac{9}{4}+\frac{1}{2}\ln(432)\right]\right\}\sigma_{\rho\sigma}\gamma_\mu\mathring{\chi}^a(t,x)G^a_{\rho\sigma}(t,x)\\
&-\frac{\bar{g}(1/\sqrt{8t})}{(4\pi)^2}C_2(G)\gamma_\nu\mathring{\chi}^a(t,x)G^a_{\nu\mu}(t,x)\\
&+\frac{1}{2\sqrt{2}}\left\{1+\frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}C_2(G)\left[-\frac{19}{4}+4\ln 2+\frac{1}{2}\ln(432)\right]\right\}\\
&\qquad\times\left(\tfrac{1}{3}\sigma_{\mu\nu}-\delta_{\mu\nu}\right)\left(P_+D_\nu\mathring{\chi}^a(t,x)\mathring{\phi}^a(t,x)-P_-D_\nu\mathring{\chi}^a(t,x)\mathring{\phi}^{a\dagger}(t,x)\right)\\
&\quad-\frac{3}{\sqrt{2}}\frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}C_2(G)\left(P_+D_\mu\mathring{\chi}^a(t,x)\mathring{\phi}^a(t,x)-P_-D_\mu\mathring{\chi}^a(t,x)\mathring{\phi}^{a\dagger}(t,x)\right)\\
&-\frac{1}{\sqrt{2}}\left\{1+\frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}C_2(G)\left[\frac{1}{2}+4\ln 2+\frac{1}{2}\ln(432)\right]\right\}\\
&\qquad\times\left(\tfrac{1}{3}\sigma_{\mu\nu}-\delta_{\mu\nu}\right)\left(P_+\mathring{\chi}^a(t,x)D_\nu\mathring{\phi}^a(t,x)-P_-\mathring{\chi}^a(t,x)D_\nu\mathring{\phi}^{a\dagger}(t,x)\right)\\
&+\frac{1}{\sqrt{2}}\frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}C_2(G)\left(\tfrac{1}{3}\sigma_{\mu\nu}-\delta_{\mu\nu}\right)\gamma_5D_\nu\mathring{\chi}^a(t,x)\left(\mathring{\phi}^a(t,x)+\mathring{\phi}^{a\dagger}(t,x)\right)\\
&+\frac{1}{2\sqrt{2}}\frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}C_2(G)\left(\tfrac{1}{3}\sigma_{\mu\nu}-\delta_{\mu\nu}\right)\gamma_5\mathring{\chi}^a(t,x)\left(D_\nu\mathring{\phi}^a(t,x)+D_\nu\mathring{\phi}^{a\dagger}(t,x)\right)\\
&-\frac{1}{4}f^{abc}\frac{\bar{g}(1/\sqrt{8t})^3}{(4\pi)^2}C_2(G)\gamma_5\gamma_\mu\mathring{\chi}^a(t,x)\mathring{\phi}^{b\dagger}(t,x)\mathring{\phi}^c(t,x)
\end{aligned}
$$

Summary

- グラディエントフローによる 4D $\mathcal{N}=2$ super Yang–Mills スーパーカレントの構成
  - スカラー場を含む超対称性理論に拡張

1. 次元正則化で正しく規格化されたスーパーカレントの計算
2. Small flow time expansion を用いた正則化に依らない表式

- 格子ゲージ理論での数値計算に応用可能

- $\mathcal{N}=4$ SYM スーパーカレント?
  - Weyl $\times 4$, real scalar $\times 6$.
  - Yukawa ($SU(4)_R$), ϕ^4.
- フロー方程式の一般化 [Kikuchi–Onogi, Kadoh–Ukita]

SUSY transformation

$$\delta_\xi A^a_\mu = \frac{1}{2} g_0 (\bar{\xi}\gamma_\mu \psi^a - \bar{\psi}^a \gamma_\mu \xi),$$

$$\delta_\xi \varphi^a = \frac{1}{\sqrt{2}} (-\bar{\xi} P_- \psi^a + \bar{\psi}^a P_- \xi),$$

$$\delta_\xi \varphi^{\dagger a} = \frac{1}{\sqrt{2}} (\bar{\xi} P_+ \psi^a - \bar{\psi}^a P_+ \xi)$$

$$\delta_\xi \psi^a = -\frac{1}{4g_0} \sigma_{\mu\nu} \xi F^a_{\mu\nu} - \frac{1}{\sqrt{2}} \gamma_\mu P_+ \xi D_\mu \varphi^a + \frac{1}{\sqrt{2}} \gamma_\mu P_- \xi D_\mu \varphi^{\dagger a} - \frac{1}{2} g_0 \gamma_5 \xi f^{abc} \varphi^{\dagger b} \varphi^c$$

$$\delta_\xi \bar{\psi}^a = \frac{1}{4g_0} \bar{\xi} \sigma_{\mu\nu} F^a_{\mu\nu} - \frac{1}{\sqrt{2}} \bar{\xi} \gamma_\mu P_- D_\mu \varphi^a + \frac{1}{\sqrt{2}} \bar{\xi} \gamma_\mu P_+ D_\mu \varphi^{\dagger a} - \frac{1}{2} g_0 \bar{\xi} \gamma_5 f^{abc} \varphi^{\dagger b} \varphi^c$$

Improvement of the supercurrent

- "Canonical" supercurrent

$$S_\mu = -\frac{1}{4g_0}\sigma_{\rho\sigma}\gamma_\mu\psi^a F^a_{\rho\sigma} + \frac{1}{2}g_0 f^{abc}\gamma_5\gamma_\mu\psi^a\varphi^{\dagger b}\varphi^c + \frac{1}{\sqrt{2}}\gamma_\nu\gamma_\mu P_+\psi^a D_\nu\varphi^a - \frac{1}{\sqrt{2}}\gamma_\nu\gamma_\mu P_-\psi^a D_\nu\varphi^{\dagger a}$$

- (Classically) γ-traceless supercurrent

$$S^{\mathrm{imp}}_\mu \equiv S_\mu + \frac{\sqrt{2}}{3}\sigma_{\mu\nu}\partial_\nu(P_+\psi^a\varphi^a - P_-\psi^a\varphi^{\dagger a})$$

- "Finite" supercurrent by adding terms $\propto$ EoM

$$\tilde{S}^{\mathrm{imp}}_\mu \equiv S^{\mathrm{imp}}_\mu - \frac{1}{2\sqrt{2}}\gamma_\mu(P_-\not{D}\psi^a\varphi^a - P_+\not{D}\psi^a\varphi^{\dagger a} - \sqrt{2}g_0 f^{abc}\gamma_5\psi^a\varphi^{\dagger b}\varphi^c)$$

Lattice energy-momentum tensor...?

- Composite operator contains further UV divergence

$$a \times \frac{1}{a} \stackrel{a \to 0}{\to} 1$$

- i.e., EMT on a lattice [Caracciolo et al. '89]

$$\{T_{\mu\nu}\}_R(x) = Z_1 \left\{ \sum_{\rho} F_{\mu\rho} F_{\nu\rho} - \frac{1}{4} \delta_{\mu\nu} \sum_{\rho\sigma} F_{\rho\sigma} F_{\rho\sigma} \right\} + Z_2 \delta_{\mu\nu} \sum_{\rho\sigma} F_{\rho\sigma} F_{\rho\sigma} + Z_3 \delta_{\mu\nu} \sum_{\rho} F_{\mu\rho} F_{\mu\rho} + \dots$$

To-be-determined coefficients Z_i (for QCD, 7 coefficients)

$\to$ Tune Z_i to satisfy the translation WT identity

- Lattice $\leftarrow$ flowed composite operator $\to$ other regularizations (dimensional)

Perturbative expansion of the gradient flow

- Gradient flow with the "gauge fixing"

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x) + \alpha_0 D_\mu \partial_\nu B_\nu(t,x)$$

- Formal solution of this equation

$$B_\mu(t,x) = \int d^D y \left[K_t(x-y)_{\mu\nu} A_\nu(y) + \int_0^t ds\, K_{t-s}(x-y)_{\mu\nu} R_\nu(s,y) \right]$$

where the heat kernel

$$K_t(x)_{\mu\nu} = \int \frac{d^d p}{(2\pi)^d} \frac{e^{ipx}}{p^2} \left[(\delta_{\mu\nu} p^2 - p_\mu p_\nu) e^{-tp^2} + p_\mu p_\nu e^{-\alpha_0 t p^2} \right]$$

$$= \delta_{\mu\nu} \int \frac{d^d p}{(2\pi)^d} e^{ipx} e^{-tp^2} \qquad (\alpha_0 = 1)$$

- and the non-linear term R

$$R_\mu = 2[B_\nu, \partial_\nu B_\mu] - [B_\nu, \partial_\mu B_\nu] + (\alpha_0 - 1)[B_\mu, \partial_\nu B_\nu] + [B_\nu, [B_\nu, B_\mu]]$$

Perturbative expansion of the gradient flow

- Correlation function of the flowed gauge field

$$\langle B_{\mu_1}(t_1, x_1) \dots B_{\mu_N}(t_N, x_N)\rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}A_\mu \, B_{\mu_1}(t_1, x_1) \dots B_{\mu_N}(t_N, x_N) e^{-S_{\text{YM}} - S_{\text{gf}} - S_{c\bar{c}}}$$

- Free propagator of the flowed field

$$\left\langle B^a_\mu(t, x) B^b_\nu(s, y) \right\rangle_0 = \delta^{ab} g_0^2 \int \frac{d^d p}{(2\pi)^d} \frac{e^{ip(x-y)}}{(p^2)^2} \left[(\delta_{\mu\nu} p^2 - p_\mu p_\nu) e^{-(t+s)p^2} + p_\mu p_\nu e^{-\alpha_0 (t+s)p^2} \right]$$
$$= \delta^{ab} \delta_{\mu\nu} g_0^2 \int \frac{d^d p}{(2\pi)^d} \frac{e^{ip(x-y)}}{p^2} e^{-(t+s)p^2} \qquad (\alpha_0 = 1)$$

- Gaussian damping factor e^{-tp^2}

  $\Rightarrow$ a simple renormalization property of the gradient flow